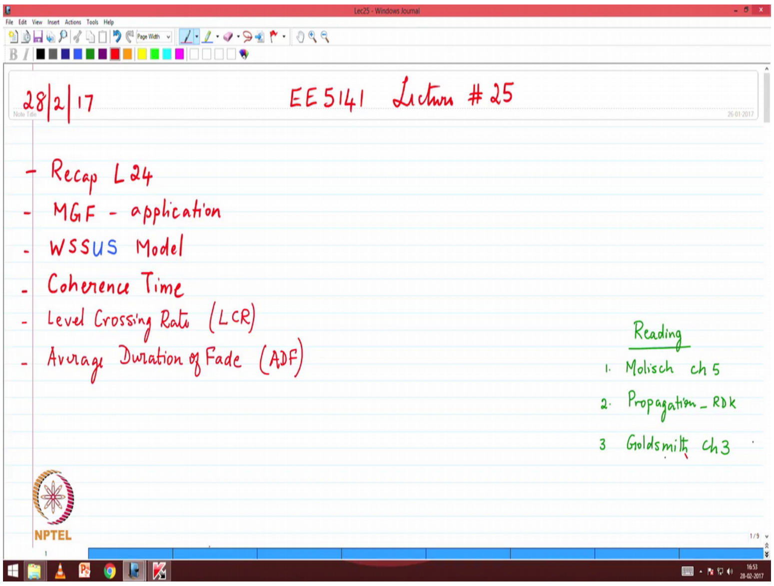
Introduction to Wireless and Cellular Communication
Prof. David Koilpillai
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 26
Wide Sense Stationary Uncorrelated Scattering (WSSUS) Channel Model
WSSUS Part II, Coherence Time, Doppler Spectrum

Good evening, let us begin. Apologize for the late start, but we will never the less cover some very interesting material in today's lecture. This is lecture 25.

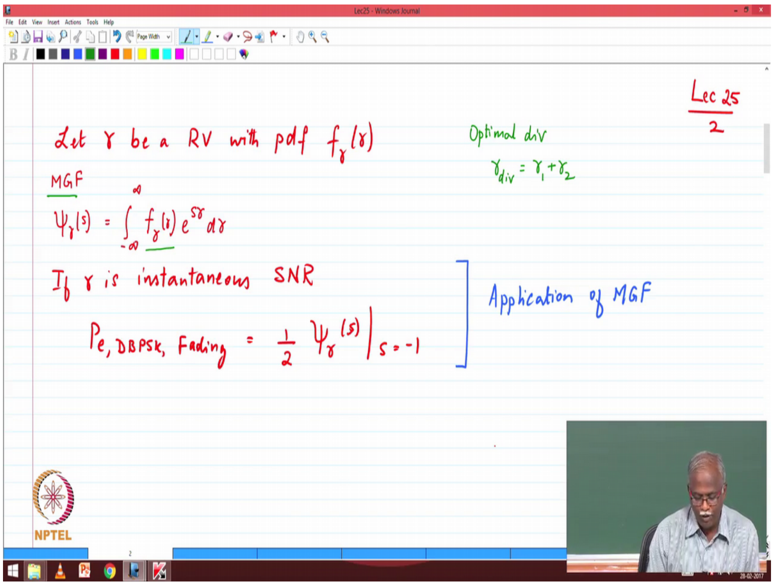
(Refer Slide Time: 00:26)



And I would always like to begin with a quick summary of lecture number 24, and give you a flavor for what comes ahead. Our focus in a today's lecture is to build up or understanding of what is the most comprehensive model of the wireless channel. It is called the wide sense stationary uncorrelated scattering model, I have used different colors to denote that WSS pertains to one dimension US pertains to another dimension. And the today's lecture will pull those things together.

Given that we have a good understanding theoretical frame work of the WSSUS model. We will then derive several very practical aspects that will help us in the design of systems. One of them is coherence time which we have already have a feel for from the say N Vishwanath example there are two more which are going to add a lot of insight into our understanding of how the fading channel has to be delta with from the aspect of design.

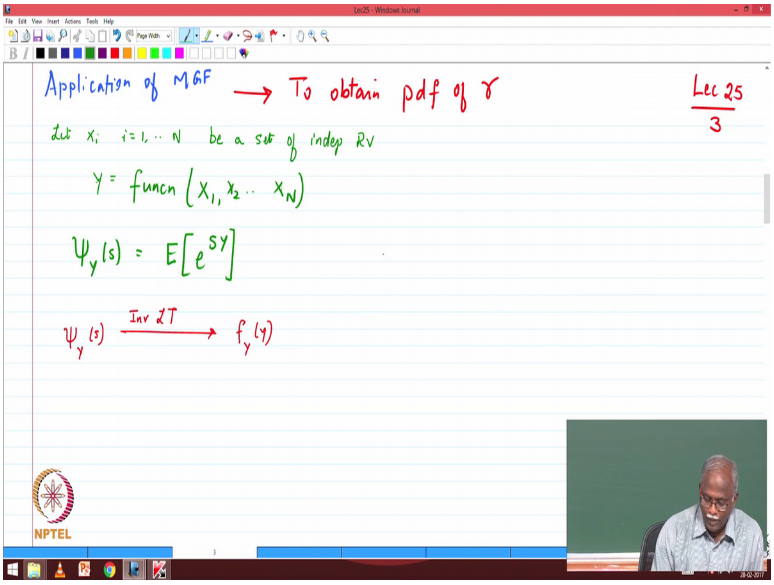
(Refer Slide Time: 01:23)



So, let us begin with a quick recap of the last lecture. We define the moment generating function our goal was to use the moment generating function to help us evaluate the probability of error. And one of the things that we need is the f gamma of gamma that is the probability distribution of the SNR. And one of the k examples that we talked about was optimal diversity combining where the resultant of the diversity was the some of the two SNRs seen by the two antennas.

