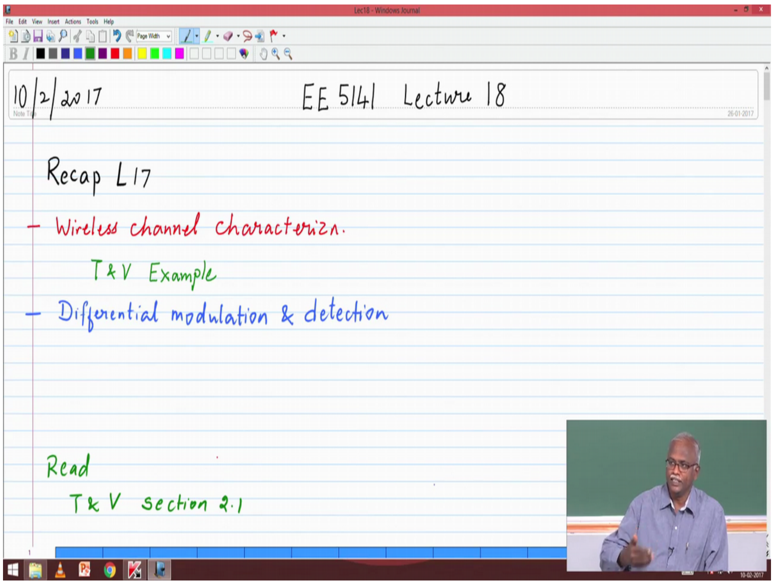
Introduction to Wireless and Cellular Communication

Prof. David Koilpillai
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture – 18
Multipath Fading Environment
Characterisation of Multipath Fading Channels

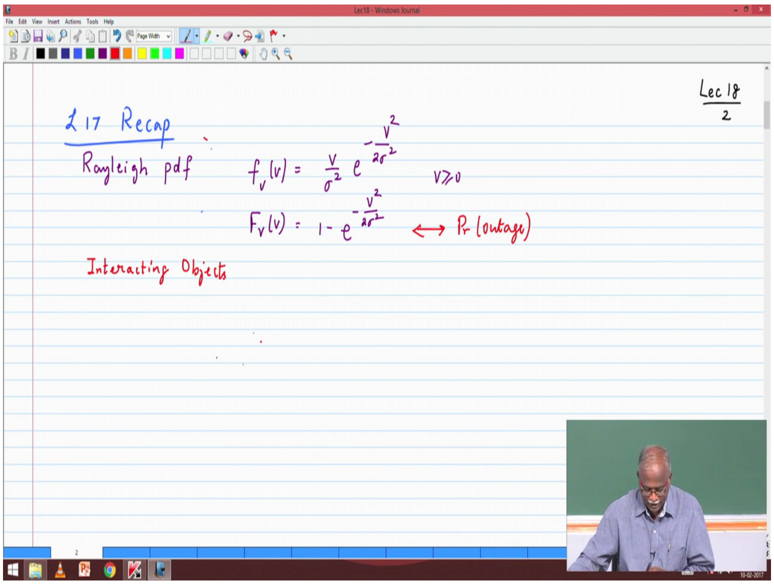
Good morning. We begin with a quick summary of lecture 17 and move on to the topics to be covered in lecture 18. The topics that we will be covering today is to understand a characterisation of the wireless channel and this will be done through a very simple yet very elegant, very deep example that is provided in TSE Vishwanath chapter 2.

(Refer Slide Time: 00:39)



Having understood, some of the characterisation elements, we then move into the section where we talked about what are some of the challenges when it comes to the choice of modulation and then, actually the task of detection in a fading channel. So, that is the task that we are working towards. So, we begin with the quick summary of the points that we have looked at in the last class and build on that.

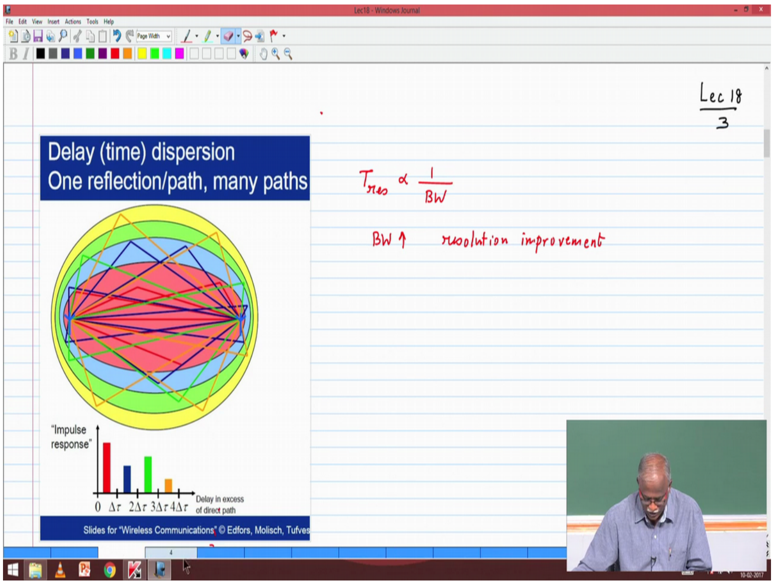
(Refer Slide Time: 01:02)



So, our main goal so far has been to characterise the Rayleigh Fading which occurs in a multipath environment when there is no line of sight component. So, we have looked at the Rayleigh pdf f v of v v by sigma squared e power minus v squared by 2 sigma squared v greater than or equal to 0. Then, the corresponding c d f 1 minus e power minus v squared by 2 sigma squared and use that to understand how we can relate it to probability of outage.

