Introduction to Wireless and Cellular Communication
Prof. David Koilpillai
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 22
BER Performance in Fading Channels
Coherent vs Differential Detection - Part II and BER in Fading

Good evening, we begin the lecture for today; little bit of explanation on the numbering scheme; last lecture was unplanned review lecture. So, we called it 21 the so this is actually the lecture 21 which follows lecture 20. So, again it is called lecture 21 B because this already another lecture 21, but this is the continuation from where we left of in lecture 20.

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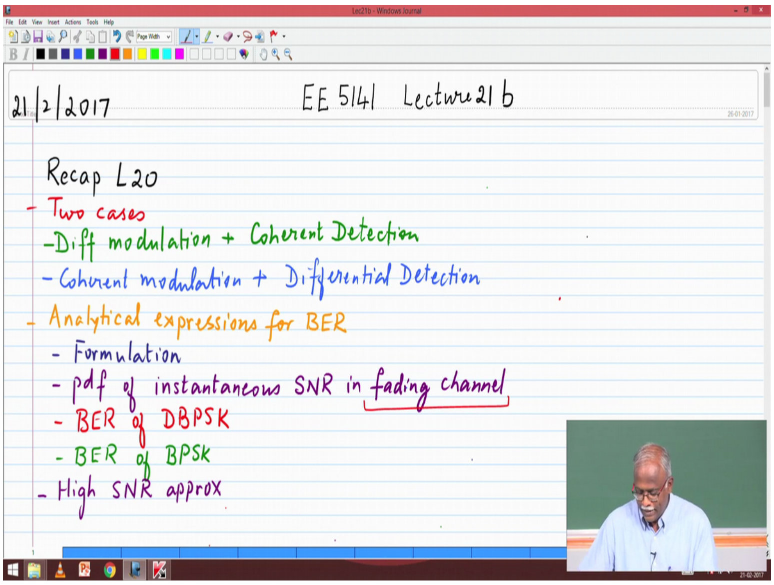

Did get some feedback from students saying that some of the material covered in lecture twenty that was Wednesday of last week was not very familiar to them again my request you is if something is not familiar to you or this material that you have not seen before please do indicate and then we can slow down I had assumed that most; most students would be familiar with the concepts of differential modulation differential detection.

But let us look at it we will we will pick it up from there and continue the discussion. So, today's lecture we will start by a quick review of what we had seen in the last class some interesting observations about the fading channel which again as I mentioned is very important that you have an intuitive feel for the aspects of fading and once we understand that we will go back to asking the obvious question we have coherent modulation coherent detection we have differential modulation differential detection what about the cross combinations does it make sense would are there scenarios in which you would want to do it and if yes or no what are some of the pluses and minuses we will take a look at it.

The purpose of today's lecture is to set the formulation for deriving the analytical expressions for BER again I am assuming that the derivation of analytical expressions for BER in AWGN is familiar to you from digital communications. So, what we are going to be looking at is the aspect that we are looking at the formulation in a fading channel and. So, far we have looked at fading channels which have the Rayleigh statistics. So, we begin by looking at a Rayleigh fading channel. So, we will look at the 2 of the most simple forms of modulation saying that once we understand this it can be easily extended to other types of modulation which belong to the this family. So, we will look at the differential BPSK and BPSK and look at some high SNR approximations, but first let us quickly summarise what is our material covered in lecture 20.

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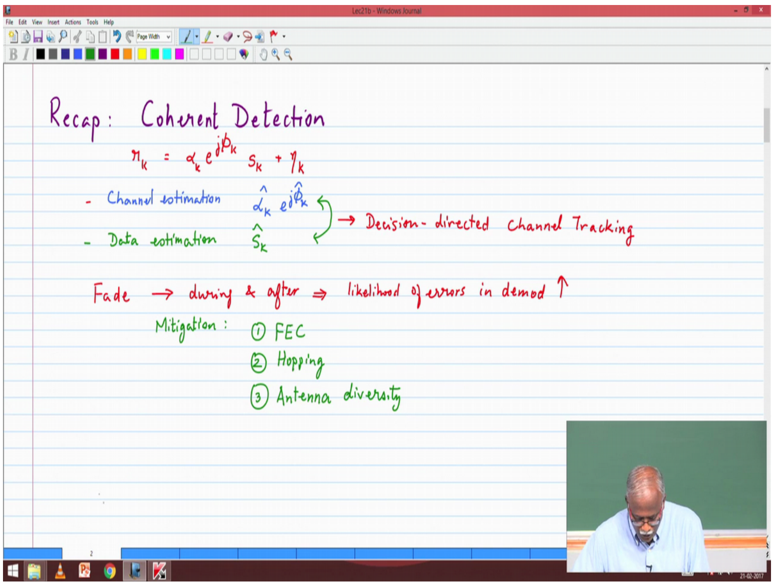

So, coherent detection if we assume that we will use the sample notation. So, coherent detection says at the k th time instant my received signal is a function of alpha k e power j phi k that is my fading coefficient times the transmitted symbol as k plus eta k the

AWGN sample and as we discussed in the last class we do need to have 2 steps one is where we estimate the channel estimation I need to know what is alpha k e power j phi k channel estimation and once we have done the channel estimation we can then do the data detection.

So, the estimates are always denoted with a hat e power j phi k hat so; that means, through some process using training symbols I have obtained an estimation of the channel and once we have obtained the estimate of the channel then we can do the data estimation data estimation that would be getting an estimate of s k hat and we also said that you can go back and forth between these 2 methods that you can do s k and then get the estimate of the channel that the next time instant and then go back and forth that would that would constitute data driven or data director now decision directed.

Based on current decisions you are doing channel tracking decision directed channel tracking and we also indicated that this is the one that makes it venerable if one you have errors creeping in either in your estimation or in your tracking it; it can lead to cascading effects.

