Introduction to Wireless and Cellular Communication
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Lecture – 25
Wide Sense Stationary Uncorrelated Scattering (WSSUS) Channel Model
MGF Part II, WSSUS Model

Good morning, we begin with a quick review of the lecture number 23 and then the new concepts that we want to introduce in today's lecture the substantial focus is going to be on a very powerful model that we have which is the model which we refer to as the wide sense stationary uncorrelated scattering model and it is one of the things that helps us completely characterized wireless channel, time domain, frequency domain, Doppler; all of the elements are captured in this model.

So, it is a very important model, we will definitely spend time on that. We have already looked at the concept of coherence time when we talked about the example and say N Vishwanath and we will now formalize it using the wide sense stationary un w s s u s model and of course, there will lots of a simple examples to help us capture the concepts even as we as we go along. So, let us begin with a quick summary of the points that we have discussed in the last class and then we will move on to the new-new points in today's class.

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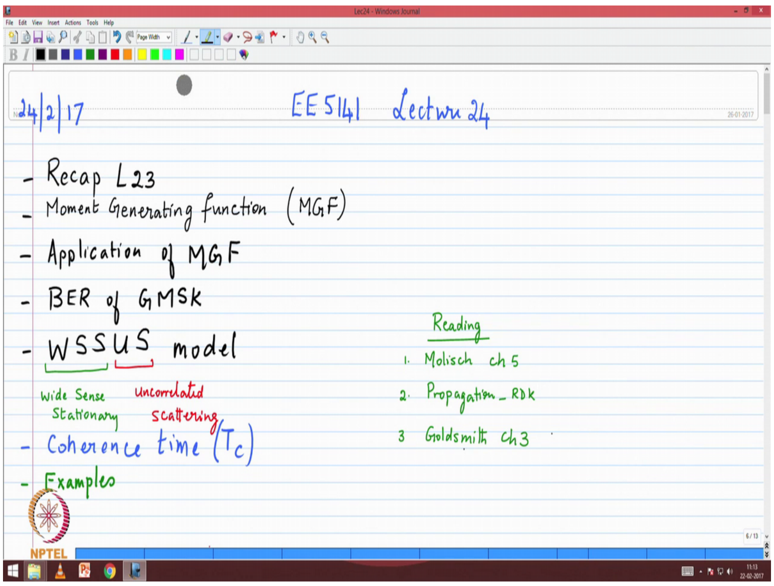

So, in the last class we have looked at the Nakagami m fading. So, let me write down that our interest is to study Rayleigh; fading Rayleigh is the case of interest of lot of interest.

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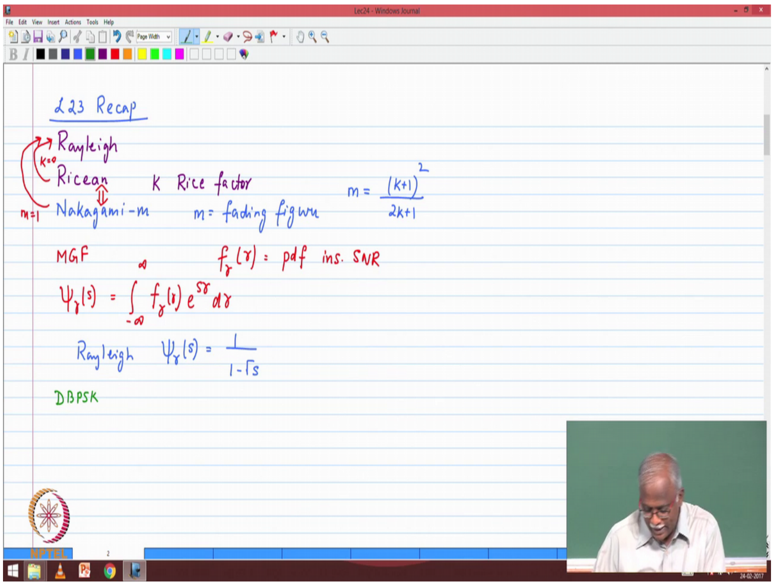

But several times the environment requires to characterize other channels as well. So, we come up with the Rician when there is a line of sight component depending upon the amount of line of sight component relative to the non line of site you get family of distributions then we also have the Nakagami m; Nakagami m distribution which is obtained from the experimental data may be its to sort of say that these 2 are in some way equivalent because they are covering the same except that Nakagami m is a broader class of a distributions. So, the Rician distribution will be characterized by means of the rise factor k rise factor k the Nakagami m will be through the fading figure m is equal to fading figure and they are related by the relationship we have given in the last class, but let me just write it down k plus 1 whole square divided by 2 k plus 1 that says that I can relate k to m and m to k that sort of makes me able to go from Rician to Nakagami.

Now, interestingly both of these have the Rayleigh as a special case. So, in the case of Rician it will be k equal to 0 in the case of Nakagami its m equal to one they give you that as a special case and is useful for us to keep that relationship in mind because when we get expressions for the for the bit error rate using the Nakagami m you can get Rayleigh as a special case by substituting these specific value of the fading figure. So, in the context of the bitter rate calculations we introduce the notion of the moment generating function.

