**Introduction to Wireless and Cellular Communication**
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**Lecture – 14**
**Cellular System Design, Capacity, Handoff, and Outage**
**Shadowing, Outage, Multipath**

Good morning, we begin lecture number 14; our plan is to complete our discussion of systems level aspects. We have talked a lot about the handoff aspect in the last lecture. We have now started to explore more specifically the aspects of the wireless channel understanding the large-scale effects, small-scale effects. So, let me just quickly run through the key points that we have highlighted in the last lecture.

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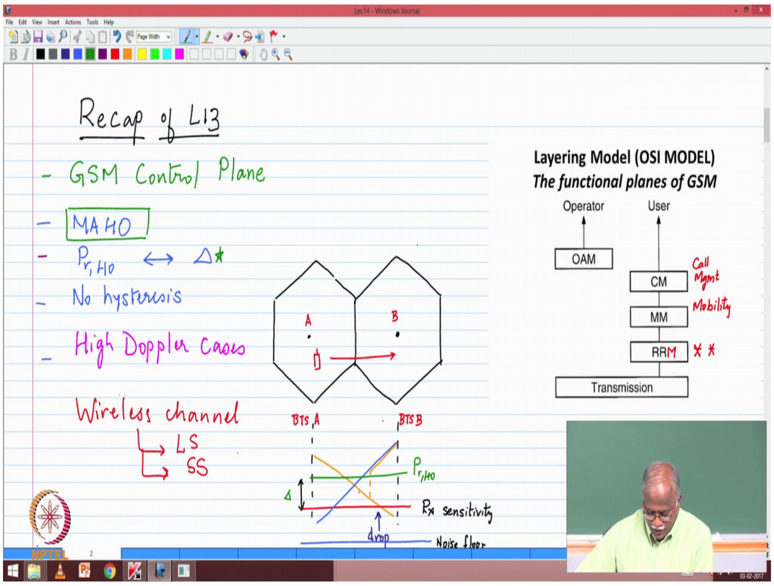


So, when we about handoffs, there is a whole range of topics that we have covered all the way from the control plane understanding how handoffs are handled by the system. The role that the mobile plays in the whole handoff process because the measurements are reported by the mobile, mobile assistant handoff. The design parameter delta which tells you how much, so this is the threshold at which you start searching for base stations for possible handover. The larger the delta the more robust your handoff mechanism is, but that also means more handoffs in the system. So, therefore, that is a trade off that you must do. So, this a key parameter that we must keep in mind. We do not want ping pong handoffs. So, we have introduced a hysteresis into the system. We also keep track of the fact that there are high Doppler users high velocity users who may need quick handoff.

And as we mentioned yesterday high Doppler does not necessarily mean that that you will have a handoff, but these are sorry the other way. Low Doppler does not mean that you may not need a handoff, but high Doppler cases definitely are the things you have to be careful with. The wireless channel has several effects that are different from the AWGN channel, which we have studied extensively in digital communications. And yesterday we started to introduce some of the aspects of the wireless channel.

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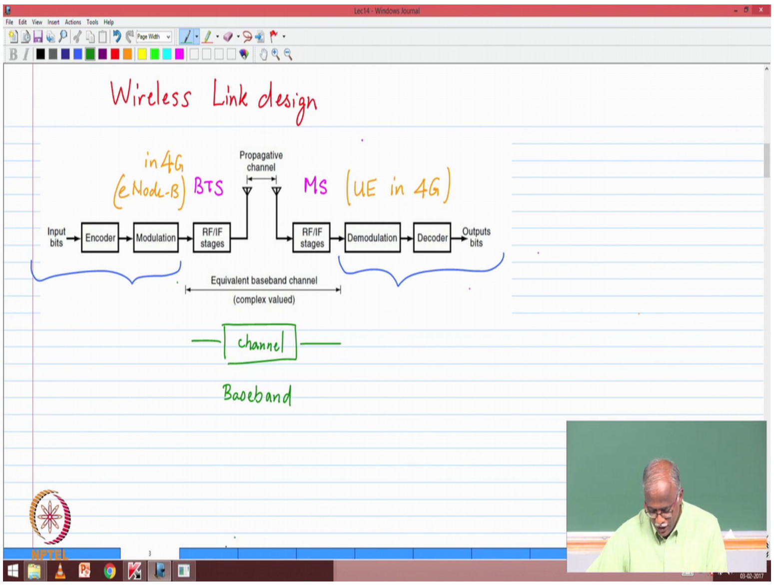


So, again to remind ourselves, we have a complex baseband representation of the transmitted signal. We have a complex baseband representation basically carried over into the receiver side. We also have this in between what we call as the channel. So, let me draw this in digital communications, this would be called as a channel. The channel is characterised by a symbol spaced equivalent filter. So, basically all of the impairments are captured. What we need to understand is how does that change when we want to go into a wireless channel, what are some of the elements that we need to keep track off.

So, we will also take the route as representing complex baseband equivalent of the propagative channel, but it is very customised to our understanding of the wireless channel. So, the link that we are focusing on is between a base station or a mobile or in the uplink, it will be mobile to the base station. We said it is a point-to-point link that is what we are interested in characterising. There are some differences, but by enlarge the behaviour of the wireless channel does not differentiate who is talking to whom the impairments are the same.

So, some notation that you will see in the 4 G literature the word base station in not used they use a acronym called e node-B, node-B, B stands for base station; and on the receiver side mobile station is called the user equipment - UE. So, in 4 G, there is some different notation. But we are looking at that aspect we would like to get a equivalent base band representation that helps us understand how the signal is impaired when it propagates through the channel, what are some of the ways in which we can rectify those and then and get a good performance out of the system. So, that is our goal for today.