So, that has been one of the elements that we have looked at in our understanding of that in the last class. We also looked at the aspects of the interacting objects. So, the notion that there are several objects that can give you multipath components that are arriving within the time resolvability window, so that you treat it as a single composite signal. So, interacting objects we say that these would be typically a long on the ellipse.

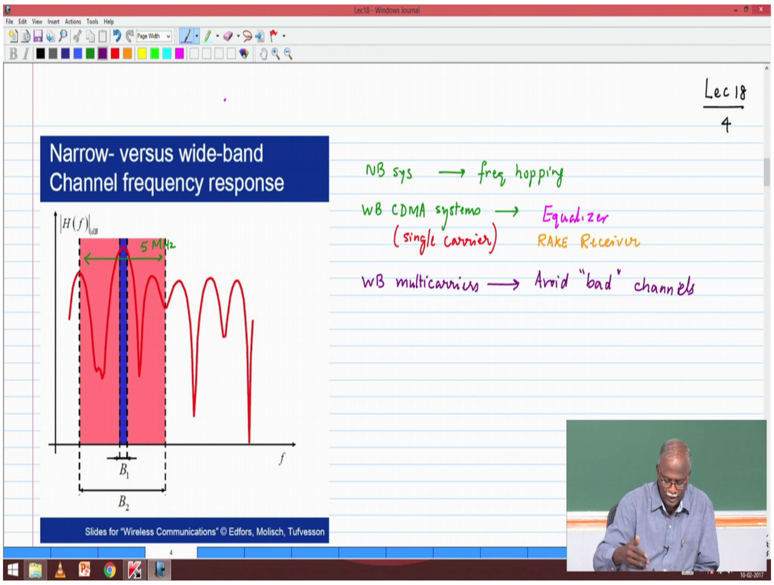
(Refer Slide Time: 02:33)



If it were a single reflection and because of the time resolution uncertainty, it becomes not just a line ellipse, but an elliptical band and all of those multipath components which we cannot resolve, we treat it as a single composite component which has the Rayleigh statistics. So, that is our way of characterising it. This was something that we had looked at in the last class. We said that the time resolvability is inversely proportional to the bandwidth. So, as the bandwidth increases, my ability to do time resolution, my time resolution also actually improves. So, we should actually say time resolution actually becomes smaller or becomes better.

So, resolution I just say has improved, so that for wide band signals, we have better understanding or better characterisation of the multipath channel, but for the narrow band signals, it looks mainly very often like a flat fading channel or mildly dispersive channel. You may see maybe one tap of dispersion is not because that the channel did not have multipath, but your resolvability is limiting that case. So, let us summarise the points that we made in this context as well.

(Refer Slide Time: 03:58)



So, if you do if you were to do true characterisation of a channel, you would have to do a wide band characterisation. The wide band characterisation says that there are lot of multipath components with which are arriving at different time instances. So, the red line gives us the frequency response of the channel and now, where does my signal lie if it were GSM signal denoted by this blue line that is the center frequency and that is the frequency response. Now, if the same GSM signal was at a different point, we would see different channel response, but notice that the spectrum itself is not distorted by enlarge what it is seeing is a flat channel, ok.

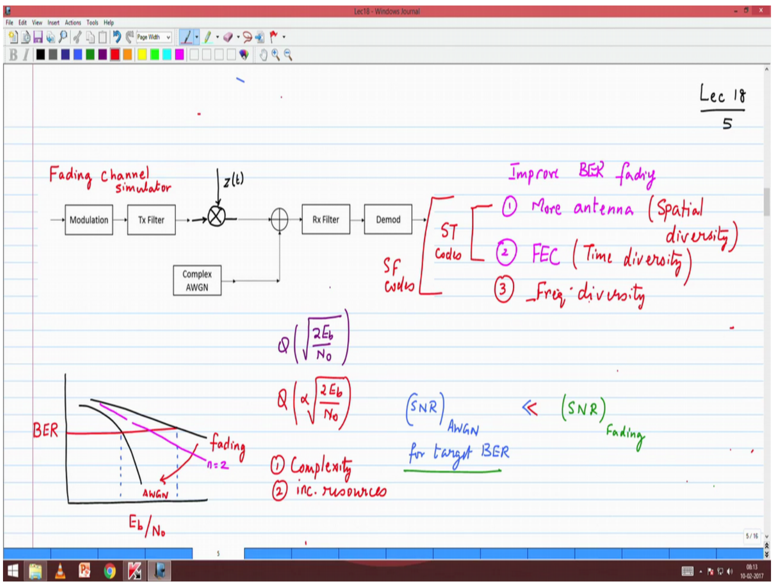
On the other hand, if I were to have a CDMA system which had 5 mega hertz, so think of this as the CDMA system with 5 mega hertz. Then, I would see some channel variation which means that I am able to resolve the multipath and if I co phase the multipath, then I will get the ability to detect the signal. So, the narrowband channels are the ones that are more affected by the fades and we talked about frequency hopping as a way by which we would deal for narrowband systems. So, for narrow band systems, we said that we could do frequency hopping. Of course, you could do the array correction also that is definitely one way of doing it, but frequency hopping is probably when you look at this diagram, you say that the best option is to do frequency hopping change your center frequency, so that you get out of the null that you are in whereas, for wide band systems, wide band CDMA systems, very important that we qualify that because wide band systems could be multi carrier.

So, CDMA systems are a single carrier wide band systems. So, basically maybe we can start using that characterisation. It is a single carrier wide band system and the wide band is obtained through the process of spreading. We will be studying a lot about CDMA systems in the future, but for now there is multipath. You are able to resolve the multipath. So, you take care of it through the process of equalisation because if you do not take care of it, you will have inter symbol interference and in the context of a CDMA system, this is called a RAKE receiver. It is nothing, but an equaliser which handles the affect of the multipath.

