Introduction to Wireless and Cellular Communication
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Lecture - 23
BER Performance in Fading Channels
BER in Fading - Part II, Ricean Fading

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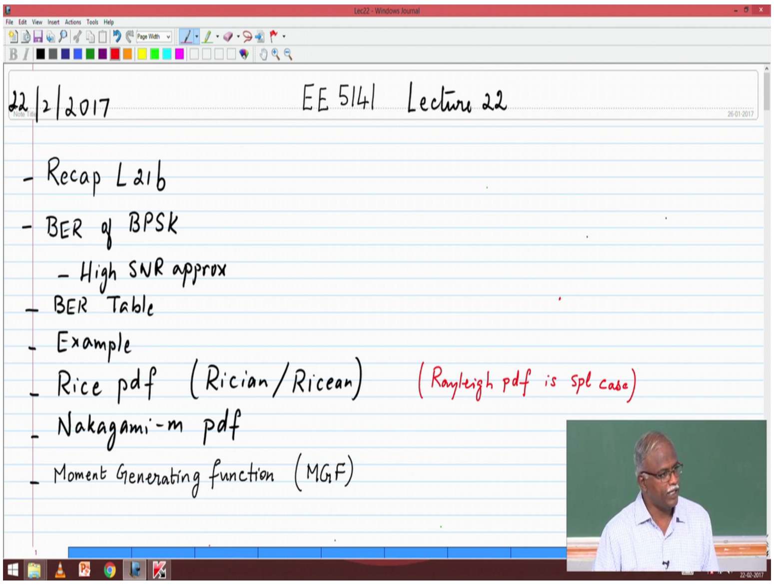

Good morning, we begin lecture 22 as always with the quick summary of lecture twenty one and some overlap with the previous lecture because some students expressed a feedback saying that some of the mathematics was not very familiar to them. So, we work a tight line between spending too much time in the mathematics and I was not doing any.

So, basically somewhere in between so that we get enough hints to solve the detailed derivations, but at the same time making sure that we are getting the intuition. So, this goal is this course will work that fine line between spending too much time the ideas not to spend too much time and mathematic though that is something that is a very valuable component. So, let us start with the quick summary of lecture 21 we were almost at the point of a completing our derivation of the BER of BPSK we will take the last step and then spend a little bit of time understanding the asymptotic behaviour asymptotic in the context of what happens at high SNR. So, that sort of tells us what is the trend that each of these modulations schemes will give us with that information we will be able to construct a bit error rate table of most of the impact of the complete set of binary modulation scheme. So, that sort of tells us this is the way to proceed this is how we would compare this is what the methods that would be followed.

We will move from the context where we have been talking primarily about Rayleigh fading to some of the other environments that are very important to us when there is the presence of a line of sight component that that scenario comes when we have a different distribution function for the received signal envelop. So, no longer Rayleigh, but a new distribution which is referred to is Ricean which turns out that it is actually a generalization of the Rayleigh PDF in other words the Rayleigh is a special case. So, we can just make a note that Rayleigh is a special case of this Rayleigh p d f is a special case of the Ricean. So, it is useful for us to understand how to characterise the Ricean p d f and today's class I will I significant portion is going to be to understand how to characterise the Ricean understand and interpret it in terms of the performance and its impact this is the special case. So, if the terrestrial wireless communications a cellular gives us Rayleigh give me a scenario that will give you Ricean not indoor.

You already told me before what else I think I here is satellite because which are the scenarios that you would you have like for example, if you are do in GPS and you are outside you know outside of the buildings there you will have a line of sight component. So, this actually comes a lot of times in the context of satellite communication. So, it is very very useful for us to understand Ricean as we will soon find out in the course of the lecture the mathematical formulation of the Ricean p d f is not very amenable for us to do the BER analysis. So, it turns out that the macadamia m is a formulation which is an equivalent formulation which tells you the similar types of distributions, but which is more amenable for the for the analytical word and in the process we will also be in using tool called the moment generating function which I am sure you would have studied either in probability when you are looking at these different distribution functions or in digital communications if not we will just use the definitions here.

So, that is the going to be the flow of today's lecture as always if there is a doubt or a clarification please do not hesitate to stop me and ask for that. So, as I mentioned in the last class our goal is to get a good handle on the fading channel both analytically as well as through simulation as well as through intuition all 3 components are important. So, let me just mention some of the points that we discussed in the last class. So, we were talking about fading channels and why we were not able use a coherent in certain scenarios.

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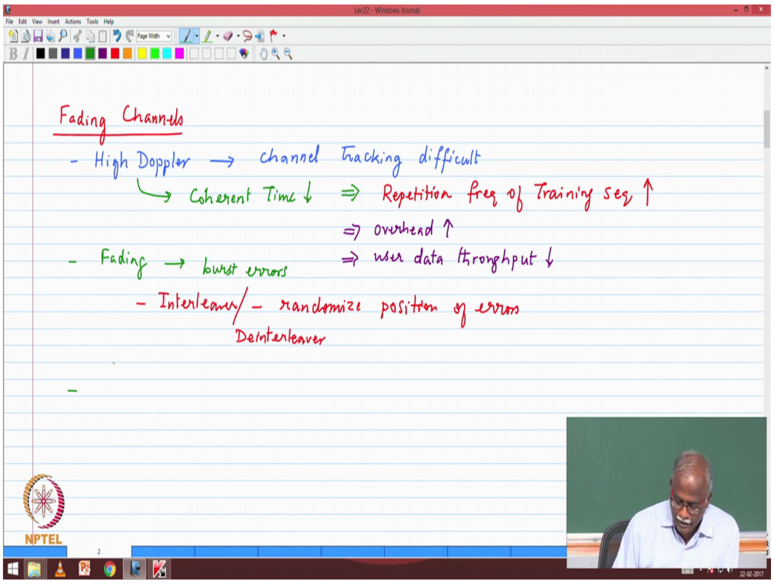

So, one of the things that we always remember in the context of fading channels is that we could have high Doppler high Doppler means that we will have difficulties in channel tracking that is what we have mentioned and again we saw it in the context of differential detection hybrids schemes all of that, but the essence of it is there is the issue of high Doppler where channel tracking becomes difficult and it may be advantages for us to go head and use a differential modulation scheme.