Now, if they were of having difference of training statistics it would be very difficult for us to calculate the pdf. However, if we are using the moment generating function.

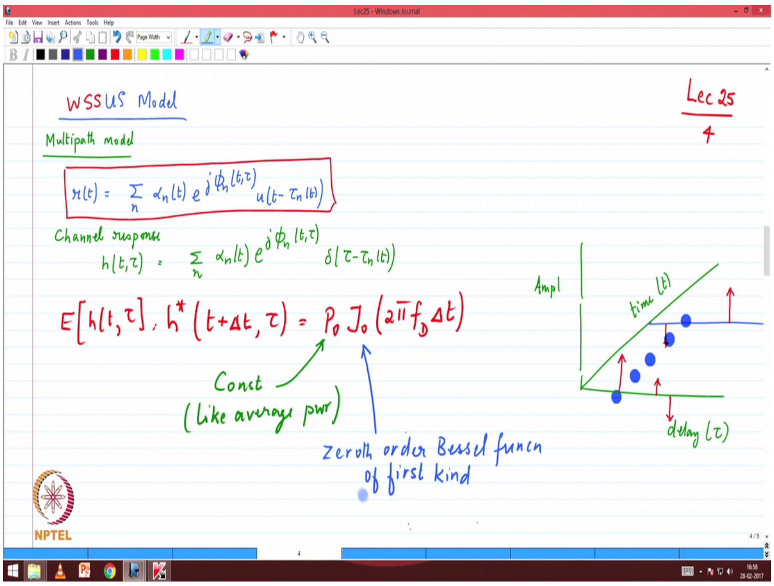
(Refer Slide Time: 02:18)



We showed that it would be fairly straight forward for us to apply the moment generating function in the following fashion. If y is a function of x 1, x 2, to x n in our case it was x 1 plus x 2 then we derived the moment generating function of y and then though the inverse Laplace transform to get the moment generating function that or the pdf that we are interested in.

So, moment generating function a tool for us to be able to calculate the bit error rate. Especially if it is difficult for us to already estimate the pdf of the SNR.

(Refer Slide Time: 02:48)

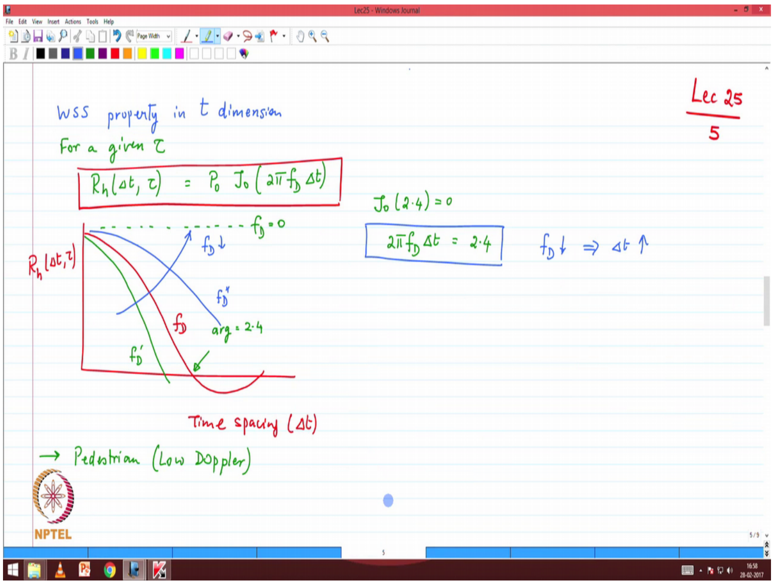


So, hopefully that is a tool that we can use through some examples and then build up or confidence of using that. From there we went back to a topic that we had covered several lectures earlier that is the multipath model. So, the multi path model the received signal is a super position of several delayed and face rotated components and that is captured by this.

They may arrive at different time instances. Therefore the response of the channel we said would be a described by means of a three dimensional plot: where one of the dimensions would be the tau dimension, the other one would be the time dimension. So, the channel response was obtained as a summation of these super positions of these phasers with the appropriate delay. So, the delta functions put the appropriate number of delays. For example, if we were to look at the channel response at the origin t equal to 0 it would consists of three coefficients with different delays tau 1, tau 2, tau 3.

Now, the bulk of the last lecture we were looking at the autocorrelation of the fading channel h of t comma tau. We were looking at the autocorrelation or the correlation between the channel response at different instances of time space delta t; characterized by a spacing of delta t. So, h of t comma tau h conjugate t comma h of t t plus delta t comma tau. So, this was the expectation that we were trying to evaluate. And we went through a series of steps to show that the resultant comes out to be a constant we can think of that adjust to the scaling term. The actual shape of the of the autocorrelation is given by a Bessel function; Zeroth order Bessel function of the first kind.

