Introduction to Wireless and Cellular Communication
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Lecture – 17
Multipath Fading Environment
BER in Fading, Narrowband vs Wideband Channels

Good morning we begin lecture seventeen with a quick summary of what we have covered in lecture 16, I would like to take a substantial portion of today's lecture to answer a question that was raised by one of the students after the class again definitely welcome questions of this nature these actually are fairly deep questions and so therefore, we will take some time to answer it in its entirety and I think the answer will become clearer as the course progresses, but I will try to make an attempt to give a as complete an answer as possible today's goal is to look at one of the most fascinating examples that we have in wireless that is taken from the opening pages of TSE and Vishwanath, chapter 2; section 2.1 definitely that is something that after today's lecture I would like you to go and read through in detail because that is sort of gives you an amazing insight into the wireless channel.

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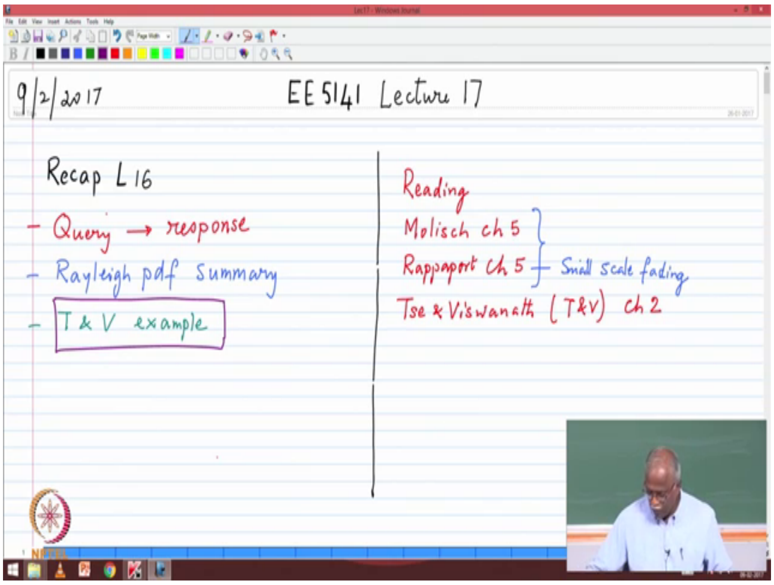

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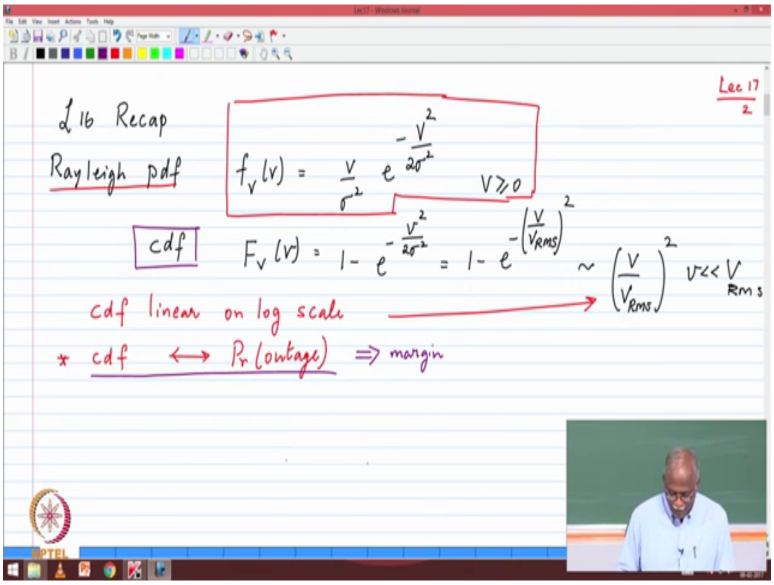

So, let us begin with a quick summary of what we have been discussing in the last class been looking at the Rayleigh p d f you have spent quite a bit of time understanding Rayleigh p d f and its properties the reason for that it is it is one of the most commonly occurring channel types that we will encounter in non line of sight wireless communications we have the probability distribution function we have also looked at the cumulative distribution function it turns out that both of them give you very useful information particularly from the point of view of understanding outage p d f is a very very good tool and an example and we showed how the we could relate the cumulative distribution function in terms of V by VRMS to the outage and this also let us to finding a good way to deal with the margins that are needed in Rayleigh fading environments.

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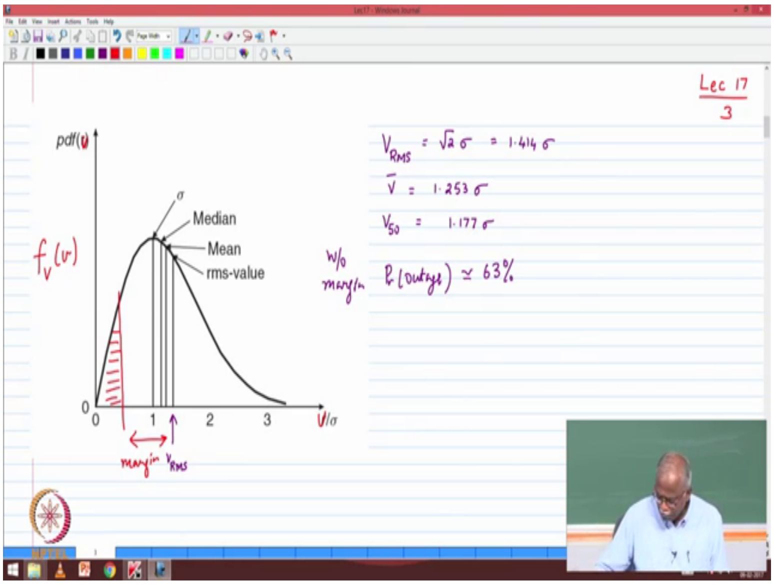

I hope this concept is clear this is something that we will definitely leverage going forward now the another element that I had encouraged you to look at is the shape of the p d f and the reason for this is because of the different values that we computed in the last class let me just write that down. So, V R M S; V R M S is a root 2 sigma or 1.414 sigma and the mean value V bar comes out to be 1.253 sigma and the median value V 50 comes out to be 1.177 sigma again 3 values you can see where approximately they lie. So, if the V R M S is a very very important parameter for us because that sets the threshold of what you will see as the average received signal power.

So, keep in mind that that is going to be a very very important parameter the V R M S is going to tell us. So, now, if I have not kept any margin the probability that I will run into fade is a probability that the V lies less than V R M S this is V R M S notice that most of the larger values lie to the left to the V R M S which means the bulk of the probability is going to be there. So, the if I do not have any margin against fast fading then the probability of outage pretty much is going to be the integral or the shaded portion here notice that is going to be a substantial part and that is what we saw yesterday without margin without margin the probability of outage was I think 0.663 percent probability of outage was approximately 63 percent.