Moment generating function; we can talk about for the any distribution, but we are primarily interested in the distribution of SNR we are also interested in the distribution of the envelop, but SNR is very useful for us f gamma of gamma this; is the instant pdf of the instantaneous SNR instantaneous SNR we would be very interested to understand what would be the 5 gamma of s which will be minus infinity to infinity the distribution e power s gamma d gamma again we derived it for the Rayleigh fading channel for the Rayleigh fading and said that the moment generating functions are for the Rician and Nakagami are given in Andrea Goldsmith. So, goldsmith books, so, therefore, this is just as an illustrative example for the Rayleigh fading channel the SNR has an exponential distribution and the moment function is given by one minus gamma s.

Now, we also showed how we can calculate the bit error rate of DBPSK; DBPSK thorough direct integration also through the use of the moment generating function.

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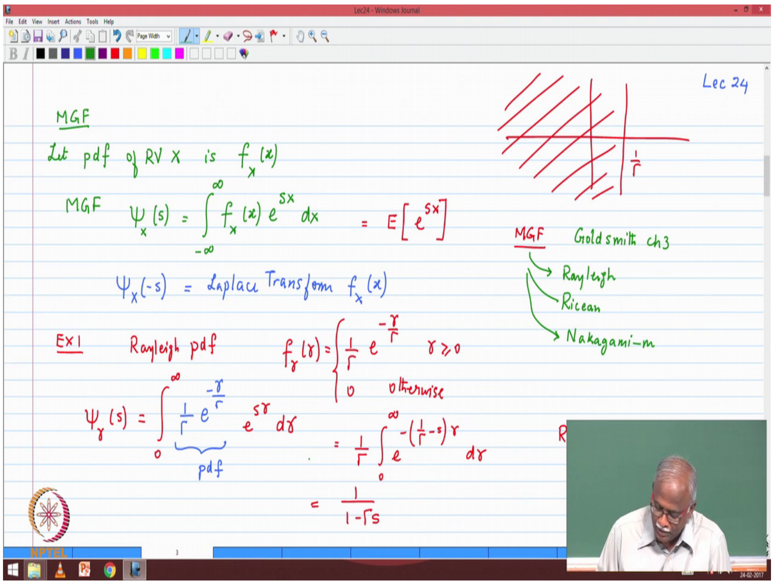

Let me just capture that again the interpretation in terms of a Laplace transform; Laplace transform and also in terms of the expected value both of them are useful for us keep in mind that we have define a or region of convergence and we just want to keep that a picture in mind as well.

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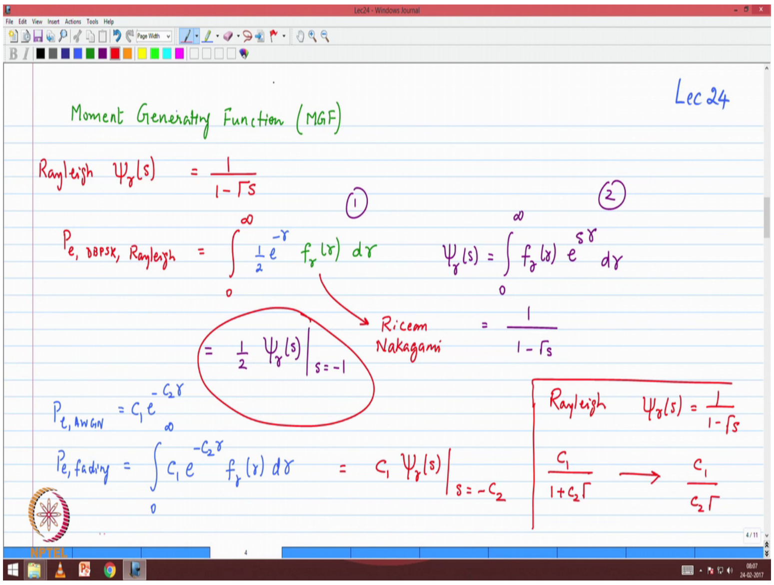

So, now the moment generating function how did we use it to calculate the bit error rate of DBPSK we said the integral of interest is a half e power minus gamma f gamma of gamma that is the expression for the bit error rate when you integrate it over the range of gamma that gives you the bit error rate in the fading channel then we rode down the expression for the phi gamma of s that is the moment generating function and saw this similarities between these 2 and then said well this is nothing, but special case of the moment generating function evaluated at a specific value s equal to minus one which lies in the region of convergence and of course, the scale factor half just comes in to the picture.

Now, I would like to extend this the; this usefulness you may think well it is only for DBPSK what about others. So, there is a very broad class of a modulations where the probability of error in AWGM probability of error in AWGM if it is of the form C 1 e power minus C 2 gamma C 1 is a constant C 2 is a constant and basically it characterizes a bit error rate depends on the modulation type that we have now if you have a modulation of this type then the probability of error in a in the fading channel in the fading channel will be integral 0 through infinity C 1 e power minus C 2 gamma f gamma of gamma d gamma that is the general expression for the probability of error given the distribution of the SNR.

Now, notice that this can also be readily linked to the moment generating function C 1 is a constant that comes out of the expression this is phi gamma of s where s is evaluated at minus C 2 minus C 2 again it is a; for broad class of transfer function as long as the error is of the form a something e power minus C 2 gamma you can quickly derive that expression. So, very quickly you know if I were to ask you to evaluate this expression in Rayleigh fading Rayleigh fading I would not even look at the integral I would straight away say that were the phi gamma of s in the Rayleigh fading is one over one minus gamma s.