(Refer Slide Time: 04:49)



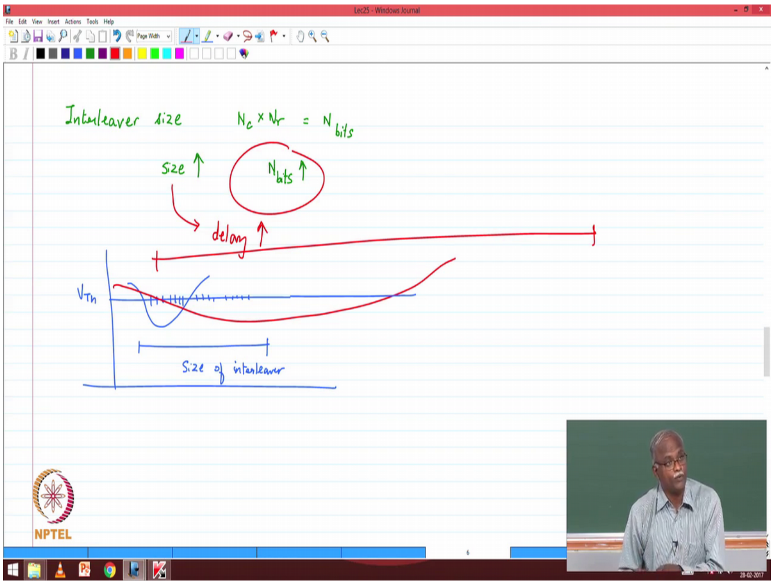
And we looked at several interesting observations from the sketch of a Bessel function. Basically the Bessel function does look like a sink function. So, as a quick approximation you can think of this as the basic Bessel function. The Bessel function crosses the value 0 when the argument of the Bessel function is 2.4.

So, based on that we also said what happens to the shape of the Bessel function when the Doppler increases, when the Doppler decreases. And basically we showed that when there is 0 Doppler there is no change. Therefore, you have constant correlation and the channel is continuously at the same level. If you are at a good level very good because it will continue to stay remain good, if you are at a bad position very very sad because there nothing can help you.

Basically, this is not a scenario that we would like to get stuck in it is always better to see some channel variation. So, we can then design the system appropriately. So, that was the summary of what we had seen in the last class. We also said that the low Doppler case is the one that we are more concerned about. The low Doppler case was the one that was easier for us to do coherent detection, but from a system design point of view low Doppler basically starts to approach that the correlation is very high. So, if the channel is good it remains good for a very long time, when the channel goes bad it remains bad for a very long time.

So, that is one challenge that we would have to face. We just touched upon that.

(Refer Slide Time: 06:34)



So, now we begin today's lecture and build on the notion of what are the some of the things that we would have to worry about. So, first one that I would like you to think about is the notion of the interleaver size. Again this is an intuitive concept we will you know give you the formal background later in the lecture.

So, interleaver size: what is the size of the interleaver? It is the number of columns into the number of rows; that is if you think of it as a rectangular interleaver. At the end of the day when you have fixed your size this corresponds to the number of bits that you going to place into the interleaver I mean that is that is the size of the interleaver.

So, now if I were to tell you that I need to increase the interleaver size; that means number of columns is going to increase or the number of rows is going to increase or the both are going to increase what essentially means that the number of bits that I am going to accumulate; basically interleaver says- take a block of data shuffle them around transmit reshuffle undo the shuffling and then demodulate. Now, you may think no that is not a problem it is very straight forward why do you have to revisit it. Now if I tell you that the interleaver size is thousand bits you have a certain amount of delay, because you have to first collect thousand bits then do the shuffling.

Now if I tell you is going to be 1 lakh bits that is a thousand times. So, basically that is going to mean that you are going to ask for a large number of data to be collected before you do the processing. So, the size of the interleaver is actually related to the delay in processing. The delay also increases the minute; you say that I want to increase the size you may have to also keep in mind that ok I am going to have to pay penalty or price in terms of the delay.

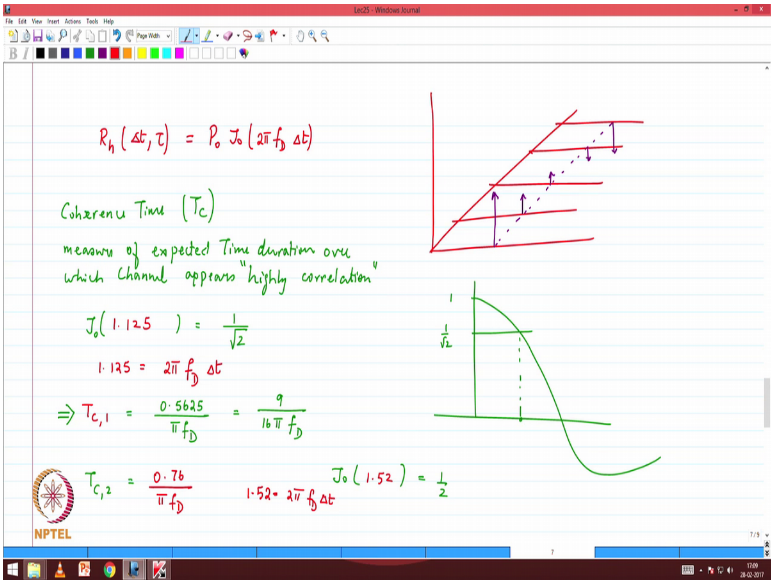
Now why is this an issue here is a visualization. So, if this is my threshold for a fade anytime the signal goes below this the fade. Now, what happens if I have high Doppler; the channel goes back and comes back up. In this duration I get certain number of bits that will be lost. Now my interleaver, remember the notion of the forward error correction says that I must have sufficiently large size of the interleaver so that not all the bits are wiped out by the fade at least majority of the bits must be outside of the fade. So, basically this should be the size of the interleaver.