Now, another element which tied to this is the following if you remember we said that if I want to do coherent estimation I mean coherent detection I must get channel estimates I must get them at least as frequently as the coherence time because beyond that the channel will change significantly. So, high Doppler is related to coherent time and its related to the coherent time in the sense that coherent time is decreasing which means that my system design is going to have a impact which says that my repetition of the training sequence the repetition frequency how often I send repetition frequency of the training sequence the training sequence is the known sequence which is used for estimating the channel. So, that I can do coherent detection this frequency will increase what is the impact of this increasing my overhead has increased which means that my user rate is going to go down overhead is increasing which means that user data rate user throughput is going to go down.

So, you see that there are linkages and it is not one without the other and all of this. So, if you could do coherent differential detection or any other form of non coherent detection which did not require the knowledge of the channel notice that your system design is not affected by Doppler you will not be spending any overhead on transmitting this training sequences. So, there are several advantages, but again there are limitations that we would have to keep in mind. So, this would also mean user data though put will go down. So, there are linkages to the system design and I would like you to always keep in mind that the different tradeoffs are happening the other element that we also mentioned is that fading which occurs when we are moving this will typically lead to burst errors and burst errors are not very effectively handled by f e c. So, therefore, the use of an inter leaver is to make; so, we introduce the inter leaver to make the data the errors appear there in random positions.

So, the notion of the inter leaver. So, at to make to randomise the I will just say randomise the errors randomise the position of the errors what in the channel occurs as constitutive errors, but when we it goes into the decoder will be in random positions because of the inter leaver; deinter leaver that that will happen. So, basically it is this is inter leaver coupled with the deinter leaver with only then you will get the randomisation we did not talk about the deinter leaver, but basically is the reverse operation that is happens the other and the other point that we discussed in the in the last class is that is the notion of the feeding SNR basically in AWGN if SNR is a constant E b by N naught.

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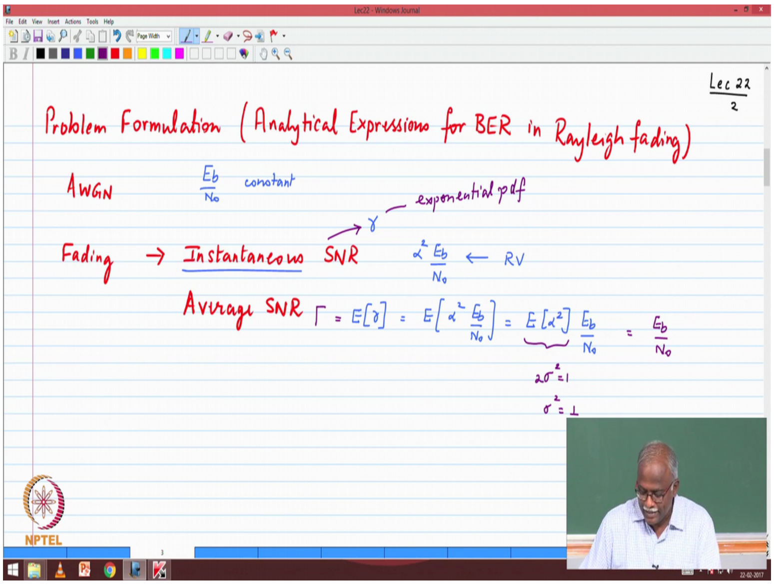

When we talk about fading it becomes alpha squared E b by N naught was the alpha was random variable alpha has got a Rayleigh distribution if you define gamma to be equal to alpha squared E b by N naught then this gamma is the instantaneous SNR. So, this gamma is a very very important quantity for us that is the instantaneous SNR and it is a random variable and we have shown that this gamma has got a exponential distribution that is what we derived, but we will just view that once more exponential p d f it has exponential p d f.

Now, since it is a random variable we can talk about average SNR expected value of gamma and this we denoted by uppercase gamma and we said that this will be equal to expected value of alpha squared d B by N naught and if you have chosen your variances of the real part and imaginary part of your complex channel gain to be such that they are each of them should be equal to one half sigma squared equal to one half; that means, the variance of the real part is one half imaginary part is one half then you will get the net variance of the 2 sigma squared to be equal to one and; that means, that your average SNR in the fading channel will be the same as the SNR in an AWGN channel again this is the environment which we will do most of our comparisons.

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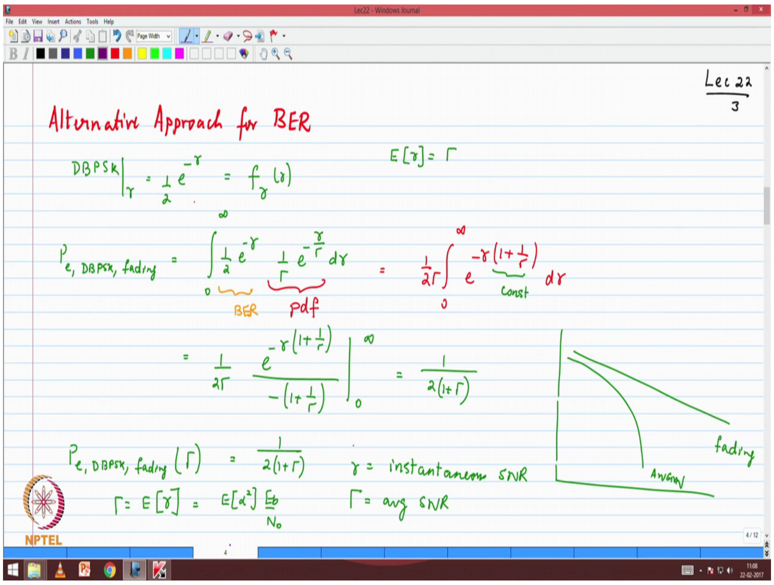

Then the next part of the maybe we can we can we can summarize; summarize this in the following manner let me just write it down just for since few people said that this is not very familiar.

