

**Functional Programming in Haskell**  
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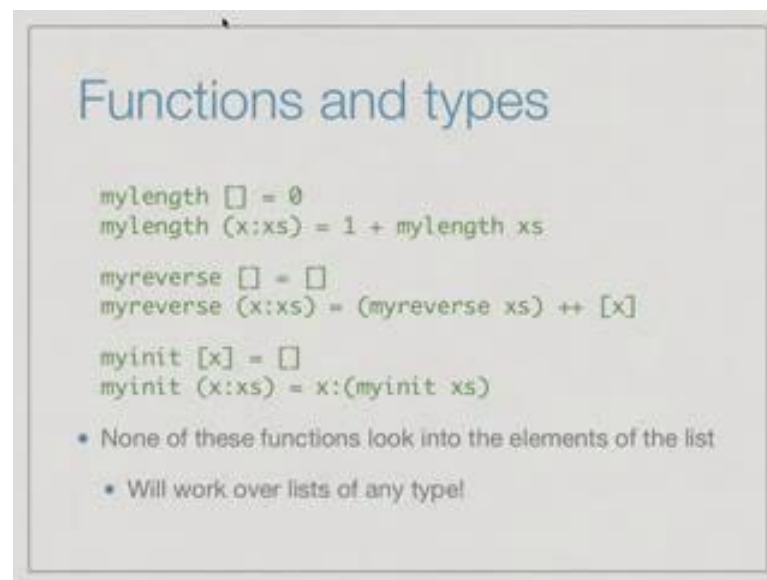
**Module # 03**

**Lecture – 02**

**Polymorphism and Higher Order Functions**

Let us take a closer look at types in Haskell.

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Functions and types

```
mylength [] = 0
mylength (x:xs) = 1 + mylength xs

myreverse [] = []
myreverse (x:xs) = (myreverse xs) ++ [x]

myinit [x] = []
myinit (x:xs) = x:(myinit xs)
```

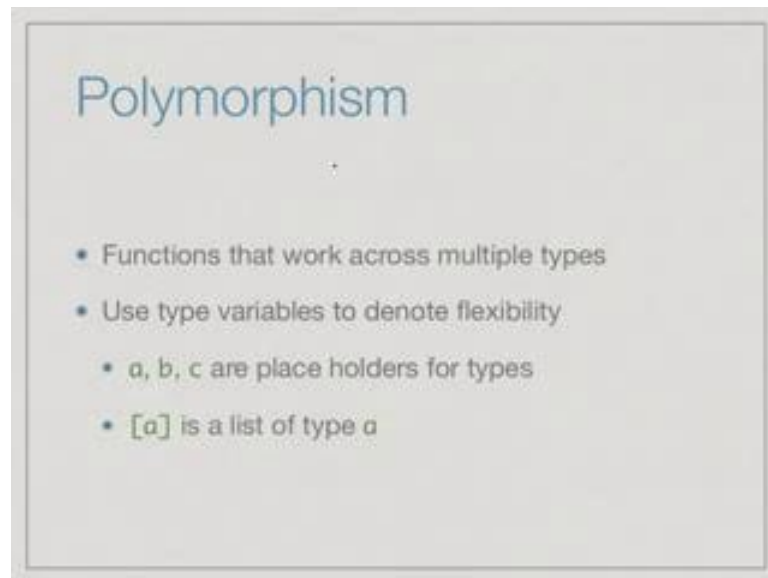
- None of these functions look into the elements of the list
- Will work over lists of any type!

Early on we wrote some functions on list, which we defined as functions over list of integers. Princess we wrote this function length, which inductively assign 0 to the length of the empty list and adds 1 for each element in the list. You also wrote this similar inductive function to reverse a list. And finally, we wrote the reverse of the head function, which takes the last element; I mean the tail function, which takes the first segment of the list, leaving of the last element.

Now, all of these functions have one thing in common, which is that they do not actually bother about the values in the list, so it is really a structural property of the list. The length of the list really just depends on how many elements there are, when you reverse the list, we just take the elements in the list and reverse the order. Similarly, when you take the initial segment of a list, you just drop the last element.

So, in all these things, you can actually assume that similar things will work regardless of the type of element in the list. So, if you think of the list as a list of boxes in some sense and then if you need a value in the list, you need to open the box, so none of these functions actually need to open the boxes, they can just move the boxes around or select few of the boxes or one of the boxes and so on.