Now, when it comes to wide band multi carrier systems, that would be systems like OFDM, where you would have large number of these narrow band carriers. Then the technique or the approach that we would take is to avoid bad channels. You can think of the wide band system as having large number of channel options for you to choose from avoid those carriers that have a bad frequency response. So, therefore do not waste your transmit power on those. Use your transmit power on those which have got good channel. So, avoid bad. Let me just say use the word channels, but you understand those are the carriers that we are talking about, ok.

So, this is a summary of the points that we had made to make sure that you know how are we dealing with the fact that there are nulls in terms of my channel response and how do we respond to it, how do we take care of it in the fading channel.

(Refer Slide Time: 07:43)



Now, I want to go back to our discussion on the context of the fading channel. Remember we said that fading is a multiplicative impairment and it occurs before the addition of the noise. Inter changing the two cannot be done because this is how you would represent it and we also characterise that the bit error rate in AWGN channel for BPSK or KPSK would be Q of square root of 2 E b by N naught. That would be for AWGN For a fading channel, you would have the fading coefficient also affecting the response and through the simulation, you will be able to see some of these affects, but I want to ask you to think about one specific aspect and that aspect is this is the affect of fading. So, if I want to achieve a target bit error rate, notice that the bit error, the signal to the noise ratio that I would need in AWGN channel.

So, I am going to write the statement while you must interpret correctly. SNR in AWGN for a target bit error rate, for a target bit error rate is less than the corresponding SNR in a fading channel. If you just say SNR in AWGN is less than SNR in a fading, that would be a confusing, probably incorrect statement, but to achieve a target bit error rate and I am sure you are now seeing why it is because if I want to achieve let us say 10 power minus 3 and in some cases, the SNR required becomes much less because further down if you take 10 power minus 4, the difference becomes larger and so on and so forth. So, there is always a concern that fading is going. I have to pay a very heavy price if I have to build a communication system that works in a fading environment. So, there is the challenge of wireless communications to see if we can somehow move this graph closer to AWGN. The rest of the course on wireless communications is to help us move towards AWGN, ok.

Now, what are some of the ways, we have already touched upon several of those elements, what are some of those ways by which you think that this graph can move towards a AWGN graph? What are some of the ways in which we can move that?

Student: (Refer Time: 10:27)

Pardon

Student: Multiple antennas

Use multiple antennas and let me just give you one result. If I were to use just two, receive antennas at the receive and choose the better of the two antennas, one of them maybe in a fade, the other one may not be and I choose the better of the two. This graph will change. This graph now has a different slope. So, this is n equal to 2 antennas. So, the slope definitely increasing the diversity is going to help. So, one of the ways in which we would improve BER in fading channels, one of them would be more antennas, but you realise that there is a limit to how many antennas we can keep on a mobile because mobiles are small and you have to have some special, but it falls under the category of special diversity.

Diversity essentially means that there is a second or alternative signal that is available to you and it comes because I use a second antenna which tells me that in space I capture a second signal and I choose the better of those two. What are some of the other methods that we have already studied is FEC way of improving the bit error rate. Answer is yes FEC is a form of diversity. Yes no maybe. It is a form of diversity and what form of diversity is, it take a very simple example. I do repetition coding every bit. I repeat twice instead of one. I transmit 1 1 or 0, I transmit a 0 0, but I transmit it separated in time. So, in case there is a fading process happening, the both of those bits will not get affected. So, if I just repetition transmit simple form of coding and that already tells you that this is a form of time diversity.

So, FEC is a form of diversity, it is a form of time diversity and one more method of improving the performance. So, now we saw that repeating the information through FEC and separating them out in time is one way of doing it. You can transmit it on different frequencies through frequency hopping.

So, there is a third dimension that you can exploit with array correcting codes that is frequency, diversity. frequency and in an OFDM system, you can transmit the two coded bits on different carriers which are separated. So, therefore, both will not get affected by the fade at the same time. So, there is special diversity, there is time diversity, there is frequency diversity and it turns out that there are combinations of these that are also existing. So, for example I can take space and time forward array correction and multiple antennas.

This is the broad class of array correcting clause called space time codes. I am sure when you study array correcting codes, you will study this as one of the important branches and you all already want you to, I want you to appreciate why are we even worried about space time codes because diversity is one that is going to pull my fading array graph towards my AWGN graph and every form of diversity that I can exploit from my system, I want to take advantage of that.

So, instead of taking a choice between antenna or time, I say I want combination of both. Now, you may be able to also do space frequency codes. Space frequency codes you do FEC, combine it with a technique like OFDM or frequency hopping. You can do space frequency and of course, space time frequency probably you exploit all the dimensions that is also possible. So, I am starting to think about the rest of the course. It is about moving this graph from the poor performance as close to AWGN.

That you will get or the price that you will pay one is the complexity. You will have to increase the complexity of your receiver that we say no problem, we will handle that. You may have to spend more resources, more power, more spectrum. So, increased resource that is also a price that we will pay in order to get the performance, but the idea is that start getting better performance out of the fading channel. So, again when we start talking about coding or when you start talking about receiver techniques, they are not disjoints. Ultimately keep this picture in mind. Everything that you do to improve the BER performance is to move the fading BER graph towards AWGN because AWGN is the best that we can do. AWGN additive by Gaussian noise is something that we cannot eliminate. So, that is our base line, that is our goal standard everything that we do. We try to move towards that and achieve the performance closed to that, ok.