So, it should be typically much larger than the duration of the fade or the total duration of the or the number of bits that are affected. Again this is an intuitive concept we formalize it a little bit later, but once you have the intuition the formalism becomes very obvious. So, this is the size of the interleaver.

Now what happens if I were in low Doppler situations? Low Doppler what it says is this is what is going to happen; that once it goes into a fade is going to remain in a fade for a long time. So, what do you going to ask me for the interleaver size? You are going to say the interleaver size must be this large; which means, if I were to design it for the worst case in the case of channel tracking I am affected by the high Doppler, in the case of the design of the FEC I am affected by the low Doppler. So, I am actually you know a lot of conflicting requirements are coming.

So, this is something that I need to keep in mind. That you know the low Doppler case is going to be a challenge, because what it will do is it will wipe out a large number of bits successively. So, keep this picture in mind we are going to come back to visit it and formalize a lot of things pertaining to this, but let us now go in to theheart of today’s lecture. So, the first thing that I would like to cover is a very important derivative from the observation that we have all already made.

(Refer Slide Time: 10:42)



So, the autocorrelation R h of delta t comma tau; this is given by some constant P 0 J 0 2 pi f D delta t. So, I am assuming all of my multi path components are arriving around the same time. So, there is nothing no differentiation in terms of the tau, tau is not in the picture I am just looking at the correlation as a function of time.

So, if we were to visualize this think of the scenario that there are several observations that are being made. There is a channel the tau is not changing, so essentially the tau remains the same, but the channel is change. Maybe it is starting to decrease than eventually becomes negative, large negative.

So, what is the time variation of the channel and the correlation? So, that is what we have captured. And we have said that the rate of correlation is defined by the Bessel function. So, the notion of coherence time we made a very intuitive description of this; earlier when we are looking a talking about the say Tse and Viswanath example today we are going to formalize it.

Coherence time, we call this as T c- subscript c stands for coherence. This is the measure of the expected time duration; it is a measure of time expected time duration over which the channel appears to be highly correlated. So, if you got a strong channel for what amount of time can you assume that the channel is still remaining strong. So, basically the expected time duration over which the channel appears to be highly correlated; over which the channel or the channel gain appears to be highly correlated.

Now, highly correlated is a very qualitative term, let us make it quantitative. So, we are going to say that the autocorrelation we want is 1 over root 2. So, J 0 of this is the autocorrelation, we wanted to be 1 over root 2. So, over what range do you expect to see a high correlation; high correlation of the order of 0.707. So, we can look at the behavior of a zeroth order Bessel function. So, we are saying at what point will it cross a value 1 over root 2 if this is 1. So, we are going to look at this point and ask for what is the argument of the Bessel function that gives that.

So, we find that the argument that gives us is 1.125. Now, you want to know; what is the time delta t that causes the autocorrelation to come down to 1 by root 2. So, in other words we say that 1.125 you equated to the argument of the Bessel function 2 pi f D where f D stands for the maximum Doppler times delta t.

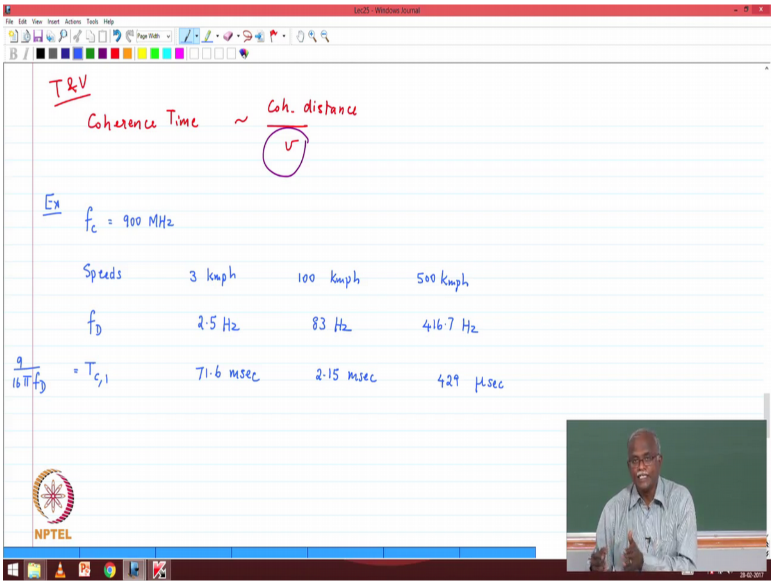
So, the first definition of the coherence time: so let me call this as T c comma 1 there are two definitions subscript one says this is the first definition it also turns out to be the more widely used definition. This is equal to basically a 1.125 divided by 2 pi f D. So, that is a same as 0.5625 divided by pi times f D. You can also verify that this is the same as 9 by 16 pi f D- 9 by 16 pi easy to remember 3 square by 4 square. So, again which ever you want to remember it.