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So, it would be nice if we could assign within the type system that Haskell uses for each function. The type which will allow a function to work across multiple input types like this. So, we would like reverse for instance to work on list of Int or list of float or even in list of list. So, long as we underline value is of a uniform type, so in Haskell, we use type variables, usually `a`, `b`, `c`; letters like these as place holders for type. So, if we say that a function takes an input type `a`, it means `a` can stand for any input. And if `a` stands for any input, then if you puts square brackets around `a`, then we denote a list of that type.

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## Polymorphism ...

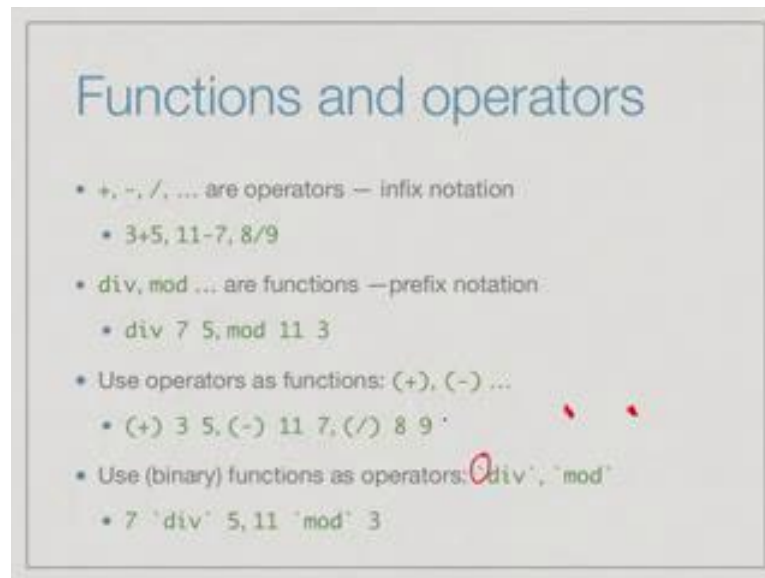
- Types for our list functions:  
 $\text{mylength} :: [a] \rightarrow \text{Int}$   
 $\text{myreverse} :: [a] \rightarrow [a]$   
 $\text{myinit} :: [a] \rightarrow [a]$        $\text{head} :: [a] \rightarrow a$
- All  $a$ 's in a type definition must be instantiated in the same way

So, if you go back now to our list functions, what does my length do, my length takes any list and produces the length, the length is a number, so the output type is Int, the input type is unconstrained, it can be a list of any type of a. So, we write the type as list of a to Int, reverse on the other hand, takes a list and produces another list. So, once again it takes an arbitrary list, but it produces a list of the same type.

So, it is important that the a on the left and the a on the right are actually the same a, it means that, if I give it an input of list of Int, it will produce the output list of Int. If I give it a list of float, it will produce a list of float as output, you cannot change the type, but the type itself is not fixed in advance. And finally, if you look at myinit, so what it does is it actually produces, so this is my mistake here.

So, it takes again a list of a and produces the list of a and if you wrote a function like heads for instance, then it would take a list of a and produce a single value of type a. So, the important thing to notice is that, if I have a variable, then wherever it appears in the type it must be instantiated by the same type.

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## Functions and operators

- `+`, `-`, `/`, ... are operators — infix notation
  - `3+5`, `11-7`, `8/9`
- `div`, `mod` ... are functions — prefix notation
  - `div 7 5`, `mod 11 3`
- Use operators as functions: `(+)`, `(-)` ...
  - `(+) 3 5`, `(-) 11 7`, `(/) 8 9`
- Use (binary) functions as operators: `div`, `mod`
  - `7 `div` 5`, `11 `mod` 3`