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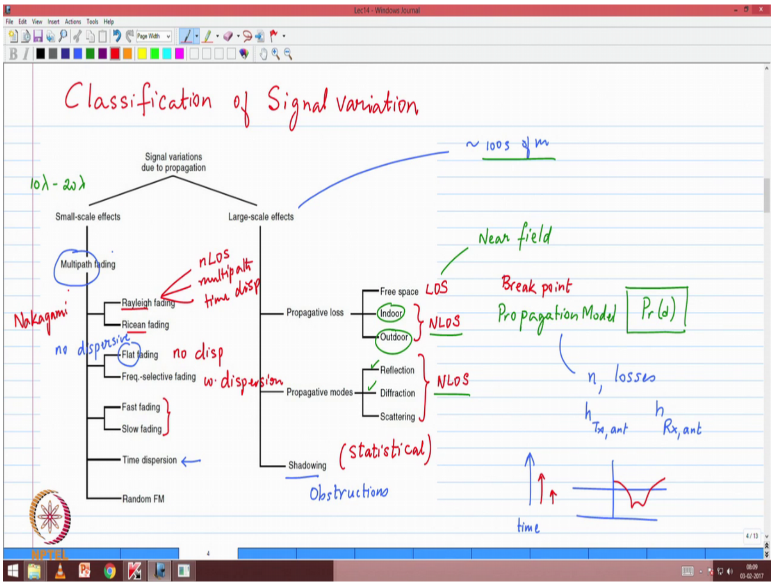


Let me quickly review the points that we have made in the last class with respect to the differences between large-scale effects and small-scale effects. Large-scale effects are of the order of 100 of meters. So, these are changes that happen when you move over substantial distances. On the other hand, small-scale effects are very much in the vicinity of the order of 10 to 20 lambda. So, again we have looked at what is the typical values of the lambdas that we are interested in it is under 1 meter. So, therefore, these are small perturbations in your position. In terms of the large scale effects we mentioned yesterday that the non line of sight transmission which is primarily what we will have in the wireless channel takes place through three mechanisms reflection, diffraction and scattering of which the more dominant ones are the reflection and the diffraction, scatterings gives you a component significant component, but it is not as large as the other two.

Now, based on what aspect of how much of reflection, diffraction, scattering is happening, we would have to characterise it for the specific environments that we are looking at either a indoor environment or an outdoor environment. And all of these are captured through propagation models. And we mentioned yesterday the Hata model is one such example. Again once you have understood one of those models it is more or less just say that I know how to apply this models. And we will take some time to look at one of the models at a later point.

But as of now at this point we already know how to handle the break point model the break point model is one of the propagation models that you can use. So, having implemented one of those you can substitute the break point model with something else because the propagation models essentially are telling you how to calculate P r of d. At a distance d if you are transmitting with the certain power, what is the received signal power because that basically tells you then the range and all of the other calculations. So, any method that tells you how to calculate P r of d is because we know how to do it for one model we can do it for any others.

But keep in mind that there is a lot of sophistication in these models, they take into account whether you are indoor, outdoor whether it is rural environment whether it is a high rise type of building whether it is a dense urban environment. So, there are lots of variation in the propagation impairments and all of those are captured in the different models. So, again those are that is an important area of study not to minimise that, but again from our approach it says it is a model that tells you how to get P r of d which is which is sufficient for us.

Now, line of sight it happens only in what we call as the near field, when there is no obstruction. And depending on the height of the transmitting antenna this could 10 meters, 20 meters may be even 100 meters, but again it is not all the way to your destination target mobile. So, therefore, we do I have designed the system to be a non line of sight system that is our basic design.

Now, large-scale effects, I believe are to a very large extent affected by one is propagation. Propagation would primarily be dependent on your propagation parameter n, what are the different losses that you have the height of the transmit antenna h of the T x antenna, the height of the R x antenna, so these are all parameters that will affect a propagation model. But at the end of the day what else affects the signal quality that is where we pick up shadowing. Shadowing is not the one created by the channel, this is obstructions that are in the path other buildings, or maybe you are in the basement of or you are going through a tunnel, so those are things that are not necessarily caused by the wireless channel, but the fact that you are in a disadvantage position. So, these are primarily obstructions.

So, today what we would like to do is try to understand a statistical characterisation of the obstructions or shadowing and then say that that is the right way to do it because we do not have a deterministic scenario, it is a scenario where there is a lot of variability you may be inside a building, outside a building. So, there are certain things that we have to characterise using statistical methods. So, with that understanding, we move over to the small-scale effects. Now, small-scale effects as we talked about earlier is caused primarily because of this phenomenon called multipath because this signal can bounce off different objects and arrive at the receiver.

Now, depending upon whether there is time separation between these multiple paths, you may or may not time dispersion if you do not have much time separation between them then all of them will arrive approximately within the same time all the multiple components then you get what is known as flat fading. So, flat fading means no dispersion frequency selective fading means that there is a time dispersion and again why frequency flat frequency selective may be you already know the answer why do you use the term frequency flat frequency selective?

Student: old frequencies (Refer Time: 10:11) dispersion.

So, if you want to have something that is spectrally flat what should be your time domain signal, it must be an impulse. An impulse in the time domain, if this is time, this in the spectrum domain will basically give you something that is flat. So, if you want to have fading that is not dependent on frequency, then your effective channel impulse response must be one tap because then all frequencies have the same gain that basically the gain of that single tap. Now, if I have a second and a third tap as part of my impulse response then what happens is the frequency response of this filter now comes into play, it may have some behaviour like that. So, some frequencies have gain some frequencies have attenuation.

So, basically when you have only a single tap, you call it frequency flat that is the same as non dispersive. So, non dispersive same as frequency flat dispersive same as frequency selective fading frequency selective meaning different frequencies experience different gains. Then there are several variations of the types of fading that we can encounter statistical characterisations, we have the Rayleigh distribution, Ricean distribution, there is also something called the Nakagami distribution. All of these are characterisations of the nature of the signal and fluctuations that you will experience in the signal that is something that we will study as well. There are some other aspects such as random f m, which are not very significant. So, we do not really focus on those, but these are the large these this is the significant ones within the small-scale fading.

