Introduction to Wireless and Cellular Communication

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Lecture - 27
Wide Sense Stationary Uncorrelated Scattering (WSSUS) Channel Model
Doppler, Temporal Characteristics of Fading Channels

Good morning. Let us begin with the quick summary of a lecture 25. So, today's lecture is going to cover and build on the Wide Sense Stationary Uncorrelated Scattering Model, we are going to add two more pieces of the jigsaw puzzle, a something called the level crossing rate and other one called the average duration of the fade. Again these are all helping us understand and characterize the wireless channel in a better manner that we can then use it to our advantage.

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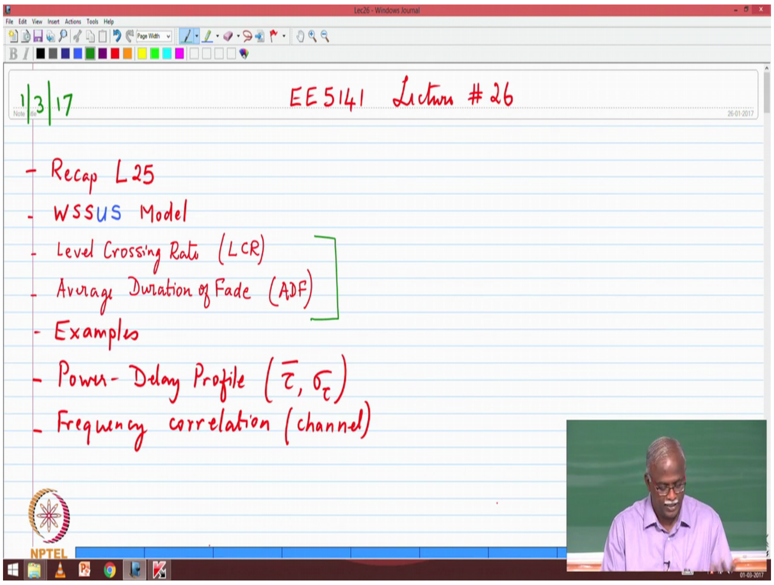

Looking at several examples through the course we will also now start talking about channels where there is time dispersion. So, up to now we have done the characterization of channels without looking at the delays dimension, only the time dimension has been done. And that with the LCR and the ADF we will build on complete the picture of the time variation.

Then we know move our focus to the delay dimension, so that will give us another type of characterization of the channel which is help full for us in fully understanding the channel. And then we look at some additional transform Fourier transform based insights that we can get from this channel.

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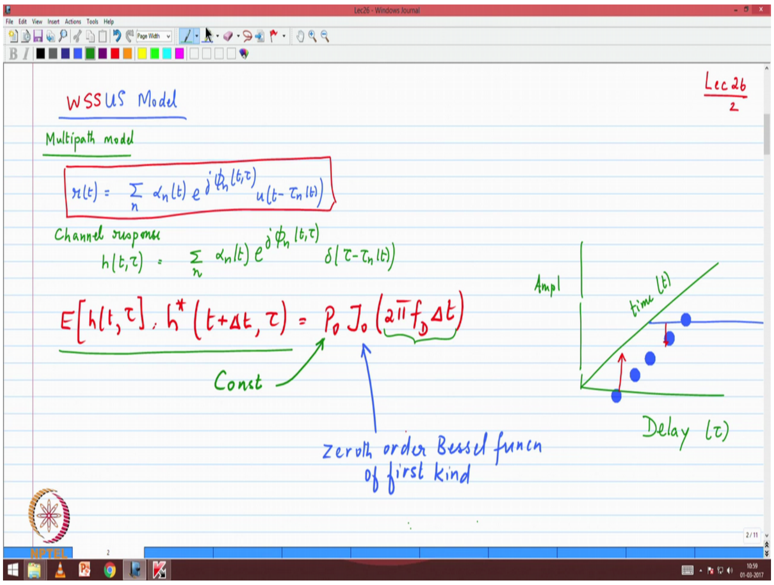

But, before that a quick summary of the points that we have discussed: if we where to look at the time autocorrelation of the complex channel h of t comma tau- h conjugate t plus delta t comma tau notice I am using the same value of tau assuming that there is no time dispersion all of the multi path components are arriving at the a delayed tau. We showed that this is constant times the zeroth order Bessel function, the argument of the Bessel function very important for us; 2 pi f D times delta t that is what we have used. And just so that we are comfortable with the visualization, basically we are looking along the dimension on the progression of the blue dots and we are looking at what is the correlation as the channel varies as a function of time.

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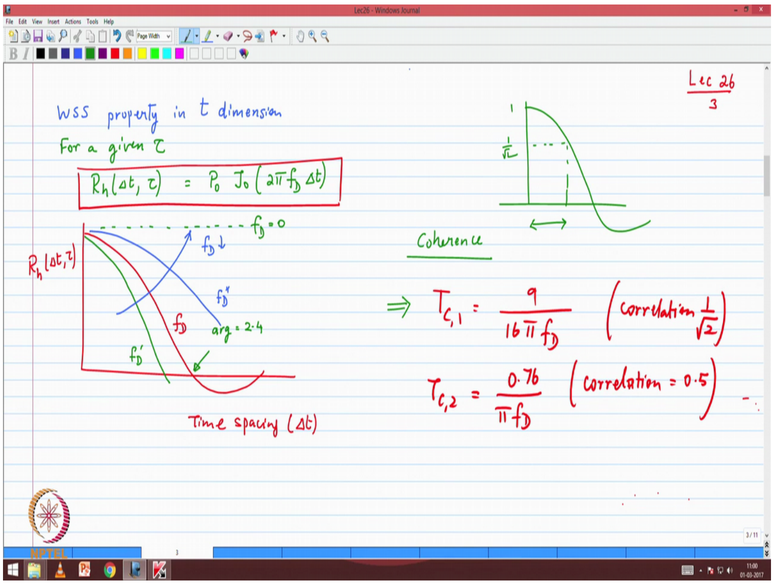

Now, the key parameter that we derived in the last class by looking at the Bessel function was coherence time. Two expressions for coherence time this gives us a quantitative measure of how much correlation is there in the channel. One of the correlation measures was to reach a correlation value of 1 by root 2. So, basically if you were looking at a normalized Bessel plot if this is 1 what is the value at which you reach 1 by root 2 and the corresponding value of the time; this would be measure of the coherence time.