This is the (Refer Time: 14:47) definition of the coherence time. So, what it says is this is the time duration over which I can expect to see worst case correlation anywhere between 1 maximum correlation to correlation of 0.707. That means the fairly high level of correlation.

Now T c 2: the second definition that is used T c 2 says I would like to look at the argument J 0 of the argument that comes out to be 1 by 2 50 percent correlation. Usually you cannot call this as very highly correlated, but that is as another definition that is given. So, this corresponds to 1.52 that is another definition that is sometimes used. So, T c basically what you are saying is 1.52 is equal to 2 pi times f D times delta t. So, the T c 2 which basically is delta t corresponds to 0.76 divided by pi times f D.

Two definitions: these are useful parameters and we will look at it a little bit more in today’s lecture. So, basically the intent is to derive the coherence time. I would like to sort of link it to the insight that we had from the Tse and Viswanath example, because always good that we are building on top of what we have already studied.

(Refer Slide Time: 16:23)



So, from the Tse and Viswanath example what did we say there- there was a notion of coherence time that was already introduced for us. Coherence time basically that is the time over which the channel will change very substantially. So, from that point of view how much time its take for the channel to change very substantially we said coherence time; this is from Tse and Viswanath if you turn back in your notes you will find that the coherence time we said is inversely proportional to the velocity with which you are moving and the numerator depends on the coherence distance.

But the important thing that was key for us was that it was reciprocal inversely proportional to the velocity with which you are moving. And is that consistent yes, because the maximum Doppler is directionally proportional to the velocity with which you are moving. So, your coherence time inversely proportional to the velocity with which you are moving. And there it was just an intuitive feel saying- if I am going very fast the channel is going to change very quickly, but here is a more formal quantitative description which says the autocorrelations are Bessel function how much time does it take for my auto correlation to come down to a value of 1 by root 2; that is obtained by this calculation.

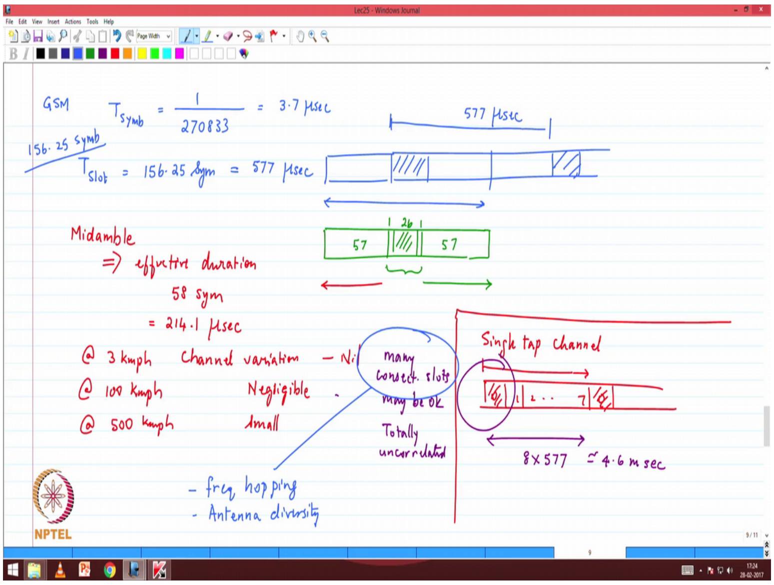
A quick point to stop and a clarify if there are any doubts. Basically coherence time is a time domain characterization of the statistics of a fading channel and it is a very useful one that that will help us in our understanding of the behavior of the channel.

Now, if there are no questions what I would like to do is again always want you to have a good feel for numbers. So we will do quick example; this is something where you should spend some time and fill out the information. But the examples says: I am looking at a carrier frequency of 900 megahertz, basically I am going to look at a different speeds I am going to look at 3 kilometers per hour, 100 kilometers per hour and 500 and case you wondering where did we get 500 from when they design their the GSM system the fastest trains where around 300 kilometers per hour they said. At some point in time trains will travel at 500 kilometers per hours, so they designed the system for 500. So, it is not an arbitrary numbers they assume that that would be fastest land travel that we probably would see.

So, the points at we want to look at is what is the Doppler at each of those speeds, and what is the coherence time at each of those. Basically we are trying to get a feel for coherence time; is it seconds, milliseconds, microseconds; what is it. That is what we want to derive. So, basically at 3 kilometers per hour if you can do a quick calculation of the Doppler; T c 1 this is nothing but 9 by 16 pi f D.

So, 3 kilometers per hour gives you 2.5 hertz, is the Doppler frequency 71.6 milliseconds as your coherence time; 100 kilometers per hour 83 hertz Doppler and 2.15 milliseconds as your coherence time; 500 kilometers per hour, 416.7 hertz, and this is 429 micro seconds- almost 430 micro seconds. So, notice that you are coherence time is definitely decreasing as the speed is increasing, that is what we anticipated. And this is something that we or able to do as a quick calculation. But as I always say the calculation is always used for a specific purpose, and here is the purpose. And here tells you a lot about a system design.