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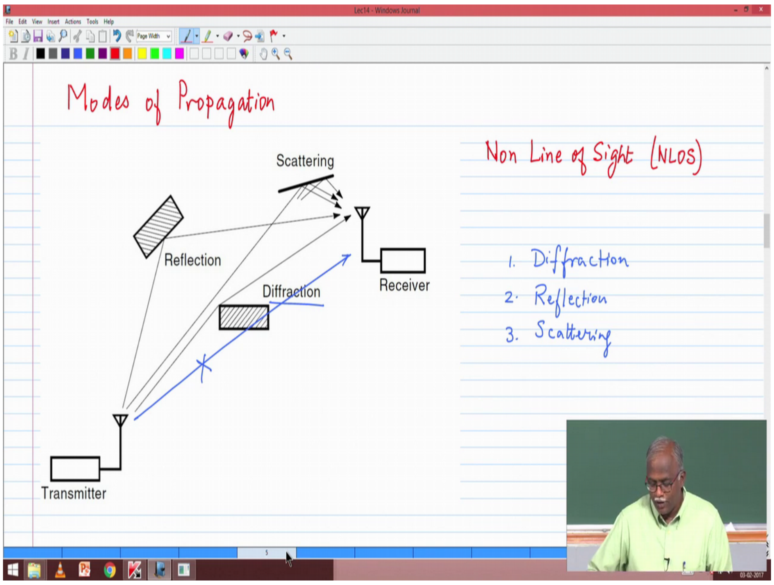


Let us move on lots of interesting ground to cover today. Now, the non line of sight basically means that there is no direct path you would have to go through one of the three mechanisms that that we have either diffraction, reflection or scattering, each of them contributes certain component to the signal. For example, reflection probably can give you a stronger component than diffraction because diffraction is based on Huygens principle it says there is a creation of secondary waves and therefore, what actually goes towards your antenna may be a smaller component. But never the less each of these can be characterised again in Rapports book its well done and may be towards the end of the course we will spend one lecture just to sort of highlight the aspects. But this is something that I believe you are most of you are familiar from studies in physics and studies in electromagnetic waves

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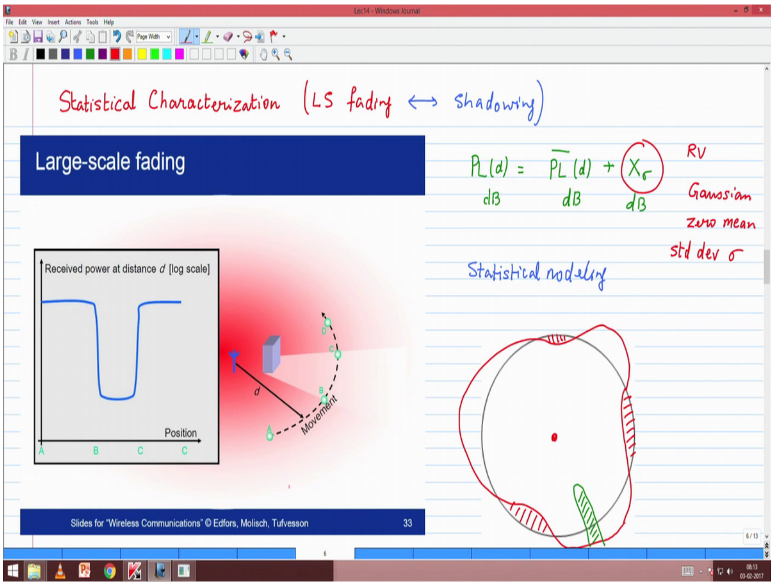


So, let us now focus on the shadowing aspect. Again this is a figure that we have seen previously. The way what we are trying to do is trying to understand what is the impact of blockages. For example, if you had a blockage of building here suddenly when you go behind the building that is the building is between you and the base station, then you see that there is a drop in the signal. So, given that there are such obstructions may be a tree may be a building or a you know bridge whatever it is that reduces the signal strength which affectively says in this along this direction your range is not the same as when there is no obstruction. So, in a way of characterising it, typically you would think of it is an omni directional antenna that your range is some average radius.

But in reality what happens is because of obstructions there may be in fact, if you have like extreme case of an obstruction. So, for example, you may even your coverage at some point may actually be a scenario where there is a significant reduction in one direction because of very big obstruction. So, by enlarge we want to understand this sort of characterization and it is helpful for us to keep that to keep that picture in mind.

So, the essence of a shadowing is captured by the following statement that the path loss at a distance d. We want to characterize it as some path loss at the distance d average value, these are all in dBs plus a random variable x sigma, we said that this is a Gaussian is a good approximation to this. So, basically it is a sum mean value with some perturbation which looks Gaussian in its behaviour. So, this is this is where we begin x sigma is the random variable it has a Gaussian distribution. It is zero mean because you can have the from the average signal point, you can have more path loss or you may have less path loss. So, basically there are some variations possible. When can I have less path loss?

Student: (Refer Time: 15:41).

If for some reason, I am at a higher height right because let us say you are in a position where there like a small hill and you happened to be there you will actually get an advantageous position. So, again shadowing can go both ways there are scenarios where you get obstructed, there are scenarios where you get benefitted. So, it is a zero mean Gaussian the standard deviation is sigma. So, our goal is to study this mechanism and to understand it fully.

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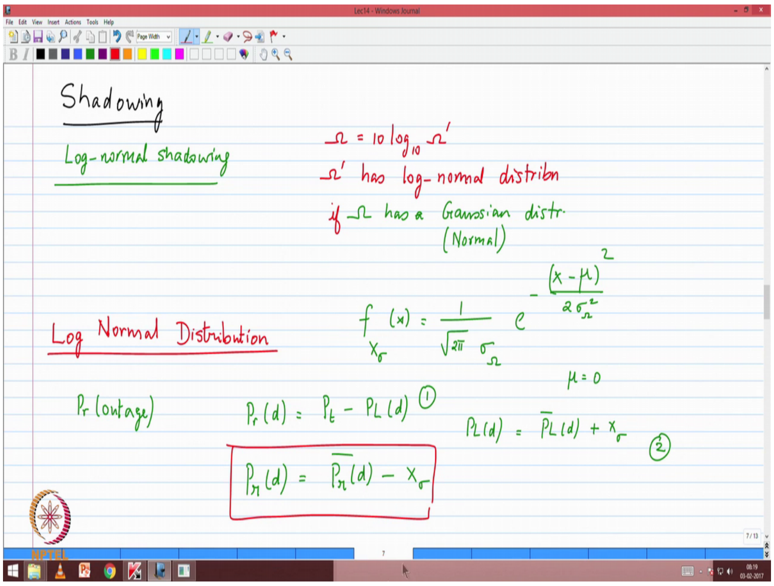