Now, I want to introduce our notion of margin I have introduced a 10 dB of margin 10 dB of margin says that I have shifted my average r m s value 10 dB above the nominal. So, now, the outage will be only much reduced.

So, this is the shift this is the margin that I have introduced and therefore, the probability that I have an outage is still that that amplitude less than this value is obtained by the that integral. So, again I hope you are comfortable with this is interpretations and where the margin comes in how the margin plays the role in terms of improving the reliability in the fading environment.

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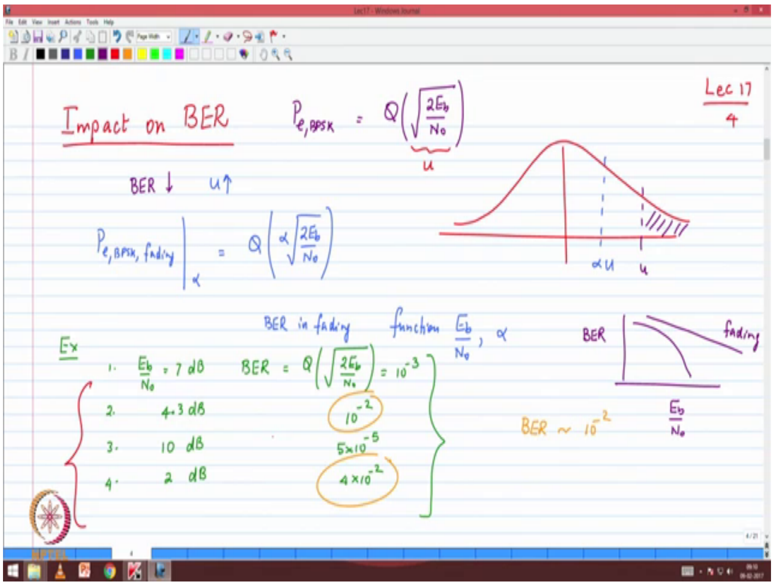

We move on next part is I wanted to look at BER again this was something that we had discussed in the last class let me just quickly summarize the highlight probability of error of the BPSK, we will discuss we will use BPSK as our baseline BPSK or QPSK because it is very useful to get insights, but of course, the comments that we are making applied to all digital modulations that we have. So, probability of error in a BPSK in AWGN result that you are familiar with 2 times E b by N naught and if we call this parameter as u this is the complementary error function.

So, basically we are looking at a Gaussian looking for this threshold u and saying that anything basically you are trying to get the integral in that in that region of the Gaussian and therefore, the natural way to interpret this result is that if I want the BER to decrease my u should increase. So, I should find ways to improve my signal to noise ratio the more u increases the BER is going to decrease.

Now, how I am going to do that I may do it by adding margin increase the transmit power increase the transmit again whatever it is basically it boils down to improving or increasing u now the situation in the context of a fading environment probability of error of BPSK in the context of fading with fading parameters specific fading parameter alpha this comes out to be we solved in the last class that this will be q of alpha times square root of 2 E b by N naught and alpha has a tendency to be greater than one or less than one with the higher probability of being less than one, so, which means that your new SNR is going to shift to the lift left. So, which means that you are going have a larger. So, this would be alpha times u which would shift your and make your probability of error worst.

So, keep in mind that the bit error rate probability in a fading environment BER in fading has 2 dependencies it has a dependency it is it depends on it is a function of E b by N naught like all modulations schemes in AWGN it is also a function of your the gain or introduced by fading and we call it gain, but invariably it is something that is going to reduce your signal level let us do a quick numerical example again this is just to reinforce what we have said before.

Supposing I had a nominal E b by N naught of 7 dB the purpose of this example is to for one is to illustrate the principle it also useful for you to have a few data points in memory because if somebody were to tell you I am designing for your BPSK system I will design for you 10 power minus 4 BER at the 0 dB is E b by N naught you say wait a minute that is not possible the; you must be able to have an intuitive feel for at what BER or at what SNR will I get a reasonable BER. So, 2 things keep the numbers also in mind at a nominal value of 7 dB E b by N naught you will get a bit error rate which is given by q of square root of 2 E b by N naught this will be 10 power minus 3 and that is a very useful number to remember 7 dB 10 power minus 3, it is a good checkpoint any time somebody shows your BER graph just you know do a quick visual check say this is correct if it is something shifted to the right or to the left you know that there is an error.

So, this is a good standard reference point now I am doing an experiment in fading channel the first experiment the SNR comes out to be 7 dB alpha equal to one just by chance it came out to be than the second experiment that I tried out the SNR came out to be 4.3 dB the BER I look at I plug into the q function it comes out to be 10 power minus 2 you can check these values you should not say that I am biased trying to prove a point taking all values below let us say that the experiment gave me 10 d B.

The next trial very fair experiment this is 5 into 10 power minus 5 excellent BER performance because that is a very steep portion of the graph and then fourth one really bad 2 dB 2 dB 4 into 10 to the power of minus 2, now you must repeat this at least a hundred times and then take the average I am going to do a little shortcut I am going to take an average of these 4 values assuming that you know I have done this experiment sufficient number of times.

Nominal value of BER was 7 dB because of the fading parameter alpha I got perturbations in my SNR somewhere lower somewhere higher again the that is based on the Rayleigh statistics once I do this using the Rayleigh statistics properties and I do an average, but you have do the average over large number in this particular example the one I wanted to illustrate was that you got a BER of approximately 10 power minus 2 why because there are couple of problematic cases which are going to offset the good cases of BER and they are going to dominate.

So, you notice that what was 10 power minus 3 in AWGN again just as an illustrative of example suddenly became 10 power minus 2 in a fading environment. So, that is what we highlighted in the in the class yesterday the BER graphs that we are used to have a very steep fall this is BER versus E b by N naught because most of the times we have been working in a AWGN where the SNR is fixed what you will find in a fading environment is something that the where the BER deduction with as a function of SNR is very slow and therefore, we just need to keep that aspect in mind any questions on what we covered. So, far this is where we ended the last class all the concepts clear.

We move on now to answer query that was raised yesterday again I definitely want to command you on asking this question again feel free to ask these types of questions because they are what hep the ability to understand.

