Introduction to Wireless and Cellular Communications
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Lecture - 16
Multipath Fading Environment
Properties of Rayleigh Distribution

Good morning. We begin with lecture number 16. Quick recap of the key points from the lecture number 16. The main concepts that we have covered is to introduce the multipath environment with the assumption of there is no line of sight path. So, all of them are through some process of either reflection or diffraction and we have looked at the model the basic assumptions, and showed how the Rayleigh PDF emerges as the resultant envelope of the signal.

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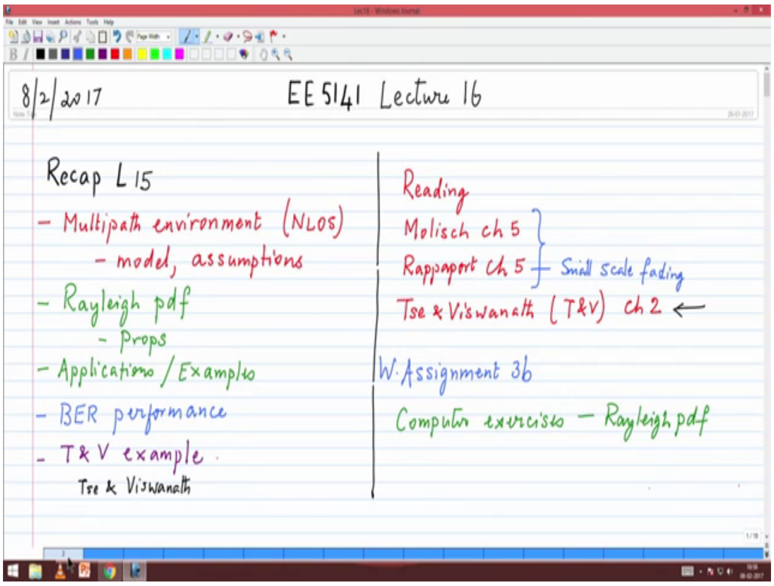

So, again we will reinforce this through several examples, and also through the derivations that we will be covering in the lectures. The goal of today is to look at some examples some applications of and the consequences of the envelope of the signal becoming a Rayleigh distributed random variable. So, once the envelope gets affected, the bit error rate performance also will get affected, today's class we will touch up on that.

There is a very classic example that is often discussed in the context of a multipath channels that is found in the book by Tse and Vishwanath. T and V is my short form for Tse and Vishwanath and the indicated that that you should have a look at chapter two especially the beginning part of the chapter two, which will give us a good introduction a very, very elegant example which helps us to understand. The rest of the material the main contents can be found in Molish chapter 5, and Rappaport chapter 5 basically the chapter in a Rappaport which covers small-scale fading again there are some chapter number differences between the different editions the chapter that contains a small-scale fading. So, we begin today's lecture with the quick review of the material covered in the last class and again if there are any doubts or questions, please do not hesitate to ask.

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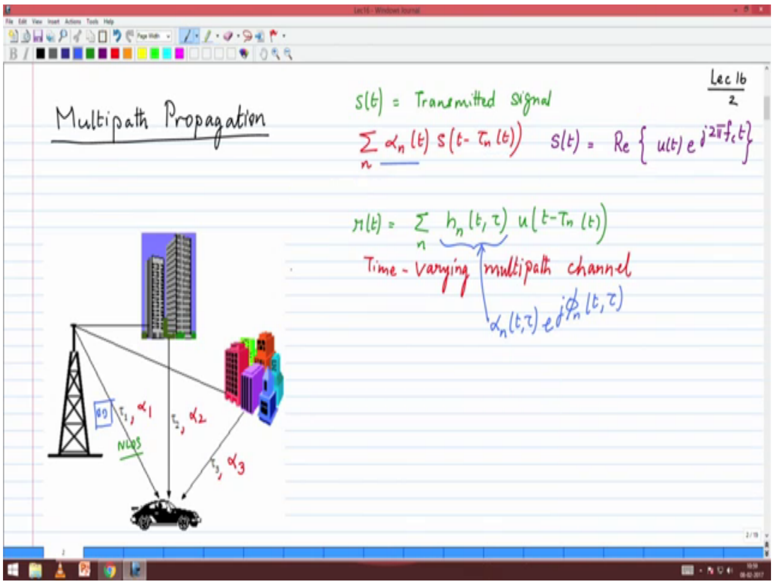

So, the multipath propagation model no line of sight. So, basically we assumed that there is some buildings here which has a affected even this path; so giving us a line of non line of sight three components in this case. We said that the if s of t is the modulated signal then we can write down these multiple arriving paths as a superposition of scaled by alpha n is that a gain of each of the multipath components with appropriate delays. If we take the form of complex baseband notation s of t can be written in this form applying this to the multipath environment, we came up with the following expression that the received signal, this is complex baseband.

Complex baseband notation says the received signal is a superposition of the complex baseband signal that was transmitted with different gains and delays and that is represented by the time varying multipath channel. And if you want to think of it in terms of discrete occurrences of these multipath components, you can think of it as alpha n which is a function of t and tau, the delays whether delay which it occurs and the corresponding phases which will also be a function of t and tau.

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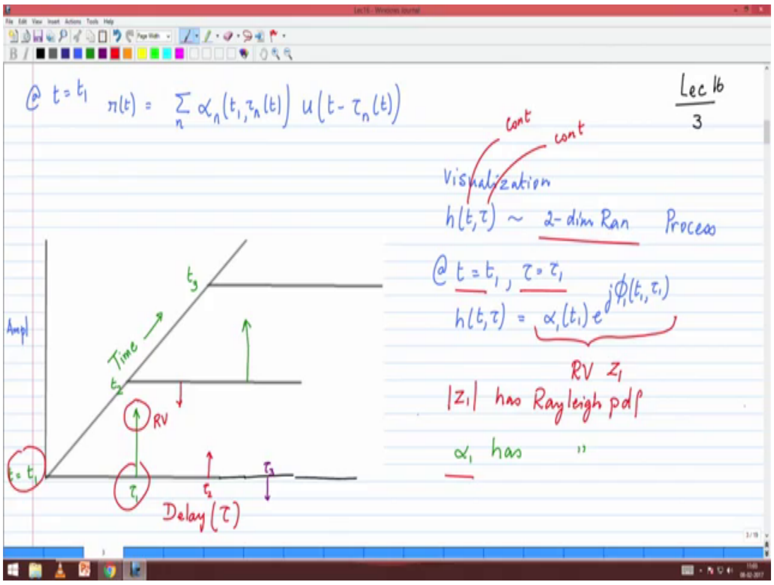

One of the very useful ways of equalization of a multipath channel is to think of it in terms of two dimensions; one dimension being the delay dimension, the other one being the time dimension. So, at any given point, if you take a snapshot, you get a certain multipath profile basically number of multipath components at the delays, the gains are occurring. We have shown a few examples. Now, basically this is a visualization of what is in reality a continuous function.