So, the bit error rate in Rayleigh fading is going to be C 1 what is that divided by one plus C 2 gamma that is it that is the expression for because basically I took phi gamma f s kept the constant and then substituted s is equal to minus C 2 high SNR approximation would be given that I have my probability of error expression as C 1 by one plus C 2 gamma what is the highest SNR approximation you should be able to get this basically says that gamma is very large. So, C 2 gamma is going to dominate over one. So, it will be asymptotically this will be C 1 by C 2 gamma that is it the high SNR approximation always says that gamma is large. So, if there are other constants it will dominate over the other constants and therefore, easy for I mean the becomes a useful way of approximating I would like to now go back and address one more question.

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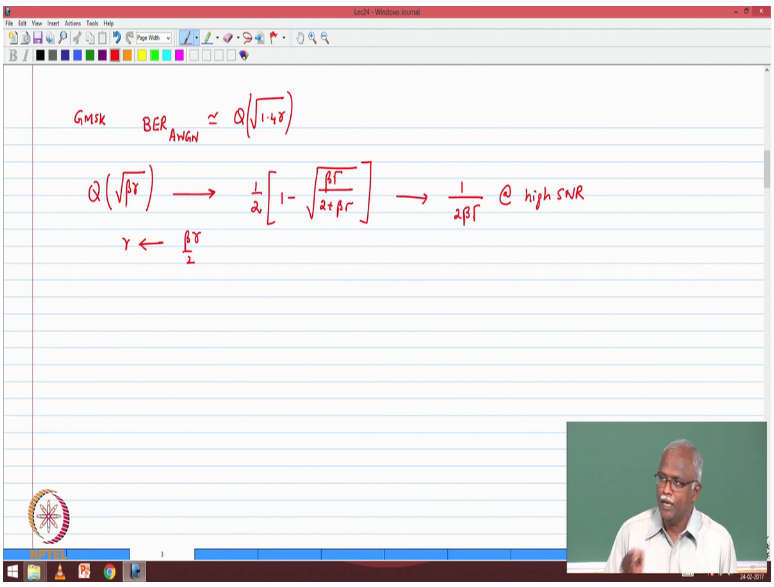

now what about if the error function is not exponential what if it is a q function and I have some arbitrary pdf and again and to anticipate that question let us look at a example again its very straight forward. So, for example, GMSK; GMSK yesterday we said that the BER in AWGM was approximated by q of square root of one point four gamma now what do I do? I go back and read do the integration no we showed that you can just substitute and get that.

So, in the general case if my error function is of the form q of root beta gamma beta some constant and many of our modulations schemes we will fall in to this picture there that constant changes, but ultimately it will be of the form beta gamma. So, this we can then through the process of substitution whatever we have done I believe we showed it in the last class or otherwise if not through the process of substitution where you would do gamma replace by beta gamma by 2 in the expressions that we have that will give us one minus square root of beta gamma divided by 2 plus beta gamma and the high SNR approximation in this case in the in the a case of BPSK was one over four gamma if you replace it with the this expression it becomes 1 over 2 beta gamma at high SNR.

So, whether it is the exponential form or the complementary error function form there are certain simple tricks that you can use to get the bit rate expression, but of course, there is a lot more to the BER analysis than just getting the basic form and that is where I would like to high light some of the applications of what we are going to be discussing today. So, any questions on what we have covered in lecture 23 basically a new; new type of fading which has a broader characterization Rayleigh as a special case of the Nakagami m with a fading figure where m equal to one then the moment generating function and its role in computing the bit error rate and also some simple techniques for getting the expressions when you have a form q is equal to root beta gamma or when you have the form that we discussed just now where you we have the probability of error is C 1 e power minus C 2 gamma.

So, these are 2 very broad categories of a expressions for the different modulations schemes that will encounter and therefore, we have some simple tools to calculate the basic bit error rate, but as I mentioned this course is not about the basic the basic is what you have done in digital communications we have just showed you that you can extend it to the fading channel, but this course is all about taking that BER performance in fading and making it go closer to AWGM that is where the trick is and that is where the challenge lies. So, that is where we pick up today's lecture.

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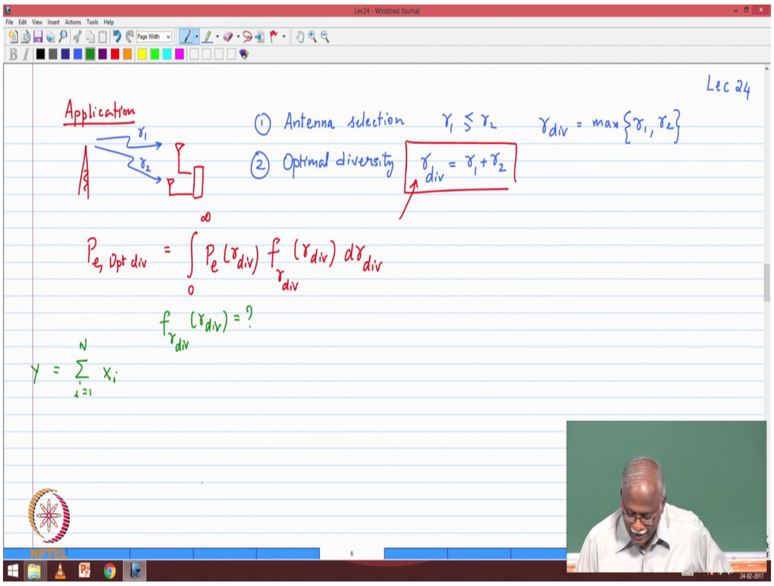

So, as a first motivation for the types of things I have going to be studying let me a write down an example or an application again its simple example, but again it gives you a lot of insight and into the motivation for the things that we are going to be doing. So, consider that I have a base station and I have a mobile which has got 2 antennas a 2 antennas and the transmission from the base station is picked up by both antennas, but because of their spacing between the antennas the channels are different and this one is seeing an instantaneous SNR gamma one instantaneous gamma 2.