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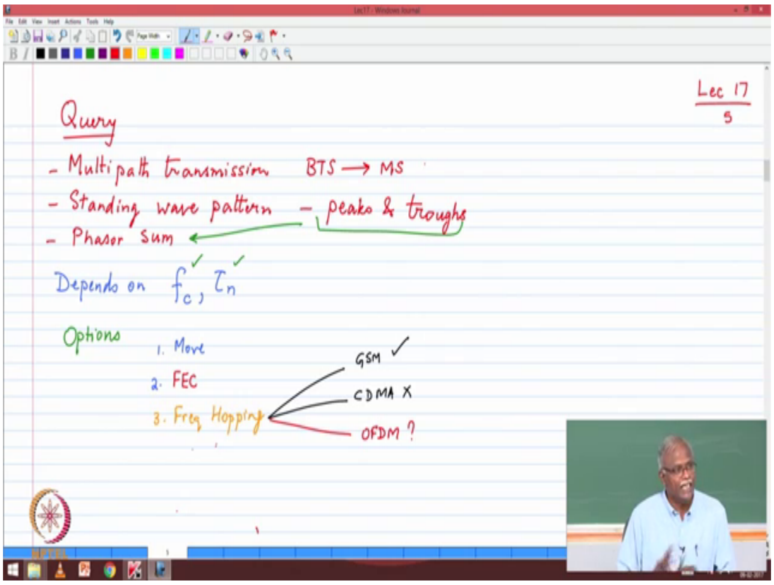

The concepts that we are talking about this is what we said yesterday base station to mobile; mobile station transmission system it is a multipath transmission system there is a standing wave pattern which create peaks and troughs again the picture I am sure is a very familiar in your mind.

Now, these peaks and troughs are caused because of the Phasor sum again because of the way these phases add constructively or destructively 2 things that they depend upon one is the carrier frequency and the other is the relative delays that come if you take into account that they are coming not exactly at the same, but within the time resolvability window there is some differences in their delays that contributes to it.

Then we asked the question what are the ways in which you can get out of a trough where you are in a bad situation we can get out of a trough. So, the options that we listed down yesterday where move to a new location one is you move second you add A f E c and third probably the most effective of the lot was to was to do frequency hopping frequency hopping. So, the question that was asked is s this for CDMA systems as well the very very valid question let me just begin by saying whatever we had said yesterday was in the context of GSM for frequency hopping is a very good method for GSM to do get around of this the bad channel conditions.

Now, it turns out that frequency is something that we do not do for CDMA. So, the question is then what do you do if you get stuck in a trough because you know CDMA user is as likely to stand in this location as a GSM user and probably the even more important question what does the fourth generation systems do because they are OFDM which are neither GSM nor CDMA it is a different option what are the ways in which we can handle that. So, next few minutes is going to be addressing this; the answer to this question because it requires 3 concepts to fully explain this one.

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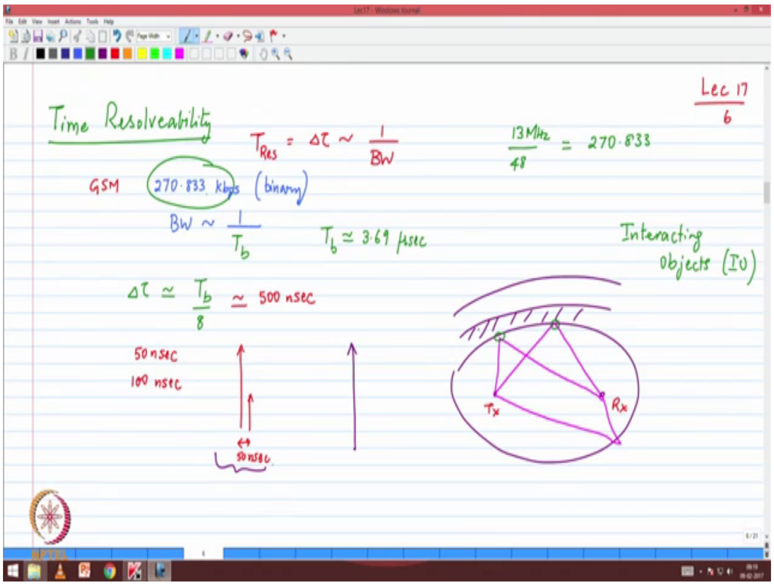

The first I want to address is the time resolvability issue we did mentioned this I thought I should highlight this a little bit more because it explains the answer to today's question.

The yesterday we said that there is a parameter called time resolvability and we said that you know usually it is given as delta tau that basically tells you what is the resolution in the delay dimension that you can resolve and I made a very generic statement that is inversely proportional to the bandwidth of the signal lets be a little more precise and answer and explain this concept little further.

So, let us take the GSM signal. So, GSM signal uses bit rate of 270.833 kilo bits per second now typically when you have a system that is using binary transmission. So, this is a form of binary modulation. So, the bandwidth required will be reciprocal of the bit rate. So, the bandwidth required is approximately one over r b, but before that just an incidental note somebody asked me one of the students asked after the class you know why this really weird number 270.833.

Now, could not have made it 271 or you know 270 that I would have sounded much better it turns out that thirteen mega hertz crystal was easily available most of you are receivers must have a frequency reference. So, thirteen mega hertz and dividing the frequency by an integer number turns out to be easier from a circuit implementation. So, thirteen mega hertz by 48 is 270.833. So, in case ever you are wondering why 270.833 it has nothing to do with what 270.833 looks like just it is a derivative of the thirteen mega hertz clock and it was something that was found convenient in terms of the design system.

Now, that is not important for us key point is this reciprocal its reciprocal you dint catch it reciprocal of the t bit not the r b the bandwidth is proportional to r b. So, this is proportional to one by T b or t s t symbol. So, the question that we have to ask is; what is t symbol or t bit in our case this is approximately 3.69 micro seconds 3.69 micro seconds. So, according to this course estimate the our time resolvability delta tau when I am using GSM as my system of transmission is approximately the reciprocal or basically it is of the order of one by T b which is of the order of T b right.

So, approximately 3.69 micro seconds, but it turns out with some very clever processing we can reduce this to approximately one eight of a symbol that is the time resolvability that we can achieve through the signal processing methods that we know. So, this would be approximately 500 nano seconds. So, anything that arrives within a 500 nano second window multipath components that arrive within a 500 nano seconds will look like the same for me it; it cannot differentiate.

So, basically if there is a multipath component that comes as 50 nano seconds with a delay of 50 nano seconds and another multipath which comes with a hundred nano seconds. So, maybe I draw it in this fashion there are 2 multipath components which are approximately 50 anno seconds apart further when I do my processing with a GSM system I cannot resolve these 2 do not look that it looks like one just one channel that is present because I cannot resolve between these 2 multipath components. So, you I get a feel for what is the time resolution and what is the impact in terms of the system.