So, it is continuous, the multipath profile is continuous in t, it is continuous in tau both are continuous. Basically, it is a best way to visualize this channel response is as a two-dimensional random process. It has some variations along the tau dimension which are dependent on the environment, there is some variations in the function along with the time dimension which also depends on the position. So, it is a two-dimensional random process is a good way to visualize this channel response. It is random both in the time dimension as well as in the tau dimension.

Now, if you take a particular realization of a random process you get a random variable. So, in this case what is the random variable if I specify t equal to t 1 and I specify tau equal to tau 1. So, which means at time t 1 around along this axis along the tau equal to tau 1 this becomes a random variable. So, particular sample or a particular realization becomes the sample function. So, yesterday we said that if all of the multipath were arriving at the same time then we could represent the entire channel response by means of a single complex term. But even otherwise each of these can be visualized as random variables which can have a PDF. And again as long as the basic assumptions are the same that you have a large number of multipath components which are coming from different directions together they are comprising this particular component then you can say that each of these random variables also behave as a Rayleigh distributed random variables.

So, the notion is that if I look at a specific time instant t equal to t 1 and the specific delay tau equal to tau 1 sampling of the random process gives me a random variable this is what I will refer as z 1 the modulus of a z 1 is what has the Rayleigh PDF. So, we have also used the notation that alpha which is the amplitude, amplitude of the random way of the random variable you can say that alpha also has the same Rayleigh PDF. So, is this is the visualization as a two-dimensional process, a particular sampling point becoming a random variable and thereby giving you a PDF that you can then interpret and apply in terms of our understanding of the multipath channel.

Is that part clear? Are any questions, I can clarify them; otherwise, we move forward into building up on this basic assumption.

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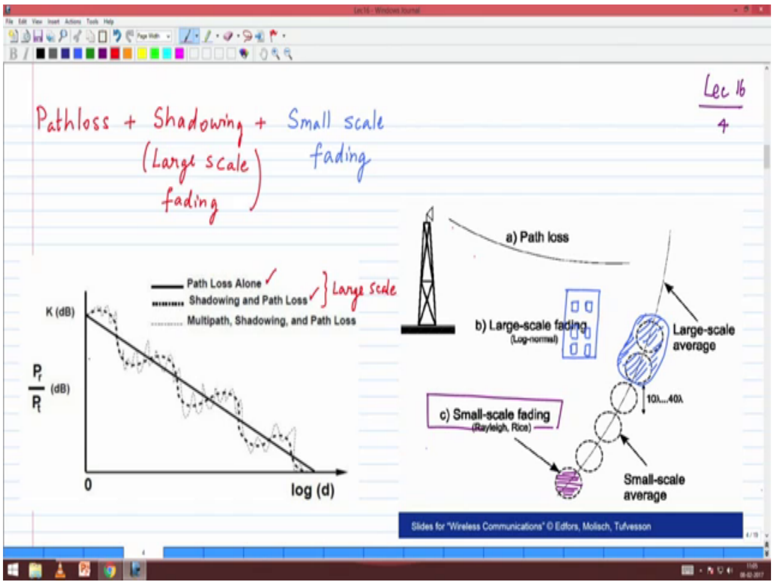

We move onto understanding of this. Again this is a repetition from last class, but for the specific purpose that we are trying to highlight where does small-scale fading come into the picture. Small-scale fading comes over and above the large-scale effects. The large-scale effects being the path loss and the shadowing loss these are the large-scale effects. So, the large-scale effect gives me the solid dash the solid the thick dashed line; and then on top of that any movement within a 10 lambda to 20 lambda space gives further fluctuations. So, basically over and above the large-scale variations, you will see small-scale variations.

Now again, so that in our minds we are very clear that this is not a different one does not occurs independent of the others, the all kind of act together we have path loss, we have shadowing you also have on top of that large-scale fading. So, again the entire picture gives us a good understanding of the environment that we are working with. So, the path loss we all can assume we understand I am the further away from the base station, I move I am going to have a degradation in my signal that is very intuitive. Shadowing also may be somewhat intuitive because you have an obstruction between you and the transmitter.

Therefore, that also is reasonable to expect. The fast fading or the small-scale effects are not intuitive until you have a good handle on what is happening the process that is happening in inside and that is where the lecture that we going to what we are covering yesterday and today is going to help us in our understanding.

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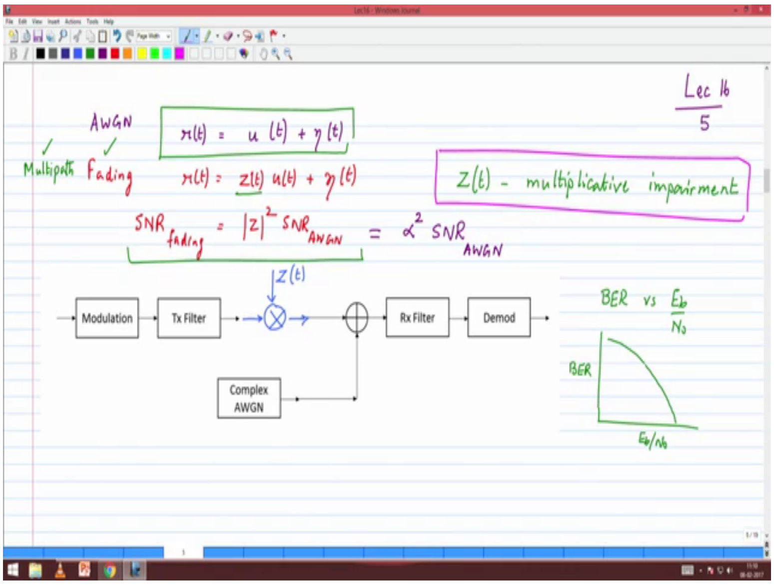

So, let me go back to another important element which helps us which reinforce our understanding of the environment the fading environment that we are working with. Now in AWGN environment suppose in you are looking at that which is the computer assignment number one that you did you have the modulated signal, baseband signal and then on top of which you add the Gaussian noise. In other words, if I have to implement this in a block diagram, you would have the modulation followed by pulse shaping and then you would pass it into the channel where noise would get added AWGN would be added AWGN white spectrum.