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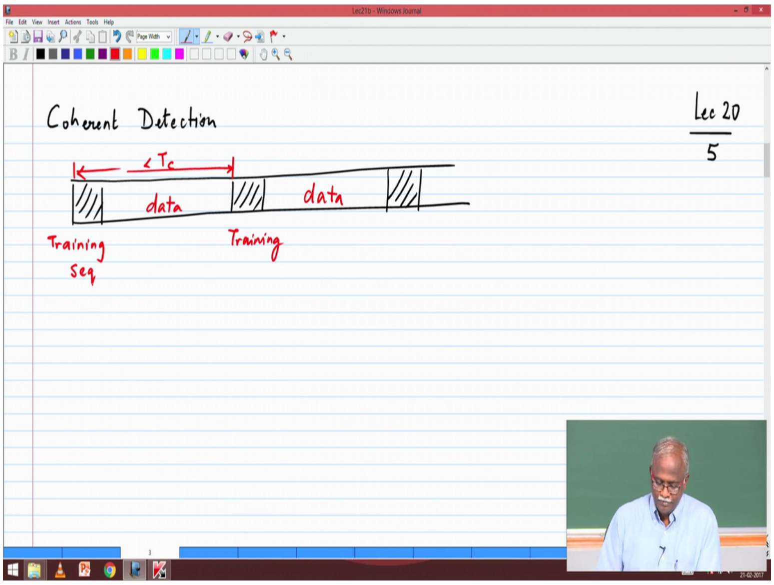

And therefore, we wanted to make sure. So, just to remind you the coherent detection structure is that we will have training sequences which are known symbols that are transmitted followed by segments where there is a unknown or user data that is transmitted using the training sequence period we will estimate the channel and using those channel estimates we will then detect the data; however, if the channel is changing then we must do a continues tracking of the channel even between these 2 training bus. So, that is the mechanism that that we have.

So, let us summarize our understanding. So, far that that we have coherent detection that is our method that we are most familiar with and some of the challenges that we will face in the use of coherent detection in a fading channel is the ability to estimate the channel and also to do a error free or you know as close to error free channel tracking. So, we can continue to make the estimates. So, what are some of the observations that we have made that if there is a if there is a fade if there is a fade then what happens is during and after a fade during and after because that is when my estimation and detection process can go wrong this could lead to errors in a likelihood of errors in demodulation likelihood of errors and keeping in mind that fades can occur even if my average SNR is quite high the instantaneous SNR can fluctuate. So, this can cause me errors to occur just suddenly with the because of the presence of a fade and we have also discussed that some ways by which we will mitigate and some of the ways that you would mitigate fading the errors causes by fading are what are the some of the ways that we have discussed.

Student: error corrections.

One would be FEC that is a very powerful method we have several a powerful techniques for doing the error correction second one.

Hopping is definitely one way of doing it yes changing your sequence, but then it must be either you must repeat the data because if you have already lost the data due to the fade you must do some combination of that if you do which means that in hopping you let us say you do how repetition of data you are losing a lot of overhead way by which we can avoid that overhead is by antenna diversity and we will actually devote substantial portion of the course to understanding antenna diversity because antenna diversity basically says that I have multiple antennas and if one of the antennas is seeing a signal which is a effected by a fade hopefully the others are not and pick the antenna or combine the antennas to get a better improvement.

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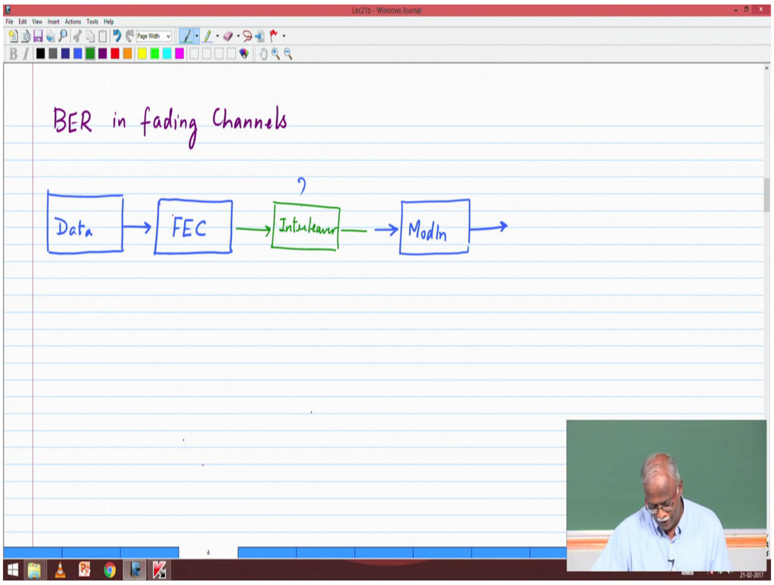

Now, I want to go back to block diagram which helps us understand the property of bit error the bit errors that happen in fading channels. So, what we said was between data and modulation between data and modulation we would like to insert an FEC because that is what will make our system robust against those sudden dips in the signal to noise ratio the question is now do I connect like this is it this connection or is that something else in between anything that you have come across well source and channel coding. So, I am assuming source coding has been done you are compressed your source and whatever is the information that you want to transmit over the channel that you have protected using a channel code there is a very important block inter leaver.

So, there is a block which has a very important function inter leaver in the context of a fading channel and let me just take a moment to explain again keeping in mind that we are building our co comprehensive understanding of fading channels and the aspects of inter leaving will be studied in the context of error correction codes, but very important for us to understand intuitively what it is.

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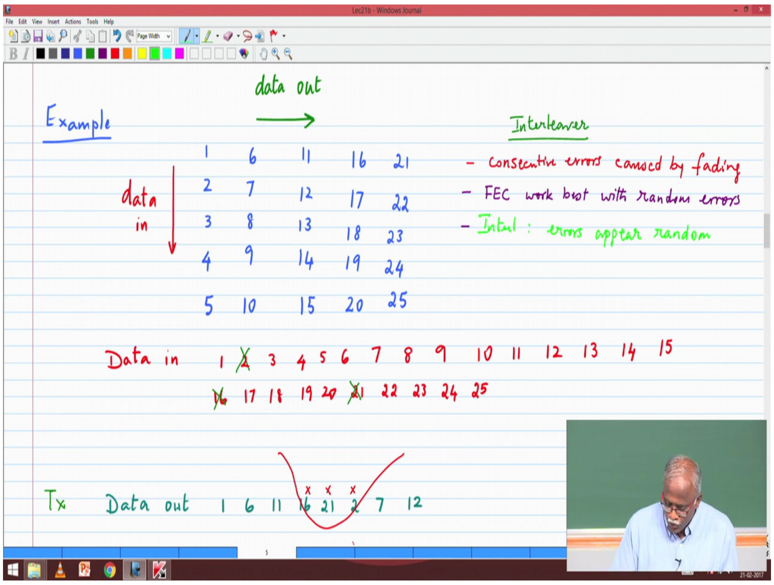

So, let us take a look at a very simple inter leavers called a rectangular inter leaver wherein you write the data in one way and then you read out the data in other way; that means, the data that is coming out of your FEC is coming out in the flowing sequence one 2 3 4 all they up to 25 lets we have 25 bits to be transmitted I have written it in the form of a square inter leaver I have written the data in a vertical form 1 2 3 4 5 6 7 8 9 10 and then I am going to read it out in the horizontal format transmit it in that direction this is the transmit direction this is the. So, the data will be transmitted not in the sequence in which it was generated, but it will be generate it will be transmitted as one bit number one bit number 6 bit number 11 and so on and so for that is the that is the nature of it. So, this is the data sequence that is being transmitted.