So, like that you can also do for a correlation of 0.5 as we mention T c 1 is the more commonly used, so 9 by 16 pi f D is a very useful formula to for us to remember.

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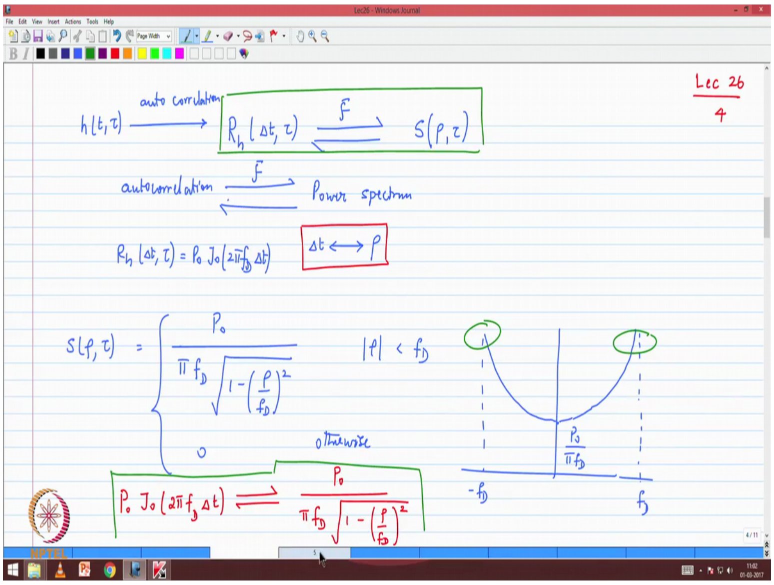

We also did the Fourier transform of the autocorrelation function. We are doing the auto correlation or the Fourier transform with respect to time. So, delta t mapping to rho as my Fourier variable, we showed that the Fourier transform pair is given by the expression on the bottom of the slide. So, by the Bessel function maps to a spectrum what you call as a Doppler's spectrum which looks like a u shape and it strictly less than f D, because at f D the spectrum this expression would go to 0. So, it is a bounded function. So, therefore, we get a very steep price as you go close to f D and minus f D.

When you see a spectrum like this how do you interpret it? What do you, how do you, what do you make of it, when you say that the specter density is of the shape. Dominant components are close to f D that is what it says basically because the density is very high for these. So, as you go closer to the f D, you know the strong components. So, those are the ones that are going to make the most impact.

So, given that scenario we also in the last class towards the end made a statement about the impact of such a channel. So, let me just repeat that in case there was some doubt about that.

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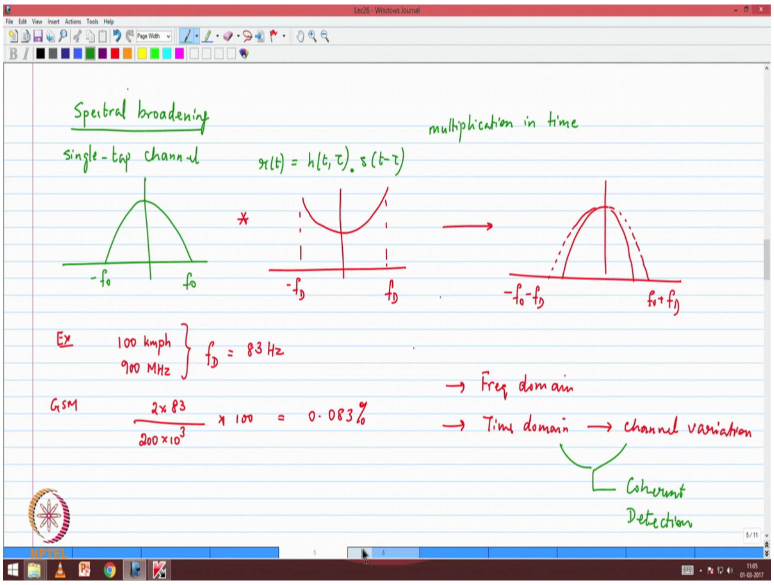

Spectral broadening was the last concept that we touched upon. I will restate the result and hopefully that will be a good insight into what we are talking about. So, we are coming to consider a single tap channel; a single tap channel which has the statistics as we have described before. So, the spectrum of the signal is from minus f not to f not, the received signal r of t is equal to h of t comma tau s of t minus tau. So, some delay it happens, but there is only one channel. Notice that this is a multiplicative process; this is where I think they may have been some confusion. So, I am doing multiplication in time.

So, if I were to try to interpret this in the frequency domain I will have to do convolution of the spectra; multiplication in time corresponds to convolution in frequency. Therefore, the frequency domain interpretation says convolve this with the spectrum of the channel response. And again from the power spectral density I know that the channel response must be 0 outside of f D. So, the likely shape is something of this type, it is not exactly the same, but it is of that nature there is a strong spectral components around f D that is h.

If I convolve these two where this one goes from minus f D 2 plus f D then the resultant spectrum that we get is a slightly broaden spectrum. So, the original spectrum is here, the broadened spectrum is this is f not plus f D. Again this is we have looked at this a couple of times minus f D. So, basically slightly wider, I believe we also done these spectral widening as a function of the of the band width of the signal.

So, just as a example you can please verify that if I were looking at 100 kilometers per hour speed at 900 megahertz that will fix for me my Doppler frequency, Doppler of 83 hertz. Now the total broadening of the spectrum is 2 times 83 hertz over a bandwidth supposing we are looking at a GSM signal, the bandwidth is 200 kilohertz into 10 power three into 100 to get percentage this comes out to be 0.083 percent. You may not even notice that the spectrum has become wider. Again we have mentioned that the frequency domain interpretation says that the impact is minimal in terms of the spectral broadening.