But then when it passes through the receive filter, you reject, but whatever is outside your band of interest. So, you have the desired signal as well as the impairment that falls within your received bandwidth that is what is passes through the receive filter and that is what we receive at the de mod and this is what you have characterized in your simulation as BER versus E b over N naught.

And very familiar graphs these graphs are quite steep. If you look at the performance the BER performance drops very fast, this is E b by N naught, this is BER and you have been able to generate these graphs. Now, what changes when you move from an AWGN environment to a multipath fading environment? So, this is a multipath fading environment. Notice, I am not calling it Rayleigh fading for a very specific reason because we have made an assumption, but there is no line of sight component. It is Rayleigh fading, if there is no line of sight component, but multipath very much present fading very much present, whether line of sight is present or not decides it.

If there is no line of sight it will become a Rayleigh fading, but in a multipath fading environment let us say that this is the environment. So, we said that the net of all the multipath components adding together gives me a random variable z of t z of t magnitude squared is what is going to affect because mod z is what effects my amplitude mod z squared is what just going to affect my SNR.

So, in the yesterday's class we said that the resultant SNR in a fading environment is going to be mod z squared times the SNR that you would see in an AWGN environment. So, now, if I want to introduce this fading into this process, then what I would have to do is introduce the fading and let me just bring in the fading part. So, where does fading occurs in the channel, where does noise get added in your receiver, so which happens first, fading happens first, so fading happens first.

Now, what is the process of the impairment of fading, it comes as z of t multiplying u of t. So, z of t we make an immediate note, it is not an additive impairment, it is a multiplicative impairment, and it happens before the AWGN comes into play. So, again it is very important that you we keep in mind the sequence because this is where z of t is happening in the channel basically z of t gets multiplied. So, all this says is when it comes to my receive side even before the AWGN gets added, what has happened my signal is going up and down and there is a possibility that the signal has gone down. Now, does AWGN care whether your signal is high or low? No, it adds a certain noise level to the signal.

So, if it is so happens that your signal has become faded and your noise as always the difference between these two actually keeps fluctuating and that is what is represented by a SNR, which is not constant anymore. It is a time varying SNR, so that is a very, very important element we will come back to discussing this little bit more towards the end of the lecture. But is this part clear that we have introduced an impairment which potentially could have very significant impact on my bit error performance because the z of t is the fluctuating one.

And we showed that there are some points where the Rayleigh fading can go quite deep which means that your SNR can momentarily go very low. So, which means that there is a impact on BER, how much of an impact how do we overcome that all of that is what we are going to study in the course. But is this basic notion of what happens when we move from an AWGN environment to a fading environment is that reasonably intuitive and comfortable to anyone; any questions?

Now, we move onto the second aspect of it how do we understand this through computer simulation. So, yesterday's class we talked about the scenario when two waves mix. And very good question was asked towards the end of the lecture, how many such waves how to mix to get a Gaussian distribution after real part and the imaginary part. So, we will do a simple computer experiment which will show that even with as little as four waves you start to see the Gaussian behavior with eight components random components, you will see very good Gaussian distribution of the real part and the imaginary part.

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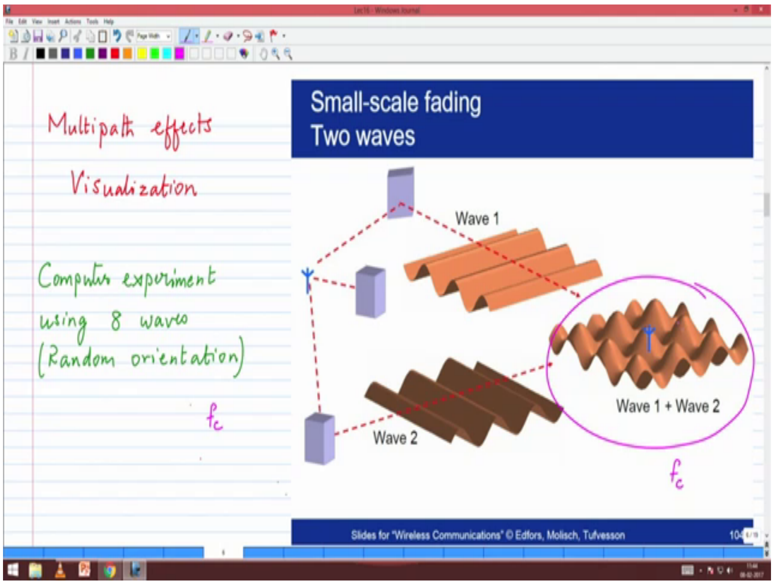

So, basically if you this is what will look like at the resultant at the end of after you do this simulation. So, this is your position of the receive antenna. If it happens to be on a peak, you get good signal you move slightly deep little further again there is a peak. So, basically this is a spatial distribution in the x and y-direction. So, in the plane, it is no longer a constant level, there is a variation in the signal level that is received again through the computer experiments we will reinforce this concept.

So, quickly let me summarize the key properties of the Rayleigh PDF. Again keeping in mind that there are people who are probably seeing this for the first time, but more importantly we just want to make sure that the basic assumptions are clear and that we are able to have a good understanding of what are the impacts will be.

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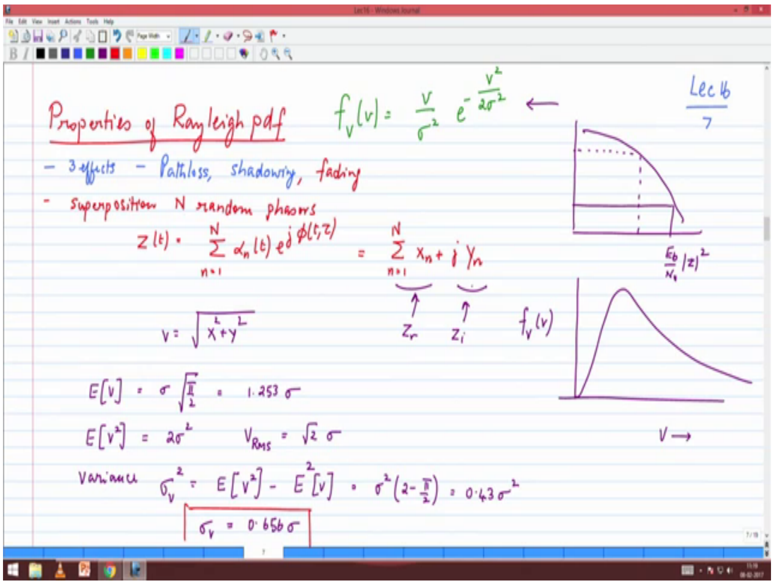

So, the key one of the key points that I want to want to mention is that there are three effects that we are looking at; one is path loss, there is shadowing, and the third one is where we bring in the effects of fading the small-scale effects. So, basically the third components are what we are looking at. And it starts off with by being by making the assumption of a superposition of a large number of random phasors. So, there is a superposition that is happening superposition of a large number of random phasors of n random phasors that is an important part of our assumption.