So, this is a quick summary of the points that we had discussed in the last lecture. Are there any questions on the point that we mentioned in terms of the interacting objects, the time resolvability, the ability to interpret the same channel as a wide band channel as well as a narrow band channel because for example, to the wide band signal, it looks like a frequency selective fading channel to the narrow band system, it looks like a flat fading channel.

Now, is there a discrepancy in our description? No, it is the affect that it has based on your channel bandwidth and that is why we are able to interpret, that is why we are able to say how does a narrow band system deal with fading, how does a wide band system deal with fading and a lot of these will become clearer as we go progress in the course.

The question is, what do we do if you have constructive interference say thank you to nature and be very happy.

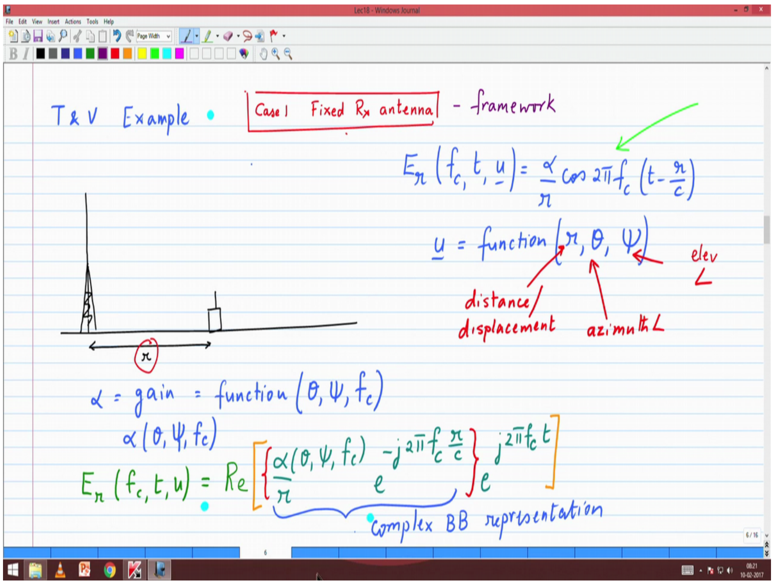
Student: Sir I mean that we are doing better than AWGN in that case, right.

Well, you will find that constructive interference will not happen all the time. There may be momentarily certain scenarios where you will do better, but remember it is a distribution. You have to keep in mind that ultimately it is the distribution that is going to affect you and though you will have instances where you may do better than AWGN, those times when you do worst than AWGN will pull you down in terms of the average.

So, the net result is that you are much worse of than AWGN, but it is a good question. There are those times when you are better than AWGN and it is very important that you capitalise on those situations because that is a time when you should be able to get the maximum data into your system. When we talk about exploiting the capacity of a wireless channel that will come into play because whenever you see that your channel is getting better, what you do is you send more data, you send more power because that is what maximises the capacity, but that is precisely the reason why we talk about the notion of maximising the capacity in the fading channel, but we will talk about that, but that is a good question. Any other questions, ok.

Then we move on to our example, the framework is that we have a base station. We have a mobile and it is at a distance R.

(Refer Slide Time: 17:59)



So, let us call this as case 1. We did not call it case 1 yesterday, but we will call it case 1 to characterise our different systems. This is case 1 fixed R x antenna at a distance R from the base station. We said that the received electric field, we will talk about only the electric field and it is more to illustrate the principles that are involved in a wireless channel.

The received electric field will be alpha which is the gain itself. When we talk about antennas, the gain of an antenna depends upon the orientation with respect to the azimuth angle, the elevation angle and the carrier frequency. All three components will have a role to play because all antennas have a three-dimensional radiation pattern. So, the combination of the transmit antenna and the receive antenna gives you a net gain. We call that as alpha.

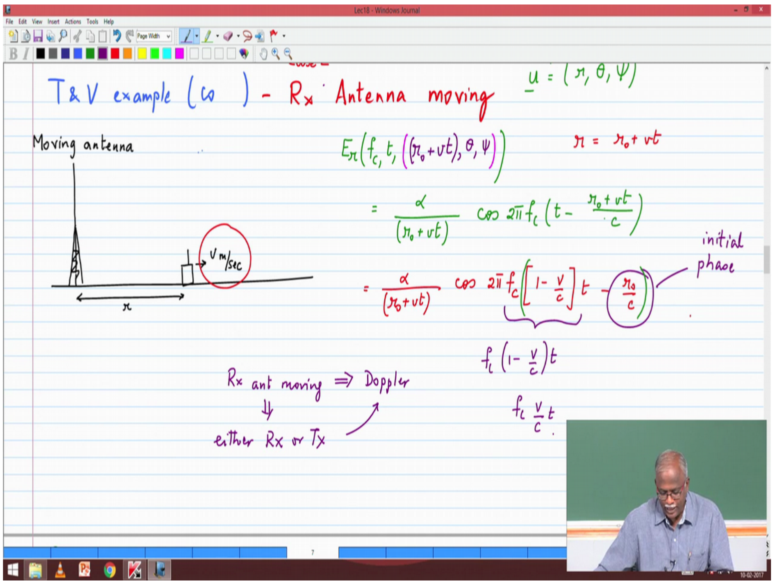
It depends on the location of the received antenna with respect to the transmit antenna in terms of theta phi and also the carrier frequency. So, alpha is a function of theta phi and f c. We have left out the functional dependencies and just called it as alpha, but keep in mind that this is something that can have an impact based on your orientation. Then, the electric field decays inversely as distance in free space propagation. So, it is alpha by R, where R is the distance. So, the intensity with which you transmitted has gone down by a factor R by the time you have traversed a distance of R.