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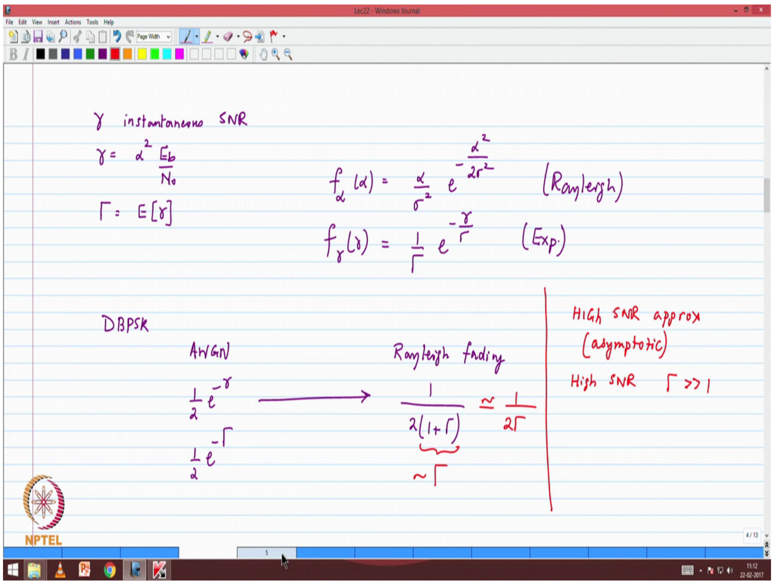

So, we have defined an instantaneous SNR gamma instantaneous SNR and this is equal to alpha squared E b by N naught and we have said that gamma is equal to expected value of the of this random variable and we have shown that if the alpha f alpha of alpha is alpha by sigma squared e power minus alpha squared by 2 sigma squared this is the

Rayleigh the envelop is Rayleigh distributed the SNR f gamma of gamma is given by one by upper case gamma e power minus gamma by gamma where gamma is equal to I will be written that. So, this is the exponential p d f exponential p d f.

The other parts that we had also revised or you know derived in the process was the bit error rate performance of d BPSK d BPSK in AWGN has a BER expression that is given by e power minus gamma half e power minus gamma we used the expression for the p d f to get the performance in Rayleigh fading Rayleigh fading the BER in Rayleigh fading we got it to be equal to 1 over 2 into 1 plus gamma and again if the average SNR in AWGN case this would be equal to half e power minus gamma because instantaneous and average are the same. So, this is a very useful formulation we derived the p d f of the SNR applied it to get the BER of d BPSK in Rayleigh fading using it is just a one step integral it was an exponential integral.

Now, I would like you to focus on how to do the high SNR analysis again its something that is very useful for us. So, high SNR approximation or asymptotic analysis sometimes we call it as asymptotic analysis asymptotic analysis basically says what happens when my SNR is high; high SNR. So, which means my gamma is much greater than one because gamma equal to one is 0 d B SNR that is that is not what we are talking about twenty thirty d B SNR where gamma is much much larger than one now under the assumption that gamma is much larger than 1; 1 plus gamma can be approximate its gamma can be approximated as gamma.

So, this actually can be approximated as one over 2 gamma it is a very simple approximation once you apply the interpretation that is available to us this is a useful aspect something that we add on to whatever we have done in the last class and I believe this is this is where we left of the differential BPSK we then picked up the. So, that was the graph we showed that the fading graph is going to be much worst it is going to go linearly with a negative slop and the that is the and slop is equal to one and therefore, this is the this is the scenario that we have derived with the with the BER analysis.

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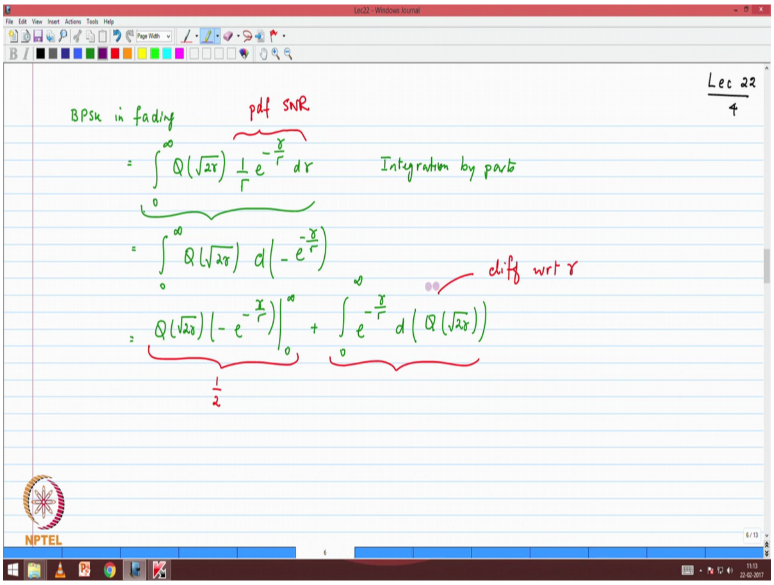

Now, we moved over from there to BPSK and we wrote down the expression for the bit error rate in fading which involved the Q function and the distribution of SNR again we will use the SNR distribution. So, that is the p d f of the SNR we did the first step of integration by parts and I hope you had a chance to complete the step that we had where we left of and we showed that the integration by parts actually results in a derivative of an integral.

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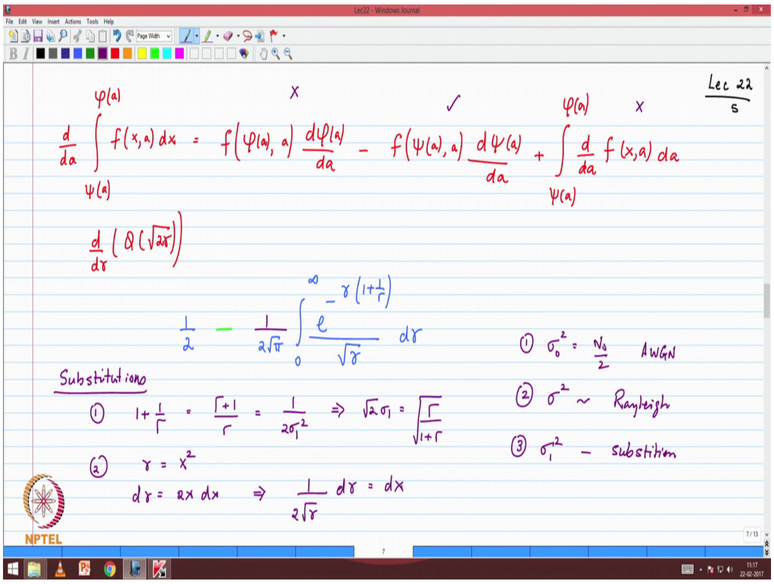

And basically we should have been able to verify this, this is basically you have been able to get one half now what I would like to do is just give you the one more step before we get the final answer and begin and move forward from there.