So, basically the channel z of t we said can be show can be approximated as n equal to 1 through n we are basically making the assumption that all the multipath components are having at the same time alpha n of t e power j phi t comma tau all of them have got the same delay. So, tau is; so this can be written as summation over n equal to 1 to N of components once we resolve them as X n plus j Y n. And each of these, this contributes two pats, one of them contributes the z r part, the other one contributes the z i part the summation of those terms and those are what give us the Gaussian distribution.

So, based on that we then did a transformation of variables and showed that the envelope where v is equal to square root of X squared plus Y squared where X and Y represents the Gaussian IID random variables that is what was the distribution. So, quickly the properties that we have already derived in the last class expected value of the random variable v expected value of the envelope. Now, you know wonder you know any PDF has got mean variance now what is the reason to actually spend time and looking at those values. One of the things that I want you to observe is the shape of the PDF are large values more likely or are small values more likely.

So, if you were to look at the sketch of the Rayleigh PDF, the Rayleigh PDF looks like this. So, that tells you that Rayleigh PDF, the distribution of the envelope this is the envelope, this is the probability distribution of the PDF, it is more biased towards small values. Now, or smaller values are more likely to happen the larger ones are part of a tail which is small. So, what does that tell me my envelope is going to affect my SNR? So, therefore, I need to be very careful with Rayleigh PDF because it is going to shift my SNR to some possibly to some lower values when fading is present.

Now, in addition to that another point to keep in mind why is that important elements go back and just quickly look at your BER graph your BER graph is a very steep graph. So, let us say that you had some value as you have defined as your E b by N naught and you are very happy with the BER that you have gotten. Now, suddenly, if I multiply it with mod z squared, and it happens to be a value that shifts it by 3 dB. What happens to your BER all of a sudden, that what was considered as a good BER suddenly got perturbed to a very bad BER, because these BER graphs are very steep? If I shift by 3 dB very easy to happen in fading, but the BER in practice is going to be totally unacceptable to you.

So, this is why we are very interested in the mean, the variance, all of these are things; otherwise we would not care too much about I like every distribution is got a mean and a variance and all of that, but we are focusing more on that. So, expected value of the distribution, yesterday we derived it, sigma times root pi by 2 also 1.253 sigma. Again the values may give some insight expected value of v squared is 2 sigma square. So, again if I wanted the RMS value, I would take the square root of this. So, it will be root 2 sigma: so the RMS. So, we can call it as V RMS will be equal to root 2 times sigma that that would be the root mean squared value.

Now, the variance, not sure, if you wrote it down yesterday sigma v squared notice you have to be very careful not to confuse this sigma with the sigma of the Gaussian random variable. Each of these Gaussian random variables has got a variance sigma squared, this is different, this is the sigma squared of the envelope the variance of the envelope. So, again keep that picture in mind, basically this would be equal to expected value of v squared minus expected value of v the whole square.

Basically, the mean squared value minus the square of the mean that will give us a expression which will be sigma square 2 minus pi by 2 which is 0.43 sigma squared and or if I have to look at the sigma v that would be 0.656 into sigma. Again, please make sure that there is a clear understanding of which belongs to the Gaussian, which belong to be a Rayleigh random variable and keep those things in mind. So, the variance of the Rayleigh random variable is less than the variance of the Gaussian that we are dealing with.

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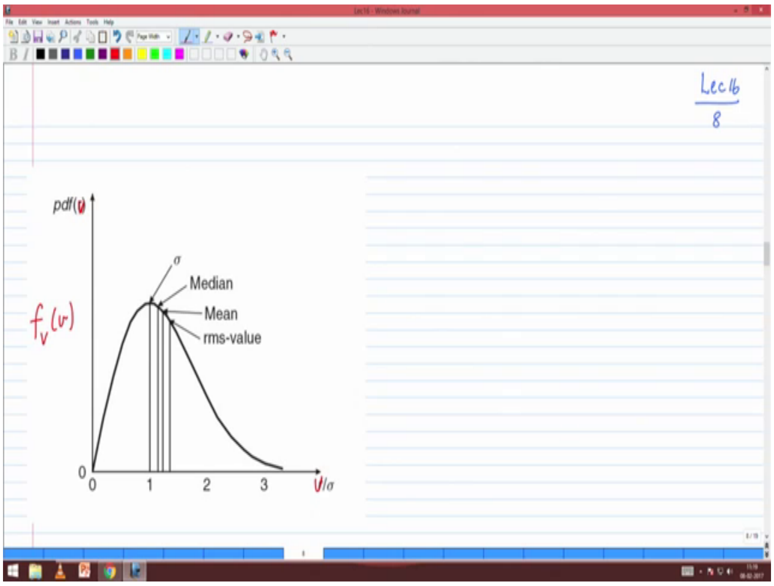

If you had a chance to look at the assignment that was uploaded just as called as assignment 3 b gave you some asked you to do some simple calculations. So, to look at the sketch of the PDF, plot that again that is fell easy for us to plot if you take v by sigma as a parameter because then you can plot that. And on this graph, we can then look at where the different points lie. So, one of the points that we are asked to check is; where does the peak of the PDF of the Rayleigh PDF occur.