Now, comes another very very important principle this notion of which paths arrive at the same time. So, there again we go back to our analytical geometry which says that if I have an ellipse and I have the foci the path between the a to b horrible straight line this path has the same distance as something like this or something like this anything which bounce is a single bounce of an ellipse will reach at the time. So, let us simplify matters we are looking at those paths which arrive with a bounce. So, now, if you if I tell you what are those paths that will arrive at the same time you will tell me those paths you know this is the transmit antenna this is the transmit antenna this is the r x antenna between the 2 imaginary ellipse you draw whichever bounces off an situated on the ellipse.

Now if you have this time resolution uncertainty then it does not have to be on the ellipse I cannot tell whether it is on the ellipse or whether it is in this inside this boundary notice it is no longer a line it becomes an annular region like basically it is a band around that the greater the time resolution uncertainty the wider this band is going to become right. So, that is a very very important concept. So, there is a term that describes those points that interact with the multi which create the multipath components they are called the interacting objects interacting objects sometimes shortened using the term I o use quite often by Molisch.

But not so, much in the other literature, but it is a correct term it says that those objects sort of creating this now let us look at what is the how does the channel look like for a system where there is time resolve resolvability uncertainty.

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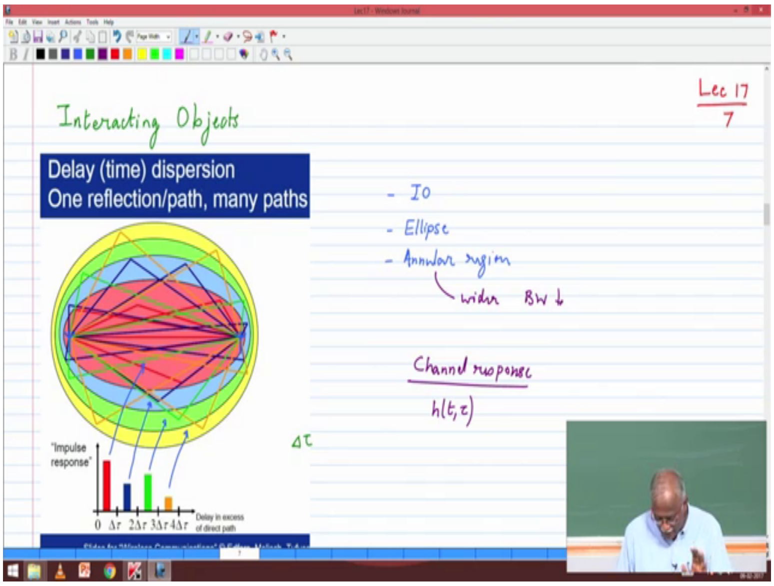

So, here it is you can think of it as several ellipses each of these ellipses has got a finite region around it because anything that arrives with an interacting object in that location I cannot tell the difference. So, if I were to characterize my entire multipath for a system though there is many different multi-paths with finely varying multipath delays if my time resolution is delta tau it will look like everything between 0 and delta tau comes as looks like one path then between delta tau to 2 tau to delta tau and another path.

So, basically this red corresponds to the portion from the inner most all those from the inner most circle the second one comes from the blue the third one from the green and the yellow is that clear. So, depending upon the bandwidth of the signal that I am working with the number of paths are going to show up now if I have GSM with a with a time resolution of about 500 nano seconds and this entire spread that I have in terms of the multipath is only 200 nano seconds does not matter how many multi-paths were there it will look like a signal multipath to me because I cannot resolve between them a very very important concept basically the notion that you have interacting objects you have the principle that if it is a single bounce these have to be on an ellipse and it is not just an ellipse it is a annular region annular region and the width of the annular region is wider this region becomes wider if the bandwidth decreases.

Why if bandwidth decreases bandwidth decreases my time resolution becomes worst. So, therefore, I cannot tell the difference the annular region becomes wider. So, this is a very very important notion for us to be able to understand and interpret because this leads to the next concept which we will refer to as the channel response channel response yesterday we said that the channel response is a describe by means of a 2 dimensional function h of t comma tau h of t comma tau.

Now, how will I characterise h of t comma tau how many taps do I represent depends on my time resolvability that will map according to this region now how does it make a difference and how does it affect me in terms of my understanding of the wireless system that is what is going to come next. So, if there are any doubts please feel free to raise it because the; we are kind of building on that.

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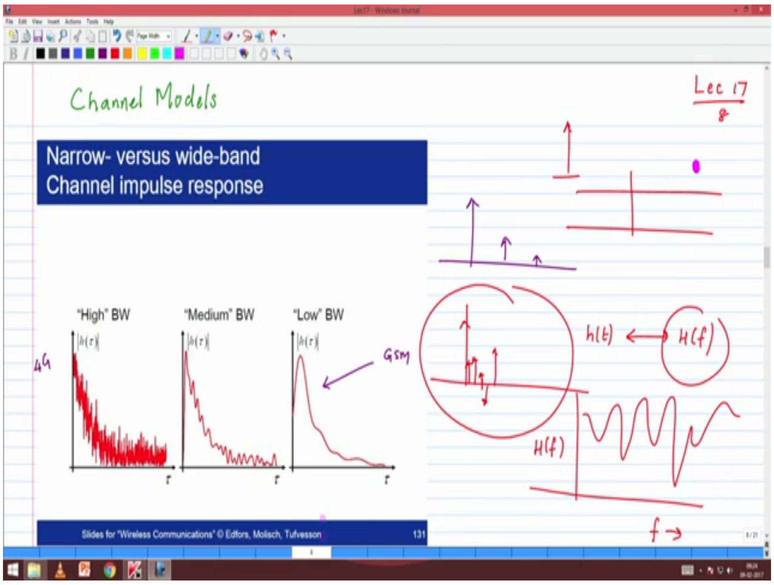

So, if I have a GSM signal narrowband compared to a 4G signal which has got 20 mega hertz bandwidth. So, the same channel looks like I have a few multipath coming the shape will look the same, but it looks like just have a few multipath components this is what it will look like for GSM this is what it will look like for a 4G system.

But basically you will have to sample it using a delta tau spacing. So, for GSM it may look like this it may look like this for the 4G system may look very very different the same channel looks like you have got lots of multipath you got lots of multipath. So, that is what we are trying to show. So, the same channel looks different for a narrowband signal versus a broadband signal or going back to the bandwidth resolvability is that is that clear. So, keep that picture in mind.