So, there are 2 substitutions that we need to do for for completing this calculation. So, let me just give you an indication of those. So, substitutions I am assuming you reached up to this point this is the expression for the probability of error of BPSK in fading and we are one step away from the answer. So, I am going to do the following approximation approximate substitution number 1; 1 plus 1 by gamma that is the that is in the exponent that is actually equal to gamma plus 1 by gamma I am going to write it as 1 over 2 sigma one squared one over 2 sigma one squared now you may wonder why it we are doing that let me explain it you once we have completed the step, but before I do that I must now tell you that we now have 3 sigmas that are present in the in our discussion the first one the different sigmas that are present the first one is sigma naught squared equal to N naught by 2 what is that that is AWGN that that is the additive Gaussian noise that is always present even whether fading is present or not.

So, this is related to AWGN then there is a sigma squared which is related to the Rayleigh fading the real and imaginary parts those 2 are already there in our discussion I am introducing a third one this is only for convenience this is only a substitution there is no other element this is only for the purposes of completing this integral. So, it does not have any interpretation like the other 2, but do not confuse it with the others because then it will actually will not give us the result that we looking for. So, basically if I went to write this down I will get root 2 sigma one is equal to square root of gamma divided by one plus gamma square root of 1. So, that is one substitution the second substitution that we will help us complete this discussion is gamma equal to x square.

Now, that my scream counter intuitive because we went from envelop to SNR saying that is an easier one, but actually for completing the integral this actually turns out. So, since I am since I am integrating with respect to gamma I need d gamma that is equal to 2 times x d x substituting for x this basically tells me one over root 2 gamma sorry, 2 root gamma 1 over 2 root gamma d gamma is equal to d x. So, I will just give you a minute just. So, that you are of with the 2 substitutions that that we have we have proposed here now if that is then please rewrite the equation rewrite the equation.

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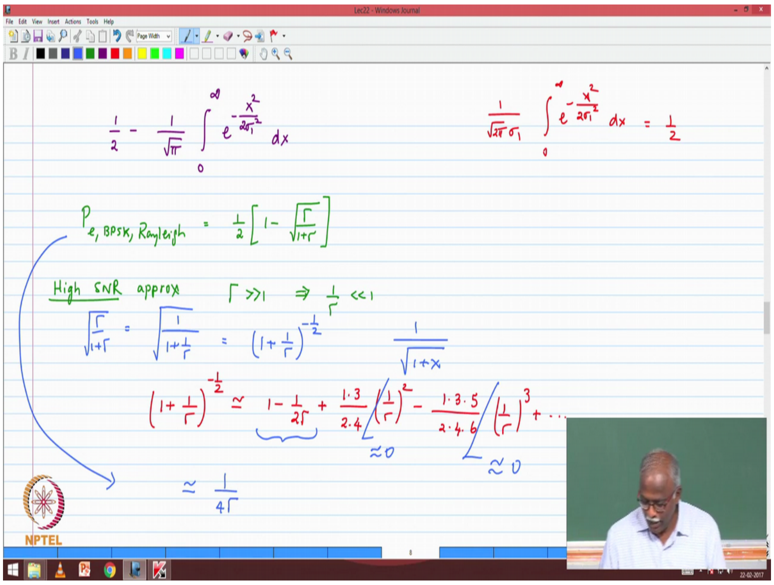

So, now we will get the probability of error expression as one half minus one over root pi integral 0 to infinity e power minus x squared by 2 sigma one squared d x I think you are already seeing why the substitutions were made and what that what that helps us with basically this is of the form of the integral of a normal random variable. So, we have the following results which says 1 by root 2 pi sigma one integral 0 to infinity e power minus x squared by 2 sigma one squared d x this is equal to one half. So, actually we will not perform the integral, but we will do a substitution that is all. So, basically avoiding the step of integration

So, as I said we are already through with the result. So, the final results says the probability of error of BPSK in Rayleigh fading has a very nice close form expression which is given by one half one minus square root of gamma by one plus gamma just substitutions I hope you will be able to do that without much difficult. So, that is our result now I would like you to do the high SNR or asymptotic analysis also because that is also very helpful high SNR approximation. So, what you would have to do is make the same assumption that gamma is much greater than one this is the same as saying one by gamma is much less than one. So, basically I would like to get an approximation for this let me again in give you the first step and then request you to complete it square root of gamma by one plus gamma if I divide numerator and denominator by gamma I will get one divided by one plus one by gamma.

So, just rewritten it, so, this can be written as one plus one by gamma raise to the power of minus half or you can treat it as one by square root do not put equal to here write a square root of 1 plus x where x is a small quantity and for that we know the series approximation and its valid because gamma is much larger than 1; 1 by gamma is much less than one please do the series expansion let me just give you the first 3 terms. So, that you can verify that, but I would like you to actually be sure that you are comfortable with this. So, this series expression can be written as 1 minus 1 half times gamma plus the series expansion goes as one into 3 divided by 2 into 4, 1 by gamma squared it is an alternating series then next term will be one into 3 into 5 divided by 5 into 2 into 4 into 6 one by gamma whole cubed plus dot, dot, dot.

Because we have made the assumption that is the high SNR I am going to ignore this which is one by gamma squared this is also I am going to ignore and all subsequent terms I can ignore. So, what I am left with is only one minus one by 2 gamma it is an high SNR approximation and it is a fairly good one. So, the high SNR approximation the probability of error of BPSK in Rayleigh fading under high SNR, I would like you to verify comes out to be 1 by 4 gamma is also a linear slope it is a goes down with a slop of minus one, but it has got a different constant with respect to differential BPSK.

What is the difference between the 2 the factor of 2 which means which is worse coherent BPSK or differential BPSK differential BPSK is worse and the shift is going to be 3 d B. So, what we can do is now draw a graph which will be very helpful for us in interpreting these results.