However, this is the frequency domain impact. But on the time domain side the story is different the channel is varying as a function of Doppler, and therefore we need to be a very very aware that the impact that it is going to have on the coherent detection aspects. Time domain there is a channel variation and that is going to impact our ability to do coherent detection. So, this will affect coherent detection.

Again the whole idea of the analysis is so that we know how to characterize and handle the impact of the channel. So, today's lecture we are going to introduce two more characterizations of the channel with the time variations of the channel.

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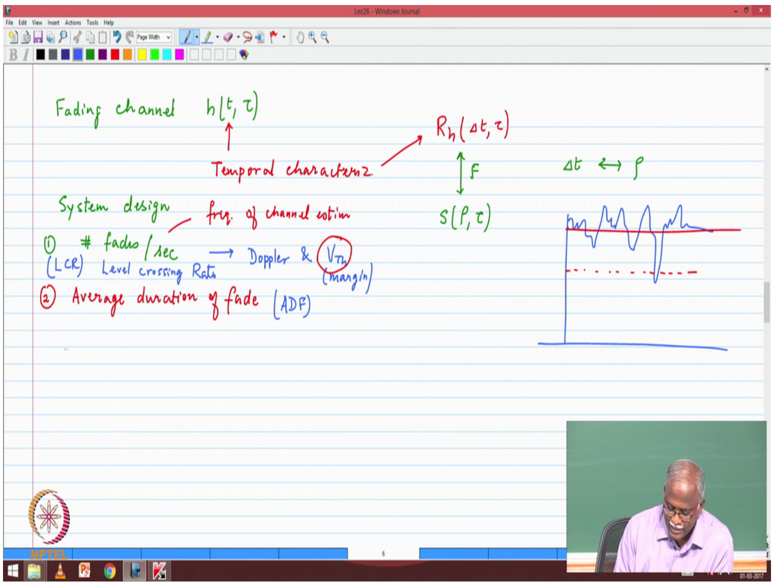

So, let me begin by introducing the notation. We are dealing with a fading channel not dispersive at this point we are not talking about dispersion. So, h of t comma tau, so basically variations along the tau dimension. So, the characterizations that we have done so far we would call it as the temporal characterization, anything to do with the time dimension temporal characterization. And temporal characterizations that we have done so far are to get the autocorrelation of the time variation. We showed that it is a wide sense stationary process does not depend on the time itself, but on the time difference R h of tau and if we did a Fourier transform of this we will get s of rho comma tau. This is the Fourier transform and the transformation variables are delta t to rho. That is the so far that is the extent to which we have obtained.

So, what we can say is we have a statistical characterization, but now I have to design the system. So, for system design I need something which is of the following form that is very useful for us. If I were to focus on the system design aspects the two parameters which we have already referred to in terms of their impact on the system: one is going to be the number of fades per second. So, what are the things that this is going to affect the in terms of system design; what would this affect in terms of system design for coherent detection.

Student: Operating.

Well, number of fades per second tells me that I have to send as that many training sequences so that I can estimate the channel, because I assume that every time there is a fade I lose track of the channel and I must need a help the receiver needs help. So, this is going to affect the frequency of channel estimation.

So, how often should I send the training sequence? So frequency of channel estimation for coherent detection. Now we have already seen; what are the things that affect the fades the number of fades that occur, what are the two one is Doppler of course very good. So, the Doppler will be an effect, what is the second one? There is another one which, see it they also has to with how much margin you have and therefore what constitutes a fade. So, there is the notion of one is Doppler of course there is no arguing, the second one is a threshold what is your threshold for a fade. And this threshold actually depends on the margin that you have built into the system.

So in other words, if I were to draw it in terms of a time variation the channel fluctuates like this. Now if I have designed my system to have this as my threshold I get a certain number of fades. If I have designed my threshold to be something else this is a fade then I get much fewer fades. So, what this tells me in the in the second case the dash line case I have more margin therefore the fades are not occurring as much.

So, an important element is going to be the threshold at which you are going to declare a fade. So, one is the number of number of fades per second. The other very important system design parameter which we actually talked about it quiet extensively in the context of the interleaver was the average duration of a fade. Once the fade occurs how long will it last and how many symbols or bits will it affect? So, average duration of a fade. So, the acronyms that are used are ADF for average duration of a fade, the number of fades per second is called LCR or level crossing rate.

So, if I set my threshold as V Th how often will I cross that thresholds; that is the understanding of the level crossing rate. So, let us just write down the acronyms. Level crossing rate refers to the number of fades per second and the average duration of a fade is a. So, what I would like to do is not go through the complete derivation, but give you enough pointers that you can then pick up and read the proofs that that are given to us.

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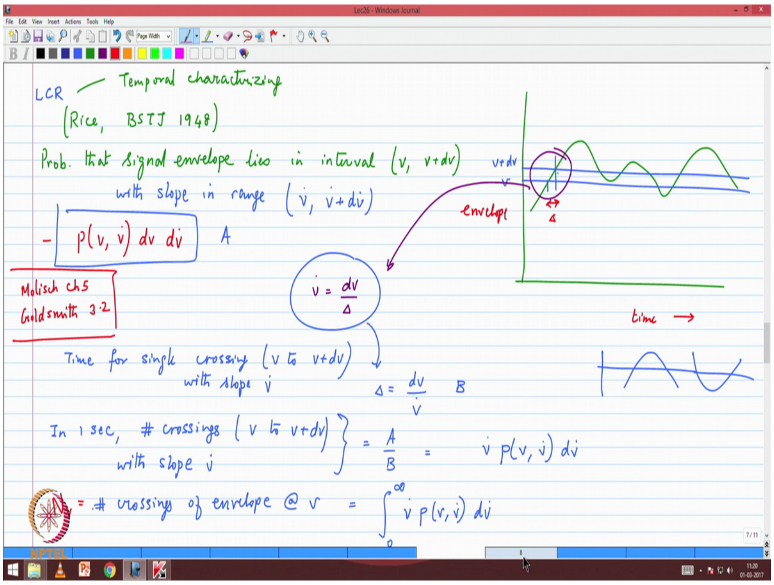

So, the LCR is the first thing that we will address- level crossing rate. The original formulation of this by the way this is another aspect of the temporal characterization, just write that down. This is also part of the temporal characterization; this also has to deal with the variation with respect to the function t of variable t and not tau this is variable t.