So, now comes the next part which is the most critical in terms of the interpretation. So, the true channel has got lots of multipath. So, when I tell you that my filter has got multipath like this let if I call this as h of t I can tell you please compute for me the frequency response it will h of f when there is a lot of multipath what you expect to see is that h of f will have lots of variations typical filter right it has got 0s, it has got poles basically there; there is a lot of variations in the frequency response this is h of f as a function of frequency if I have only one tap in the time domain frequency response is flat. So, remember we talked about flat fading versus frequency selective fading this is flat this is flat this is frequency selective fading this is very important for us to interpret the next slide.

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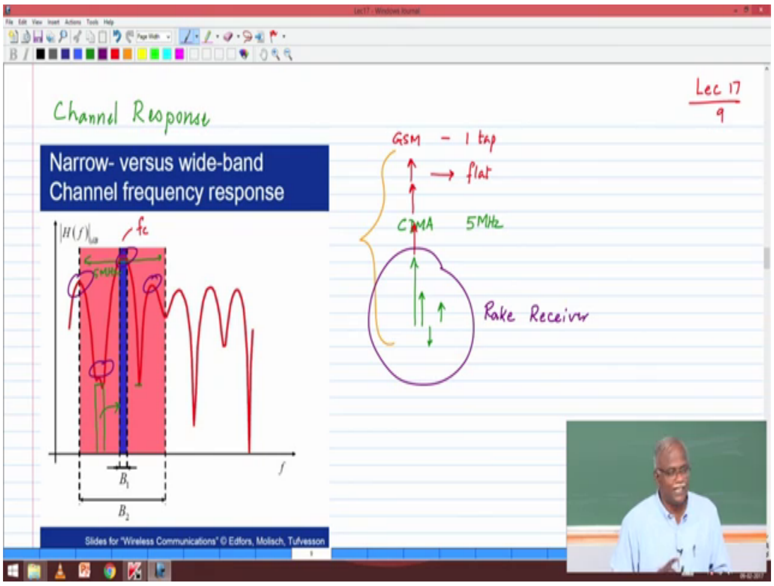

So, now I have a channel which has got lots of multipath and I have a certain frequency response that is given by the red signal my GSM signal has got 200 kilo hertz only therefore, its resolvability is very less and let us assume it is such that the resolvability of a of the GSM multipath says that has only one tap that is the because of the resolvability. So, the GSM channel GSM channel sees a single tap channel which means in the frequency response it is going to be flat am I right and that is what we are showing here the blue line is the is the GSM signal the its only sees that it looks like that portion of the based on the carrier frequency if this is my carrier frequency GSM will see a channel that looks very good and it looks flat as for as the signal is concerned it, it is easy for the GSM signal to detect the signal.

Now, if your carrier frequency unfortunately had been located somewhere slightly differently supposing if this or this had been your GSM location unfortunately you will be in a fade it is a flat channel as for as GSM is concerned though the true channel has got a lot of variation, but to you it seems like a flat channel and because you are sitting in at in a particular location where the channel response is bad it to you it looks like a bad channel which is which you cannot detect.

So, this is the scenario where we said frequency hop maybe you go from a bad channel to a good channel you are happy and then you keep doing that therefore, you do not get stuck in these bad channels you come out of it. So, why does the frequency hopping work for GSM is that clear now remember the CDMA system CDMA system has a 5 mega hertz bandwidth so; obviously, it will see a different channel in terms of the time resolvability because it has got much wider bandwidth and therefore, much higher resolution.

So, as for as the CDMA is concerned it says it has not a flat channel definitely not there is a lot of variation in the channel. So, this is the bandwidth of the this is the 5 mega hertz signal 5 mega hertz and what the GSM what CDMA signal says is a its not flat at all there is a lot of variation what is the variation says oh I can see several multipath components they are sitting in different slightly different delays and they are actually arriving slightly staggered time and I am able to see that.

Now, what is the advantage that the CDMA system has that the GSM system does not have the CDMA system can take these multipath components and align them in a co phase manner and actually get a good signal to noise ratio out of it because there are multipath components if you co phase the multipath components it actually gives you ability. So, in an intuitive way is this portion of the signal bad of the spectrum it is bad for the CDMA system also, but there are enough good portions in the CDMA signal that I can actually detect the signal that is an intuitive way of looking at it the signal processing interpretation is that there were multipath components if I dint do anything they will cancel each other, but if I can align all of them to becomes co phased then I can take advantage of that this is this is what is done in a technique called the rake receiver.

I am sure you would have studied it in the context of CDMA and that is why CDMA does not need to hop provided you have sufficient bandwidth why because sufficient bandwidth means you can resolve enough multipath you will take those multipath components and you will align them. So, that you can get a good signal to noise ratio and then be able to detect these signal any questions on these again we have not yet talked about CDMA, but just the notion that it is a wide band signal it has got better time resolution and the receiver algorithm aligns the multipath components as long as you are comfortable with those concepts this is good.

So, GSM frequency hopping CDMA rake receiver basically you combine the multipath you; you resolve the multipath and combine you do not need to do frequency hopping it does not matter where you lie as long as there are enough portions of good signal you will recover your you will be able to detect the signal now here comes the important question what about OFDMA it is not a spread spectrum signal. In fact, it is narrower than GSM; GSM is 200 kilo hertz now what is OFDMS bandwidth typically a 10 kilo hertz. So, it will almost look like a line or maybe as narrow as I can draw 10 kilo hertz.

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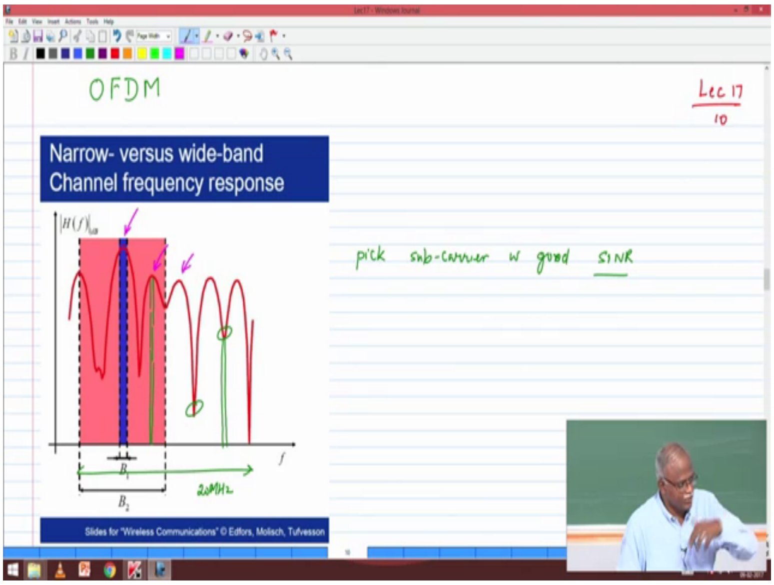

Now, you say well what do you do you; obviously, have to do frequency hopping because there will be enough of these bad channels right there is a bad channel here there is a bad channel here. So, several of them; however, if you if you recall OFDM is in the context of a ultra a wide band signal typically OFDM systems may be 20 mega hertz wide 20 mega hertz wide. So, there are large number of these narrow band carriers where do I hop I cannot hop I cannot hop all of them because I do not have enough spectrum to do hopping of a 20 mega hertz signal.