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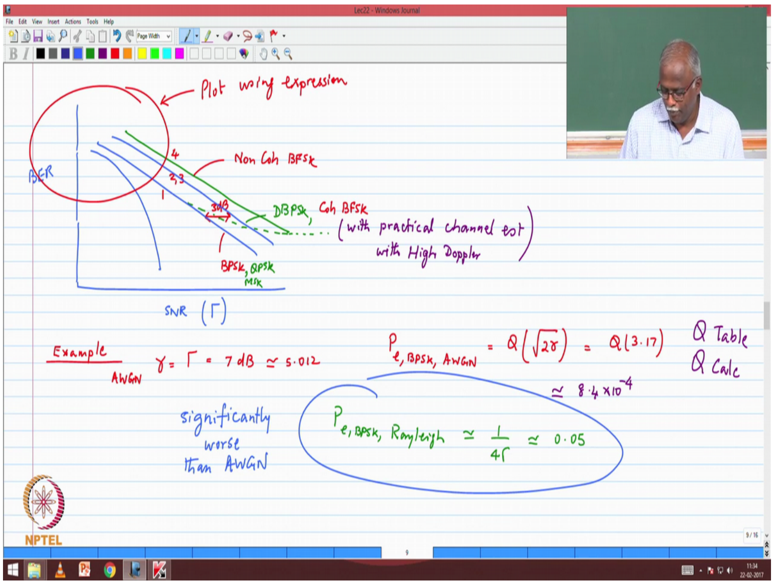

So, here is my bit error rate as a function of SNR, notice that I can now write it in terms of the SNR as gamma or E b by N naught average value AWGN graph is going to be steep this is BPSK; this is differential BPSK. So, this one is BPSK the shift of 3 d B and this is going to be differential BPSK and why is d BPSK worse than BPSK because of the.

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2 noise terms and that is where it is showing up and this is assuming making an assumption of this is this graph is correct under the assumption of ideal channel estimation that is your you have ideal channel reference for doing the detection otherwise what will happen to your differential BPSK this 3 d B advantage will at some point be lost where you will get an error floor whereas, the SNR of the d BPSK will continue to degrade. So, this is with channel estimation with practical channel estimation with practical channel estimation at high Doppler practical channel estimation at high Doppler. So, we have the complete picture good understanding of what the fading we will do what is ideal what is high SNR approximation and with high Doppler make sure that you mention this because at load Doppler's the channels the graph should be exactly the same as what you have seen you will be able to do the tracking very nicely with high Doppler this is the good point to answer any questions that you may have we have done 2 modulation schemes basically this tells us that regardless of which modulation scheme

we are given once you know the bit error rate performance in AWGN you can then generalize it to what will be the performance in the fading channel of course, there are a little tricks in the integration, but nevertheless with experience you will be able to handle these things without any difficulty any other any questions any clarifications that that will be helpful for the students.

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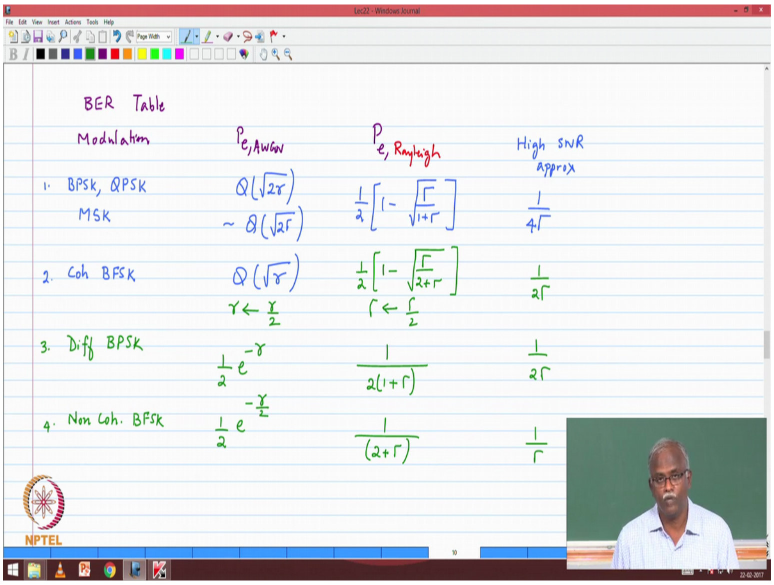

If not, let us now complete what I believe is a very useful table. So, this is the BER table BER table it has got 4 columns the first column is the modulation type modulation type the second column is the probability of error in AWGN third column is probability of error in fading well let me write it as Rayleigh fading we have not talked about any other I will write it in red. So, that you will be Rayleigh fading and this is the last one is the asymptotic analysis high SNR approximation what able to do is write it down write down this table which will be which will answer all the modulation cover all the modulation schemes that are binary. So, the first entry that we make is probably the most useful entry which says BPSK and whenever we talk about BPSK the modulation QPSK is also covered because both of them have got the same BER performance in AWGN and then there is a constant envelop scheme called minimum shift scheme that is also a binary scheme which is used in several modulation several communication systems. So, BPSK QPSK and MSK all of them have got the following expression Q of root 2 gamma that is the expression in AWGN.

When we take the average SNR and exp express it in a fading channel this becomes one half into one minus square root of gamma by one plus gamma I am deliberately using different values in AWGN and Rayleigh fading because once you move to Rayleigh fading this lower case gamma represents instantaneous SNR and the expressions that we get are based on average SNR. So, again its deliberate you could also write this if you wanted to as Q of root 2 gamma there is nothing wrong in the in the case of AWGN instantaneous SNR and average SNR are the same.

So, no, no, no problems about that and high SNR approximation we just now derived this is 1 by 4 gamma now there is another modulation scheme which is called coherent binary frequency shift key 2 frequencies f one and f 2 the separation between frequencies is different from minimum shift key minimum shift keying is the is the one that gives us the optimum performance coherent BFSK slightly worse than MSK performance this is given by Q of root gamma Q of root gamma that is the BER in AWGN now do we need to re do the derivation for Rayleigh not Rayleigh because if the instantaneous SNR is divided in half the average SNR is going to divide in half.

So, basically this can be obtained using the following substitution you take gamma and replace it with gamma by 2 that also tells me that average SNR in the previous expression will be replaced by gamma by two. So, if you do that it this will come out to be one half minus one minus square root of gamma by 2 plus gamma. So, basically just a substitution and of course, you carry over the analysis all the way to the high SNR side replace gamma with gamma by 2 this will become one over 2 gamma. So, this is actually the asymptotic analysis says that this family of modulation schemes are actually shifted by 3 d B, but we know that already differential BPSK also as is in the same location.