So, temporal characterization of fading, this is one more element that we are utilizing. And it basically adds to our understanding what we already have in terms of the autocorrelation and the Doppler spectrum, so this is a very useful one. So, the original formulation was made by a communication engineer by name Rice- the person after whom Rician fading is given its published in the journal called bell systems technical journal I must review already used this when we introduced the MacDonald's paper; that is where a lot of the original work came from 1948 archives are present if you would like to take a look- definitely you should.

So, basically Rice posed what a problem that looked like a very mathematical problem, but it actually has a very useful systems levels interpretation. So, the problem that was posed by Rice is what is the probability; probability that the signal, signal envelop always we are talking about the envelop here we are not talking about SNR signal envelop which is got its a complex Gaussian random process, it is got a Rayleigh statistics, but now we are looking not at the envelop, but we are looking at the envelop of the signal. Probability that the signal envelop lies in an interval; lies in interval which is bounded by v and v plus dv. So, a picture may help. So, basically there are the signal is fluctuating and there is a level which we have identified or we have specified as v and dv is slightly higher that. So, this is v v plus dv.

So, what is the probability that the signal lies in this region. Basically you would look at the time duration for which the signal lies in this interval. So, this would be a part then again you will see what are those times in that you will have. Now keep in mind that this is the fluctuation of the envelop; the fluctuation of the envelop depends on several parameters. So, the slope of this fluctuation is random process, therefore I cannot predict. So, the slope can theoretically be positive or negative: if it is increasing it will positive, if it is decreasing its negative, and it can go all the way from being almost flat which means very low Doppler or very high Doppler. Again instances of both can happen.

So, he also bounded the rate at which it crosses so with slope in the range v dot to v dot plus dv dot. So, some slope and within a certain margin. So, what is the probability that the signal envelop lies in this region v plus v and v plus delta v. And its slope lies in this range. And as you would expect v and v dot are two different things: one is the distribution of the envelop, the other one is the rate of change. And the assumption that Rice made was that they are independent which is a reasonable assumption. What we know about v is that it is really distributive. Already we know one of the two variables.

The second one is; what is the distribution that you can assume for a v dot and Rice assume that it was a Gaussian distributed, but that that is secondary. Basically what he said was the probability that it les in this range can be given by the joint pdf of v and v dot multiplied by dv by dv dot; so the based on calculus. So, this is the probability that you will lie in this region.

Now, if I were to define this as this transition period as delta, this is your time axis and this is the signal envelop; amplitude of envelop. So, the slope at which this particular crossing is occurring, this particular crossing let me highlight it in a different color. So, zoom in on this part it says- that the slope that I am encountering here is dv is the change in amplitude that occurs in the time delta. So, the problem formulation is clear. There may be other parts where the crossing is happening with the negative slope somewhere is a steep slope. And so the question is what is the probability that the signal level lies in this region and how do we capitalize on this particular aspect.

So, here are the steps that are involved. And again I would encourage you to go through and read the; by the way the places where you will find this derivation are Molisch chapter 5 and Goldsmith section 3.2. So, please do look up, but most of it is the final result is very important to us, but the method of deriving it is also important. So, the next step from this formulation is the following: it says- what is the time for a single crossing of the from v to v plus delta v v plus dv with slope v dot, but basically is this asking the question again which says the time taken will be delta that was given by dv by v dot.

So, if the probability that it lies in this range is given by this expression. So, in 1 second what is the probability that it lies in this? I write this expression for 1 second. And the time taken for a single crossing with this slope v dot is also given, then the number of times you will cross with the slope. So, in 1 second, so just follow this argument in 1 second the number of crossings of the same interval from v to v plus dv with slope v dot; with slope v dot is given by if this is A this is B its given by A divided by B which if you write it down it says it comes out to be v dot p of v comma v dot dv dot. So, that is the number of crossings.

Now v dot can be positive or negative and keep in mind that any crossing of a threshold can be positive or negative; a single crossing or if he could look at it on the other side it can be. So, you do not count is as two crossings. Though there are two crossings you count it as a single, because basically it is crossed the threshold once and it will also come up. So, by convention we will take only the positive going crossing; that means, you are below the threshold and crossing up. So, which means my slope can only be positive. Therefore, the number of crossings of the envelop at a value v is given by integral 0 to infinity; basically the slope can be completely flat or it can go very close to 90 degrees of this parameter v dot p of v comma v dot dv dot. Basically now it is a matter of a doing the integral, but the most important thing is this is; what is the level crossing rate N v.

Now, let me just you a minute to sort of assimilate this. The formulation had two steps: one is what is the probability that you lie in a certain range with a certain slope, and then if you are at a given slope what is the time that it takes for you to cross. Then using those two results you say how many such crossings can there be within a unit time. And then with all the possible slopes and that gives me the, because my crossing can be occur at any slope I do not mind, I just need no other crossing has happened and I crossed it in the positive direction.