Now, at the receiver I can do one of 2 things I can do what is called antenna selection antenna selection would be a very simple method which says compare gamma one and gamma 2 if gamma 2 is higher pick gamma 2 if gamma one is higher pick gamma. So, that is what we call as antenna selection. So, you compare gamma one is greater than or less than gamma 2 and then you pick the pick the stronger antenna that is one way of doing it and this would also shift the changes statistics. So, there is selection diversity.

Now, there is an even better form of diversity which we; I will for now call it optimal diversity optimal diversity which says you do not need to pick the larger of the 2 you can actually do a form of combining. So, that your resultant diversity. So, in this case the gamma diversity is equal to the max of gamma one comma gamma 2 correct that selection diversity optimum diversity says you can actually achieve gamma 1 plus gamma 2 this is very very powerful because even if both antennas are in a fade you may actually be able to combine them and detect the signal because the combination of 2 SNR may be above the threshold that you can detect. So, this is very very useful for us.

Now, notice that gamma one and gamma 2 are the instantaneous SNRS both of which are which are random variables both of which we will have their own distribution and there is no way we can make the assumption that they are identically distributed because one antenna may be in the top of the phone one antenna may be inside one antenna may be behind the battery. So, the we cannot make the assumptions that both of them are identically distributed now that are exactly where the challenge comes. So, if I have a new random variable which is gamma one plus gamma 2 where I do not have the guarantee that their identically distributed then I have a problem because I know if I if you tell me give me the pdf of gamma diversity I have a problem because most of the familiar methods that we have worked with says assume that the the variables that you are adding are identically distributed.

So, here is the challenge that is before us. So, we want to estimate the probability of error under maximal ratio combining or optimal diversity optimal diversity. So, this is going to be integral 0 to infinity probability of error under gamma diversity and I will ask you to give me the pdf of gamma; gamma diversity. So, this is going to be an absolutely essential requirement for me to characterize the behavior under these types of scenarios and. So, the question is what is gamma f gamma diversity gamma diversity what is this equal to and I am sure you are familiar with the in probability statistics we often do this type of a problem I is equal to one to N x i and we find out that this is equal to y and find out, but we often make the assumption that they are identically distributed. So, that sort of mix the, but it is definitely doable if you have a different-different distribution, but not straight forward. So, this is where moment generating functions come and play an important role.

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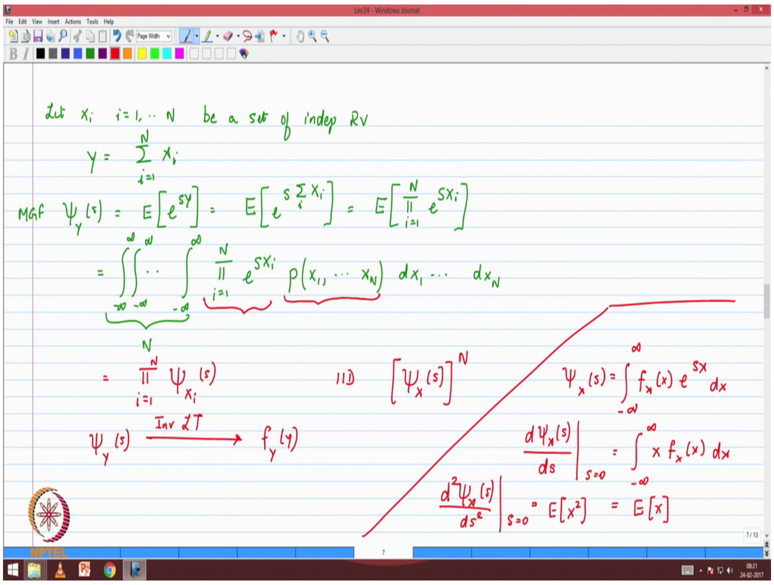

So, let me just define the method and then we will do. So, let x i is equal to one through N b a set of independent random variables that is all the assumptions that we are making be a set of independent random variables not identically distributed. So, y is equal to summation I equal to one to N x i the moment generating function of y is given by expected value of e power s y correct that is the expected interpretation as an expected value the moment generating function.

So, this the moment generating function that we are interested in, so, now, substitute for y this becomes expected value of e power s times summation over i x i basically your replaced y with summation of x i basically expand the summation and rewrite the expressions this becomes expected value your adding the powers. So, therefore, it is like multiplying the exponents multiply the exponentials. So, I product of i is equal to one through N expected value of s times x i i have just rewritten just a simple rewriting of this expression. So, the right hand side is a expectation of N random variables of a function of N random variables. So, I would now get a integral situation where there are N integrals each of which going from minus infinity to infinity whatever is the range of the these general definitions.