(Refer Slide Time: 20:45)



So, the GSM system: as I told you we are let us say we are designing the GSM system here is the thought process. What is the symbol duration? Symbol duration is 270833 kilo symbols per second. So, this is 3.7 micro seconds. The slot structure that we have a for GSM, number of symbols in a slot; number of symbols is 156.25 symbols in a slot. I think if you go back and look at the diagram and why that 0.25 is because of the god time that is allowed. Basically this corresponds to 156.25 symbols that is a same as 577 micro seconds.

So, what this says is that- if I were to look at the slot structure; time division multiple access slot structure here is a training there is some data on both sides and then the next this the end of the slot, the next slot begins and then there is a training sequence that is provided. So, the duration of one slot basically is this; from start to finish of the time slot that is the duration of a slot notice that that is also the same as the duration between the start of the training sequences basically it is a.

So, this duration is 577 micro seconds correct; that is the duration of the slot. Now I want to go in and look at it a little bit more closely so that we can draw the insights that we are looking for. So, from the point of view of one slot this is the training sequence on either side there is a flag the training sequence is 26 bits, that flags on either side are 1 bit and then there are 57 data symbols on either side and then there are some tail bits and guard bits and other things.

So, from the point of view of the receiver design algorithm I will get a channel estimation using this point. Basically, I have a channel estimate and I using the channel estimate I have to demodulate the 57 symbols that are going to follow. Using the same channel estimate I will also be doing the demodulation of these 57 bits on either side.

So, effectively what is the time duration of my coherent detection block? So, because I have my training sequence in the middle it is called a midamble, because of the midamble the 577 microsecond times slot now became split into two parts each of which consists of 57 plus 158 bits; so effective duration of the burst; effective duration for coherent detection was 58 symbols which is the same as 214.1 microseconds ok.

Now, why is all of this important? Now correlate this to our understanding of the channel variation. How much channel variation, is the channel estimate that I get is it valid across the entire burst. So, take the question like user moving at 3 kilometers per hour, for how long is that channel estimate valid in terms of the coherence time? 71 milliseconds. What is a duration of the burst that I am detecting? 214 microseconds. So, the channel is absolutely stationary; once you done the estimate your good to go.

So, basically at 3 kilometers per hour the channel variation is I can say is almost nil; channel variation is nil. Because it is much much less than my coherence time, because that is on the order of milliseconds I mean the micro second regime. Now, what about 100 kilometers per hour? 100 kilometers per hour my coherence time is a 2.5 milliseconds; still in the millisecond range much less than that. So, the channel variation is negligible; it is not very substantial.

However, once I get to 500 kilometers per hour I must be a little bit careful, because I am coming into the domain where the correlation is going to happen. And at 500 kilometers per hour may be I must do some channel tracking so, but still the channel variation is small. And this is a important aspect that we want to want to keep in mind.

Now, I also want to tie it to one more concept which is very important for us. Now add to this same scenario the following condition. Assume that you have a single tap channel; so which means if this channel tap goes high, you get good SNR channel tap goes low your basically going to be wiped out in terms of the fade. Now this is just one time slot, but if you are when you look at the TDMA slot structure, basically you get 0 1 2 all the way up to 7 and then you get times slot 0. So the duration of this, because your data will come on time slot 0 then there will be some gap it will come on the next time slot. So, if this is your some of your data is on this slot then the next slot.

So, now the question is what happens if this slot is in a fade; what happens if this slot is in a fade? Can I say anything about the next slot that is going to happen, it is actually tied to your interleaver design argument, but let us see if we can and we can make a statement? So, this duration is the duration of 8 time slots. So, 8 into 577 microseconds this is approximately 4.6 milliseconds.

So, I tell you that the currents burst is affected by a fade, what can you tell me about the next a slot; am I likely to have a good channel in the next slot. At 3 kilometers per hour what is a coherence time is 71.6 milliseconds. So, which means that if one slot is affected you might as well write of the next one; that is very high likely hood that the other is also. So, this basically says at 3 kilometers per hour many consecutive slots will be gone will be wiped out; consecutive slots effected by the fade.

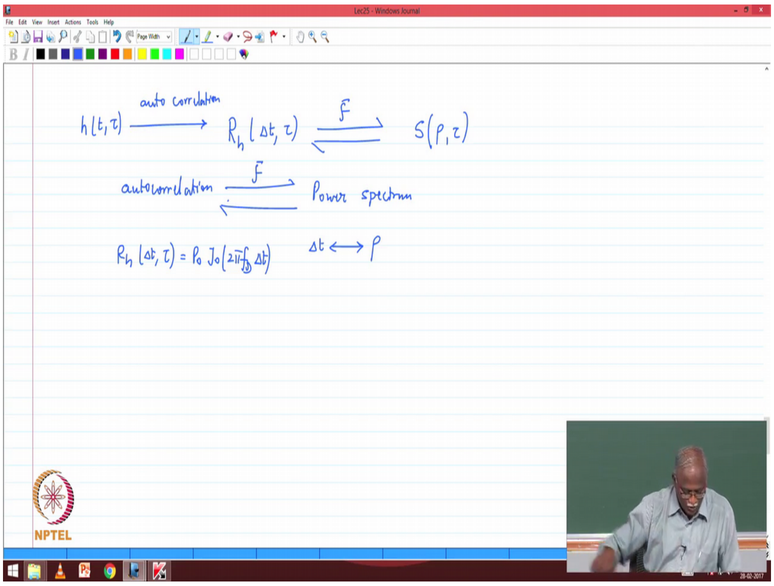
So, again we are we are tying concepts together so that everything sort of makes sense when we put it together. So, when the case of 100 kilometers per hour, what is the likelihood that the next slot is affected. 100 kilometers per hour 2.15 milliseconds and you know the separation is 4.6, so even if it was a bad slot the correlation period has gone. So, there is a good chance that may be next slot may be 500 kilometers per hour they are totally uncorrelated. So, the basically in this case the two slots are because this is completely outside the zone of correlation, because the coherence time is only 427 microseconds you are you know way above that so totally uncorrelated. So, one burst to another it is totally uncorrelated.