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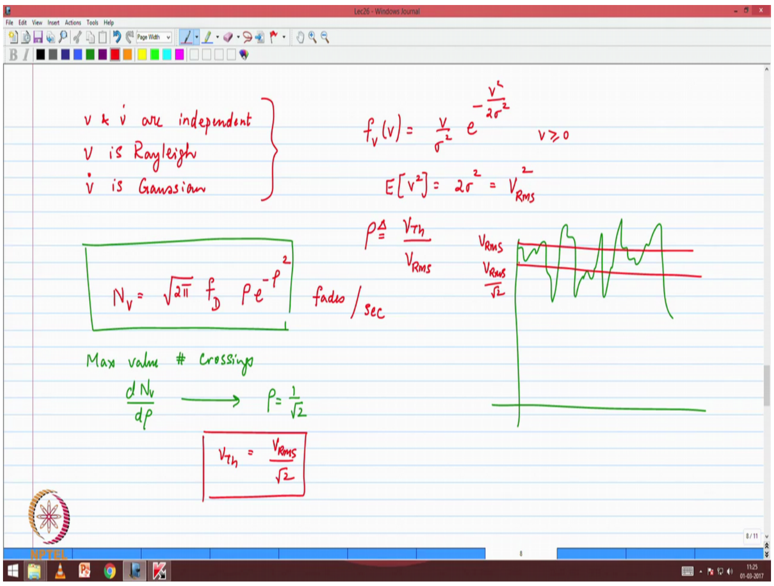

Here are the assumptions made by Rice seems very reasonable: v and v dot are independent, that means the joint pdf can be split as a product of the pdfs are independent; v is Rayleigh distributed that is already known to us is Rayleigh and v dot again through a argumentation of the lightly hood of a certain types of crossings. He has made the assumption that this is Gaussian depends on the rate of crossings.

So, using these assumptions comes a very elegant result which says that of course, the level cross then the number of crossings depends on the threshold so let us go back and define our threshold. So, if the Rayleigh pdf is given by f v of v v by sigma square e power minus v square by 2 sigma square v greater than or equal to 0. We know that expected value of v square is equal to 2 sigma square that is also this RMS value V RMS square. So therefore, if I define a normalized value of the threshold let me call it rho this is defined as the threshold relative to the RMS value. In other words we have already seen that that is the measure of the margin that you have.

So, if my V threshold is what going to determine my a fade the rho is a normalized parameter then what Rice showed was the result of the integral that was there in the previous chart is given by a very compact form root 2 pi f D very good, because higher the Doppler more frequency more crossings and rho e power minus rho square. So many fades per second was the result shown by Rice. Again the integration is not difficult, but nevertheless the result is more important. So, I have chosen to focus on that; the expression is very important.

Now this is a function, if you look at it carefully is a function that will first increase and will then decrease. Therefore, you will always useful for us to find where the maximum occurs; maximum value of the number of crossings. So, if you differentiate N with respect to rho N v with d rho and then set it equal to 0 then you should find that the maximum occurs at 1 by root 2. Now is that a reasonable answer we need to validate.

So, here is the fading channel. If you remember our understanding and description of the RMS value your likely hood of being below the RMS value is quite high 60s plus percent. So, this is approximately where your RMS value is. Now if you have the threshold value very small then you will have very few level crossings; if you have that very high again you will have level fewer level crossings. So, it turns out that if you are somewhere slightly below where V RMS by root 2 V RMS by root 2 that is where you are seeing the maximum level crossings, which is correct because the likelihood of you being below V RMS is much higher and once you are in that region you are likely to see lot of level crossings in them. So, it is a reasonable thing, but again it can be validated through experimental verification.

So, V threshold is equal to V RMS by root 2 that is when you will have the maximum number of crossings now. If you have only such a small buffer margin then you are actually going to be in a fade quite often, because basically the number of crossing means the number of fades that are occurring. So, that is not a good point to be in you must have a sufficiently more margin so that you can be robust.

So, but that is one useful characterization of the time variation of the system. So, let me just give you a minute to sort of a digest that aspect and then we can pick it up from there and build upon that. So, the understanding of what is the level crossing rate and how it is going to impact and what sort of insights that we can get from here. The level crossing rate depends on the threshold, depends on the on the on the Doppler frequency, both of which we have already even before we started the discussion we had already looked at it and understood that impact. If there are any questions I can answer them otherwise we move on to the next important aspect of our study.

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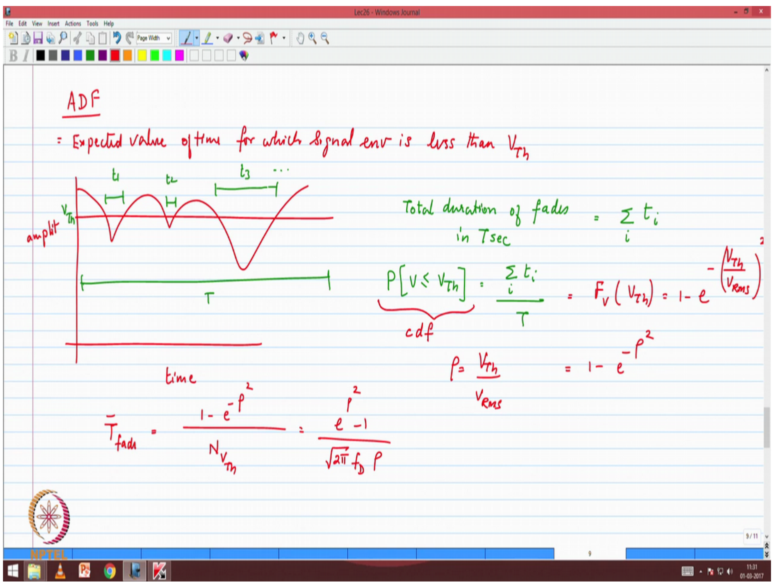

If we then move to the next concept that is also in the temporal characterization space it is the average duration of a fade. You will find that actually this one may be the previous one was not as intuitive in his formulation, but this one happens to be very very intuitive in its formulation.

So, the average duration of a fade this is nothing but the expected value that the signal envelop expected value of time for which the signal envelop is below a specified threshold is less than V Th. So, that is my fade how much time I am below that problem that is going to determine the probability that I am in a fade. So, a picture is actually very very helpful for this one. So, this is the time axis. Notice I have shifted from I mean there is no tau dimension here. So, I am using the x axis as my time dimension this is the amplitude dimension.