So, shadowing is what we are trying recognise, and notice that when I write the equation on a dB scale, I got the following expression. So, this x sigma has got a very unique name, it is a random variable that is Gaussian in the log domain, because basically it is a x sigma is represented in dB. Such random variables are called log normal because they are normally distributed, but in the log domain. So, let us quickly characterise the log normal random variables. So, if I have a variable sigma which is 10 times log base 10 of sigma dash sigma prime that is my so sigma prime is random variable I have taken 10 times log base 10. Now, the random variable sigma dash is said to have a log normal distribution log under the following conditions if the when you go to the logarithmic domain omega has a Gaussian distribution, Gaussian or normal distribution, so that is the basic definition.

So, I am looking at a random variable which is Gaussian in the log domain, therefore, it is underlined random variable is a log normal, so that is why whenever we talk about shadowing we actually you will hear the term log normal shadowing. So, it is Gaussian in nature, but it is not Gaussian in the natural scale, it is in the logarithmic scale. So, we always refer to as log normal shadowing and the reason and the underlining explanation for that is given in this fashion.

So, given this understanding, what is the distribution probability distribution of f of sigma, it is Gaussian. So, it is 1 over root 2 pi sigma omega e power omega minus mu whole squared divided by 2 times omega 2 times sigma omega whole square basically a Gaussian distribution. So, this is a general one, but normal basically means that mu equal to 0 sigma x is the only one that is that remains. So, I think I should write it here with x, mu it is a zero mean Gaussian. So, therefore, what you are left with is only the standard deviation expression in the equation.

So, now, I want to translate it into a meaningful a useful tool. So, the tool that we are going to focus on is probability of outage. Now, how does the fact that I have characterised this particular impairment as a log normal impairment, how is it going to help me or help me understand the aspects of the impairment. So, here is what we would like to look at. So, the received signal strength at a distance d is given by the transmitted signal power minus the path loss at a distance d that is the. But we also have an another equation which says that my received signal power is equal to the path loss at a distance d is the path loss at a distance d plus x of sigma.

So, using these substituting the expression for path loss, so if I have this as equation number 1, equation number 2, I substitute it into this equation, you can see that we can write down the following expression. The received signal power at a distance d can be written in terms of the received signal power at a distance d average that means, with the average path loss minus x sigma. So, path loss had, if you think of it as a positive addition because of shadowing that means, your received signal power is going to be less by the same amount and that is what this equation tells us. Now, you may wonder you know really it does not seem like we have done a whole lot but this is a very, very useful equation and a useful expression.

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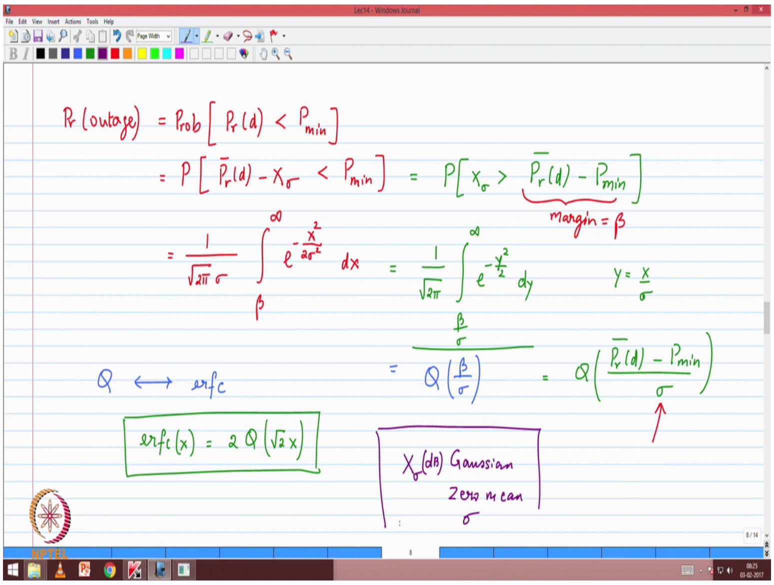


Now, let us move into the aspects that we want to explore. So, the probability of outage as we had described earlier is a probability that the received signal at a distance d well may be I will write this as prob. So, do not confuse between. So, this is the probability that the received signal strength at a distance d is less than the threshold P min. Now, which if I substitute using the equation from the previous page, this is the same as the probability that the average signal strength P r of d minus x sigma that means, because of the shadowing is less than P min. Now, this gives a very, very interesting rewrite of the equation this basically says x sigma is greater than P r of d average minus P min I have just rewritten the variables, but the insight comes from this because what is P r of d minus P min that is my margin.

So, actually this equation has not given us anything new, all we are saying is if you are shadowing is greater the reduction of signal due to shadowing is greater than the margin of course, you are going to go into outage. So, it is a very simple equation, but here the all the things that we have done now comes into play because x sigma is a Gaussian and for a Gaussian I know how to characterise. So, if I call this as beta, so if I ask you what is the probability that has zero mean Gaussian is greater than beta, then you will say that this is this probability can easily be done 1 by root 2 pi sigma. You will integrate from beta to infinity e power minus x squared by 2 sigma squared dx that is your cumulative value starting from beta if you go all the way to infinity.

Now, one more step and then we have a useful tool for us to understand and implement that is basically a change of variable where I want to introduce y is equal to x by sigma. So, if I introduce that notation this becomes 1 by root 2 pi is do the change of variable, this becomes beta by sigma, this will become infinity remains as infinity e power minus y squared by 2 dy. And this is a well known expression, this is the Q function at the value beta by sigma, Q functions are well known you have used it in digital communications, it is something that we are quite familiar with there are tables that are available to us or you can also compute this expression.