So, from minus infinity to infinity there will be N such integrals product of I is equal to one through N e power S x i i must have the joint pdf of these random variables x 1 through x N then integration over the variables d x 1 to d x N this is what will give me the expected value of the quantity that is within this bracket. So, again I have just rewritten the expressions for the moment generating function of y now this is these are assumption is that these are independent random variables. So, the joint probability is the product of the probabilities which now says in one step you have the final answer I is equal to one through N phi of x i not i phi of x i of s. So, it is a product of their moment generating individual moment generating functions and as long as you know what the distribution of those variables are you can write down the expression by inspection there is absolutely no problem for that no if its. So, happens that they are i i d which is very good for us then it becomes phi x of s raise to the power N once again a very easy thing for us to work with.

Now, what is the benefit of all of this notice that we said that the moment generating function can be interpreted as a Laplace transform. So, which means that if I have the moment generating function I can do the inverse Laplace transform and get the pdf very straight forward. So, once I get the phi y of s then I can do the inverse Laplace transform to get the f y of y. So, what may have been difficult for me to do directly in the with the pdfs it becomes a very easy problem for us when we work with the moment generating function and then do the inverse Laplace transform notice that we have used both the mathematical interpretation of m g f as well as the Laplace transform interpretation in to our advantage to make sure that we are able to get.

In case you are wondering why the name moment generating function I probably you are familiar with this phi gamma of s phi x of s definition is minus infinity to infinity f x of x e power S x d x we talked about differentiating an integral. So, if you can differentiate from me phi the moment generating function with respect to s differentiate it with respect to s first term will be with respect to the upper limit second term with respect to the lower limit both are constants are not in the picture then you have to differentiate the integrand.

So, basically differentiate the integrand and set s e equal to 0 if you do the differentiation should be straight forward what you should get inside the bracket is e power minus integral minus infinity to infinity x times f x of x d x which is nothing, but expected value of x and once you have the moment generating function it is very straight forward you take the second derivative d square phi x psi x of s divided d x square set s equal to 0 you will get expected value of x square and if you want the higher order moments is just more difference additional differentiation.

So, moment generating functions have a very useful rule in the context of mathematics you may not have seen it, but for us its very very useful because, but for the moment generating function some of these pdfs would be a quite difficult for us to work with any questions.

So, moment generating functions I will assume that we are familiar with it that you can apply the moment generating function to get the more difficult pdfs and then use that to get the bit error rates of the modulations schemes that we will encounter that something that is useful tool for you to have once you as we study this course.

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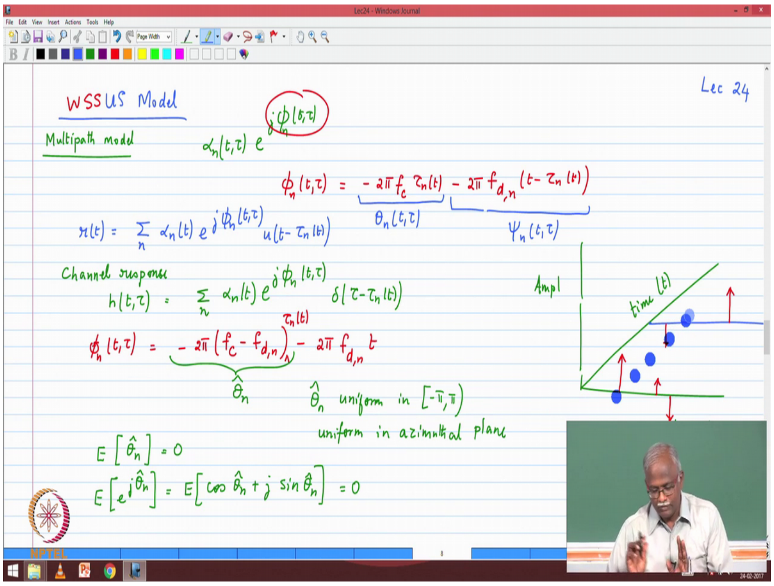

Now that is has set the stage for us to talk about the important model that we are going to working with wide sense stationery uncorrelated scattering model for a fading channel. So, for this I would like us to revisit the multi path model that we had derived earlier when we first talked about Rayleigh fading.

So, go back turn your pages in your nodes I do not have the lecture number, but I am sure you will see that this is just before we derive the multi path model. So, we said that each of these multi path components there are large number of multi path components which are going to get super posed each of them has got a angle of arrival based on their the direction of their multi path component; that means, their Doppler's are different. So, based on that we said that there is a channel gain alpha N and there is a phase term phi N E t comma tau e power j where I should write it. So, alpha N t comma tau e power j phi of t comma tau. So, I want to focus in on the part phi phi of t comma tau there should be an N here.

So, phi N t comma tau again please if you can refresh just turn your pages back to that, but if not I will just write the expression minus 2 pi f c tau N of t minus 2 pi 2 pi f D n f D n t minus tau N of t this was the expression for the resultant phase of that particular component and we actually gave them some names this one we called as theta N t comma tau and the next one we labeled it as psi N t comma tau again the exactly copied from the previous page.