Now supposing, this is my V Th, the durations that we are interested in this is t 1, I am in a fade duration t 2, I am in a fade this happens to be a slightly longer fade t 2 and so on. In a overall duration of uppercase T. So, here is a just a one step insight that we can get from here. The total duration of fades is summation of all the t i's duration of fades in t seconds is given by summation over I all the t i's.

Basically you measure each of those. Now, ADF refers to the probability that v is less than or equal to V threshold. So, from this figure it is easy for us to write that down; its summation over i t i divided by upper case t that is. Events that we are looking at the event is you are below a certain threshold in a in a time span of t. But this is a quantity that we already know is the statistically it is a cdf this accumulated distribution function. And we know that we have a way of characterizing that. So, this is nothing but f v the c d f using V threshold as the as the parameter this would be given by 1 minus e power minus V threshold by V RMS whole square. This is something which we know from the statistics of. And if rho is V threshold by V RMS this is nothing but 1 minus e power minus rho square.

So, if I take a 1 second duration of the fade that I am in. So, in that 1 second how many fades occurred? I already have the expression for that. So, the average duration of a fade says- take the total period for which you were in a fade in 1 second which is given by 1 minus e power minus rho square that is a probability into 1, if you want to take it as 1 second divided by N V threshold; the number of zero crossings that you will expect for that given V threshold for the Doppler's channel that you are looking at.

So, if you just substitute for the N v from the previous example this comes out to be e power rho square minus 1 divided by root 2 pi f D times rho. That is the average duration of a fade. So, two expressions that we have that are very useful for us. Again if there is any questions? Basically what we have said is; what is the probability that you are below a certain threshold multiplied by time to get the duration in seconds. So, then divided by in that same unit amount unit time how many zero crossings you will likely to occur and on average that is going to be your fade duration.

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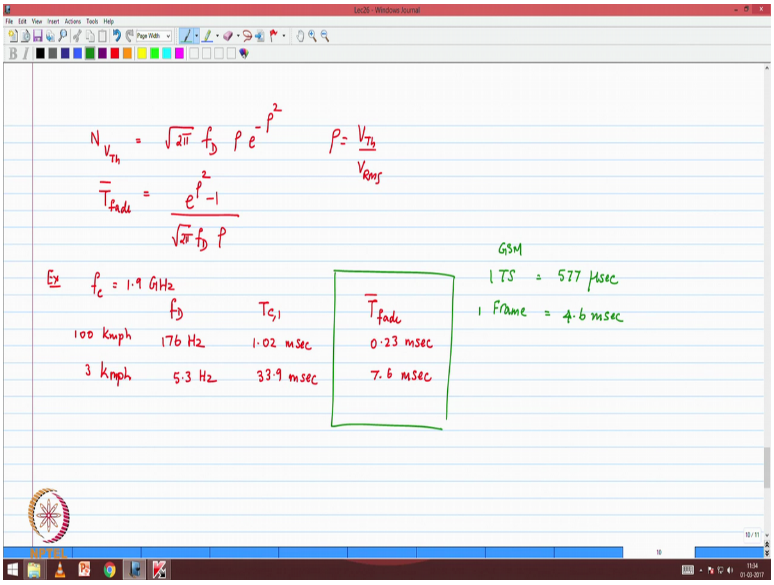

So, the two expressions that we have N V threshold is given by root 2 pi f D times rho e power minus rho square where rho is given by V threshold by V RMS. The other one is the average duration of a fade which is given by e power rho square minus 1 divided by root 2 pi f D times rho.

That is a useful parameter, but may be let us make it even more relevant and practical. So, take an example, slightly different example: carrier frequency of 1.9 gigahertz, just for a change. Look at two speeds 100 kilometers per hour 3 kilometers per hour one is pedestrian other one is vehicular the corresponding Doppler's; corresponding Doppler frequencies in this case is 176 hertz, this is 5.3 hertz. The coherence time T c 1 if you calculate this will come to be 1.02 milliseconds. In this case it is 33.9 milliseconds- lower Doppler, longer correlation time, very very useful for us to keep that picture in mind.

Now alongside the coherence time, previously using the coherence time we were making some qualitative arguments about how to do the system design. But now all alongside if you now write down T fade, this comes out to be 0.23 milliseconds. So, which says that coherence time is 1 millisecond, but if you are in a fade the duration of that fade is likely to be around 0.23 milliseconds. Likewise, the coherence time at 3 kilometers per hour is 33 almost 34 micro second milliseconds, the average duration of a fade is 7.6 milliseconds.

Now, the design of the interleaver and the FEC needs to take into account; probably a more closely what is the duration of a fade- average duration of a fade and then correspondingly work with that. And always good for us to keep in mind if you are talking about GSM what is the duration of one time slot, one time slot corresponds to 577 microseconds. So, if a fade occurs what is the likely hood that you will wipe out the entire slot very easy to see. What will happen if you have a fade at 100 kilometers per hour? The average duration of a fade is 200 and.

Student: 30 micro.

30 microseconds, the duration of a time slot is 577- so likelihood that a part of your slot will survive. Now on the other hand if you had the lower speed no chance it will wipe out the entire. Now what happens to the time frame 8 time slots? One frame of GSM, because that is when your next time slot is going to come, so one time frame is 4.6 milliseconds, definitely not affected by the duration of a fade because the fade would have already have gone if you are at 3 kilometers per hour very likely that you may have problems with your next time slot also. Just need to be careful how you design your system so that we can build on that appropriately.

So, this is just a simple example how to tie these concepts together. I would now like you to sort of put these 2 piece of the jigsaw puzzle also adjacent to it, so that we can now add to our understanding of that.