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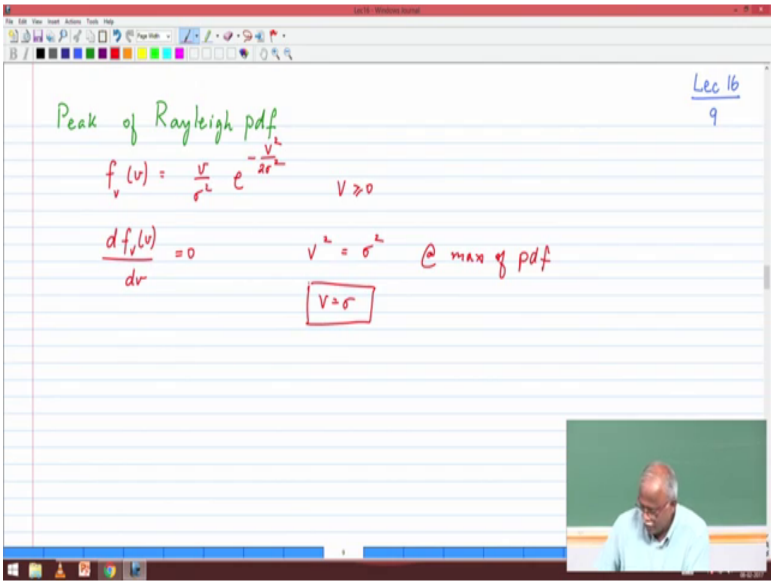

And again just so that you are comfortable with this let me just mention that the way to be done and I will just leave the steps to you. The Rayleigh PDF is given by v by sigma squared e power minus v squared by 2 sigma squared; for v greater than or equal to 0. The peak of the PDF basically says that I want to differentiate with respect to v, set it equal to 0. Find out what is the condition that comes out to be. So, basically the point at which you get maximum PDF, v max squared is equal to sigma squared or in other words or the maximum value of the PDF it may be should not call it v max. The place at which the maximum occurs, at maximum of PDF so basically v is equal to sigma is where the peak of the PDF occurs, which is what you can see where v by sigma is equal to 1 that means, v is equal to sigma you will see that the PDF will peak.

Now, what happens at the value of the mean? At the mean you are already the PDF is dropping at RMS value it is further dropping, again it starts to give some insight as you work with this more and more. So, please do make sure that you are comfortable with the shape where the peak occurs and what these values are.

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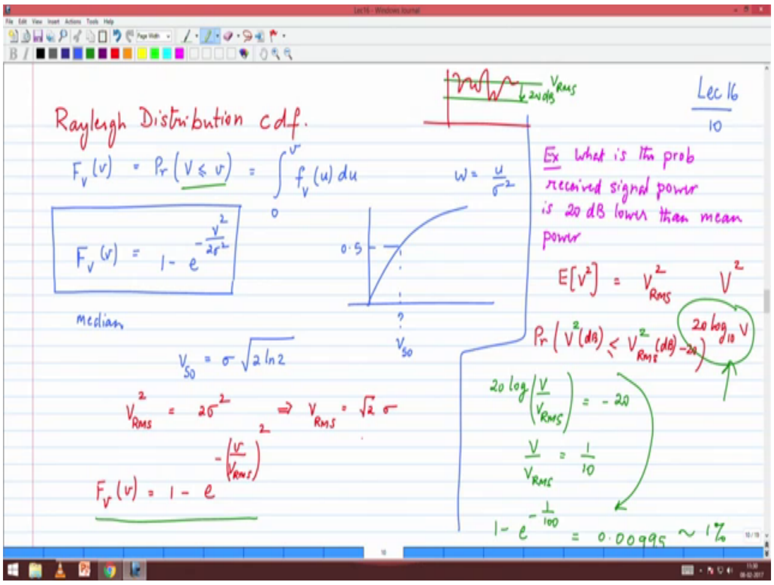

Another very, very important off shoot of the Rayleigh PDF is the cumulative distribution function. Now, here again from a mathematical viewpoint as a given a PDF I know how to compute CDF, you know why do we why do we labor over this one, but it turns out that there is a lot of useful information that we get from the CDF from an engineering prospective. So, therefore, we will spend a minute to highlight this. So, given the Rayleigh PDF, I wanted to compute the CDF.

So, the Rayleigh CDF is the cumulative distribution function F v of v, this is the probability that the random variable is less than or equal to some threshold value v. So, which means that cumulative distribution function my variable goes from 0 to this value uppercase V. I have to integrate the variable just avoid confusion; I am going to use u as my variable of integration. So, the PDF integrated from 0 to V is the answer that that we are looking for.

So, again please do change of variable w is equal to u by sigma squared. Again I will assume you are very comfortable with these types of operations, this will give me the cumulative distribution function as 1 minus e power minus v squared by 2 sigma squared. And once you have the CDF there are some interesting observations that we make from here, but let us take it one step at a time. The first one is the median value. When does your cumulative distribution function when does your cumulative distribution function cross the value of 0.5? So, what is this value that is we call that as v 50 that is your median value in your distribution? So, you can substitute F v of v to be equal to 0.5, and basically verify that v 50 is given by sigma times square root of 2 times natural logarithm of 2, 2 ln 2. Again very simple manipulation I am assuming it is very straightforward for you to look at that.

Now here comes another important observation or again inside start to come from the equation once we take the following steps. We had mentioned earlier that V RMS squared is equal to 2 sigma square expected value of v square. So and this also means that V RMS is equal to root 2 sigma. So, the cumulative distribution function can actually be written very nicely in terms of the V RMS 1 minus e power minus v by V RMS whole square minus V RMS the whole square.

Now, why is that an interesting form is we are going to see it in a minute, but basically I hope the steps I mean we have skipped steps, but those are all simple ones that I am sure you will be able to fill in. So, the reason for this is the following examples where the question that is going to be posed to us is that; what is the probability that the instantaneous received signal power. So, that the received again the instantaneous is understood it is a time varying at a given time instant, the received signal power is 20 dB lower than the mean power mean signal power.

Very valid question, because you told me that the signal is fluctuating, and I want to ask the question what is the probability given that you have now understood what a PDF is, what a CDF is all the different characterization of the mean, the variance, the RMS. So, first question what is the mean received signal power proportional to, expected value of v square because the signal is not changing that is normalized. So, the mean signal the received signal power is going to depend on expected value of v squared expected value of v squared which is the same as V RMS whole square, so that is the reference point.

Now, the question that is being asked is: what is the probability that the received signal power, the received signal power instantaneous received signal power is proportional to V squared. So, the question that is being asked is: what is the probability that V expressed in dB what I mean by that is 20 log base 10 of V that represents the mean received signal power. V is instantaneous received signal power is less than or equal to V RMS expressed in dB minus 20. Am I correct? That is the question being asked. What is the probability that the instantaneous received signal power is less than the mean signal power by 20 dB; it is very, very useful question.

Now, if you want to see where this is leading to, if you go back and look at the signal fluctuations, the question that is being asked is if this is V RMS that is your mean received signal power. What is the probability that your signal exceeds at 20 dB margin basically what is the probability that it goes below 20 dB with respect to the mean signal power and that is: what is the question that is being answered here. So, basically rewriting this, it would be 20 log of V by V RMS, this has to be let set it equal to minus 20 that basically tells us that V by V RMS is equal to 1 by 10, 10 power minus 1. So, what is the probability that this condition is satisfied is the same as going back to the CDF function because this is the condition, where I substitute V by V RMS equal to 1 by 10.