Now, some people are not as familiar with the Q function as the complementary error function. So, Q function and the complementary error function are actually related and you can map them. So, the complementary error function is erfc the basically if you look at the definition of the Q function the definition of the complementary error function, I would like you to please verify the following result, again more as a refresher of what you have already studied. So, basically the complementary error function for a parameter x is the same as 2 times the Q function with root 2 x. So, if you calculate the function with root 2 x and multiply it by 2, you get the same as complementary error function.

So, as a function these two are more or less representing the same. The Q function is very intuitive for us because it goes from one threshold to infinity of the normalised distribution. Now but what did all of this give to us the following this is the Q function of P r of d bar minus P min divided by sigma. Now notice that we said that the shadowing is described by means of a log normal distribution that means, in the Gaussian domain, it is zero mean it has got a standard deviation you always have to specify. So, you will never find a statement which says the shadowing is log normal and that period, they will never specify the mean because its assumed it is zero mean, but the standard deviation is always specified why because standard deviation is one that is going to play a very key role sigma has to be specified.

So, now, we have the understanding that the perturbation of the path loss is a random variable x of sigma which is Gaussian if I write it in the dB domain it is Gaussian it is zero mean and it has got standard deviation sigma. And these are important parameters for understanding the shadowing aspect.

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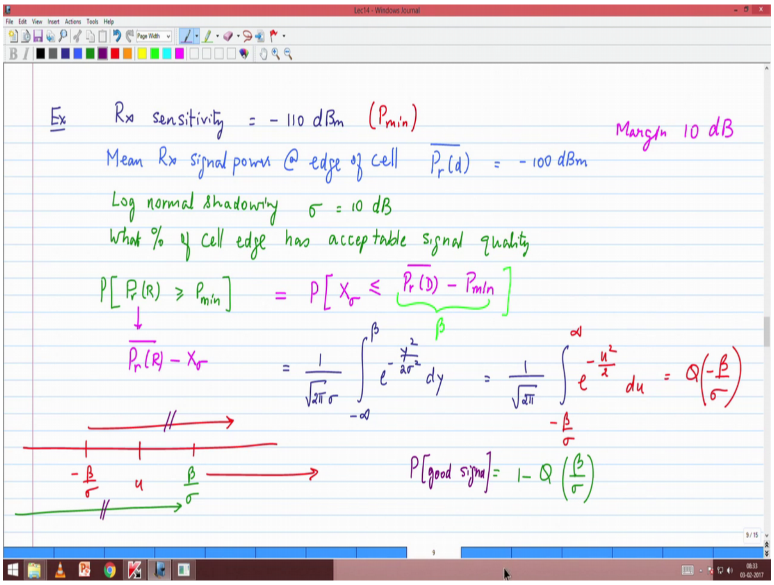


So, now may be to anchor all of what we have said so far. Let us look at an example. The example will help us consolidate whatever we have said so far. So, the example says that the received sensitivity that my system is designed for is minus 110 dBm. So, this my P min I should make sure that I do not exceed I do not go below threshold. I have designed my system with the margin. So, the statement the next statement, I would like for you to interpret my statement is that the mean received signal received signal power at cell edge of cell or whatever I consider as the edge of my coverage of my base station is minus 100 dBm. So, this is on average, this is the P r of d bar. What have I done what how much margin have I given 10 dB margin.

So, the margin that I have designed my system for is 10 dB. Now, this 10 dB good enough or 10 dB is not good too much too little the important question you will be what is the standard deviation of your log normal shadowing. So, log normal shadowing we are given is has a standard deviation in this particular example just for illustration purposes has got a sigma of 10 dB. So, standard deviation is 10 dB I have given myself a margin of 10 dB. Now, the key question is what is the probability that I get good signal quality, what percentage of my edge of cell gets good signal power?

So, basically this is a question that. So, what percentage of the cell edge has good coverage has acceptable coverage, so that is the problem that we have to design for. And again it is a very, very practical computation that you want to want to understand has acceptable signal quality. So, we just are one step away from the answer because the probability that you have good signal is the same as probability that your received signal at your cell edge. Let us say that is at a distance I think we have been using r as our let be consistent of your cell edge the average of that or no, the received signal at your cell edge must be greater than or equal to P min that is the condition that we have been asked to characterise.

Now, P r of d at a distance d we know is given by P r at a distance R average value minus x sigma. So, again use this information which says this is a same as saying the probability that x sigma in the previous case I was talking about outage this time talking about good signal that your shadowing is less than the margin basically we are asking the complementary question. So, this less than or equal to P r of d average minus P min, basically this is still my margin. So, the question is being asked in this particular problem is a complementary one, but exactly the same thing.

So, the answer would come along the same lines as before it would come in the form of an integral 1 by root 2 pi sigma. Now, what are the limits of integral minus infinity to beta e power minus y squared by to sigma squared dy. Now, if I wanted to convert it into a Q function then I would do a change of variable and get it into the normal form 1 by root 2 pi minus infinity to I can I can do two things, I can just do the scaling, I can also do a change of the sign. So, I can also do a minus y by sigma.

So, if I do both those changes, again just I am sure you are very comfortable with those types of manipulations, what you will get is minus beta by sigma going all the way to infinity. So, minus infinity becomes plus infinity, beta becomes minus beta by sigma e power minus u squared by 2 du. So, basically if I were to look at the continuum covered by the variable u this is 0, we are asking for the integral to go from minus beta by sigma.

So, this integral is Q of, so basically we are asking for this integral, so this integral is nothing but Q of minus beta by sigma am I right?

Student: Sir.

Yeah.

Student: (Refer Time: 33:39).

Yes so that is the way to interpret Q of a negative value you interpret it in the following way because the Gaussian the normal distribution is symmetric. There is a point which is beta by sigma, it is a same integrating from minus sigma beta by sigma to infinity is a same as integrating from minus infinity to beta by sigma that is another way of doing that, and integrating from minus infinity to infinity is 1. So, therefore, what we say is the integral from minus beta by gamma all the way to infinity can because of symmetry I can take it as 1 minus this, so which basically says that its 1 minus Q of beta by sigma.