So, what this model told us was that the received signal r of t can be written as a super position of many components where this is resultant of alpha N of t that is the time varying gain and e power j phi N t comma tau times u of t minus tau N of t. So, basically you will get shifted versions of the transmitted signal transmitted signal is u of t u of t and they are going to be scaled by these complex numbers and then added together to produce the resultant signal.

So, we said that the channel response not impulse response; channel response because it is not a LTI system. So, I cannot call it an impulse response, channel response at a given time can be characterized as a 2 dimensional variable h of t comma tau, this is given by summation over N alpha N of t e power j phi N of t comma tau and delta of tau minus tau N of t this was the channel response and again if you recall we explained this or a used a 3 dimensional representation. So, if you think of this as the amplitude this as the delay dimension which is indicated by tau this is the time dimension which is indicated by t.

So, at a given time instant; a snapshot, I may get some number of multipath components. So, we said that I could get a multi path component like this some distribution then at another time instant; another time instant which is a different snapshot, I may get a channel which is very different. So, for example, the first path may be one, second path may have gone off and then you have a second (Refer Time: 28:19). So, basically that this is what it looks like when you look at a channel response at different instances of time and these are the multi path characterization that we have obtained I hope you are familiar with it you are able to recall whatever we have discussed in the last class in the and the when we talked about the multi path model. So, start with using that as my starting point I want to build the w y w s s u s model and therefore, that is why we are or rewriting this equation.

So, I would now like you to focus on the phase of the multi path component. So, that is phi N of t comma tau that is the phase response now the previous characterization was for the earlier discussion, but the same thing I want to write it in a slightly different manner for the particular analysis that we are going to be doing now. So, I am going to rewrite it in the following form again you can verify that is exactly the same 2 pi f c minus f D comma N minus 2 pi f D N times t and I would like to label this as theta N hat there is an amusing theta N hat is because theta N I have used it for something else now going through is a missing something or tau.

Thank you I missed a tau N of t that is a; that is multiply; it is not exponent its multiplying that bracket term in the bracket using the same arguments as before because these Doppler shifts are can be based are based on the angle of arrival; these this term theta N can lie any were between minus pi to pi there is no preferred direction. So, therefore, we can say that you know if there are large numbers of these multipath components then this theta N hat can is basically there is no preferred direction it is uniform in minus pi to pi minus pi to pi I have no preferred direction.

So, in other words this is this now captures our understanding based on the angle of arrival this theta N hats are uniform in the azimuth plane that is the horizontal x y direction in the Azimuthal plane. So, that is a observation that again based on the earlier reasoning same reasoning applied again in the Azimuthal plane hell very useful and a very important thing that is going to come about. So, the first thing that we ask is; what is the expected value of theta N expected value of theta N 0 uniforms in this distribution. So, I there is no can be minus plus if you take it as minus pi to pi the expected value 0 now think and answer this question expected value of e power j theta N hat what is that right a next step and then you may change your mind cos theta N hat plus j times sin theta N hat is that correct and uniform distribution.

Student: Sir what is that c to the 0?

So actually 0 is the correct answer. So, so the of course, you can write down the expected value, but basically you are integrating sign or cosine over an entire period. So, therefore, it will go to 0. So, this is a very useful result for us to keep in mind and extend.

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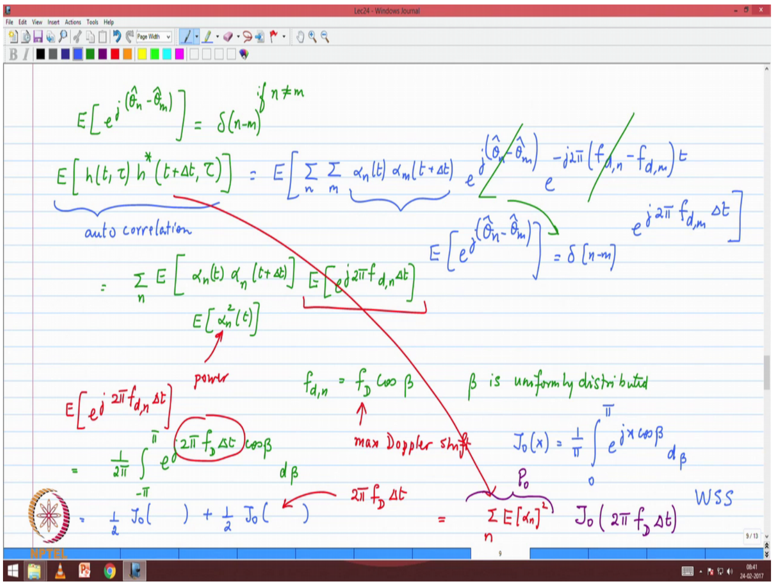

Now let me extend this to the following. So, what is expected value of e power j theta N hat minus theta m hat where N is not equal to m; if N is not equal to m and if theta N and theta m are independent right there is no; that means, your basically looking at 2 multipath components there is no relationship between their angles of arrival (Refer Time: 35:53) independent of each other.

So, then this also becomes delta of N minus m it is equal to one if it is if E; E power j theta N is z otherwise it will basically it is non zero only when it is only when its equal in this fashion. So, that tells me that I can now write the following expression I want what am I trying to do may be even before we before we do that see this red; red in channel response now I want to know how this red channel response changes as a function of time. So, I am not I am not worried about the delay dimension I am worried about how this is going to change at different instances of time as I you know along these dots that is my a intent for task that I was doing.