So, this probability using the CDF can be given by 1 minus e power minus 1 by 100 which is approximately 0.01 let me write the exact value you can approximate it 0.00995. So, it is approximately 1 percent. So, it is a good tool for us to have. So, why did I compute CDF because I am always interested to find out what is the likelihood of my signal going below some thresholds. And the one that you want to look at it is not the PDF you would want to look at the CDF because any envelope which is less than this also is going to be a problem.

So, using the CDF, CDF is a threshold by which we ask the question what is the probability that V is less than or equal to some threshold. Now, we are talking about a threshold where V squared is less than the corresponding RMS corresponding value of minus 20 dB. So, I hope this example gives you an indication of the usefulness of the CDF; any questions?

Student: Probability of V square expressed in dB.

Same thing, basically v squared means you will take 10 log V squared 10 log 10 that is why I have said I have taken it as 20 log.

Student: The probability of V square.

V square, I do not think it matters because I have given you the expression that I am using because if it is v expressed in dB it has to be 20 log as the same as v squared expressed in dB which will be 10 log 10, 10 log v. So, the both mean the same thing if I did not give you this expression then there is a ambiguity, but given this expression that is how we are writing V in dB, if it help us avoiding confusion, no problem, let us write it as v squared. So, basically this is a good way for us to visualize the benefits of working with the CDF.

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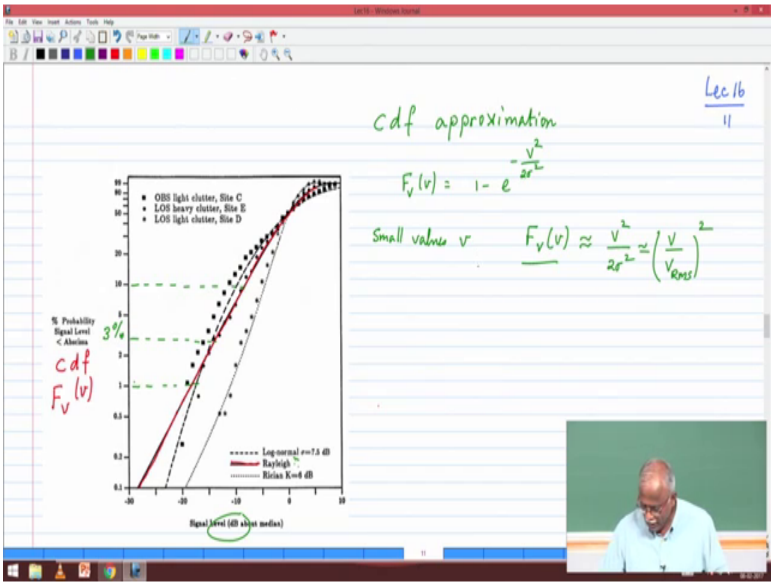

Now, there is another very important element that comes out from the CDF which I would like for us to highlight and again work with work with that as we move forward. So, the other element that we noticed is that the cumulative distribution function F v of v is given by 1 minus e power minus v squared by 2 sigma squared or this the expression that is given to us. Now, for small values of v, v squared by sigma squared also is small. So, therefore, we can approximate this F v of v during the expansion series expansion of e power minus x that this is approximately given by v squared by 2 sigma squared.

Now, check for this approximation would be go back to the because the previous case we were looking this is the same as V by V RMS whole square correct, and the V by V RMS that we got in the previous case was 1 by 10, square it 1 by 100 – 0.1 percent. So, again it is a sanity check that this is a good approximation. What does it tell you, it tells you that in the end entire range where V is small this approximation is going to hold? And if I plot it in the dB, the CDF as a function of V by V RMS in dB then what I will see is a linear line because it is a squared term slope is equal to 2.

So, basically this is the linear portion the red graph shows you the CDF drawn on a dB scale and you can see that the portion is very, very linear. Now, I would like to also introduce you to another very important aspect and that is the notion of how do we how do we work with this basically I want to relate the factor. Now, we understand that the small-scale fading is going to cause dips in the signal. How bad are the dips we can look at the CDF and we are going to tell? Now, the question is previously we allocated some margin in my link budget for the shadowing. Now, the question is should I allocate margin for this fast fading or small-scale effects of fading or not what are the implications, if I do not and if I do. Basically, it is building on a understanding that we have we have already developed, but we want to take it to the next level.

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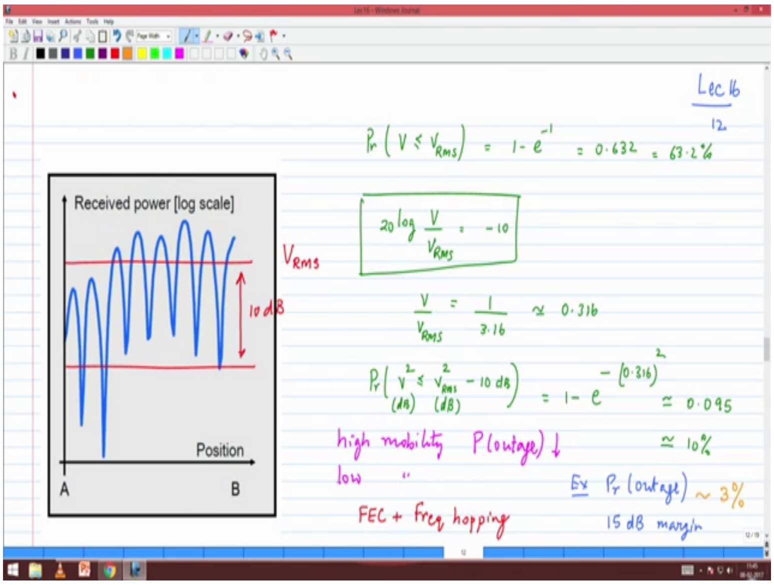

So, let us look at an example which should give us some insight. So, keep this picture in mind. This is the signal fluctuation. Now, there is V RMS that is your mean received signal power, you can draw a line and because of signal fluctuations, your x-axis is positioned; that means, displacement with respect to some reference point which you call as 0 comma 0. You move in the in a certain direction the signal is fluctuating that is that we will come back to interpreting it in a minute. So, basically that is that is my threshold level.