So, let us quickly bring up the rest of the description. The rest of the description says that we have a electromagnetic wave which is defined by cosine 2 pi f c t. The received signal will be cosine 2 pi f c t minus t minus R by c, where R by c represents the time lag for the electromagnetic wave to propagate a distance of R something is I am getting lot of points that I did not write down. So, theta is called the azimuth angle, the psi is called the elevation angle and again that is the representation of the displacement vector. So, the received electric field is a function of carrier frequency is a function of how much time has elapsed and where is the reference point that we are looking at that is given by R which is a displacement and theta and phi to anchor your position in a three-dimensional space. So, that is our representation. So, we can write down the received signal vector.

The received signal vector Er f c t, u will be alpha by R alpha by R, where alpha is theta psi f c again just to represent that to capture the e power 2 pi minus R by c. That is the first term. The f c happens to be the second term and if I take the, write it as the real part, take it as a real part that will be my representation of the signal and if I leave out the f c part that gives me the complex base band representation which will be given by this. So, case 1 basically tells us that if I have an antenna at a distance R from my transmitter, the signal that is going to be received, there is going to be of the form given by this expression alpha by R cosine 2 pi f c t minus R by c.

Now, this R by c shows up as a phase term which is caused by the delay in terms of the propagation of the electromagnetic wave from the transmitter to the receiver. That is basic. It gives us a framework. So, case 1, the role of case 1 is to give us a framework for understanding this example the basic notation that we have introduced. So, now we move to the second case.

(Refer Slide Time: 22:22)



Second case is the same scenario except that the mobile is actually moving away from the base station at V meters per second. The mobile and it is currently at a position r. So, actually at r like the previous example, previous case, but it is moving to the right with a certain speed and that is what we are going to capture and move forward with that illustration.

So, the R x antenna is moving and now, we need to see how will we represent the received signal. So, the received signal electric field Er is a function of f c. It is a function of t, it is a function of the displacement vector u, but u is a function, the u displacement vector is a function of r. It is a function of theta and psi theta and psi are basically says that along the x y plane, it is moving in a certain direction. R is the one that is of interest to us. So, let us capture that element in more detail.

So, the portion that is represented by R, we write it down as some starting point r naught plus v into t starting point r naught plus v t. That is my displacement, theta, psi . So, that is my displacement vector and that is what we are writing here. So, this is my representation. I just need to make sure that it is not a static quantity. It is something that is going to be changing. So, let us write down the expression for that. So, it will be alpha divided by r is now v naught r naught plus v t r naught plus v t. Notice I just changed wherever there was r, I am replacing it with r naught plus v t and this becomes cosine 2 pi f c into t minus r by c r. I am again going to replace it by r naught plus v t by c. So, basically in the previous equation if you replace r as r naught plus v t, then you get basically other change in this one, but the reason why we do is not just a simple substitution. It is what we can get from this equation. So, let us re-write this equation in the following fashion.

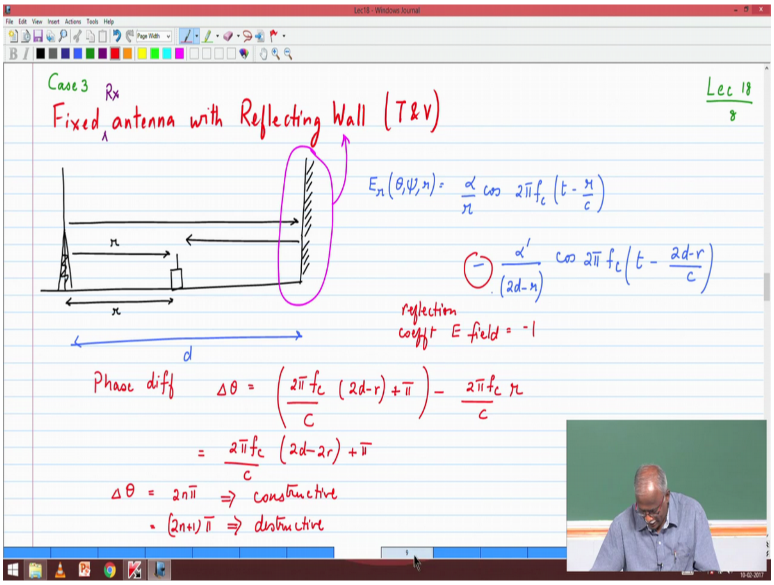
Alpha by r naught plus v t no insight, there the second term is where I would like you to focus on. It is 2 pi f c basically combine that t terms together. So, the t terms if I want to combine, it is 1 minus v by c into t minus R naught by c and then, I will have to draw brackets around that. Let me draw the brackets in green. If I have made some mistakes in terms of the sign or something, just catch and let me know is this basically we just rewritten this expression, two observations. This one, this second term r naught by c, what does that indicate? That means it was your starting point. So, it is like a starting phase off set. So, there is an initial phase that is present, that is represented here, but I want you to now focus on this particular portion of it. So, it is f c into 1 minus v by c into t or the second term is f c v by c into t. What is that?

Student: Doppler shift

Doppler shift. So, that means the minute the antenna starts moving, you do not longer have a constant carrier frequency, but the Doppler will start shifting. Now, of course the Doppler can be in either direction moving away. We basically drew it as moving away, otherwise you will be moving towards the base station. It will become R naught minus v t and therefore, it will be a positive Doppler. So, this simple illustration says that once the antenna is moving, you will have the Doppler.