So, the thing that I going to do now is calculate expected value of h of t comma tau h star of t plus delta t just a small delta increase in the time, but I am not changing the tau dimension not worried about the tau dimension right now I can even assume that there is no dispersion there is only one single tap is that if that is helpful you can take it up in that in that fashion this is an important expression. So, what are we trying to do I am trying to get the autocorrelation of the channel response basically I am trying to see between time t and t plus delta t is there any correlation between them. So, this is autocorrelation expression for the autocorrelation of the channel impulse response at a given tau notice that tau is the same for both I am just changing time dimensions. So, I am going along the time dimension at a given tau and trying to see if there is time correlation.

So, this is basically we will write down expression may look a little messy when you write down, but actually keep in mind I am trying to get an understanding of how this channel changes is it totally uncorrelated from time to time that is you know channel at this time instant is very is a or how does it change. So, that is what we are trying to get. So, this is written as expected value of write down the expression for h of t comma tau that is the first summation, let is call it as summation over N then h conjugate that will also give you another summation we will have to keep different variables of summation independent variables.

So, we will call it N and m. So, the expression inside will be alpha N of t alpha m of t plus delta t that is part from the second summation alpha is a real valued a parameter because it is a amplitude. So, therefore, there is no conjugation there the all the phase terms will see a conjugation e power j theta N hat minus theta m hat, if I miss something please catch it let me know e power minus j to pi f D comma N minus f D comma m times t run out of space, but let me just write the last term e power j 2 pi f D m delta t.

So, I got all of the terms with the correct signs and this is within the brackets. So, basically this is the; that we are interested in looking at now of course, the alphas are completely independent of the angles of arrivals. So, basically we will quickly try to segregate and try to get the things of interest. So, this is a parameter of interest for us. So, expected value of alpha N alpha m is an important a important parameter. So, let is quickly analyze that.

So, if you if you expand this expression notice that you will get a expected value of e power j theta N minus theta m hat. So, that is equal to delta of N minus m which means the double summation will collapse to a single summation will work with a single summation; summation over N I am going to take the expected value inside it will now be alpha N of t alpha N of t plus tau alpha N t plus delta t.

So, that is one of the terms then notice that where ever this N and m are the same. So, some of the terms have cancelled out and what we are left with is another term which is expected value of e power j 2 pi f D N delta t this term gave us the delta function. So, which means this one goes away living you only 2 terms in the expectation one of them is the amplitudes the other one is the with connected to the phase. So, there are 2 things that we will try to capture and very quickly get the key results that that we have.

So, if you make the assumptions that delta t is small. So, this can be written as e power alpha N t wholes magnitude square. So, in some sense this is like the power of the received signal. So, this has the notion of the power some measure of power amplitude squad is coming into the picture expected value of that, but the second term may be not. So, obvious let us take a closer look. So, the second term is what we are going to focus on expected value of e power j 2 pi f D N times delta t actually the answer comes in just one step.

So, the insight comes from this particular observation f D N is maximum Doppler times cosine of the angle between the direction of propagation and the direction of motion. So, and because there are large number of Scatruss from our model we have always being saying that beta is uniformly distributed uniformly distributed the only thing that is random here is f d. So, therefore, I know that it that depends on beta and beta is uniformly distributed. So, I can calculate this expected value.

So, the expected value here will be minus pi to pi is uniformly distributed. So, its one over 2 pi e power j to pi its now f upper case d because its d we writing it as maximum where f upper case d is max Doppler; maximum Doppler shift or maximum Doppler frequency f D times delta t delta t times cosine beta d beta that is the random variable there we got the pdf and we have written down this expression.

Now, this is also of the form of another known function 0th order Bessel function let me just write down the definition J 0 of x is one over pi integrals 0 to pi e power j x cos beta d beta. So, notice that it is of the form. So, this is. So, this particular one is going from not from 0 to pi, but from minus pi to pi split it as 2 parts you can you can show that each of them is equal to one half of a Bessel function and one half of this same Bessel argument the argument that is of this is the expression that is here which is 2 pi f D times delta t 2 pi f D delta t now if you take a minute to sort of write it down systematically I am sure there is nothing complicated here, but you may be wondering why I am doing this because you know what at the end of the day what did.

So, basically this tells us that the autocorrelation the auto correlation is equal to equal to expected value summation expected value of alpha squared alpha N squared where the N is over all the. So, if I if I call this as my some measure of power let me call it as p not its some average value that is some p naught. So, my autocorrelation is equal to p not which is a constant times j naught 2 pi f D times delta t the most important thing is the auto correlation is of the form of a Bessel function that is interesting enough, but look at the argument of the Bessel function it is does not depend on t it depends only on delta t. So, this is what makes it a w s s process the amplitude the or the channel response as it varies as a function of time looks like a stationary wide sense stationary random process.

So, therefore, there is correlation between any 2 adjacent points it does not depend on t it depends on the time separation between those 2 points and that is consistent wherever you observe it is a very useful parameter for us to characterize.