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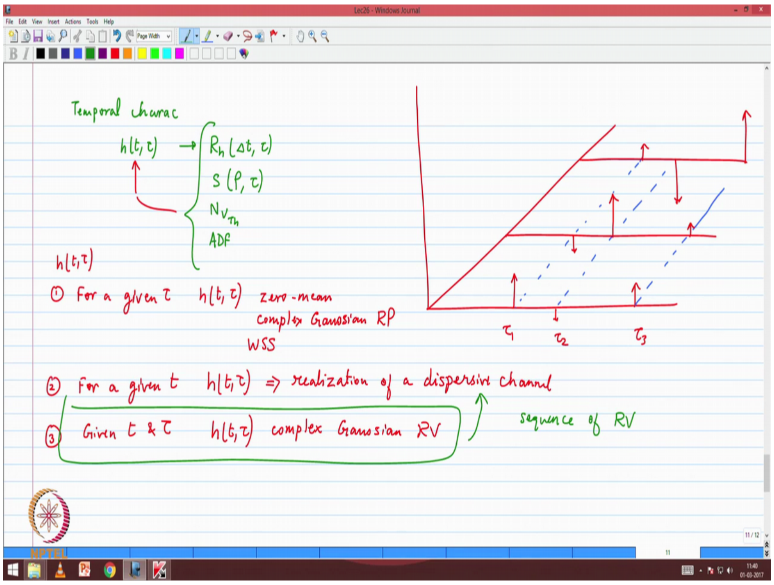

So, so far the temporal characterization: temporal characterization of h of t comma tau has given us sort of four aspects: one is R h of delta t comma tau, S of rho comma tau, the number of level crossings at a given threshold, and the average duration of a fade. So, all of these are related to the temporal aspect; all of them are connected to the temporal aspect. So, I want you to know shift your thinking to include the delay dimension, therefore the following simple exercise. So, far we have been looking at just one delay now a channel where there is time dispersion, anything.

So, now if I want to look at the different time instances of time at instances of time, so basically I will have to look let me just quickly draw this so that you can then visualize what we are trying to. So now, if I where to look at it at different instances of time another snapshot may be at this times a snapshot this channel is negative, this one is becomes something very large, this one is something small, may be a third window where once again your positive this one has become negative some variation.

But notice that you are interested in variations in time dimension for the different delays. Let me call this as tau 1, tau 2, and tau 3. So, this is what we are interested in characterizing. Now of course, these tau's can also change, but in a in a reasonable window of time things are you can assume that tau's are remaining study and therefore you can do the following analysis.

So, here is our understanding of this picture. First one: I have h of t comma tau. So for a given tau, that means I am only looking along the time dimension; for a given tau h of t comma tau is a zero-mean complex Gaussian random process. Because it is something that has some variation along the time dimension, it is a random process we do not have control over it, but basically it changes as a function of time.

However, there is a correlation, so it is wide sense stationary. So, if I forget the delay part and only look along these time dimensions for each of these delays, each of them will look like a random process. So, given a tau h of t comma tau is a zero-mean complex Gaussian random process. For a given t, h of t comma tau basically tells you where the multipath components are (Refer Time: 39:51) the clusters of multipath components.

So, this is nothing but a realization of a dispersive channel, for a given time at if you take a snapshot; realization of a dispersive channel. There are certain delays, certain gain coefficients, but at the end of the day it is a realization of a time dispersive channel. Now very important: if I specify both t and tau given t and tau; that means, I have specified both t and tau- h of t comma tau is complex Gaussian random variable, basically it is a sample of a random process; so is a complex Gaussian random variable.

So, if you now accept this can you redefine what is the snapshot at a given time? It will be a sequence of random variables; if it is discretized. It will be a sequence of random variables if you were to discretize it.

So, from the time dimension where we have gotten the following four characterizations we are now going to shift over to the delay dimension and see what sort of insights we can get from there. So, let me build on this particular element in a very quick and simple fashion.

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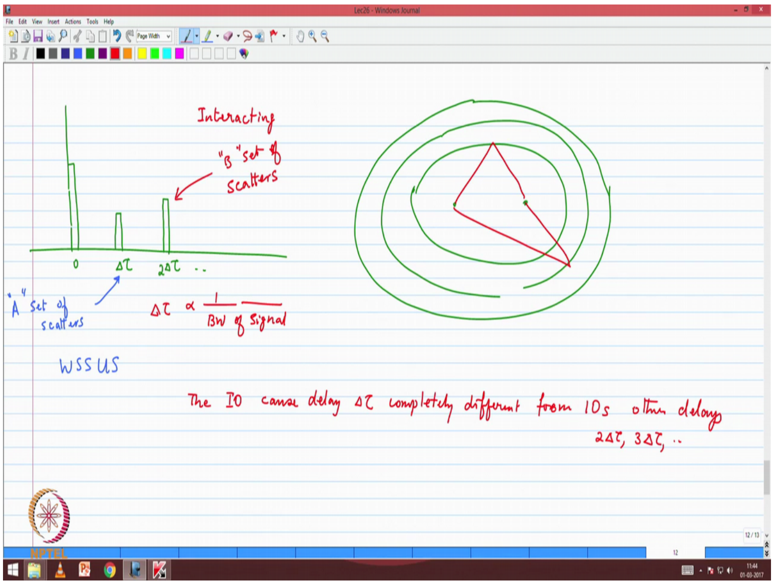

If you remember we introduced the notion of interacting objects. So, you have a ellipse and the anything that lies within an annular region of the ellipse will arrive at a certain within the certain time uncertainty window. So like that there are several of these annular regions that we have.

So, we said that all of those that are coming from within a particular annular region can be thought of as a single coefficient then because of my resolution bandwidth the next block I would see would have a distance of spacing of delta tau. So, this is 0 delta tau 2 delta tau and so on. So, basically coming it from the different annular regions. And this is from our earlier discussion of what we call as the interacting objects; objects that create the multipath and because of the multipath then. So, basically you are looking at the multipath components causing the different reflections that then eventually give you the different delays.

So, again this is a reuse of the earlier expression we also at that point in time also said that this time resolution delta tau which tells me when I can I differentiate between the multipath is inversely proportional to the bandwidth of the sigma. So, this was something that we had already planed and we have understood. Now I would like to incorporate this concept into our wide sense stationary uncorrelated scattering model in the following fashion.