So, basically Q of beta by sigma will go from here to here to infinity 1 minus that is the integral value that we are interested in because this and this are in the same value. And 1 minus the Q of beta by gamma gives us the same value also. Now, these are just stational things in case you are not familiar with the Q function just spend a few minutes and you should be able to get the following expression. So, this is the probability of good signal the probability of good signal is given by this number.

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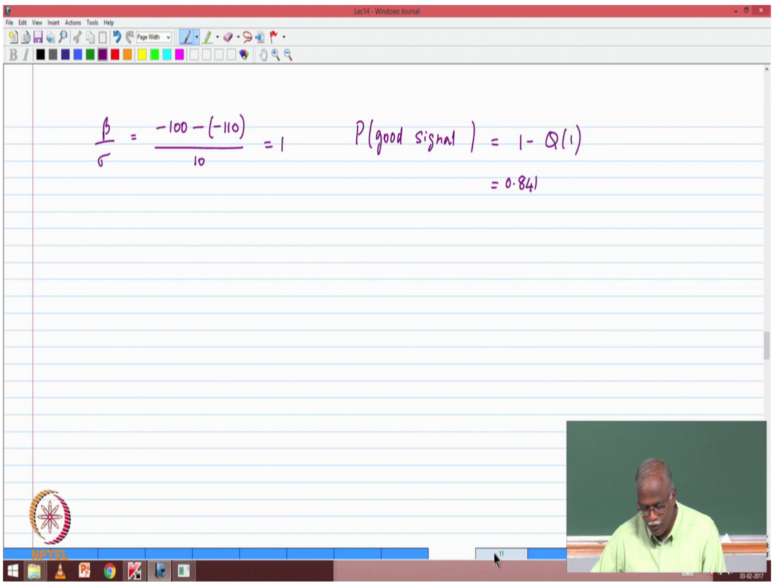

So, let us try to interpret it and try to apply to get some insight out of it always our examples are intended to produce some good understanding. So, what is beta by sigma in our case, beta by sigma beta says it is the margin my minimum signal strength that I can I will I have designed my system for is minus 100 dBm, my P min is minus 110 dBm. So, my margin is actually 10 dB minus 100 plus 110 divided by sigma the sigma is also 10, in this case my beta by sigma came out to be 1. So, probability of good signal under this condition under the assumptions that we have made good signal, good signal is given by Q 1 minus Q of 1. So, basically this is given this you can you can look it up in tables this comes out to be 0.841. So, 84 percent of your cell boundary has got good coverage.

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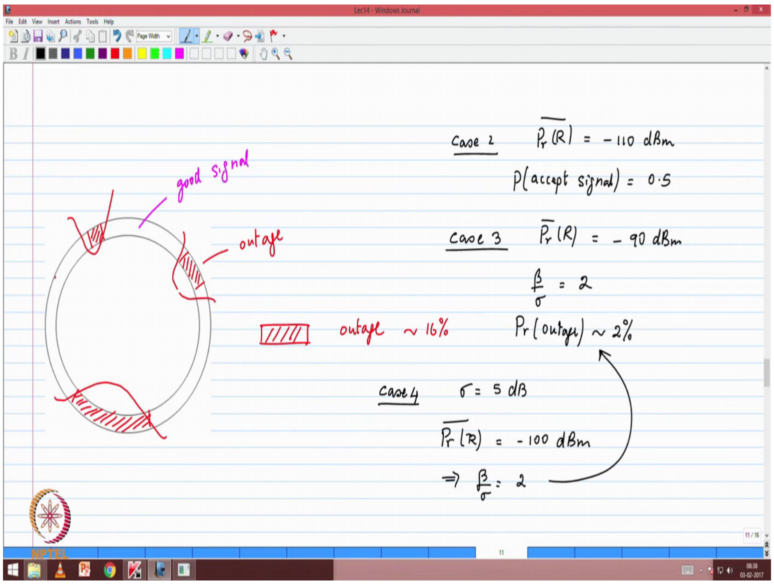

So, how do we visualise it for intuitive purposes. Think of the cell boundary as a small annular region. We do not know exactly where these outages will occur, but what we are saying is the shaded portion, those portions are shaded are places where you have outage, it is approximately 16 percent of the cell boundary has got the outages. And the rest of the parts are good signal. This is outage portions. And again that is why at these points if you want to actually look at the coverage, the coverage will actually go in because at the boundary you do not have good coverage. So, actual the coverage actually ends before you reach the boundary. So, at these points you will find that these cell coverage is actually going in.

So, the intuitive understanding and the mathematical characterisation come together to help us understand what is the impact of shadowing where will it affect you, it can affect you anywhere in the cell. But where do you want to characterise it, I want to definitely characterise it at the cell boundary because I want to know what percentage of my cell boundary is going to have acceptable signal strength. So, let us try a few variations of the of the example. Again this is more for practice suppose my received signal P r at the cell boundary P r of r bar I had designed it for minus 110 dBm is this a bad good design bad design, what is my margin.

Student: (Refer Time: 38:29).

Zero.

Student: Zero.

Zero so that means, what do what do will I get I will get Q of 0. So, what is the probability that I have good signal 0.5, only 50 percent of my area is going because I am not designed for margin, because it is a Gaussian perturbation. So, half the time it will be higher half the time it will lower probability of good signal. Well I should not say good signal it should be acceptable signal, probability of acceptable signal is only going to be 0.5. Now, let us push this a little bit more for clarity and for reinforcing what we have studied. So, if I knew this was the problem and I actually took extra precaution to design it for minus at minus 90 dBm then my beta by sigma comes out to be 2. And in such a case, the probability of good signal is 98 percent or let me write it the other way the probability of outage in this case is approximately 2 percent, again a very good design.