Let us say that fade occurred and affected 3 symbols. So, sixteen twenty one and 2 got affected in the process of a fading the fading process. So, in order for my FEC to work let us see this inter leaver actually made any difference. So, when it goes when the data is fade back into the decoder which are the bits that are drawn basically you have to the sequence in which it was generated that is the same sequence in which you have to give it back to the decoder encoding decoding has to be in the same sequence, but it got transmitted over the channel in some scrambled form. So, if you look at this sequence number 2 bit number 2 was in error bit number sixteen was in error bit number twenty one was in error. So, it looks like they were scattered about and it looks like the positions are random. So, here is the here is observations regarding the inert leaver why we want to use and inter leaver not so much because we want to understand inter leavers, but because we want to understand the pattern of errors that happen in the fading channel.

So, typically in a fading channel in a fading channel you will get consecutive errors because when the channel goes into a fade it is likely to affect multiple symbols. So, consecutive errors are likely to happen in a fading channel cause by fading. So, this is something that is we recognize. So, therefore, now another important element which you may have already studied is that most of the error correcting codes works best if the errors are random work best with random errors. So, string errors or a chain of errors generally actually can cause failure of the error correction mechanism work best with random errors. So, FEC is our technique to work against fading, but FEC works well with random errors does not map with what happens in a fading channel because what will what will happens is exactly like what is what we see here consecutive channels. So, this basically tells us that the inter leaver plays a very important role and that inert leavers role is to make the consecutive errors look random.

Inter leaver the job is to make the errors appear random we cannot change the fading, but you can make when it comes us for as the when the decoder sees it looks like some the random there errors are and the code my actually be effective in. So, errors appear random because of the presence of the inert leaver and the de inter lever on the other end the pattern looks. So, what do we wanted to take away from here errors will occur as burst in a fading channel. So, that is why coherent detection has its limitations because if I have consecutive errors then my likelihood of me loosing track of the channel is high and once I lose track of the channel then I will have performance degradation in my errors.

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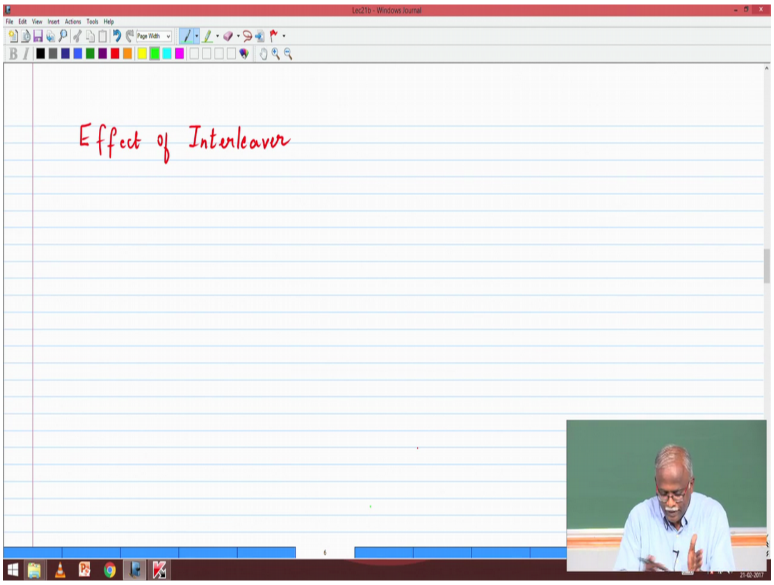

So, now let me move into the effect of inter leaver we have already said it will make the consecutive errors appear random.

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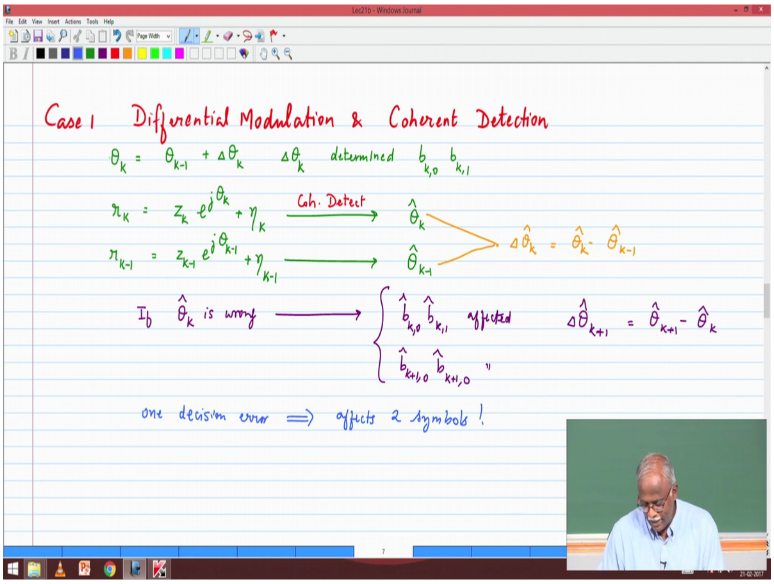

The 2 questions that we wanted to look at coherent and differential comb cross combinations coherent co modulation with coherent detection differential modulation differential detection have already been studied now let us look at the cross combinations and again this is more for us to make sure that we have a good handle on the fading channel and you know our ability to work with this type of environment. So, differential modulation basically says that my current symbol that I want to transmit let me call that as theta k is given by theta k minus 1 plus delta theta k the information is contained in the delta theta k and delta theta k is determined by the bits that I want to transmit if I take a QPSK conciliation of 4 level conciliation there will be 2 bits deciding delta theta k lets me call that as b k 0 b k 1 and of course, if it is higher level you would have more bits that are coming that.

So, given this scenario basically the coherent detection mechanism says what is the received symbol at time instant k at time instant k it is a complex scale factor again write as alpha e power j phi I am just writing it as j k for simplicity e power j theta k plus eta k that is my received symbol at time instant n what does coherent detection do coherent detection says I will tell you what zee k is z k is you tell me what theta k is. So, the coherent detection mechanism will tell help me get an estimate of theta k at the time instant k now the at the time instant k minus 1 it is z k minus 1 e power j theta k minus 1 plus eta k minus 1 that is the equation for the received symbol at time k minus 1 basically this link is coherent detection because that is what we are coherent detection coherent detection and let me write down; this would have given me an estimate of theta k minus 1 hat let me call that I would through the process of coherent detection.