So, what else can I do what can I do I cannot hop because it is a wide band signal there are lots of these narrow band carriers which will be seeing flat channels and many of them are going to be in bad conditions what do I do equalization is needed if I am seeing a time dispersive channel, but if it is a flat fading channel; that means, the time dispersion is not bothering me it is basically a single tap channel o OFDM is a very very flexible because those portions which have got avoid using those carriers because you have got enough further carriers which are good channels.

So, out of 1024 maybe 100 are bad channel conditions; you use the remaining ones. So, you pick those sub carriers pick those sub carriers which are seeing good channel conditions with good SNR that is it SINR it cannot not only be due to fading it can also be that there is interference you will pick those with the good SINR. So, whether it is a narrow band system a wide band system like CDMA which is doing spreading or a wide band system a CDMA system which is constructed using a lot of narrow band carriers such as OFDM all 3 of them we have ways of handling the channel, but it goes back to our understanding of what the channel resolvability is and how do we deal with channel resolvability issue.

So, this is to answer the question frequency hopping is it only for narrow band systems the answer is yes it is for narrow band systems single carrier narrow band systems multi carrier you just avoid using those bad channels if it is a single carrier wide band system like CDMA then you have time resolvability you take advantage of the multipath and therefore, are able to get a good performance in the channel any questions.

Student: In the yesterday’s class.

Yeah;

Student: If the x axis was position.

Position;

Student: Now you came to frequency.

Correct.

Student: How; how it is.

The question is yesterday’s class we were looking at a spatial distribution. So, here is what we would the way I would explain it there is multipath occurring and the multipath when I look at it in a time perspective there is multipath components that are arriving now that is one perspective now my if I move from one position to another position my multipath profile changes the multipath components changes. So, at each position that I am likely to find myself in there is a set a channel impulse response that is present it is dependent on where I am located in terms of the multipath components it is dependent on the carrier frequency because that is what tells me whether it is going to be in a trough or not.

Now, the multipath components that I see at my current location multipath components respond look when I look at it in the frequency domain there is a frequency response and if that frequency response happens to be very bad at that point it basically means that the Phasors are adding in a destructive fashion. So, that is that is why it is corresponds to another.

So, it is a physical location in the x y plane at each location there is a multi set of multipath components that I receive and the multipath components that I receive has got a certain frequency response and depending upon my carrier frequency I may see this channel or this channel or this channel depending if I was in GSM the multipath components based on the location are fixed, but my carrier frequency can change if I change my carrier frequency the response of that particular multipath components to that carrier frequency changes. So, that is what we are we are showing here.

So, you are you are right there is a spatial distribution of peaks and troughs each position has a distribution of peaks and troughs in the frequency domain for different frequencies there is a different gain and. So, the; if you are unlucky to choose the frequency which had a bad frequency response then you actually you are affected in the; you can interpret. So, you are giving an explanation both in the time as well as in the frequency.

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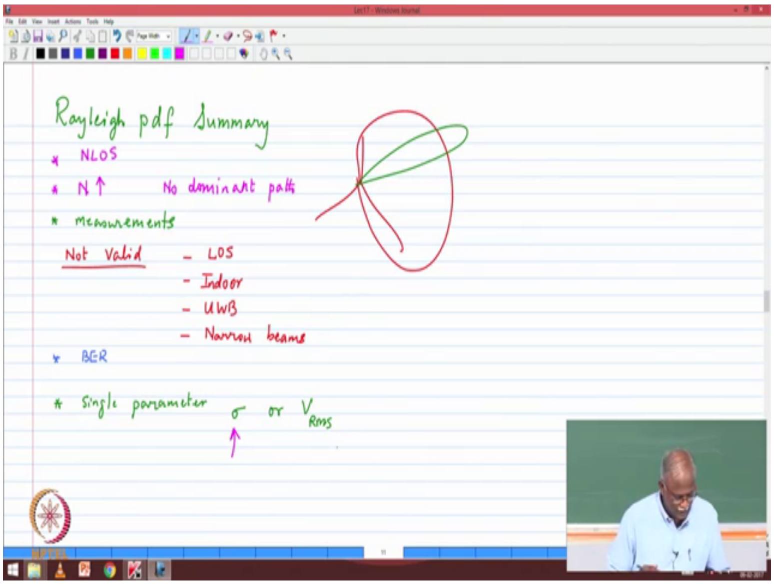

Let me move on I would like to do a quick summary of the Rayleigh p d f maybe you should be helping me with that, but let me just in the interest of time we are looking at it primarily in non line of sight this occurs in non line of sight components it also is a typical phenomenon when there is a large number of multipath. So, the number of multipath components n is large and there is no dominant path what happens when there is a dominant path we will be studying, but right now the assumption is that all of them are approximately of similar power in terms of their strength and this has been confirmed by a lot of measurements to verify that in practice something like Rayleigh fading actually occurs.

So, it has been validated through measurements. So, there is no doubt about the usefulness of this model maybe an important question is when is it not valid again it is important to sort of say the negative side is saying that you do not use it where under these conditions first and foremost if there is a line of sight it will the yours the channel will not be Rayleigh statistics it will not be there.

Second when you are in indoor environments it may not have Rayleigh distribution why number of scatters because indoor there are not as many scatters as you would have the other one is ultra wide band systems if I had a there is a there are modulation methods which will use 1 Giga hertz bandwidth why would not ultra wide band have Rayleigh fading ultra wide band has got extremely high time resolution. So, it will say that at this delay only one path came if only one path is there; there is no Rayleigh statistics you must have averaging of a large number of path. So, ultra wide band systems have got very high time resolution which actually you know changes the nature of the channel.

So, the other one is if you have very narrow beams remember we talked about sectorization; sectorization typically is a 120 degrees, but you can also think of systems where you have very narrow beams. So, if you have narrow beams it does not match Rayleigh why again number of scatters because you are limiting the scatters that you will you will pick up.

So, therefore, and also remember we made the assumption that the scatters are coming in all directions now when you have a narrow beam its very likely that your scatterers are in a particular angular section not all around. So, therefore, it will again violate that. So, there are it is something that we need to keep in mind the impact of Rayleigh fading very critical for us because in the in the way that affects the BER it affects the BER and it affects it in a very significant manner, but the good thing about Rayleigh p d f is what is a what is the way how what characterises a Rayleigh p d f what parameter characterizes the Rayleigh p d f V R M S.