So, third one; differential BPSK differential BPSK this is one half e power minus gamma that is the expression for the BER in AWGN we have derived the probability of error in Rayleigh fading as 2 into one plus gamma and the high SNR approximation turned out to be one over 2 gamma and then the last one of the binary modulations schemes that we have is non-coherent binary phase shift key BFSK and that has a BER performance which is given by e power minus gamma by 2 do the same substitution method I do not have to re do the calculations this comes out to be 1 over 2 plus gamma one over 2 plus gamma and of course, again substituting gamma with gamma by 2 this comes out to be one over gamma. So, this is basically all the binary modulation schemes that we have of course, g MSK is not there I will we will I will show you in the minute how to get the expression for g MSK, but before that all of the standard modulation schemes coherent non coherent all of them are on this on this graph any questions.

So let us move to the previous graph and then say that BPSK is correct also add QPSK and MSK all of them have got the same performance this is the graph of deferential BPSK it is also the graph of coherent BFSK and then there is a third graph which is also shifted by 3 d B and other d B and that is the graph of non coherent BFSK, so, all the modulation schemes. So, if you want to look at into in terms of the in the table this corresponds to entry number one this corresponds to entry numbers 2 and 3 the green the last graph corresponds to number 4.

So, 2 of them have got the same asymptotic behaviour. So, you know there is a little bit of variation in this region you have to actually plot the functions to see how they get the asymptotic analysis is applicable in the high SNR region. So, this you would have to plot using the expressions what I have done is not correct you have to actually plot it using the expressions. So, again keep in mind that this is the. So, it is good for us to have a picture of what happens with Rayleigh fading how do we deal with Rayleigh fading what are the characterisations how do we get the SNR approximations and then also be able to do the high SNR approximation any questions on what we have done.

Always good for us as engineers to have a feel for the numbers now if I were to be given that the average SNR in a AWGN channel AWGN channel gamma is equal to gamma is equal to 7 d B this is approximately this is approximately 5; 5.012; what is the probability of error in the AWGN channel probability of error of BPSK in the AWGN channel is given by Q of root 2 gamma which is Q of 3 point one 7 you can either look at the table or the calculator. So, using the Q table or the Q calculator you can get the expression important thing is to get what is the order of magnitude we are talking about 8 point 4 into ten power minus 4 this is very good 8 bits out of ten thousand bits its; that means, a most of your frames of hundred bits duration are going to go through without any error. So, this is a very good performance very easy for us for if we see to make it you know almost error free.

Now, same SNR if I were to be asked to operate in a Rayleigh fading channel probability of error of BPSK in Rayleigh fading again useful for us to get a feel for it you can substitute in the expression that we have, but I am going to use the approximation of one over 4 gamma and you can verify this is approximately 0.55 bits out of 100. So, which means that this is quite error prone channel for the same channel on average in the AWGN case was giving you almost error free now you have a. So, the performance of Rayleigh fading is significantly worse.

So, let me just write it saying it is significantly worse. So, this is something that we always have to keep in mind that we need to make our design sufficiently robust when we are working in a significantly worse than AWGN that is something that I always want you to keep in mind that minute it becomes wireless and you have multipath and you have Doppler you say now I need to re look at my system design because this is an important element another useful aspect that I always want to keep in mind is what if you do not have the Q tables available to you or you do not have the Q function calculator with you.

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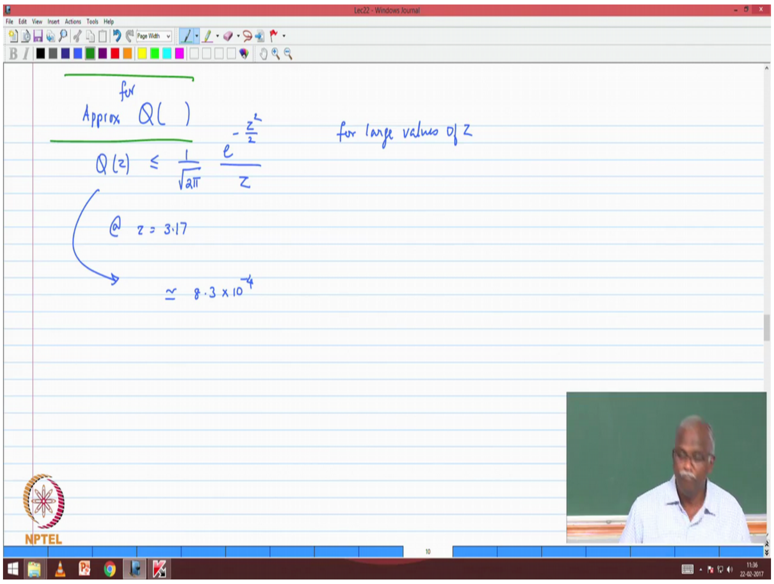

So, as engineers we also always have approximations for the Q tables approximation for the Q calculations which basically says that Q of z is upper bounded by 1 over root 2 pi e power minus z squared by 2 divided by z for large values of z and again most of the time we may be interested in the high SNR analysis. So, the argument of the Q function will be something which is you know greater than one for large values of z please do verify that if I substituted at z is equal to 3.17 this would have told me that my bit error rate is

8.3 into 10 power minus 4 the Q function told me it was 8.4 into 10 power minus 4 its almost negligible difference. So, again keep this in the back of your mind that there is a good approximation for the Q function if you do not have axes to the tables and useful for us to work that any questions, alright.

To summarize Rayleigh fading means no line of sight now comes the question what happens if I happened to have line of site component present how is it going to affect my statistics how is going to change my BER how is going to affect all of the analysis that we are gone through. So, what we will do is just give you a flavour for what are the aspects that we need to we need to work with and then we will draw the intuition from there.