Now, because I have done differential modulation I need to combine these 2 pieces of information to get delta theta k delta theta k hat which will be theta k hat minus theta k minus 1 hat. So, there is absolutely no problem if you have encoded the information in a differential form I can do coherent detection by detecting the received phases at each instant of time and then taking the phase difference because that will tell me what is the differential phase between the 2 instances of time. Now here comes a very important element when you have when everything is error free no problem you know your things are going fine, but what happens if theta k is wrong if delta theta k or if theta k hat is wrong; that means, the coherent detection process because of noise made an error if there what is going to happen it is first going to affect b k; b k 0 and b k 1, these are going to get affected these will be affected why because delta theta k is wrong means theta k is wrong is delta theta k will be wrong and therefore, these 2 will be affected not a problem in a whenever you make a wrong decision it is going to affect the information bits that you transmitted.

However, notice that the next symbol delta theta k plus 1 delta theta k plus 1 k plus 1 also depends on theta k because that is theta k plus 1 minus theta k hat. So, it is going to affect. So, which means that b k plus 1 comma 0 hat b k plus 1 comma 0 hat or also affected; that means, one wrong decision is going to make wrong decisions make errors occur over 2 symbol duration which we actually information transmitted now that is that is an undesirable effect because coherent detection what does it do if I make a mistake in my current decision it is going to be affect only my current bit it is not going to affect future or past bits. So, keep in mind that if I try to do this method differential modulation and I do not do differential detection, but I do coherent detection then the penalty that I pay or the consequence is that one decision error one decision error affects multiple symbols affects at least 2 symbols that is an undesirable property and we will probably do not want to use that and therefore, this combination is not a very commonly encountered one.

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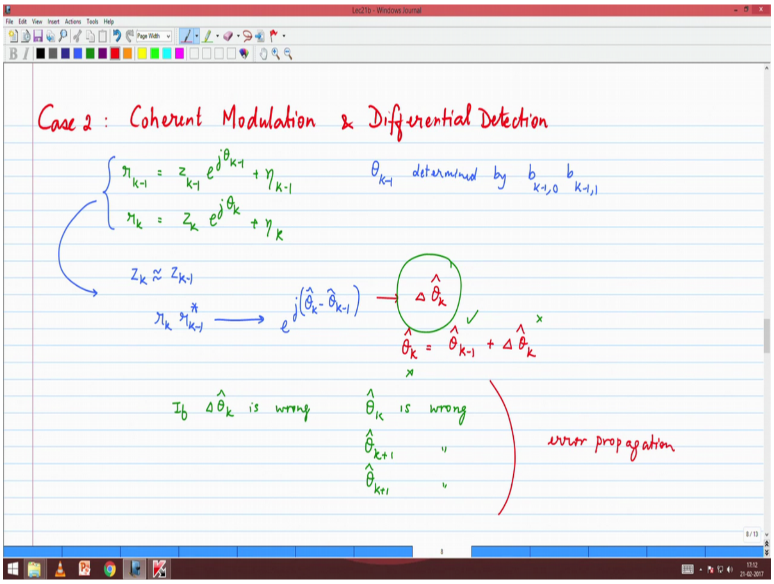

Now, let us look at the second combination where I do coherent modulation and I want to do differential detection lets quickly look at this combination as well. So, this combinations says that the received symbol at time k minus 1 will be z k minus 1 the fading channel e power j theta k minus 1 plus eta k minus 1 are k would be z k e power j theta k plus eta k now since its coherent modulation let us come make sure that we are clear theta k minus 1 is determined by directly not through a differential phase is determined by b k minus 1 comma 0 and b k minus 1 comma 1. So, directly it maps to the transmitted phase. So, the differential detection mechanism says r k r k minus 1 I make the assumption that z k is approximately equal to z k minus 1 and then I will do the following operation where it says I will do r k; r k minus 1 conjugate that is the differential detection operation again exactly like before this will eventually get for me the metric e power j theta k minus theta k minus 1 this is what I will get through the process of differential detection.

Now, what I am my int I am. So, basically this effectively tells me what is delta theta k, but delta theta k is not what I am interested in I am interested in theta k. So, theta k estimate will be theta k minus 1 hat estimate plus delta theta k. So, you in order for you to may to get the information that you want that you actually transmitted which was delta theta k theta k you would have to get theta k minus 1 and then add to it the differential phase that that you obtained through the differential detection operation is that clear basically you have done differential detection differential detection will give you the phase difference between the 2 symbols that you are which you are using in the differential operation differential detection operation that is delta theta k and this is what is. So, now comes the important question where can I make a mistake in differential detection I can make a mistake in the estimation of delta theta k. So, if delta theta k is wrong delta theta k is wrong then definitely theta k hat is going to be wrong even if theta k was correct if this is wrong theta k is going to be wrong what about theta k plus 1.

Theta k plus 1 is wrong tell me what theta k plus 2 is also wrong be why because you have driving a reference wrong reference your reference got affected and therefore, even if you are making correct decisions on the delta thetas your when it maps to theta k its actually wrong. So, this coherent modulation differential detection is actually has catastrophic error propagation. So, this is the very dangerous one definitely you do not want to try this though it looks like you can do detection without doing channel estimation this is not a good way because there is error propagation and its good for us to know some of these just as at an intuitive level because these are the things that help us make understand a big picture of about the differential detection any questions about the 2 cases that we have looked at one with differential modulation and coherent detection the other one is coherent modulation and differential detection both of them do not give us an advantage.

If you doing coherent detection best is to go for coherent modulation if you doing differential detection do it with differential modulation that is the right way to do it because you intended at the transmitter you undo it at the at the receiver and get the information that is needed any questions.

Student: Sir.

Question is how do I start the decision process. So, you always assume if you remember we transmitted a transmitted a set of known symbols. So, you assume that a few symbols are known then the training sequence. So, you can use the last symbol of the training sequence as your starting point because that theta k is known to you ahead of time and from there on you do the differential modulation. So, yes the; unless you somebody told you what was the starting point for the theta k is you cannot really start your detection process. So, good point. So, maybe to summarize when would I use differential detection I am along with differential mo modulation with the definitely I am not going to use it separately. So, the question would be is when would you use differential detection with differential modulation differential detect with differential modulation.