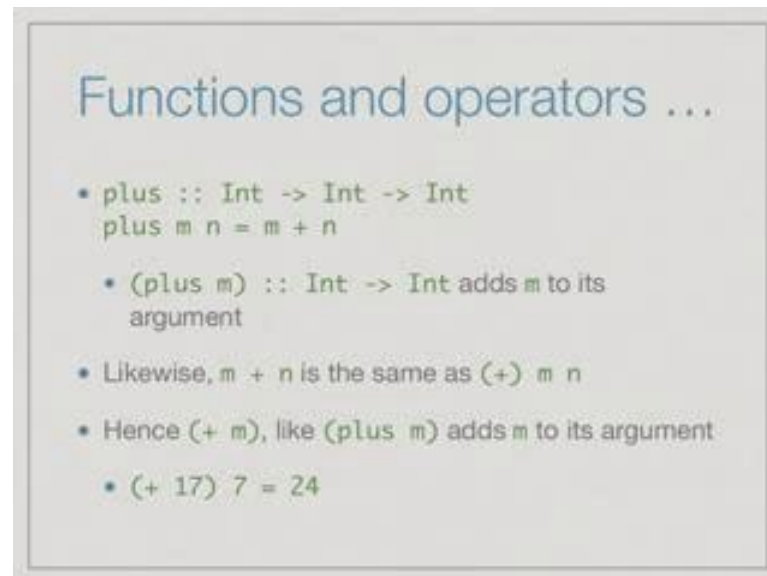
So, let us digress a little bit about notation of operators and functions in Haskell. So, we have seen arithmetic operators like plus, minus, divide, we also have I know Boolean operators like OR and AND, so we could have we know  $x$  and  $y$  and so on. So, these are operation switch operate on two values, but we write them in this infix notation; that is we put the operation in between the arguments. So, we have two arguments and the operator comes in between, so the operator is said to be in fixed.

On the other hand, we have seen functions, which are predefined like `div` and `mod`, which are written as functions. In other words, you write the name of the function and then, you write the arguments. So, this is what is called prefix notation, because the function comes before the arguments. Now, sometimes you may want to switch between these two notations or two views.

So, in particular, we can take any binary operator which we have defined and make it in to a function just by putting round brackets around it. So, the operator plus can be written as a function by writing round bracket plus. So, this expression plus 3, 5 is exactly the same as 3 plus 5 and so on. So, in this way, we can use this notation to convert operators to functions and we can also do the reverse, provided of course, the function takes two arguments, because otherwise it is not clear, what it means to be infix, infix means it is between two arguments.

So, we have binary functions like `div` or `mod`, then we can use this reverse code, this is the back code key, this is not the normal apostrophe but the back code, which we will find on your keyboard. So, if we enclose the name of the function with it is reverse code, then we can use them as operators. So, if you want to say `11 mod 5` and not `mod 11, 5`; then we write `11 back code mod 5`, so here we have written `7 div 5`, `11 mod 3` and so on.

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Functions and operators ...

- `plus :: Int -> Int -> Int`  
`plus m n = m + n`
- `(plus m) :: Int -> Int` adds `m` to its argument
- Likewise, `m + n` is the same as `(+) m n`
- Hence `(+ m)`, like `(plus m)` adds `m` to its argument
- `(+ 17) 7 = 24`

So, this is useful in the sense that we have seen, suppose we have written a function `plus`, which takes two integers and adds them. What we saw is that because of currying `plus` actually consumes the arguments one at a time. So, in an intermediate stage, when it has consumed the first argument, we have a new function which we can write call `plus m`, which actually takes, whatever it takes as second argument, we named with the `n` and it will add `m` to.

So, `m` is now fixed, so in the same way, we can think of this using the normal `plus` operator. So, `m plus n` is the same as this round bracket `plus m, n`. So, this round bracket `plus` is the function which exactly like the `plus`, we wrote above, which takes two arguments and if you take one of them, then it becomes the functions `plus m`. So, `plus m` adds `m` to its arguments.

So, you can try this out in the Haskell interpreter in `ghci`. So, if you take something like `plus 17` and you applied it to `7`, then `plus 17` will add `7` to `17` and give you `24`. So, using the fact that we can convert operators to functions by using the round bracket, we can

also get partially instantiated functions, where one of the arguments has been fixed and use these, when we manipulate functions in Haskell.

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**Higher order functions**

- Can pass functions as arguments
- `apply f x = f x`
  - Applies first argument to second argument
- What is the type of apply?
  - A generic function  $f$  has type  $f :: a \rightarrow b$
  - Argument  $x$  and output must be compatible with  $f$

*Handwritten:*  $f :: (a \rightarrow b) \rightarrow a \rightarrow b$   
*Handwritten labels:* "apply" under the first  $f$ ,  $f$  under the first  $a$ ,  $x$  under the first  $a$ , and  $f\ x$  under the second  $b$ .

Now, manipulating functions in Haskell takes as to this notion of a higher order function. So, in Haskell, there is no particular problem or specific thing that we need to do to pass a function as an argument to another function. So, here is the most obvious function, we could write like this. So, we can take a function called `apply`, which takes two arguments, when applies the first one to second one.