So, basically or sigma basically the there is a only single parameter that characterises the Rayleigh p d f single parameter you can think of it as sigma or as V R M S both are equivalent once you specify sigma the p d f is fully specified and once that is given to you know the p d f you know the c d f you know the median value everything gets fixed based on the description of sigma. So, sigma is very critical for us very important and using that we use the ability to work with this.

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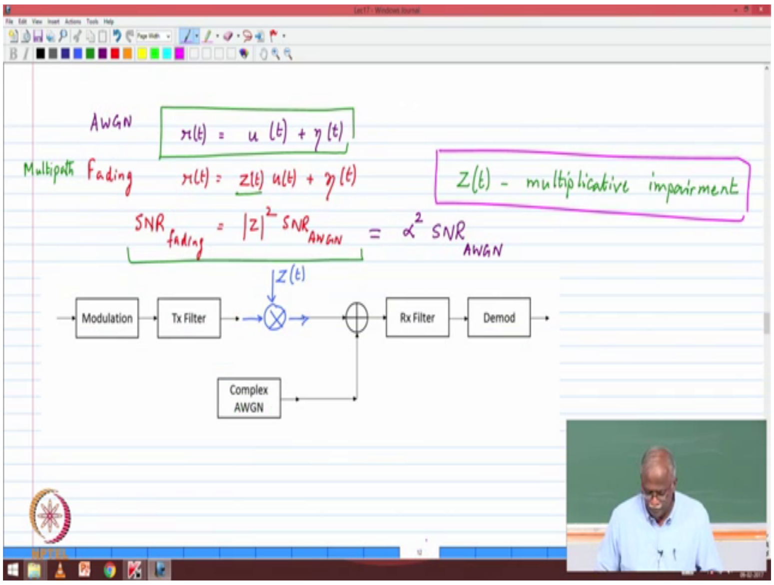

let me move on, let me quickly mention something about the assignment number 2 assignment number 2 more or less follows the discussion that we had in class yesterday keeping in mind that the fading environment is a multiplicative environment it will affect your SNR and the SNR will be affected by mod z squared or if you think of alpha as the amplitude of the of the of the Rayleigh variable Rayleigh random variable then the modulus of z or alpha is what is going to affect me.

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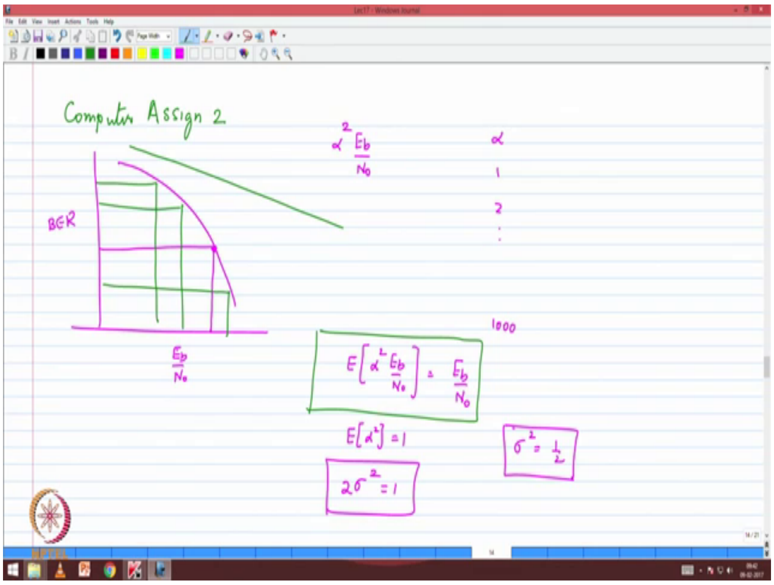

So, the experiment that that you have been asked to do is the is the following you have been asked to look at the theoretical BER of BPSK BER versus E b by N naught you got a graph like this and nominal value is here now about the nominal value keeping that as your nominal value you are asked to generate different values of alpha squared E b by N naught. So, basically you are represent alpha trial number trial one trial one trial one 2 like that all the way up to thousand or 10000 whatever you are asked to do you generate enough of these alphas for reach of these alphas according to the Rayleigh distribution and what should be the value of sigma because if do not specify sigma then it is not useful.

So, very important for us to answer this question I in the fading channel also I want the average SNR to be the same as AWGN. So, expected value of alpha squared E b by N naught if I wanted to be same as E b by N naught n which says that expected value of alpha squared must be equal to one expected value of alpha squared is 2 sigma squared equal to one and what is sigma? Sigma is the variance of the Gaussian. So, basically you must generate 2 Gaussians which have each one equal to sigma squared equal to one half the x Gaussian has got variance one half why Gaussian has got variance one half when I add them together I will get a 2 sigma squared equal to 1.

So, basically I have to generate it in that fashion if I generate it in that fashion I am guaranteed that on average my average SNR in fading is going to be the same as the average SNR in AWGN. So, now, all the perturbations in because of alpha you can try you will do the measurements remember the first example that we did you will get several of these BERs then you average it and then you will find that your average is actually coming out to be the;

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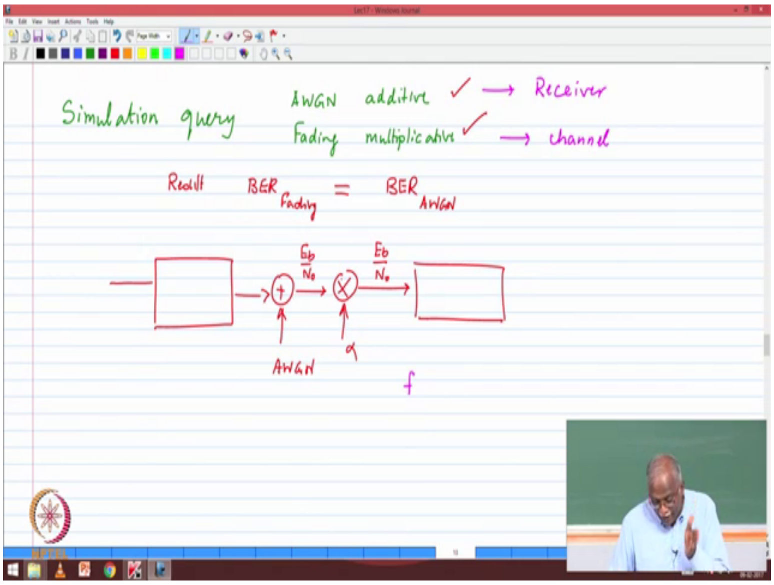

So, basically whatever we introduced at the start of this lecture is what we are going to be doing in computer experiment number 2.