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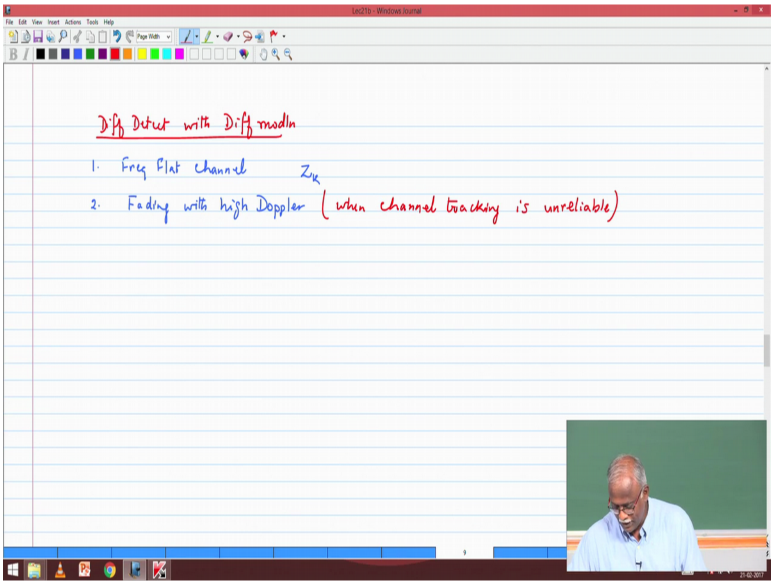

That is the when would I use that when would use that when first of all basically must make sure that there is no multipath because otherwise differential detection will fail. So, I must have frequency flat channel frequency flat channel; that means, there is only one channel coefficient z k that I need to estimate at every given instant of time now that happens to be a very easy problem for coherent detections so; obviously, if you assume frequency flat channel then coherent detection would be the obvious choice right because one channel to estimate you can track it, but would there be any condition under which you would prefer differential modulation any condition under which you would prefer differential modulation high Doppler because that is when the tracking of the channel may becomes difficult.

So, flat fading and the presence of high Doppler fading with high Doppler under this condition you may be better off with differential detection. So, basically when channel tracking becomes unreliable when channel tracking is error prone or unreliable then under these 2 conditions then differential detection with differential modulation would be a preferred choice because you may do better than the than the coherent case. So, that is just a piece of information that is to help you piece together the complete understanding of the fading channels that we are working with we move on. So, the heart of today's lecture is our goal to obtain analytical expressions for BER in Rayleigh fading now BER is always connected to SNR signal to noise ratio.

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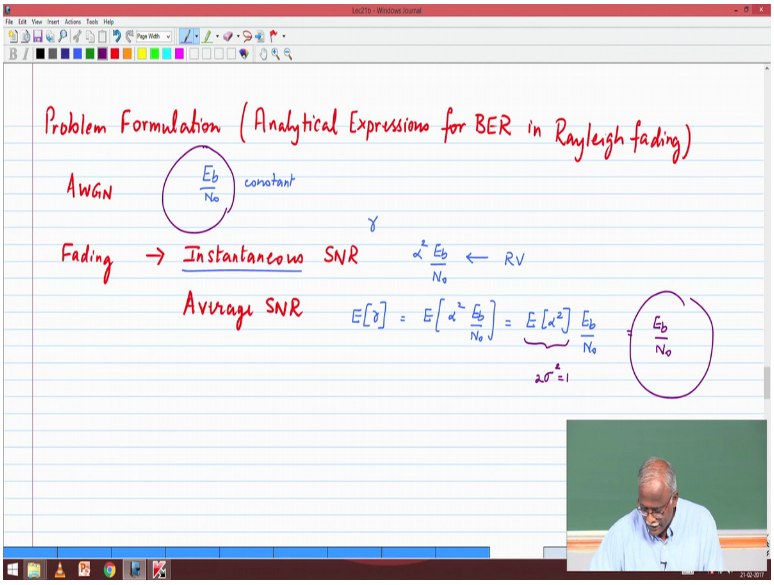

Now, in the context of AWGN channel; the E the signal to noise ratio is E b by N naught and it is a constant now the difference comes when we work with fading channels because there is no notion of a constant SNR because your signal is fluctuating because the SNR that you have is actually alpha squared E b by N naught E b by n naught is a constant, but alpha squared is a random variable alpha is a random variable. So, alpha squared is also random variable. So, therefore, this is not a constant this is a random variable and. So, we do not call it as a single number SNR we just do not call it SNR we call it as instantaneous SNR what is the current channel conditions which is depends on alpha and that will affect my SNR. So, there is an instantaneous SNR.

Now, this also tells us that if it is a random variable I can talk about meaningfully talk about the expected value. So, the average value of the instantaneous SNR will be expected. So, if I denote this by gamma then a average value of the SNR is expected value of gamma that would be expected value of alpha squared E b by N naught that would be equal to expected value of alpha squared because that is the part that is random times E b by N naught and if I have done my scaling such that my Rayleigh statistics has got a u unit variance basically the real part and imaginary part have got appropriate variances. So, 2 sigma squared equal to one expected value of alpha squared will be equal to 2 sigma squared then this would be equal to E b by n naught.

So, the average SNR you can talk about an instantaneous SNR in a fading channel you can talk about an average SNR in a fading channel and the case that we would like to be able to compare is when the SNR in a fading channel in a AWGN channel and the average SNR in a fading channel are the same because then you can make a fair comparison between the 2 of them. So, this is the starting point for our discussion of a analytical expressions for BER we will move fairly rapidly through.

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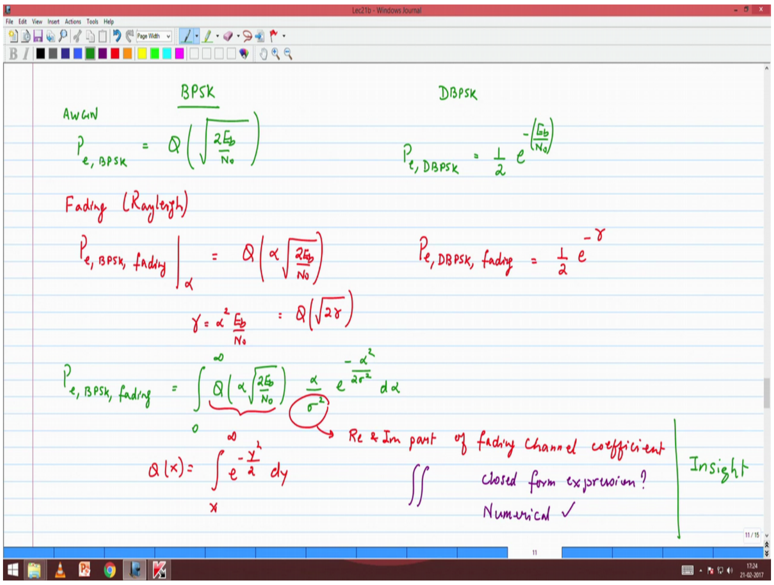

The next section, but again if there is anything that is not clear please fulfil to ask and clarify yourself so, the probability of error in AWGN.