So, what does it tell me make sure you know what is the standard deviation of your shadowing and accordingly design your margin. So, the last example actually reinforces the statement that I just now made, because if I knew my sigma was 5 dB standard deviation of my shadowing was 5 dB, then I know that P r of dr bar equal to minus 100 dBm will give me beta by sigma equal to 2, which will give me the same outrage probability. Why did I do that because I know that my standard deviation is only 5 dB. So, therefore, I can be a little bit more aggressive in terms of my margin and reduce my margin and still expect to have good signal quality.

So, as I mentioned all the mathematical tools that we are going to develop is to make sure that it actually addresses the physical problem that we are we are working with. So, intuition and the mathematics has to go hand in hand to help us understand and work with the mechanisms that we are facing. Any questions on the aspect of the characterisation of the shadowing and also how do we interpret the margins in light of the standard deviation, if you anything which is puzzling or not clear please feel free to ask. So, now we addressed the issue of how the outage probability gets affected by shadowing. Again we will come back to talking about the propagation models, propagation models are more of a somebody gives you a formula or a graph and you use that to calculate P r of d bar

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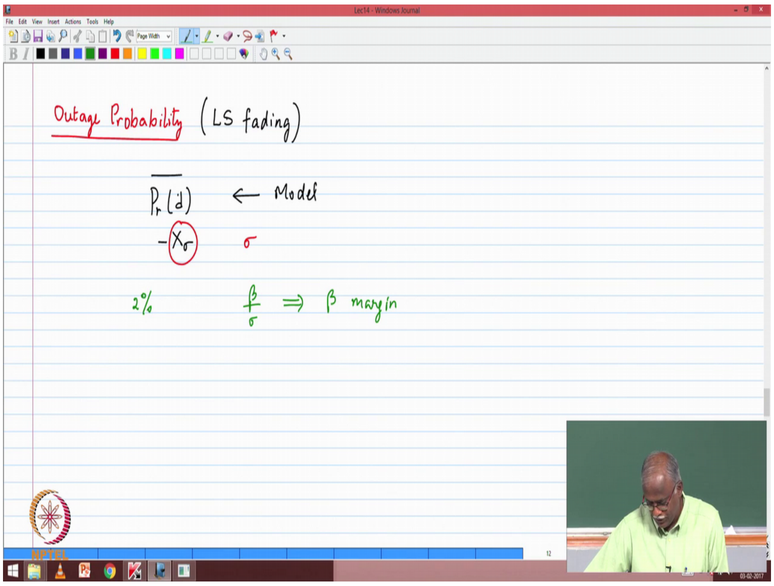


So, basically all the propagation models are giving you is the received signal strength at any distance d average value. This is what the model is going to give you. Now, on top of that if you have to say that I am going to have P r of d minus x sigma, I need to know what is the standard deviation I need to know. Based on that based on what is the quality want is satisfied let us say you want to have 2 percent outage or 5 percent outage at you cell boundary you then know how to design you beta by sigma which then tells you what should be you value of beta which is your margin.

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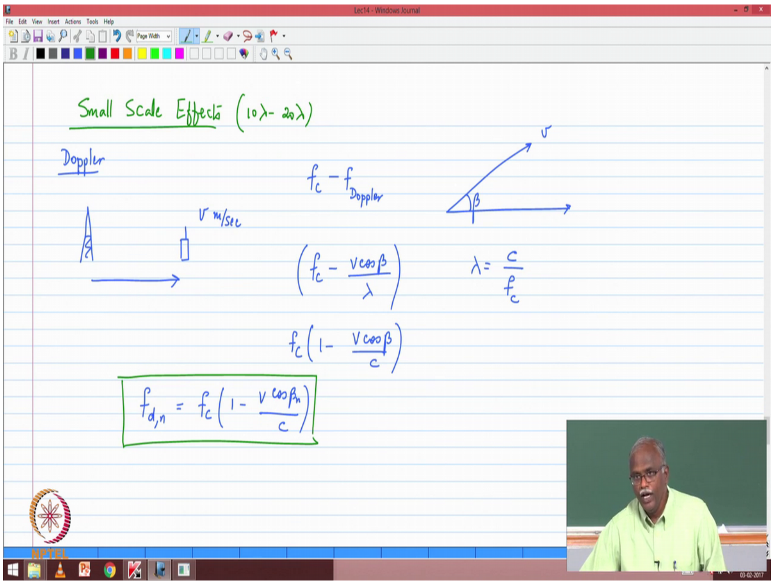


So, it is a very, very useful tool it is a combination of path loss model and our understanding of what shadowing is doing in terms of the additional impairment that it introduces to the path loss. Now, we go back to our small-scale fading, we move over from the large scale to the small-scale effects, so small-scale effects. These are the perturbations of the signal that that comes about because of small movements of the order of 10 lambda to 20 lambda. And obviously, these are things that are affected by two things one is multipath the other one is my mobility the Doppler.

So, let us get a basic common understanding of Doppler's, so that all of us are on the same page as to what we refer to by Doppler and this impact on us. So, here is the base station the mobile. This is the direction of propagation. Now, if it turns out that my mobile is actually moving away from it radially in the direction of the propagation of the electromagnetic wave. And it is moving with a velocity v meters per second then I get a reduction of my carrier frequency, because I am moving away by the Doppler f Doppler is my shift in the career frequency. It looks like I have a frequency offset because my Doppler got shifted, but in most practical scenarios I may not be going radially away I may be going at some angle. So, this is v, this is the direction of the radiation the radial direction and angle beta. So, the direction or the shift in the frequency in this case becomes f c minus v cos beta by c v cos beta by lambda that is the shift in the frequency we will incur.

Now, lambda is nothing but the speed of light divided by the career frequency. So, I can rewrite this as f c 1 minus v cos beta by by c. Now, if I were going towards the base station cos beta will become negative, and therefore I will get a increase in or positive shift in my carrier frequency. So, now, I do not know which direction I am moving, I could be moving towards the base station, I could be moving away from it. So, for an arbitrary is system this is what we write down saying, I do not know the direction. So, the Doppler shift for the nth component of the multipath n stands for the nth component of multipath. This is given by f c into 1 minus v that is a velocity with which I am moving cos beta n the direction of beta n and again it could be positive negative everything determined by this divided by c.