Now, if I do not have any additional margin, if I do not have any additional margin, what is the probability that I will have outage? Let me say that I am at the edge of coverage with respect to path loss and shadowing, I am at edge of coverage. Now, fast fading occurs over and above that, but I have not allowed for any additional margin. What is the probability that have will have outage? So, the question now becomes what is the probability that the received signal V is less than or equal to V RMS correct, because that is where I have received. If it goes anything below because that is the threshold of my receiver sensitivity, if it goes anything below that then I am in trouble.

So, this from the CDF tells us is 1 minus e power minus 1 because V by V RMS is equal to 1. This is 0.632. What is that? Bad situation, 63 percent of the time you are going to be in a outage. So, this is the same as 63.2 percent outage, the same, my goodness I cannot tolerate that please tell me what is the margin I should put down. So, now, comes the interesting trade off, how much margin should I put into the picture then you say what is the outage that that you can tolerate, 1 percent? How much margin should I give if I want to have 1 percent outage? 20 dB that is what we already calculated remember the likelihood of going below threshold of 20 dB is 1 percent.

If you want your outage to be 1 percent then what should be the margin that you should provide 20 dB, because with respect to V RMS. 20 dB if it goes to no problem you will still survive because that much margin you have provided. If it goes beyond that if the fade goes beyond that only then you will go into outage and that is precisely what the calculation tell us. Now let us say that you know say 20 dB you know that is ridiculous how can I add 20 dB margin over and above I already added 10 dB for shadowing that means, you know this is a huge amount of cost that you are going.

So, now, let us look at tradeoffs. Then you say give me 10 dB margin what if I give only 10 dB margin I cannot afford 20 dB margin. So, the interpretation of that will be in the following form 20 log V by V RMS this is equal to minus 10 that is basically what you are saying is I am if I have V RMS and I am going to have 10 dB margin. That means, only time I will go into outage is v exceeds that basically falls below that 10 dB margin. So, this is the threshold that we have which basically says that V by V RMS is equal to 1 by 3.16 which is approximately 0.316, and the probability of this happening. So, probability that we will write it as V squared to avoid confusion is less than or equal to V squared this is all in dB. This is V RMS in dB minus 10 dB. This one is given by 1 minus e power minus 0.316 squared, which is approximately 0.095, which is approximately 10 percent. Good.

We accomplish something very significant in today’s lecture in understanding what is the price that we have to pay to build a system that is robust in Rayleigh fading what is it true what is its secret margin? Margin is now this one which says 10 dB I am likely to have 10 percent outage no problem we are going to live with that, but now the engineering intuition starts to kick in. Say I have 10 dB fading margin shadowing margin; now I have 10 dB for fast fading two together I have 20 dB margin. So, if I do not have the effects of fast fading I can use 20 dB for shadowing or if I do not have shadowing I have 20 dB for fast fading. So, basically you are trading off between shadowing and fading hoping that you are you are not unlucky and you get hit by both at the same time again.

So, now comes a very interesting question, what is our x-axis? X-axis is position so that means, the faster I move the more fades I am going to encounter is that correct it is fair enough because as the faster I move if I am going to move slowly then my signal is going to slowly go it will still good, good, good, good, good, good then it goes to bad. When it goes into bad it is a problem, because bad, bad, bad, bad, bad for quite some time and then it comes out into good. Whereas, if you are moving fast good, good, bad, bad, good, good, bad is basically shuffling very fast between good and bad which is better, which is better? Fast is better everybody agree fast is better because you do not get stuck in a bad situation, because what would happen if you sat in a bad situation is bad, bad, bad, bad, bad, call drop gone.

Why because speech coder cannot survive beyond a certain point and beyond you have already allowed for 10 dB margin and this is gone below the 10 dB margin and therefore, you are killed. So, the thing is fast move fast do not do not walk slowly move fast no would you cannot control the user behavior that they want to walk they will walk they will stand in one place the problem comes when your users stands in this place and starts talking because then is not moving therefore, the call is going to drop. You know that you know the secret behind that.

Student: If you move fast the main (Refer Time: 43:21).

That is good observation because what we said was that in order for us to track or to do coherent detection of the received signal, you must estimate I will just rephrase that he said you have to calculate very often. What is it that you will calculate you are calculating the phase, because this is not only an amplitude, there is also phase associated with the fading. You have to constantly keep computing the phase, so that you can coherently detect your signal, otherwise what will happen you are you phase reference will be wrong, the BER will once again get affected. And therefore, so fast is not necessarily good from a receiver point of view, it actually adds complexity to your receiver.

So, the key question is well one point of view says you know go for high mobility that is the good thing to do. Yes, it is good because the probability of outage decreases correct that is the reason why we would like this. On the other hand, low mobility probability of outage will go up. So, what if and just say that you know the statistic about 80 percent of India's mobile traffic comes from pedestrian users, which means that they are not moving very fast, 80 percent of our traffic. So, then what do you tell those 80 percent of Indian subscriber say too bad you are sitting in a bad spot, call drop I cannot do anything about it what do you do you have to find ? You have to solve the problem eighty percent of your traffic is you know moving very slowly not moving fast.

They will once they go it to bad situation, it will take some time for them to come out of it FEC, when say- oh good because in a FEC is good because it will it will protect you against some number of errors. It turns out that again even FEC will run out of correction capability; if you start giving whole sequence a series of frames that are in error. So, FEC to some extent will protect you, but will not you know bail you out, you need something else, what else you already studied that.

Student: (Refer Time: 45:36).

Hand off maybe ok, but you may be close to the base station you may not be in the situation where you really need a hand off.

Let me go back to this graph that is why I thought I does I mention these pictures all have a purpose. Now, this carrier frequency and this carrier frequency are related they are shifted by Doppler, but they are they have a basic underline carrier frequency f c. This pattern that you have seen here in some sense depends on f c, because the peaks and troughs are going to be. So, this depends on f c. If I want to disturb this pattern, what should I do, frequency hopping just change the frequency because then what happens this pattern gets shifted then even in the same location where you are now it may be a better frequency in terms of multipath fading. So, FEC is good, but frequency hopping is better.

So, we just make an observation that we will use a combination of FEC to protect you against some number of fades plus frequency hopping is probably a very good mechanism for us to take care of the multipath fading. So, again hope that you are starting to see the benefits of this type of understanding. So, let me give you one exercise as always something for you to for you to try out. This is an exercise. Find out what is the probability of outage, what is the probability of outage if I have 15 dB margin power margin that is I have 15 dB margin for fast fading. So, if this is the margin what is the; obviously, it should be somewhere between 1 percent and 10 percent. Please verify that this comes out to be something very close to 3 percent.