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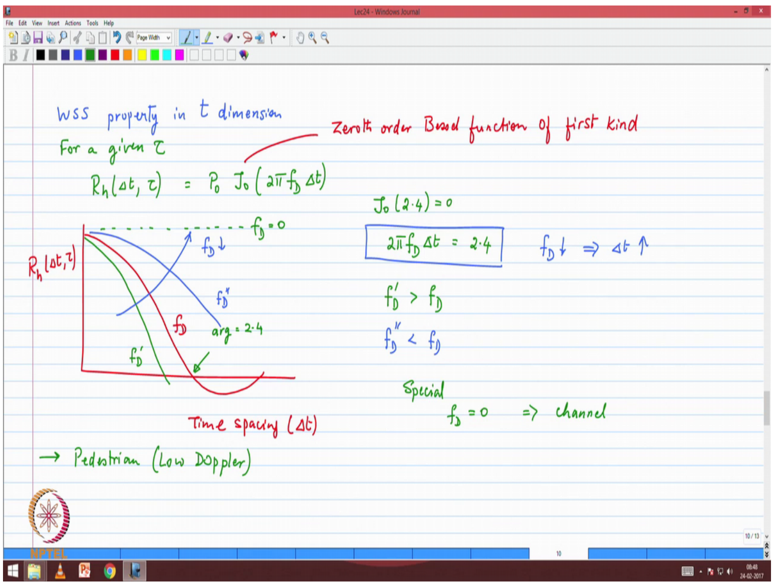

So, let me write it down. So, this is the wide sense stationary property w s s property in the t dimension in the time dimension in other words for a given tau if I observe the time variation or the auto correlation of the channel response R h is actually a function of delta t comma tau the auto correlation function and this is equal to a constant p 0 times J 0 2 pi f D times delta t.

That is the first part of the w s s u s model and this is going to give us a lot of very useful insights in a few minutes we will quickly capture that. So, I hope you will be able to go through the derivation look at the definition of the 0th order Bessel function may be we just write this down J 0 J 0 is a 0th order 0th order Bessel function of the first kind again that is a standard function not defining something different Bessel function of the first kind and it has got certain properties which we would like to like to understand and to exploit the first of it is going to come immediately. So, let is quickly look at the behavior of this J 0 function the J 0 function if you plot or if you look at it looks like a you know like a sink function looks like a sink function. So, it basically starts of at high value then cuts at 0.

Now, notice that your argument. So, this is your time spacing time spacing which means this is a function of delta t this is your R h of delta t comma tau tau is not in the picture your only plotting the variation as a function of delta t now if my Doppler increases if my Doppler increases; that means, if I have. So, notice that the. So, the point at which this one crosses the Bessel function crosses is when the argument of the Bessel function is 2.4 in other words J 0 of 2.4 is equal to 0 that is from the plot of the Bessel function. So, which says that basically the 0 crossing will occur when 2 pi f D delta t is equal to 2.4. So, I have drawn this for some f D this red is corresponds to some f D.

Now, if I have another Doppler which is f D dash which is greater than f D notice that f D is larger. So, delta t where the 0 crossing occurs will occur earlier. So, the green line is going to look like this. So, this is for a higher Doppler. So, this is f D dash and by the same token if I have a f D double dash which is less than f D the graph is going to be like this that is f D double dash.

So, in general this is the direction of decreasing f d. So, as I decrease f D it will become shallower and shallower the as you increase f D it will become steeper and steeper. So, the basic observation is that I get the intuition about these graphs by saying given this equation if f D decreases; that means, delta t will increase and basically vice versa you can than interpret this case, but insight is most important for our discussions. So, let us take the following special case there is a special case special case is f D equal to 0. So, what happens the channel is not changing at all no Doppler right basically f D equal to 0 means there is no mobility no change.

So, channel is not changing; how should it reflect in my autocorrelation it should look like a straight line which is what it is J 0 of 0 is equal to one and its equal to constant all the way along. So, basically you can draw another line which says this corresponds to f D equal to 0 that is again very consistent when what we would expect now if. So, the observations that we are making is that low Doppler the channel is highly correlated basically you have to the delta t for which the channel remains correlated is much higher.

So, when you look at pedestrian scenarios pedestrian is low Doppler low Doppler and this is gives us a very good a structured frame work to make statements remember previously when we said low Doppler means your fade will last a long time now you can tell exactly how long the fade is going to last based on the Doppler because I now know how long is because the channel is going to change very slowly if I am a pedestrian user and I go into a situation where there is a fade I am going to be stuck in the fade for quite some time except if I use frequency hopping or some other method I am going to be in trouble.

So, this actually now starts to give us a very solid way of characterizing the system and ability to work with that, so, what I would like you to do is read up the corresponding portions either from Molisch or from Goldsmith both of these are equally good because this is now the frame work on which the we have we have we have addressed the first aspect wide sense stationary part of it.

Now what happens to the uncorrelated scattering and what happens to the interactions between the time and frequency domain and what does the what are the implications in terms of the system design now those are the key things that we would like to develop and like to build up and understand that we know can make very care full and correct statements about saying what should be the size of the inter-leaver when there is low Doppler how do how do I design that this is going to tell me how long I am going to stay correlated in a fading channel and therefore, my inter-leaver must be accordingly designed my coding system must be designed if I do antenna diversity I know how to do that.

So, basically all of the pieces now have to come together. So, the whole discussion about w wide sense stationary uncorrelated scattering think of it as a jigsaw puzzle each day or you gone to get a few pieces you got to put them all together and then finally, you see the now the grand painting and you say oh everything sort of falls in to place. So, we have done few pieces you need to fill in the rest in the next lectures.

Thank you.