So, R x antenna moving means that you will have Doppler. It can be positive or negative depending upon your orientation, but you have to take into account, you cannot ignore Doppler. One of them is moving and the important thing to note is that if it comes to a scenario where you do not have to necessarily say that the R x antenna has to move. So, if we look at the uplink, the mobile is transmitting, the base station is receiving. Base station is the receiver. It is fixed, but as long as the mobile is moving, you will still have Doppler. So, whether the transmitter or receiver is moving, so either R x or T x can move. So, this is what we have observed, but actually either R x or T x can move. You will still get the affect of Doppler. It is very important that we note and again it took was for us to just substitute the linear motion and the Doppler comes out in a very elegant fashion. That is case 2.

(Refer Slide Time: 28:16)



We now move on to case 3, where we change the set up very slightly. We now are going to say that the Rx antenna is fixed like case 1, but the scenario is that you now have a reflecting wall. This is the reflecting wall. That means, any electromagnetic wave that hits, it will get perfectly reflected without any reduction in intensity. The only thing when you have reflection of this form is that there will be 180 degree phase change that has to be reflected in our signal.

So, fixed receive antenna with a reflecting wall at a distance r from the transmitting station, please help me write the received electric field at a receive point. This would be at a distance of r. It would be a function of theta psi and r. This would be two components. The first one will be alpha divided by r cosine 2 pi f c t minus r by c like the first case that we looked at and then, there is a second term that will come out because it will come out as alpha dash because it is traversing extra distance. So, again we will think of, we will account for this alpha dash. The denominator will be the total distance. If the total separation between them is d, what is the distance travelled by the second path? It is 2 d minus r. So, the denominator becomes 2 d minus R and again this alpha dash is to indicate that there can be reduction in the intensity based on reflections, but if you make the assumption that there are no reduction, it is a perfect reflector. Then, we can simplify. We will make the corresponding changes in a minute and the cosine part, it will be cosine 2 pi f c t minus 2 d minus R divided by c, ok.

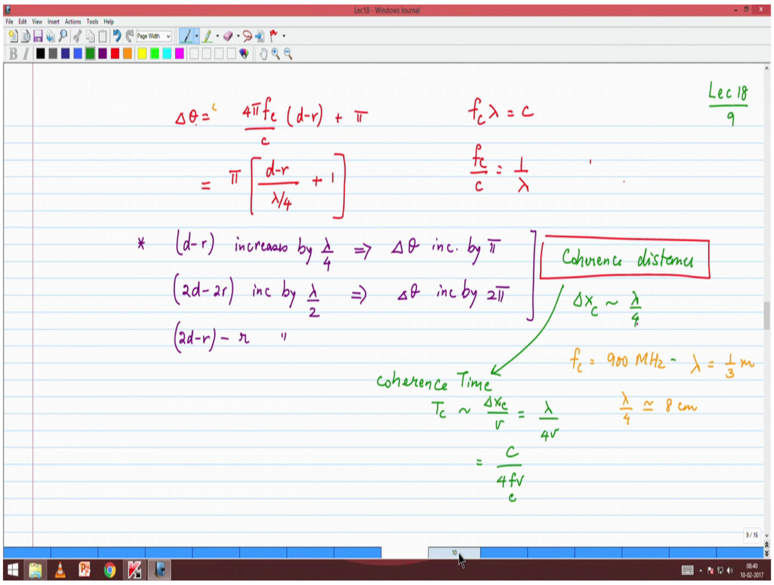
Actually I should be adding a plus sign, but because of the reflection coefficient, I have to indicate with a minus sign because there will be 180 degree phase change that has to be reflected in terms of a minus sign. So, take a careful look at the expression. It is very simple. Whatever was case 1, I replicated it twice by just using the distance as the difference and the only thing being that there is a reflection which may be just, I will highlight that this is because of the reflection of the electric field; reflection coefficient for the e field, for the perfect reflector is minus 1. So, therefore we have the expression there.

Now, the most important thing starts to emerge. Now, I want you to look at the phase difference between these two components. So, phase difference if I were to tell you, tell me the phase of the first term and the phase of the second term and let us compare the differences 2 pi f c t is present in both. So, there we will leave that out of the equation because the phase due to the carrier frequency that part is common to both of them. The phase difference let us take the one that has got the larger phase. Delta theta will be 2 pi f c divided by c into 2 d minus R. That is the phase term contributed by 2 d minus r term. In order to account for this minus sign, I will put a plus pi. That is the phase of the first term, actually it is a second term.

So, the phase difference will be this minus the phase of the first term which will be 2 pi f c divided by c times r. Please write it down because this is a step that is useful for us. Now, I would like you to simplify this expression. It is 2 pi f c divided by c, it is 2 d minus 2 r plus pi 2 d minus 2 r plus pi. Lots of very interesting things can come out of a very simple equation. Now, if the net delta theta is a multiple of 2 pi 2 n pi, then you will get constructive addition of the two phases, constructive interference. If it is an odd multiple of pi, then they will cancel each other. So, this is going to give me destructive interference just by looking at the phase and if the two components are of equal amplitude, it will actually go to 0, but otherwise it will reduce in terms of the amplitude, ok.

So, keep in mind that we are very interested in this particular equation and the interpretation of this equation.

(Refer Slide Time: 34:15)



So, let me write down this equation once more. The delta theta is 4 pi f c divided by c d minus R plus pi. That is the expression that we have now. In order for us to gain the insight that we want to let me rewrite this expression in the following fashion. I take out pi as a common term, then what I will have is d minus r. What is f c by c? It is the lambda of the carrier frequency. So, this will be d minus r divided by lambda by 4. I have taken 4 to the denominator f c.

Student: one by (Refer Time: 34:58)

No no is it one by lambda.