Now, you are very worried about this one, as we said- either have to design a very large interleaver or I have to do something else as always the options are frequency hopping. Frequency hopping means you can effectively break the fading pattern or you can use antenna diversity the same, you can choose the better of the two antennas. And therefore, you can get around it.

So, you can see that; as I said understanding the wireless channel is like a jigsaw puzzle, there are different pieces that you have to pick up you have to say- this piece has got certain behavior properties it effects in some ways and then put the pieces together to get the larger canvas. So, that is one aspect. And what we have done today is to give you a flavor for the coherence time and what its impact is on the overall system design. Any questions on what we have said. Basically, its a derivation of a formula using the Bessel functions property you have derived it, but more important is the insight that you gain from low Doppler to medium Doppler to high Doppler.

We move on the next important concept in our characterization of the fading channel and this is part of our building up of a very good understanding of the system.

(Refer Slide Time: 30:23)



So, now what we want to do is to use this observation. We have h of t comma tau using which we did the autocorrelation in the time dimension, and we observe that it is a stay wide sense stationary process because it only depends on delta t. Now I want to gain some additional insight a lot of times insights come from moving from one domain to another. So, I would like to move from the time domain to the frequency domain.

So, autocorrelation again this is a general question; autocorrelation if I do the Fourier transform gives me.

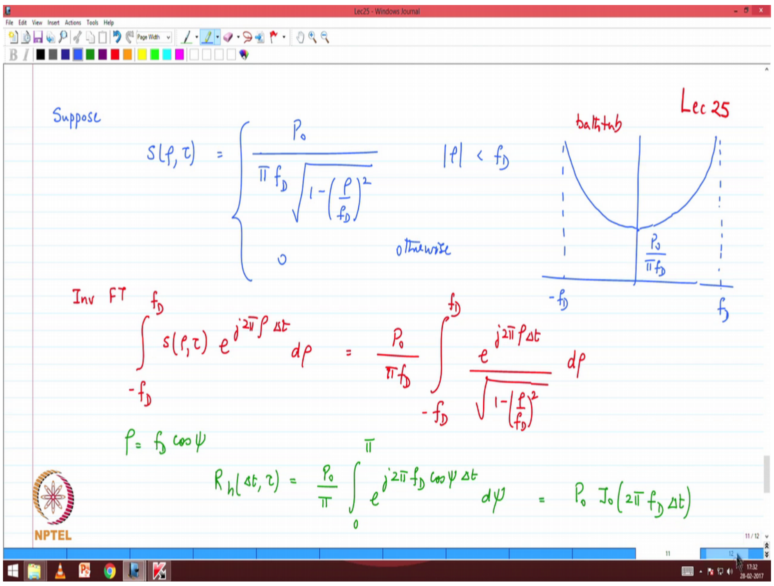
Student: Power spectrum.

Power spectrum, and therefore I am going to take the power spectrum of the time dimension. Basically, I am going to look at it not in the tau dimension I am looking I am still working the time dimension. So, this is the power spectrum. Now, of course you can do the inverse Fourier transform and go back to the auto correlation that is not a problem.

Delta t may cause some confusion if we use it when in order to do the transform. So, I am going to do a variable change to basically call it as; delta t is my time domain variable in the frequency domain I want use f I am going to use rho. So, R h of delta t comma tau through the process of Fourier transform is going to give me S- I am going to use that because it is the notion of power spectrum, delta t maps to rho and tau I have not touched so leave tau alone tau is some fixed value.

Now I know the value for the R h of delta t comma tau this is equal to P naught J naught 2 pi f D times delta t. Now I need to know how to do the Fourier transform of the Bessel function. Some of these things are little cumbersome in one direction turns out to be their fairly easy in the other direction. So, I am going to take the easier option out show you that I am going to go backwards; what power spectrum will give you a Bessel function in the time domain, because the convert transforming the Bessel function is a little bit trickier.

(Refer Slide Time: 32:50)



So, here is a model power spectrum, suppose s of rho comma tau is of the following form. P 0 divided by pi f D square root of 1 minus rho by f D whole square. This is for mode rho less than f D its 0 otherwise. So, basically what is the shape that we are seeing? As rho approaches f D denominator becomes smaller and smaller, so therefore the power spectrum is symmetric with respect to the values of the rho. It starts at a certain value,

value will be P 0 by pi f D, that is rho is equal to 0. And then keeps increasing as it approaches f D. At f D it will go to infinity, but the f D is outside the; that is f d.