And again you can see that the CDF has played a good role for us to understand, the various elements of what we want to study in terms of the effects of the fast fading.

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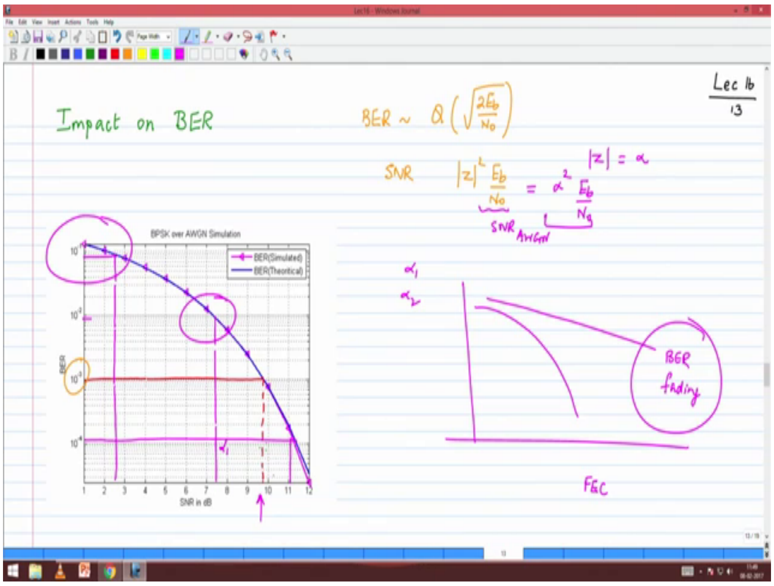

Now, comes a very, very important part of the lecture I hope this graph is very similar to all of you, as computer assignment number one. You do the theoretical graph q is equal q function of root two a square root of 2 E b by N naught BER is Q function of square root of 2 E b by N naught correct. Now, the basically let us say that you are operating somewhere close to 10 dB, you have 10 power minus 3 E BER you are you are very very happy. Now, you change it from a AWGN channel to a fading channel. So, the SNR in a fading channel is given by mod z squared times the E b by N naught that you will have in AWGN channel. So, this is the SNR in an AWGN channel, we had already introduced how we are saying that. Now, mod z we also have use the notation alpha in our expression. So, mod z or alpha is the same. So, this is the same as alpha squared E b by N naught.

Now, if I told you without telling you anything about whether it was fading or AWGN, I told you that my SNR is alpha square E b by N naught asked you to compute the BER, you will basically say now where is this alpha square E b by N naught, let us say it is somewhere here. So, this corresponds to alpha 1, alpha 1 is squared E b by N naught is somewhere between 7 and 8 dB you will look at this graph and you will tell me it is close to 10 power minus 2 not bad, but it is. Now what if so alpha 1 is specified I give you an alpha 2 which is horrible somewhere between it takes the SNR to 2.5 dB. So, now it is gone close to 10 percent of errors.

Then another scenario where it actually got better than 10 dB, it is around 11 dB, it is 10 power minus 4, that is really good because the this is what fading is doing to you sometimes it gives you a SNR that is bad sometimes it gives you an SNR that is good. And then you have to average all of them because it is the average of over all the SNRs that you will encounter. Now, if you look at the averaging process, which are the events that dominate your bit error rate, resultant bit error rate. It is not the 10 power minus 4 because 10 power minus 4 does not add much errors what adds is this is going to cause a big problem this is going to cause a big problem because these are.

So, the resultant BER in a fading environment the average SNR threshold is actually here it is just that from AWGN, I move to a fading environment and then you ask me to look at the BER this is AWGN BER the BER in fading is this BER. In a fading environment it is much, much worse; it does not even you know some takes a long time to even hit 10 power minus 2. So, which means that I actually I am working in an very very error prone environment and therefore, I have to pull out all the technique that I know in digital communication, FEC equalization all of the elements that I know because this is going to be a huge challenge for me to solve.

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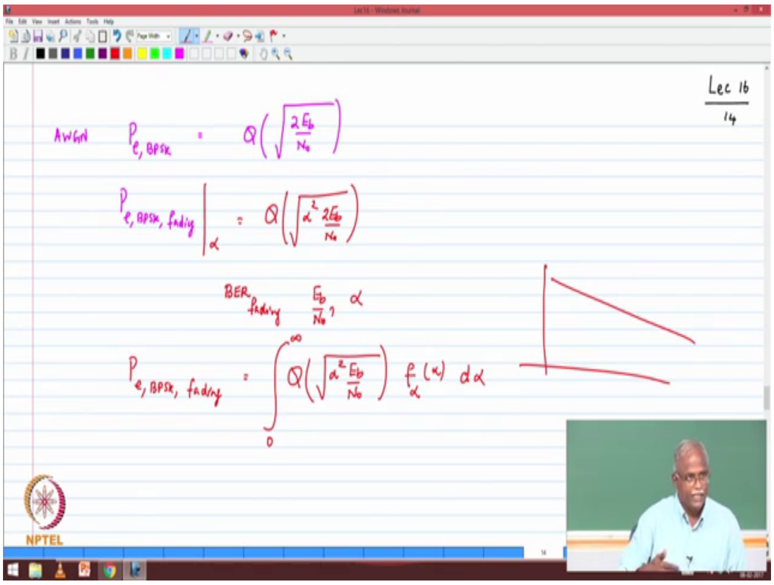

So, the theoretical expressions, I will write it down and then I will conclude today's discussion. Probability of error of BPSK this is performance in AWGN channel. This would be given by this would be given by Q of square root of 2 E b by N naught. Now, the same thing in fading environment, probability of error in BPSK in fading immediately you should ask what is the value of alpha. Given a particular value of alpha that is introduced by the fading this would be given by Q of square root of alpha squared 2 E b by N naught in most cases alpha is going to be less than 1; and therefore, the bit error rate is now a function. So, BER in fading in fading is actually a function of the E b by N naught, yes that is your nominal value where you have set, it is also the function of the instantaneous value alpha.

So, if I were to ask you; what is the probability of error of BPSK in fading, what would you have to do, you would have to take the integral of Q of square root alpha squared E b by N naught times the probability distribution of alpha. And alpha will go from 0 to infinity; this would be your fading environment. And this is what this is the graph that we showed was really bad. How do we get more intuition about this type of system, how do we deal with this kind of a system, those are things that we will build on in the lectures to come. Let me stop here.

Thank you.