Student: (Refer Time: 35:01)

Student: (Refer Time: 35:03)

One second one second. It is f c lambda is equal to c. So, f c by c is 1 by lambda. Please if I make mistake like that do catch it. It is a lambda by 4 plus 1. Now, you may say well you know this is just a simple rewrite. What is the importance of this? Basically what this tells me is a very important observation. It says if d minus r increases by lambda by 4, what will happen if d minus r increases by lambda by 4 increases by lambda by 4? That means, whatever was the original phase is now going to increase by pi because that is the term that is going to so delta increase if d minus r is some initial value and if it changes by lambda by 4, this means that my delta theta is going to change by phi pi increases by pi. That is an important observation for the following reason because if d minus r changes by lambda by 4, then you also know that 2 d minus 2 r, just double that it increases by lambda by 2, ok.

Now, what is 2 d minus r? It is 2 d minus r. That is the distance of one of the paths minus r, that is the distance of the second path and increases by lambda by 2. So, what that says is I have two paths that are coming. If the path net difference in the paths length increases by lambda by 2, something is going to change in my system. Something big is going to happen in my system. So, what that tells us is that this lambda by 2, what will happen in this case, this will result in delta theta increasing by pi. So, I have to be very careful when the path lengths change of the order of lambda by 4 because that will cause an increase in pi, a change of phase of pi. So, we have a very important phenomenon here which is called coherent distance, coherence distance and this coherence distance in this particular example basically says that if my path length delta x if it changes of the order of lambda by 4, then I am going to see something very different. I am going to see a channel that is going to be very different. So, we call it as delta x c that is coherence distance.

So, this lambda by 4 becomes an important parameter for us in terms of seeing a very significant change and this also tells us that if I move by lambda by 4 to lambda by 2, then I will see a very different channel which is what we have said very intuitively in our explanations as to when you move your head, the channel seems to changed because the coherence distance is a function of the lambda and lambda by 4.

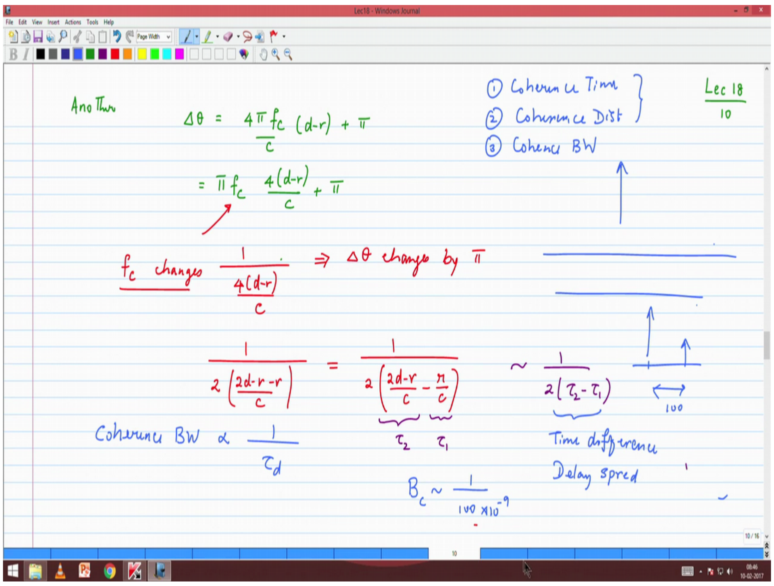
So, coherence distance is the distance over which you can assume that the channel is not changing or if you say it in other words, if I go beyond the coherence distance, the channel will change. There will be a significant change in the channel. So, if you want to just take a numerical example, if I took a carrier frequency of 900 mega hertz, the lambda is one-third of a meter and lambda by 4 is of the order of 8 centimetres. So, it is a very small movement that is your coherent distance. If you are within the coherence distance, then your channel is not going to change much. There will be some changes, but the minute you cross coherence distance, then it is going to be a substantial change.

So, one of the very important principles that you will need in terms of understanding and characterising a fading channel no matter how complex it is, the one of the things that you will ask is what is the coherence distance in this channel because how long can I go before the channel changes or in other words, by how much distance should I move to see a different channel? So, again both those questions are answered by the coherence distance. Now, coherence distance very naturally is just related to another phenomenon which is more commonly used that is called coherence time because rather than think of it as distance, this is the time varying phenomenon.

You also ask the question given that I am travelling at a certain speed, what is the time I have to wait before a channel, my channel will change. So, coherence time is usually denoted by t c. This is of the order of delta x c divided by v whatever is your velocity. So, this is typically of the order of lambda by 4 lambda by 4 times v or you can also write it as c by 4 f v in terms of the carrier frequency, the velocity on which you are moving. So, basically there is a coherence distance, there is a coherence time and any time you perturb your position of by coherence distance, you are going to see a very different channel.

Simple equation just the interpretation of that mix a good insight for us, another insight from the same equation.

(Refer Slide Time: 41:16)



So, another view of the same equation helpful for us. Notice that the equation itself was that we had written down delta theta. The phase difference between the two signals as 4 pi, the equation was 4 pi f c 4 pi f c by c d minus R plus pi. That was the basic equation.

Now, I want to rewrite this equation one more in a different manner. So, it will be pi times f c, take the other terms inside four times d minus r divided by c plus pi. This is the equation that we have this now tells me another observation by how much should f c change in order for me to produce a phase change of pi, by how much should f c change. So, if f c changes, the increment in f c is of the order of 1 by 4 d minus r by c because if there is a change in f c of this form, it is getting multiplied by pi. So, I will produce an f c chain and if the change in f c is of this order of magnitude, this will imply that delta theta changes by pi. Why are we so interested in pi because pi pretty much says that you will go from constructive to destructive or destructive to constructive.