So, let us take for example, the modulation as BPSK that will be our reference modulation until we specify we know that there is a change. So, probability of error of BPSK in a fading channel Q function root of 2 E b by N naught again something that we are familiar with from our earlier studies now by the same token if I went to look at differential BPSK probability of error of d BPSK this is something that you may studied it will be half E power minus E b by N naught I am I again this is this is material that is from books like Proakis E b by N naught. So, your instantaneous SNR or the SNR in an AWGN channel determines the performance in a now if I move from here to a fading channel fading and I am going to use the term Rayleigh fading as my environment and what I mean by Rayleigh fading is that it is a fading channel in which the real part and the imaginary part are complex gauss are Gaussian distributed 0 mean with a certain variance and the envelop is Rayleigh distributed. So, again I use the term Rayleigh fading, but I mean a very specific environment where the envelop of the signal received signal has a Rayleigh statistics. So, again interpret Rayleigh fading I just use the word Rayleigh fading, but interpreted in the correct statistical context. So, if I when if I want to ask you for the error probability of BPSK in fading.

We have already mentioned I must tell you what the alpha is the instantaneous value of the SNR is. So, this will be Q of alpha root 2 E b by N naught now I just want you introduce the notation that we are going to be using and basically if we introduce the notation that the SNR is equal to alpha squared E b by N naught now what would be the expression for the error function BER instantaneous value root 2 gamma. So, basically the expressions instantaneous values given by this correspondingly probability of error of d BPSK in a fading channel is given by half e power minus gamma the that would be my expression that we would have to work with.

So, if I went to say now give me the general expression for fading. So, this is something that we have already written down earlier, but let us just for completeness write down, probability of error of BPSK in fading have to depend remove the dependence on alpha. So, this will be integral 0 to infinity alpha is the amplitude of the envelop it can go from 0 to infinity Q of alpha root 2 E b by N naught times now I have to multiply by the p d f of alpha; alpha by sigma squared because the Rayleigh distribution E power minus alpha squared by 2 sigma squared d alpha.

Now, keep in mind that this sigma refers to the real and imaginary part imaginary part imaginary part of the fading coefficient of the fading channel coefficient fading channel coefficient and individually they have both identical 0 mean Gaussians. So, fading channel coefficient. So, that is the expression that that we have now I would have to write down the expression for Q because I cannot integrate Q directly. So, Q of x is as you know integral from x to infinity e power minus y squared by 2 d y. So, it actually becomes a double integral just please substitute for Q in that expression it. So, becomes a double integral. So, it becomes a double integral the question is does a close from expression exists close form expression exist if. So, how do we get it and what is the methods usually when close form expressions do not exist you can do numerical always that is an option you can do numerical integration close form expression actually does exists, but whenever you come across the integral you do not know a priory whether the integration and integral exists, but most importantly out of all these I must get inside no point getting some complex integral in feeling very good you know you go feels very happy that you know solved the very complex integral, but the end of the day if you do not have intuition there is really no not much benefit from that.

So, the observation that we can make from here is that the this double integral actually does not lend itself very easily for the unless you do some cleaver substitutions which we will we will we will sort of take you through that in a one of the assignments, but now as for as the class is concerned I would like to take the very clever and intuitive approach which gives us a lot of inside.

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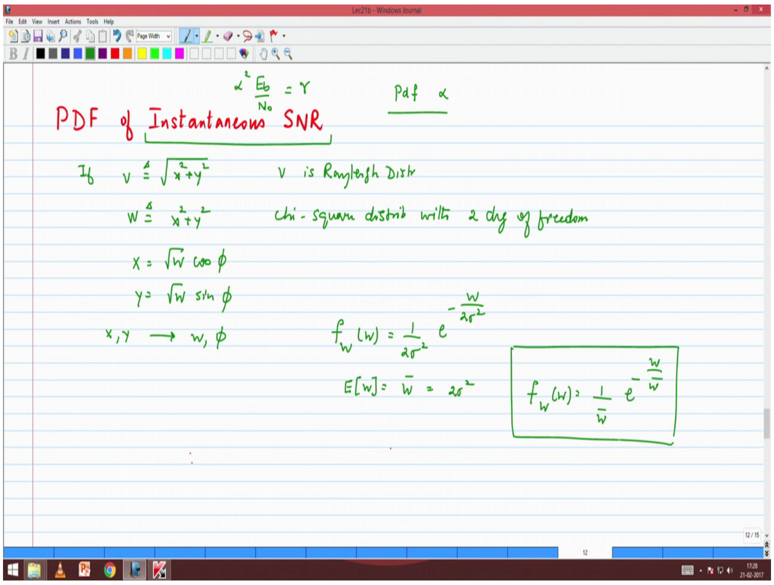

So, let me define rather than looking at the p d f previously we were looking at p d f of alpha is the envelop of the Rayleigh faded signal. So, instead of doing the alpha p d f of alpha I want you to look at the p d f of the instantaneous SNR instantaneous SNR will be alpha squared E b by n naught. So, if I denote this as gamma, now I want to find out the p d f of gamma the distribution of gamma. So, this is where we come across this problem which will be very familiar to you v is equal to square root of x squared plus y squared then where x and y are 0 mean I did not I Gaussian then we get the fact that v is Rayleigh distributed this is well known.

Now, what happens if I have v squared is equal to square root of x squared plus y squared or sorry not v squared v is equal to square root of x squared plus y squared now this is a chi square distribution chi square distribution with 2 degrees of freedom 2 degrees of freedom again the statistics and all of that you would have probably have studied when you looked at transmission of variables. So, here is the goal that we want to want to obtain want to obtain the let us; let me not confuse you let me call this as w. So, w is equal to x squared plus y squared now I want to know; what is the distribution of this way variable. So, basically x is equal to root w cosine phi y is equal to root w sin phi again this is something that you would have seen in the second quiz first quiz basically we want to do the mapping of transmission of variables from x and y to w and phi and you should go through the process what you will get is the probability distribution of w comes out to be one over 2 sigma squared e power minus w by 2 sigma squared it is actually a compact form it is it does looks like the exponential, but we will see it in the moment.