Now, here are the here is a important sort of question for you to for you to think about and answer if you know what would be a reasonable explanation for this. So, a student was told to simulate the modulation system in a fading environment and was told that AWGN has to be present and AWGN is an additive impairment and fading must also be introduced Rayleigh fading and this is a multiplicative; multiplicative impairment and the student said yes I did this I did this and the result is BER in fading is equal to BER sorry BER in AWGN and since I have checked it hundred percent sure I have implemented fading I have implemented AWGN there is no difference in the BER what happened.

Student: BER after fading after AWGN.

If he applies the fading after AWGN let me take a quick look at it. So, this is the transmitter added the noise AWGN so; that means, you have a certain E b by N naught that is happening here or in other words there is signal coming in noise coming in and this is the multiplicative factor alpha it is multiplying the signal and the and the noise. So, the resultant SNR at this point is E b by N naught because it, it multiplies the noise also and therefore, at the receiver do not find any difference in the BER performance because the noise also got scaled, but keep in mind where does fading occur fading occurs in the channel.

So, fading occurs in the channel this occurs in the receiver we cannot interchange the 2 please the never be tempted to do that because you will get the wrong answer the fading affects your signal noise gets added regardless of what your signal resolution therefore, that is why the fading becomes a problem. So, again you got it right away it because the student was correct they had implemented fading and AWGN, but if you interpret implemented in the wrong order you get the wrong conclusion.

We just have a few minutes let me just introduce the next topic that we would like to study.

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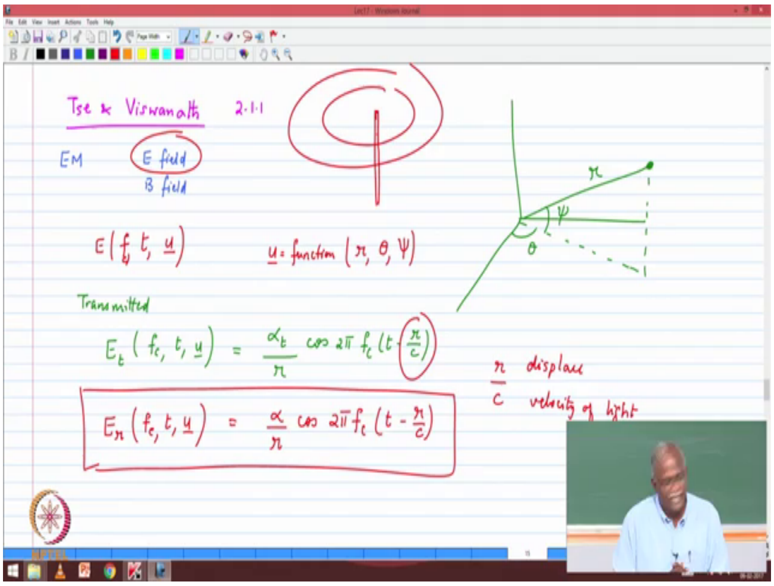

This is an example from TSE and Vishwanath section 2.1.1 again please do read that section 2.1.1 let me just introduce the problem statement to you this is one of the most classic examples that we encounter in wireless communications one of the most simple examples, but gives you almost all the necessary understanding of wireless and again that is a beauty of this example.

So, some basics in electromagnetic fields we have an electric field and a magnetic field and the direction of the electric and magnetic field I do not know you have right hand rule and all E cross b. So, both of them are perpendicular to the line of transmission. So, keep that in mind E field is perpendicular to the direction of propagation and E and b can be you know related to each other. So, we always more or less look at only the understanding with respect to the E field now E field has dependencies on the following functions let us say that you have a transmitter in free space the electric field that is generated by this transmitter in free space is a function of f the frequency the carrier frequency or may be call it as f c the time that has elapsed basically there is a dependence on time and there is a dependence on the displacement.

So, basically if I have a transmitting antenna in I can think about electric field in different locations and I can then characterise it in that fashion now the displacement u is a function is a function of 3 parameters it is a function of the distance r, it is a function of angle in the azimuthal plain it is a angle in terms of the elevation angle. So, again this is diagram that we have drawn previously. So, if I were to look at a point in 3 dimensional space this is the displacement r the angle with respect to the azimuth is theta the angle with respect to the horizontal is the elevation angle that is phi using this we can you can pin down the position.

So, I have a an radiating element that has the radiation depends on the carrier frequency and the and the and the function of time and depending upon where I am situated I will get a certain intensity of radiation. So, the transmitted signal transmitted signal. So, basically we will call that as E transmitted it will be a function of carrier frequency of time and of the displacement again it is a simplified example. So, keep that picture in mind.

There is a gain term alpha and that depends on the antenna gain and all of those parameters. So, there is a gain term the electric field decays in free space reciprocal to the distance that you have traversed. So, it is that gain whatever is there initially will decay as a function as 1 by r and the rest of the dependence it is basically a sinusoidal function cosine 2 pi f c t and how much distance have you covered traversed it is r divided by c the speed of light that tells you how much time lag has occurred. So, if I am at sitting at a distance r at a particular location in 3 dimensional space the received signal strength can be described by a gain term and a reciprocal of and one by r and times cosine 2 pi f c t is 2 pi f c t at the transmitter by the time it reaches me there is a slight lag. So, that is the lag that is described here again this is something that we have seen before the reason I introduced it using the notation of TSE and Vishwanath is that it will enable you to understand.

So, why this why is this r by c important r is the displacement this is the displacement or the distance and c is the velocity of light. So, that is the time that it takes for the electromagnetic wave to reach you at the receiver location, so, velocity of light. So, now, the next step is I am going to write the received electrical vector nothing changes here expect that I may have a received antenna gain.

So, basically it will be depend on carrier frequency is a function of time is the function of displacement I am at that location I will just make it as alpha by r whatever is the gain of the transmitter I have added to the gain of the receiver and it is cosine 2 pi f c t minus r by c this is the electric field that is received by me, it is a simplified model it assumes that it is free space propagation the electric field decays inversely as a function of the radius and.

So, this is the basic formulation you may say nothing you can gain out of this; this is there is nothing new in this formulation, but the beauty of it is using this TSE and Vishwanath have constructed one of the most elegant examples to explain all of the concepts that are in wireless things that we took quite a bit effort you know remember we derived all these multipath components Doppler shift and nothing everything comes in very very simple elegant fashion we will cover that in tomorrow’s class.

Thank you.