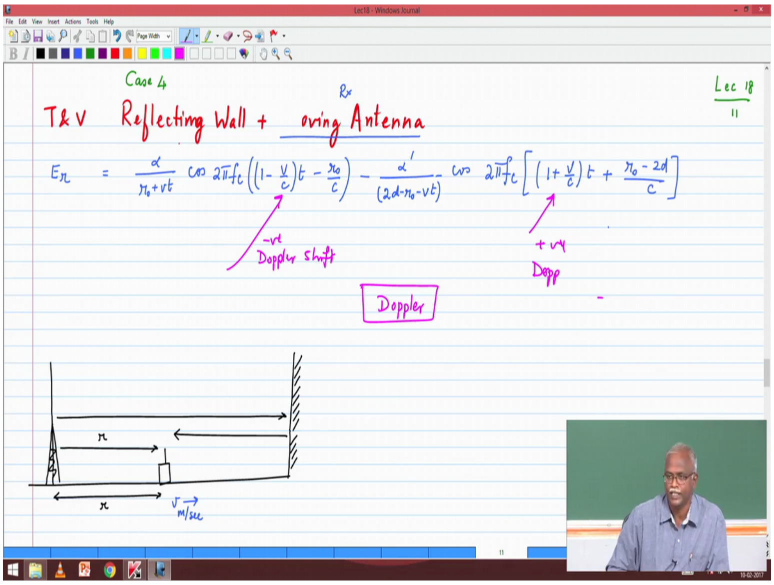
So, that means your channel is going to change very substantially. Now, I want to write this, I want to interpret this equation little bit more carefully. 1 minus r 2, this can be written as 2 d minus r minus r divided by c. This whole thing and the denominator can now be written as 2 into 2 d minus r divided by c minus r divided by c. So, basically what it says is, this is the time taken by the first, the second arriving path. This is the time taken by the first arriving path. So, we can call this as tau 2; this as tau 1. So, it is of the form 1 by 2 times tau 2 minus tau 1. Tau 1 is the first arriving path; tau 2 is the second arriving path. So, this is the time difference between the multipath components and this is called the delay spread how much delay differences between the two multipath components or the different multipath components. So, you always define you are delay spread in terms of the first arriving multipath and the last arriving multipath and then, you interpret. So, what this says is if my f c changes by a reciprocal of the delay spread, that means I am going to see a different channel all together.

So, my coherence bandwidth now it is coherence band of distance in frequency. We call it as coherence bandwidth. Coherence bandwidth is proportional to 1 by tau d, where tau is the delay spread of my channel. Very important question when somebody says I am going to do frequency hopping, you must tell them by how much they should hop in order to see a different channel and that has to be greater than your coherence bandwidth because then you are guaranteed that the channel will be different. Now, that coherence bandwidth dependence on the delay spread between the two channels happens that there is only multipath component, what is your coherence bandwidth delay spread is 0, it is infinity.

So, frequency hopping will not help in a flat fading channel if there is no multipath, then this is the channel response. The frequency response is completely flat. Frequency to hopping does not make any difference, but on the other hand if I have two multipath components which are separated by 100 nano seconds, so you say that your coherence bandwidth, let me introduce the notation right here itself. We call it as BC coherence bandwidth is of the order of the delay spread between 200 into 10 power minus 9. You can say that you know I have to jump by 10 mega hertz to get the bandwidth to get an uncorrelated channel, ok.

So, it starts to tell you, so what are the things that this case has given us? Several things, one it told us what is the coherence time and coherence distance. Those two are related. Coherence distance and the third one is coherence bandwidth. Just an interesting observation that the coherence bandwidth depends on the delay spread of the time. Basically it depends inversely on the delay spread. The coherence time inverse basically depends on the lambda that we are working with. So, basically the inter link in between the time and the frequency domains is something that we will start to work with and appreciate as move forward. So, basically this is the case that gives us these two elements, ok.

(Refer Slide Time: 47:37)



So, now taking it forward I would like to now quickly summarise the third, the next case. The next case is going to give us a slightly different perspective. We will have a moving antenna, moving R x antenna moving to the right at V meters per second base station as reflecting wall.

So, previous case had only the moving antenna. Now, we have the reflecting wall as well. So, now what I would like you to do is, let us quickly write down the equation for the received electric field e r is equal to basically wherever there is distance, now we need to write it in terms of the alpha by r naught plus v t cosine 2 pi f c t 1 minus v c 1 minus v by c into t minus r naught by c. Keep in mind the reflection coefficient. It has a minus sign alpha prime divided by 2 d minus r naught minus v t. That is the displacement cosine 2 pi f c. Notice that Doppler will show up now 1 plus v by c into t because this reflecting wave has got positive doppler. It is very important that we capture that plus r naught minus 2 d divided by c. Please keep this equation with you and notice that this has got a negative Doppler because of the negative Doppler shift. So, negative Doppler shift because it is moving away from the antenna, this has a positive Doppler shift, ok.

Now, what we would like to understand is an interpretation of what this equation can give us and what are some of the elements that we can understand from the doppler, understanding the impact of the Doppler. So, basically this particular case is going to give us more insights regarding the Doppler and how this Doppler will manifest itself in the context of a fading channel.

So, basically when does fading occur, when I have movement and there are reflecting objects. I have taken the simplest of scenarios. There is one reflector and my antenna is moving. So, interpretation of the Doppler impact will come from this equation. If you can read it ahead in from TSE and Vishwanath that would be good, otherwise we will pick it up from there and compete it in the next lecture.

Thank you very much.