So, basically our understanding is that each of the multipath components is a signal at a carrier frequency, but because we are at a sufficiently high frequency these Doppler shifts must be accounted for. And therefore, we look at the direction of propagation direction of motion of the vehicle and then say that this particular multipath component which is coming at an angel beta to my direction of propagation is going to have a Doppler shift given by this expression.

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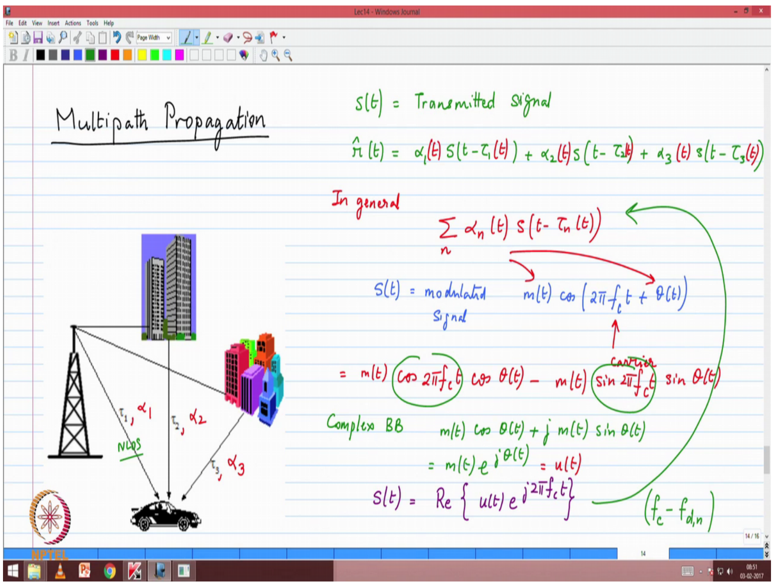


So, now going back to a very familiar diagram. So, the expression that we have for this is r of t - the received signal. Let me write it as r hat of t, r hat of t is the received signal. Again this basically showing that there is a line of sight, think of this as non line of sight also. There are three paths each of them have got gains alpha 1, alpha 2, alpha 3 and because of their propagation distances, they have different time delays tau is the delay of the first path tau 2, tau 3.

So, how would I write down the received signal for this particular system? So, it will be alpha 1 s of t is my transmitted signal. Transmitted signal s of r hat of t is alpha 1 s of t minus tau 1 plus alpha 2 s of t minus tau 2 plus alpha 3 s of t minus tau 3, so that is my way of characterising again this is a modulated signal. So, amplitude and the delay is what I have captured in this system. So, in general the alpha one and tau one may not remain constant because I am moving, so things could change. So, this actually becomes all of them become functions of time.

So, therefore, the gains are the function of time the delays are the function of time, the number of multipath components is also a function of time. So, the general expression for in presence of multipath in general the expression that is that is given is summation over n where n is the number of multipath components. It is alpha n of t s of t minus tau n of t. So, the this is basically the super position of all the multipath components that we are encountering in the multipath channel.

Let me quickly get into the heart of our discussion. I want to introduce the complex baseband form. So, S of t is the modulated signal or the transmitted signal modulated signal. And this will be typically of the form a message signal m of t, message component comes out m of t and theta of t given by where m of t is carried by the amplitude and then theta of t is carried by the phase. So, cosine 2 pi f c t plus theta of t. So, the information or the message part is carried by m of t and theta of t and the career is f c, this is my carrier frequency.

So, the base band representation that we develop must not have f of c, it should have m of t and theta of t I do not want f of c because f c can be 950mega hertz it can be 2 giga hertz, it really does not matter. So, the essence of it is the baseband representation. So, how do we translate from the modulated signal into the baseband? That the complex baseband is a one step process, but you need to do that little bit carefully. So, let me just highlight the method. So, expand this expression m of t cosine 2 pi f c t cosine theta of t minus m of t sin 2 pi f c t the quadrature carrier sin theta of t.

So, now what I want to do is I want to leave out the terms containing the carrier frequencies. So, what I am left with is remove that from the equation then I get the following complex base band representation which I am sure you are familiar with. Complex base band representation says that my complex base band signal is m of t cosine theta of t carried on the i branch plus j times m of t sin theta of t this is carried on the Q branch. So, basically the I component and the q component of my signal. So, this can be represented as m of t e power j theta of t that is my complex baseband signal. If I call this as u of t is I call this as u of t, then the relationship that I am sure each of you is familiar with is S of t the modulated signal is given by the real part of the complex baseband signal u of t e power j 2 pi f c t. This is the complex baseband representation of the modulated signal.

So, basically what I need to do is to take this S of t and substitute into my multipath expression. Remove all traces of the carrier frequency then what I will be left with is a complex baseband representation of my modulated signal in the presence of multipath. So, I would very much encourage you to try that substitution, it is not difficult, but you have to be careful in making the substitutions. And one more thing that you have to keep in mind is that when I substitute back each of these multipath components, they are coming in different directions.

So, therefore, for each of them, you must actually apply the Doppler correction minus f d n. Please make sure that it is no longer just the basic carrier frequency it will be the carrier frequency shifted by its Doppler. So, the complex base band taken back applied in a multipath environment keeping in mind the Doppler shifts of each of those components. It is a simple step of substitution, but we have to do it carefully because once you do that then you get a very rich insight into what exactly is the impairment that you are fighting against. Quick question yeah.

Student: Sir, what are explaining while you do those red arrows I do not know (Refer Time: 53:32) the relation between.

This red arrows.

Student: Yes sir.

Basically the information user specific information is carried in m of t and theta of t. So, the information content is carried in m of t and theta of t f of c is the f c is the carrier. So, that is the general representation of. So, if it is only an amplitude modulated signal then it will m of t cosine 2 pi f c t; if it was only a phase modulated signal, it will be cosine 2 pi f c t theta of t. But you have both amplitude and phase information it will be n m of t component and a theta of t component.

Thank you, we will pick it up from here in the next lecture.