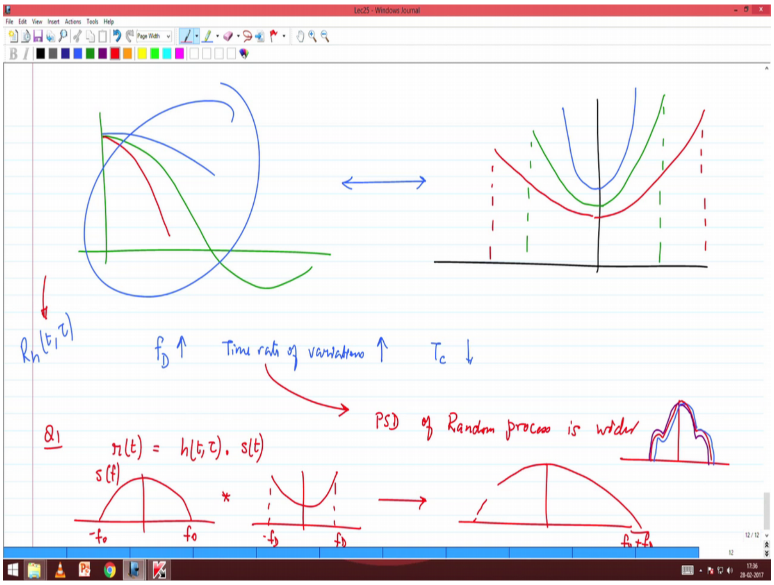
So, basically it up starts becoming very large. And likewise it is a symmetric shape and this is minus f D. So, this is the shape of the power spectral density that we are considering. And, its got a very layman's term it is called the bathtub shape. Those are first in wireless communication think of it as the Motorola symbol m is the Motorola symbol. So, in bathtub seems to layman Motorola seems a little bit more sophisticated. But basically it is the shape that we have as the power spectrum.

Now, I need to see if this maps to the Bessel function. So, I would like to take the inverse Fourier transform. So, inverse Fourier transform is minus f D to f D because that is the range in which it is defined s of rho comma tau e power j 2 pi rho delta t d rho; that is the inverse Fourier transform mapping from rho to delta tau. So please do the substitutions, we will just get the answer in one step. P 0 divided by pi times f D minus f D to f D e power j 2 pi rho times delta t divided by square root of 1 minus rho by f D whole square d rho. And basically one step and we get the answer.

So do this following substitution: rho is equal to f D cosine psi. Do the substitution and a verify that what you will get is R h of delta t comma tau is given by P 0 divided by pi integral 0 to pi e power j 2 pi f D cosine psi delta t d psi. And if you look at this form this is nothing but the definition of the Bessel function: J 0 2 pi f D delta t. So, did we achieve or what is the outcome of our observation.

I got the time domain auto correlation as a Bessel function; I want to understand what happens in the frequency domain. So, I said I will do the Fourier transform and therefore get the power spectrum. The power spectrum that is of the form of inverted u shape with the limits being minus f D and f D; that shape is the one that maps to the Bessel function.

(Refer Slide Time: 37:25)



Now the important link to what we are going to concluding today's session with is: I have different Doppler's. That is a medium Doppler, this is a high Doppler, blue gives me a low Doppler. I would like to now look at the corresponding frequency domain expressions notice in all cases it will be a bathtub shape. The shape would be if it is low Doppler I get a bathtub that is steeper, if it is a medium Doppler little bit wider goes a little lower, and then if it is a basically the high Doppler; in all cases it is a and notice that the limits are always the corresponding Doppler maximums.

So, what we have find is that the power spectrum is wider. So, if the Doppler increases, so here is a summary statement- as the Doppler a frequency increases the time rate of variation, the rate at which the time domain fluctuations a time rate of variations this increases the time correlation period that decreases that we have already seen. Now what we have finally made the observation as the Doppler increases the power spectral density of the random process is wider.

So, the last observation that I want to leave you with is the following is a question, that I want you to think about so that we can pick it up in the next class. Now, what is the process that we are looking at? The received signal r of t is h of t comma tau multiplied by s of t. Notice that it is a multiplicative process.

So, if I want to do a frequency domain interpretation what should I do; its multiplication in time it will be convolution in frequency. Therefore, if my signal spectrum has something like this, this is the power spectrum but its 0 outside of minus f D to f D. So, the Fourier transform or h of t must also lie within the same range. So, this is has to be convolved with this is s of f; this has to be convolved with something that is nonzero only in the range minus f D to f D.

Now, I need you to tell me if this is minus f naught to f naught what will be the resultant spectrum. It will be f naught plus f D minus f naught minus f D on the other side: f naught plus f D. Now as we have already done a calculation before you can verify that f D is very small compare to the bandwidth of the signal. And therefore, what you will find is a GSM type signal, GSM type signal looks like this, the variation that you will see is just a very slight shift on the other side and a very slight shift on the. So, it looks like spectral widening. So, the frequency domain part is not a big issue for us because that is not going to be the 1. What is going to be the challenge for us is to building the coherent detector for this.

So, what we have done in today's class- been able to characterize the t dimension in a very comprehensive manner. R h of t comma tau we have fully or at least as in a very good way we have understood the impact that how the channel is varying, what is the statistics, what is a power spectrum, how is it going to affect my signal, all of that we have been able to characterize. So, the next class we begin by looking at the tau dimension. And in the mean time we will also be looking at the next written assignment and next computer assignments as well.

Thank you very much.