Now, if I went to write down the expected value of w I call it as w bar now the expected value of w is expected value of x squared plus expected value of y squared that is 2 sigma squared. So, f w of w is one by w bar e power minus w by w bar and this is the general form of an exponential p d f and you can verify that it is a valid p d f by integrating it from 0 to infinity, but the imp important element is that the SNR has got a expression that looks slightly different from the p d f of a Rayleigh distribution, but it is form that is probably helpful for us in understanding or deriving the analytical expressions let us see if we can quickly apply by the any questions on this we are not looking at the p d f of alpha, but we are looking at the p d f of the instantaneous SNR which is alpha squared. So, basically I want to treat alpha squared as a random variable. So, called it as w and I have derived the p d f because that this will be the form of the p d f of the instantaneous SNR.

So, the approach would be as follows that rather than writing down the expression for the envelop of the signal we will now write it in terms of the SNR of the signal.

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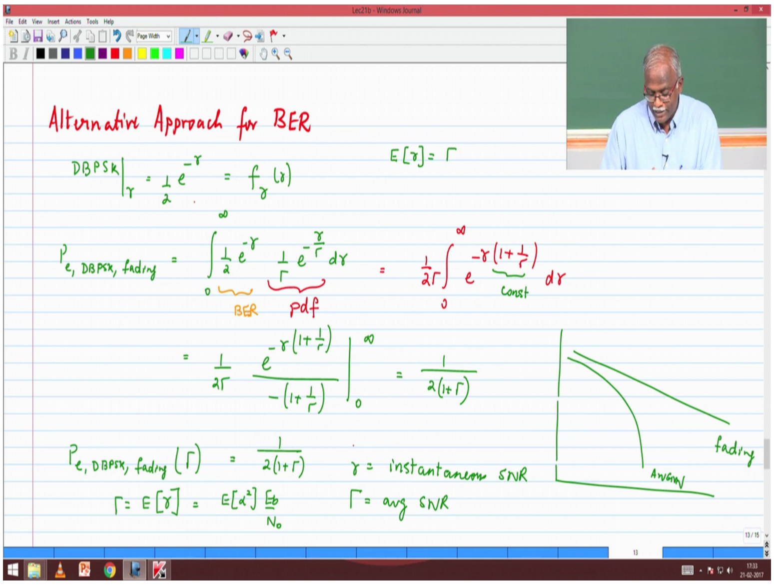

So, let us take d BPSK as a case study the probability of error of d BPSK in an AWGN channel is e half e power minus gamma right I think that is what that is what we had derived. So, now, we want to write down the probability of error in fading. So, this is for a given value of gamma I can tell you that this is the probability of error. So, probability of error of differential BPSK in a fading channel will mean that I have to integrate over the range of alphas using the p d f of gammas you see.

So, this would be integral 0 to infinity half e power minus gamma. So, this if you remember is the p d f of gamma I am going to write down the p d f of gamma p d f of gamma is 1 by let me just I need to introduce one more notation expected value of gamma I am going to use the upper case gamma as. So, my p d f is one by gamma e power minus gamma by gamma d gamma. So, this is the BER portion this is the BER portion this is the p d f portion and I do the integral to get the get the expression in a fading channel. So, I would like to now combine these 2 terms one half integral 0 to infinity gamma can be taken out that is the constant e power minus gamma into one plus 1 by gamma d gamma.

Keep in mind that one plus 1 by gamma is a constant. So, there for its of the form e power a x very easy for us to integrate please do the integration I will just write down one by 2 gamma e power minus gamma one plus 1 by gamma they divided by the constant which is minus 1 plus 1 by gamma that is the and this has to be evaluated from 0 to infinity and please do confirm that you get one over 2 into one plus gamma.

So, the complexity of the integration has been taken out because it is just a simple exponential integral which most of us are comfortable doing the integration. So, this is what is the average performance or the performance of d BPSK in a fading channel. So, notice that the BER basically goes as a one over 2 into one plus gamma. So, basically it falls in a; it falls in a linear fashion. So, if I want to write it in a logarithmic scale the; my slope will be a constant a negative constant. So, the BER plots will tell us a lot this is the BER in AWGN a w g n the corresponding graph in fading channel which we have already drawn before for the first time where actually showing that this is indeed the case.

So, what would be what would how the way we would write it down is the probability of error of d BPSK in a fading channel as a function of gamma where gamma is the expected value of the instantaneous SNR e power lower case gamma that is equal to expected value of alpha squared times E b by N naught this would be given by one over 2 into one plus gamma that is the net results this is the gamma is the average SNR the lowercase gamma is the instantaneous SNR.

So, that is the derivation of the p d f of the SNR of the SNR and an application in terms of computing the probability of a of error of d BPSK basically this lecture and you know the material that we have been covering in the in the subsequent few and the minutes also does involve integration I would definitely encourage you to write it down and try out the integral these are simple integrals, but I would almost always give you the hints as to we will be able to solve that.

If you are comfortable with that I believe we should move on to understanding the exponential p d f.

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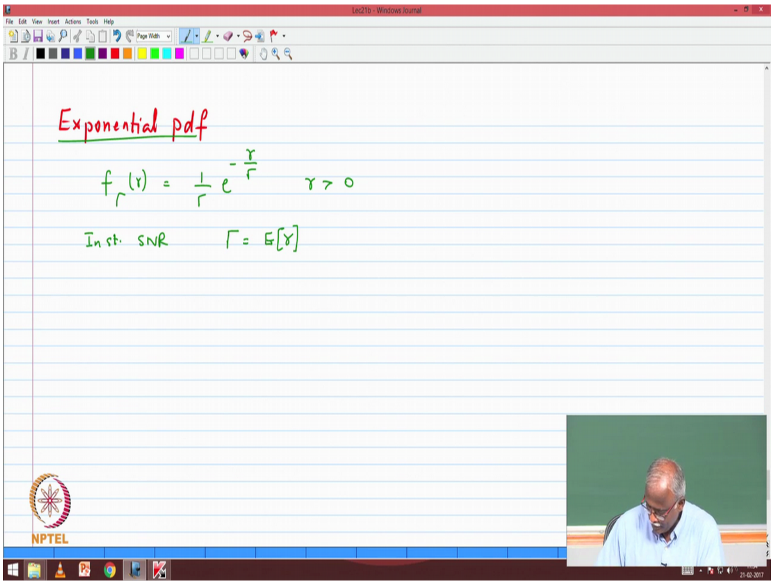

So, the probability distribution of gamma is one by gamma e power minus gamma by gamma this is the exponential p d f that describes the instantaneous SNR instantaneous SNR is the exponential p d f and this is the very useful expression for us to keep in mind where this uppercase gamma is expected value of gamma. So, I it is a very compact form it something that we will use quite extensively in our in our discussions.