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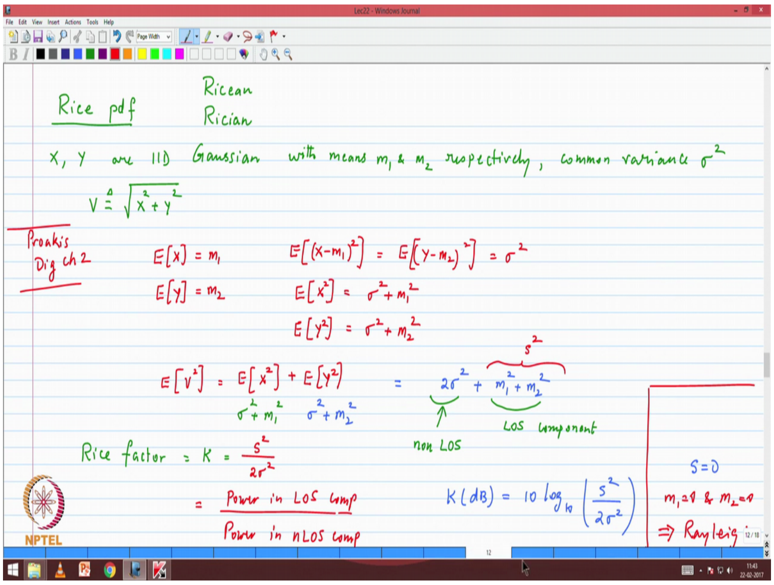

So, we are going to look at the following aspect let me give you the type it is the distribution where there is line of site component. So, it is going to result any different p d f, it is called the rice p d f after the famous communications engineer rice. So, sometimes we call it Ricean p d f or we may even spell it as Rician all are rice Ricean all of it is, but ultimately it basically covers the case when there is a line of site component. So, x comma y are yiid Gaussian just like in the case of the Rayleigh p d f no longer 0 mean, but with means m 1 and m 2 respectively why is that because if the Gaussians are 0 mean; that means, on average your expected value is going to be 0, but if you have line of sight component no matter what happens to the fluctuating component the line of sight will always be present. So, which means that there is a non zero component at all times which is captured by the means that are present.

Now, the means do not have to be the same, but in many cases they will turn out to be that the means are the same, but m 1 and m 2 and they have a common variance sigma squared common variance sigma squared now then we also now we want to look at the envelop which is defined by squared root of x squared plus y squared then we are told that this has a Ricean distribution the envelop will have a Ricean distribution which is different from that of the Rayleigh p d f, now very quickly like to cover some of the background information again the ideas not to spend lot of time in the mathematics, but it is important for us please refer to proakis digital communications chapter 2 where the Ricean p d f is derived and it is you know the complete description is given for us, but we will cover it to get to the inside. So, here is the important aspects of the description and the process that is given for us in the proakis. So, expected value of x is equal to m 1 it is no longer 0 expected value of y is equal to m 2 now the variance expected value of x minus m 1 whole squared this is equal to expected value of the variance of the x and the variance of y minus m 2 whole squared this is both of them are equal to sigma squared. So, that tells us that expected value of x squared is going to be equal to sigma squared plus m 1 squared again these are results that you will be easily able to verify and expected value of y squared is equal to sigma squared plus m 2 squared now comes a very very important why this is a the difference between Rayleigh and Ricean p d f. So, expected value of v squared is equal to expected value of x squared plus expected value of y squared again like before for the case of the Rayleigh case in Rayleigh case this was equal to 2 sigma squared because the means where 0 mean.

So, now what we will get is sigma squared plus m 1 squared and this one is going to be sigma squared plus m 2 squared this will be 2 sigma squared plus m 1 squared plus m 2 squared and we know from before that this part is what contributed to the Rayleigh. So, this is the non line of sight component basically now you have a line of sight component and a non line of sight component and this is what covers the line of sight component notice that m 1 and m 2 are non 0 values and therefore, there will be some contribution due to this in the case of the Ricean distribution.

So, this is a very important element now how do we characterise this type of environment rice proposed using something called a rice factor rice factor is a ratio of the of the 2 it is the given by the constant k uppercase k this is given by these if you call this the notation used in the Ricean distribution is called as s squared k is defined by the ratio of s squared by 2 sigma squared. So, it is nothing, but the ratio of the power in the line of sight component power in the l o s component divided by the power in the non line of sight components and most of the time when you are dealing with Ricean distributions you probably would have encountered that is always given in d B which is ten log ten of s squared by 2 sigma squared its always given in d B you know that is usually the practice that is it is a Ricean channel with 8 d B k equal to 8 d B. So, the interpretation it is the following very important.

What happens when s is equal to 0 special case s is equal to 0 means m 1 equal to 0 and m 2 equal to 0 k will become 0 and you will get Rayleigh distribution. So, this means you will get Rayleigh distribution we should verify that now the next few minutes that is what we would like to do very quickly look at the interpretation of the p d f. So, as I mentioned the goal is not to spend a lot of time in the mathematics, but the hopefully the inside that we have already obtained tells us. So, go back to our description. So, if this is the scenario x comma y i i d Gaussian with means m 1 and m 2 common variance sigma squared v is equal to square root of x squared plus y squared this p d f is given by f v of v equal to v by sigma squared.

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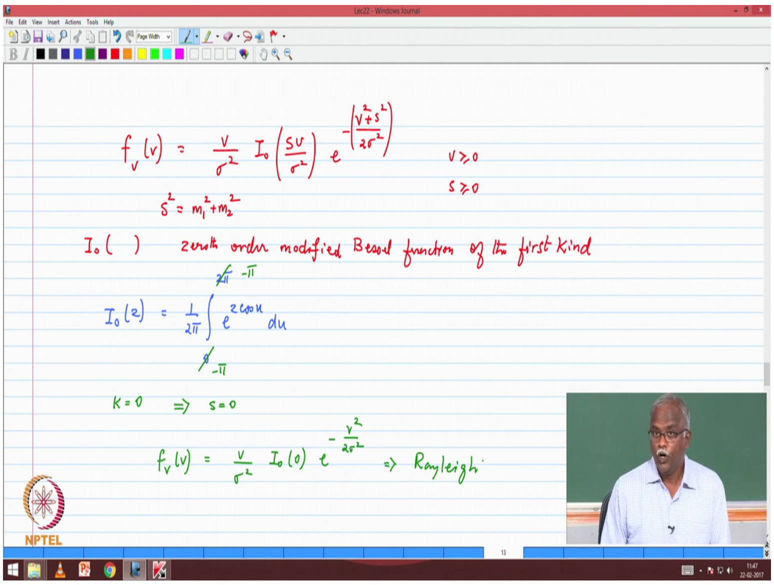