So, the statement that we can make about this is that the first path is coming from a set of scatters; let me call it set A, set of scatters. This is coming from set B, it is a completely difference set of scatters- b set of scatters. So, each of these effective multipath components that I can resolve are coming from different set of multi path component that is why they are coming at different delays correct. And again there is no correlation or link between where the set A is located and where set B is located except that the distance at which they are located.

So, what we are making the following statement is that the IO's; the Interacting Objects that cause the delay delta t delta tau are completely different it is a non overlapping set completely different from the IO's causing other delays just 2 times delta tau 2 delta tau 3 delta tau and so on. And likewise amongst themselves also they are a different.

Now this basically says that whatever happens with the IO's in the region annular region 1 is not going to affect or does not have any bearing on what happens with the IO's. So, in other words what happens with this delta tau does not correlate with what happens with two delta tau.

Therefore, there is a notion of uncorrelatedness between the IO's; if one increases the other one may decrease there is no correlation that is present. Now how do you incorporate this particular aspect? Very very important for us. So, expected value of h of t comma tau not h conjugate t plus delta t comma tau not this is a result that we have already derived. This is the autocorrelation function with the spacing of delta t being the one that is determines it and tau not being the other parameter.

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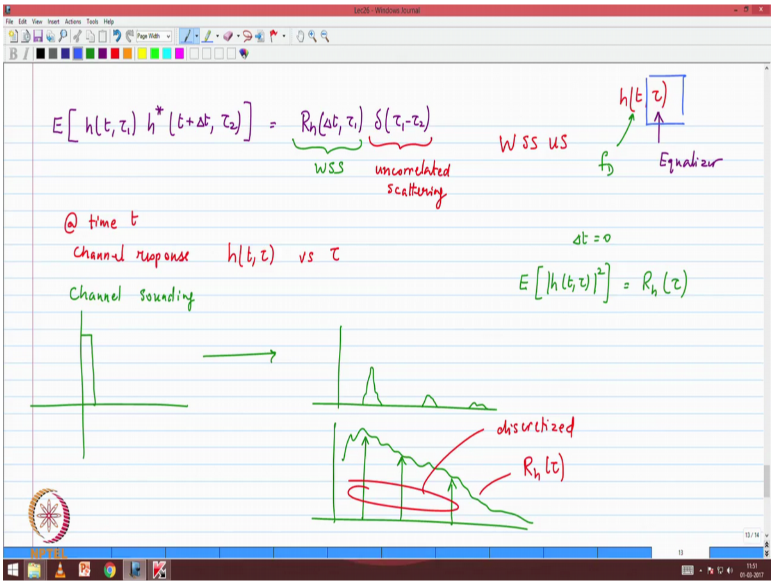

Now, comes the important extinction that we have. If I do expected value of h of t comma tau 1, h conjugate of t plus delta t and tau 2 what is the result. The result says that if tau 1 and tau 2 are the same there is a correlation function that we know; it R h delta t comma tau 1 if both are tau, but if they are not the same there is no correlation between the two. Therefore, the correlation is 0. So, I multiply this by delta of tau 1 minus tau 2.

So, the correlation exists only if you look across the same delay, if you try to correlate a channel at tau 1 with channel at tau 2 there does not exist any correlation, because those two are coming from different sets of interacting objects, and therefore no correlation exists between the two.

So, this is the wide sense stationary part, this is the uncorrelated scattering. Uncorrelated scattering is the scattering caused at different delays are uncorrelated with each other; uncorrelated scattering. So, this together gives us the complete picture of what we have to work with which is called the WSS us model. WSS comes from the time variation, the uncorrelated scattering comes from the tau dimension you put both of them together gives a complete picture of what we are trying to work with.

So, now comes a important part of the study. So, we have finish studying the tau t dimension, now we are going to study the tau dimension. So at a given time, at a given t snapshot what is the channel response, what is the channel response? Channel response is h of t comma tau versus tau, I am interested in that. And this is a well known problem in communications, it is known as channel sounding. Some of the early methods of channel sounding says- transmit a pulse, narrow pulse and see what happens at the output.

So, at the output let us say this is what you observed. This says there are three copies of the pulse that you transmitted and their coming with different gains and different delays. So, this is the channel response of the system. Or in other words you transmit a pulse and do not look for pulses, you look for energy as a function of time then what you observe is that there is some power that comes out and eventually the power dies on. Effectively when you transmit a pulse you inject some energy or power into the system and then you observe you put a power measuring device at the other end and as a function of time. So, as the time goes the effect of the pulse will decay.

So, you can think of it as a continuous profile of the power received power or you can think of it as a discretized model of this which is the description in the tau dimension. So, what is it that we are interested in we are going to set delta t equal to 0? So, my expectation that I did in previous graph becomes expected value of h of t comma tau magnitude square. This is nothing but the autocorrelation function at tau. As a function basically I am looking for the power as a function of tau. So, this is the autocorrelation or R h of tau, these are this is the discretize version, that is the continuous version. So, that is the discretized version.

So now, how do we characterize the channel? Most of our digital communication says give me a discrete time equivalent of the channel. So, we will discretize it according to our symbol duration or according to which is also related to the bandwidth of my transmitted signal. Now you see the link to the digital communication spot where it comes in how do I discretize this channel.

Now, when is it considered very time dispersive, when is it considered not so time dispersive, how much dispersion is there, do I need equalization or not those are the questions that we will now answer because our study is now going to be in the tau dimension. So, h of t comma tau this is the part that is going to tell me whether equalization is needed or not, this is the part that is going to tell me how fast my channel is changing. So, if I have an equalizer how fast should my adaptation be, because this is going to be dependent on my Doppler. So, what we need in the wireless communication system is an understanding of what is the dispersion how fast are my channel coefficients changing they are two dimensions, when you put them together then you have the complete picture of what the communications receiver is required to do.

So, we will now take up this portion, the tau portion of it and characterize and also decide when is it considered a dispersive channel when is it considered. And of course, just like we did for the t dimension the Fourier transform may give us some additional insights. So, we look at that as well. That we will do in the next lecture.

Thank you.