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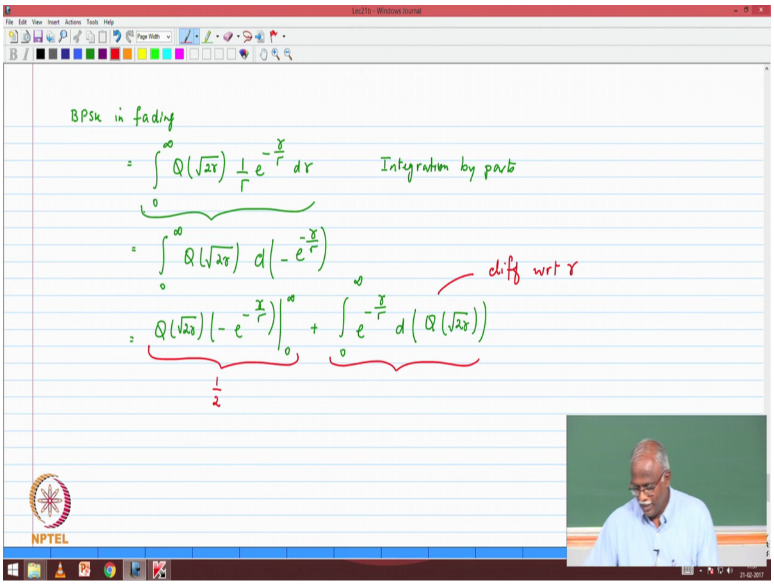

So, now what I would like to do is address the bit error rate probability of BPSK having done one other can be 2 difficult right so; obviously, you should be able to try this one out. So, the expression for this probability of error will be 0 to infinity the Q function I will not write in terms of alpha I will write it in terms of gamma root 2 gamma and the p d f one by gamma e power minus gamma by gamma d gamma looks a little bit easier than the than the p d f of using or using the Rayleigh p d f.

Let us do the steps let me just indicate and then complete the discussion. So, this you would have to re write using the integration by parts the hint is integration by parts. So, which says that this can be written as 0 to infinity Q of root 2 gamma d of minus e power minus gamma by gamma. So, basically if I write it in the u d v form you can you know how to do the distribution the integration by parts the integration by parts gives u v as the first term Q of root 2 gamma minus e power minus gamma by gamma that is u and v evaluated at 0 and infinity the limits minus integral of v d u and the when you when you work out the sins this minus sin with the with the minus sin coming out basically the second term becomes integral 0 to infinity e power minus gamma by gamma derivative of Q basically u d v root of 2 gamma. So, basically I just rewrote the integral that we have in the form of integration by parts I hope you will not find it difficult to evaluate and confirm that this value comes out to be one half.

Now, second term is little bit tricky because you have to differentiate an integral Q is actually the Q function is an integral and you are in differentiating with respect to what you respecting this is the differentiation with respect to gamma differentiating with respect to gamma. So, you are differentiating the integral with respect to gamma.

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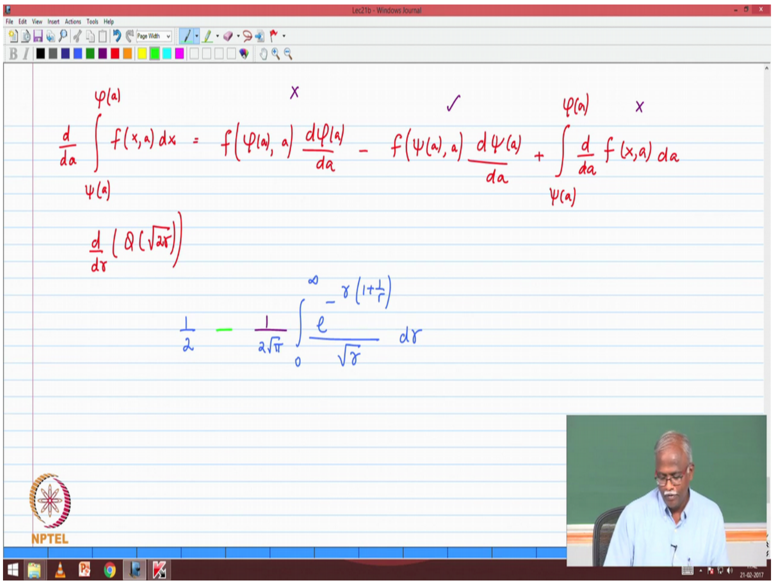

Again this is something that we do not encountered 2 frequently, but it is an important result that we should know how to differentiate and integral if I differentiate and integral it can be the variable on which is present in the lower limit of the integral this case psi of a some function or it can be variable that is present in the upper limit of the integral or it can be in the integrand itself my integral just forget about the d by d a is an integral of the function f between the limits psi of a and phi of a.

Now, I am differentiating with respect to a variable it this variable can be present in the lower limit upper limit or in the integrand or in all 3 or any combination of these. So, this is given by the upper the function evaluated at the upper limit with the with the value phi a of a d phi a of a that is the if the variable is present in the upper limit this term would be present if the variable is present in the lower limit it would be the integrand evaluated at the lower limit psi of a comma a d psi of a by d a this term would be present if it is present neither in the upper limit or in the lower limit its present in the integrand then you have the third term the limits are the same psi of a phi of a you would now have to differentiate the integrand d by d a f of x comma a d a now do not forget that these first 2 terms also exist if your variable of integration is present either in the upper limit or lower limit do not just jump in and integrate with respect to the integrand and say you are done.

Now, we are in differentiating d by d gamma of Q of root 2 gamma where all is the gamma present is it present in the lower limit Q function.

Student: Yes

The argument of the Q function will be present in the integral. So, gamma is present in the lower. So, this term will be present upper limit is infinity. So, this is this is not there and the integrand is e; e power minus x for basically it is a Gaussian. So, with the only the variable of integration is present in the integrand. So, therefore, this is also not there. So, we just need to evaluate the middle term. So, I hope you will be able to evaluate the middle term and then come back to tell us that the that the integration that we have to do basically gives us the following expression I will I will pick it up from here, but please do derive that the expression that you should get when you do the middle term will be equal to there is a one half already present.

So, it will be 1 by 2 root pi integral 0 to infinity e power minus gamma into one plus 1 by gamma divided by square root of gamma d gamma and I believe there is a one half up front and that is equal to plus. So, please just make sure you make an attempt to o get to this point and it is not difficult, but just want to make sure that you are doing it in a in a systematic manner and then we will we will pick it up from here and actually it comes out to be minus we will pick it up from here because this is an integral that you may able to solve if you are able to solve very good if you not able to solve its just one step through substitution we will we will show you how to do that in the next class.

Thank you very much.