So, it really needs the function as this first argument, it reaches the value and second argument and applies the function at gets to the value at gets. So, just says given  $f$  and given  $x$  return the value  $f$  of  $x$ . So, here for instance, if we say something like `apply function plus 17, 7`; then this should reduce by the application of `plus 17, 7` to the value 24, this is the intention.

So, what is the type of this function, well the first argument is the function and we wanted to be possible to send all possible functions to `apply`. So, if we make no assumption about the function, then it takes an input type and it takes an output type, which may be different. So, generic function  $f$  as type from  $a$  to  $b$ , where  $a$  and  $b$  are possibly different types.

So, this no constraint a and b may be the same type, may be different types, we do not constraint it in any way. Now, given that f takes an input of type a and produces an output of type b, this constraints the type of x, because x is now going to be pass of f. So, f takes a is, so x must be an a and the result must be a b. So, the argument x and the output must be compatible with the choice.

So, this must be of type a and this must be of type b. So, this gives us the following type, so at this function, so this is not f but apply. So, apply takes as it is first argument, some unknown function f and it takes as a second argument, a value to be pass this function. And it produces an output, which is the application of x, f to x, which because f has type a to b, this is of type b. So, apply as type from a function a to b and an input of type a, produce an output of type b.

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The slide is titled "Higher order functions" in blue. Below the title, there is a diagram with handwritten text. It shows "fn" in green, followed by "|| compare" in red, and "list" in red. Arrows point from "fn" and "list" to a box labeled "Sort" in blue. Below the diagram, there is a list of bullet points:

- Sorting a list of objects
  - Need to compare pairs of objects
  - What quantity is used for comparison?
  - Ascending, descending?
- Pass a comparison function along with the list to the sort function

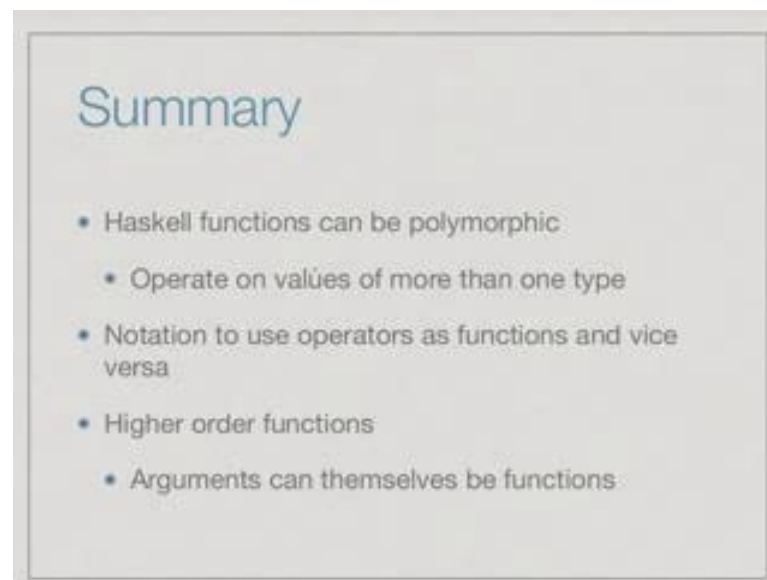
So, higher order functions will turn out to be very useful. So, here is a very familiar thing that we would have seen before. So, suppose we want to sort a list of objects, that the heart every sorting algorithm is a comparison between objects. So, we need to take pairs of objects and order them in the right way. So, in some situations, it might be important to know how to compare these objects, if we have pairs, then we use only one component, we use dictionary ordering and so on.

And also we might want to know and which direction to order, we want to order them ascending order and descending order and so on. So, to be fully flexible, what we would

like is that, we have the sort function and it should take actually two inputs, it should take the compare function and you should take the list and then, use this compare function to sort this list, according to the sorting algorithm.

So, by changing the compare function for instance, we can make it go from ascending to descending and so on. So, this is a natural situation in which we pass a function to another function and we will see other example of this.

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So, to summarize Haskell functions typically those we are seen, which are structural functions on list like things like take the length or reverse it or take the initial segment or a final segment can be polymorphic. In other words, they can operate on list of more than one type. We will also see that we can use this round bracket in the backward to convert operations to functions, operators to functions and vice versa.

And finally, we have seen that in Haskell, it is no big deal to pass a function to another function. So, we have higher order function. So, to speak for free and this is very useful and we saw an example in the context of sorting and we will see more examples in the lectures to come.