There is a Bessel function that comes in I naught times s where s is equal to s squared is equal to m 1 squared m 1 what I will say yeah m 1 squared plus m 2 squared I hope I wrote this is squared yeah s squared is equal to m 1. So, s times v divided by sigma squared e power minus v squared plus s squared by 2 sigma squared v greater than or equal to 0 s greater than or equal to 0 s is the po s squared is equal to m 1 square plus m 2 squared. So, therefore, s is a positive parameter and v is envelop of the signal again a positive parameter. So, s squared is equal to m 1 squared plus m 2 squared and I naught of the argument is a 0 th order Bessel function again does not matter it is a de function that is well defined its 0 th order modified Bessel function modified Bessel function of the first kind Bessel function of the first kind. So, the derivation is s given to us in proakis and again time permitting or maybe as part of a assignment we will we will work out the derivation of this again not difficult though you may have never seen 0 th order Bessel function live alone modified Bessel function I have no idea what it is does not matter do not worry about it something that we will handle without any difficulty so, the definition of the modified Bessel's function.

is just an integral function just if you handled Q this is this is like the Q function its one over 2 pi integral 0 to 2 pi e power z cos u d u and you can cha you can do the integration from 0 to pi or you can do it from minus pi to pi both are equivalent distribution again it is an integral form of a variable. So, the basically this is this. So, now, the one thing I want you to do is very quickly substitute k equal to 0 k equal to 0 implies s equal to 0 if I substitute f equal to 0 what I will get is f v of v is v by sigma squared I naught of 0 e power minus v squared by 2 sigma squared easy to verify that I naught of 0 is equal to one. So, therefore, what we get is the Rayleigh distribution Rayleigh distribution.

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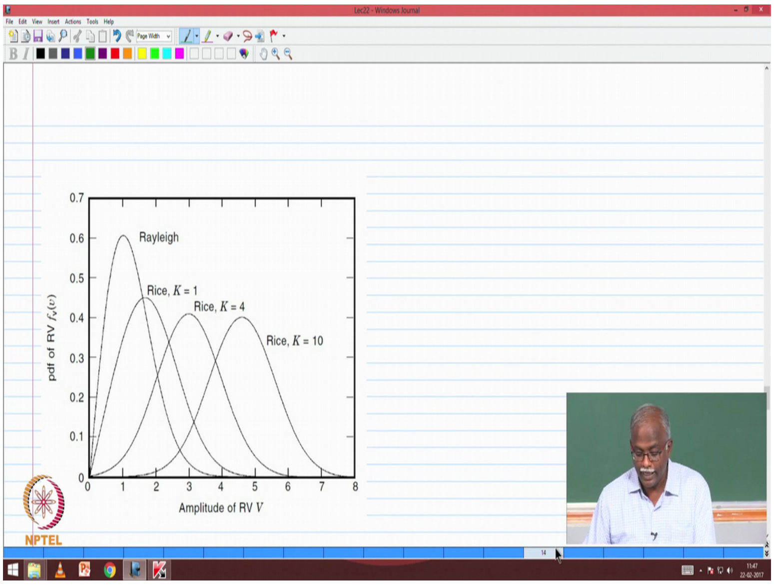

Now, very very important before we conclude to see what the Rayleigh Ricean p d f looks like for different values of k now Rayleigh we know what it looks like basically it is it is highly concentrated towards the lower amplitudes it hits a peak at equal to sigma and v is equal to sigma and then d k is that is long tail, but you know these are the worry something is that this is going to be the where the problem is going to be lie.

Now, if I introduce a line of sight component what happens the line of site component is always present even when the non line of sight component becomes very small. So, therefore, the likelihood of the envelop being slightly larger sort of sort of shifts to the right the stronger the line of sight component the more it is going to shift to the right because it is not going to be look like the Rayleigh it is going to look more and more is going to look different from that.

Now this is a graph that we will generate through mat lab assignment, but please do that by the several people I have ask for postponement of the assignment I have no problems with postponing the except that you know you it is going it is a going to a bunch of because we have going to do a number of computer assignments and a written assignments to keep in track. So, as much as possible keep up with the assignments if you need extension not a problem. So, this is one aspect that I want you to keep in mind the second one which I hope will give you a lot of inside forget the 3 lower graphs.

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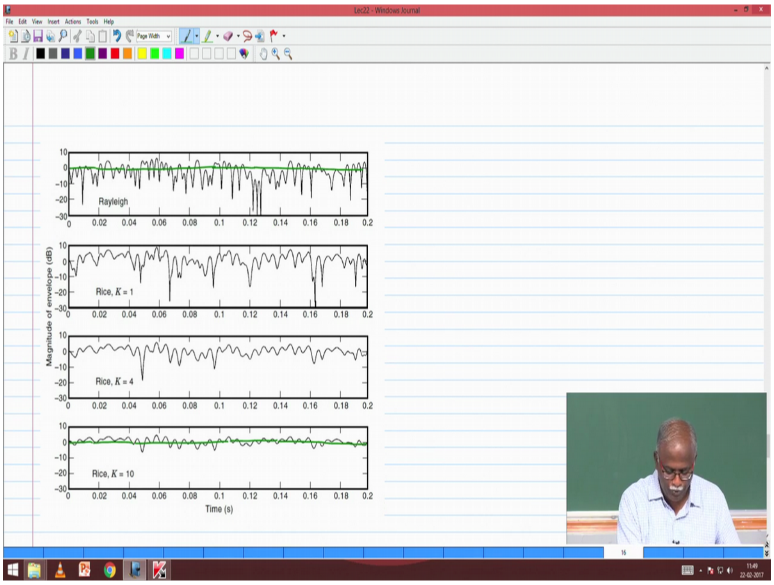

The first one is Rayleigh distribution you will have to when you go back you will have to draw this line that line basically says that is your average received signal level that is and you can see that is the type of fluctuation you will see when it is a Rayleigh distribution remember the 3 Ricean distribution that we will looked at take the last case k equal to ten. So, which means that there is a line of sight component which is a strong and there is a non line of sight component now draw now draw the graph you can see that the fluctuations about the mean level have significantly reduced.

So, your BER is going to be much much better than a Rayleigh p d f. So, this is the inside that once you move away from non line of sight to having even a small line of sight component being present you will see that there is a improvement in your BER performance. So, lot of insides that still need to be obtained from the understanding of the Ricean p d f, but by and large most of it will be through assignments I hope you have got the basic handle of what happens when the line of sight component is present we will build on this very quickly and then draw more and more inside from the performance in a Ricean system and then how to interpret it in terms of our system design.

Thank you.