Model Checking
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Lecture – 41
Tree View of a Transition System

Welcome to Unit 9 of this course. In the last two units, we saw what is called Linear Temporal Logic- LTL and we also saw algorithms for model checking LTL on Transition System. In this unit, we would see a different logic called Computation Tree logic.

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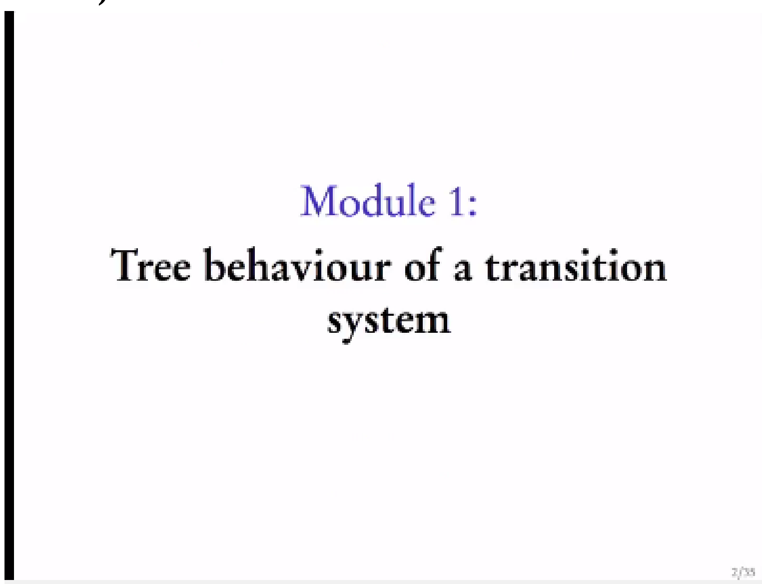

The first task would be to look at a tree behavior of a transition system this will be explained in this module.

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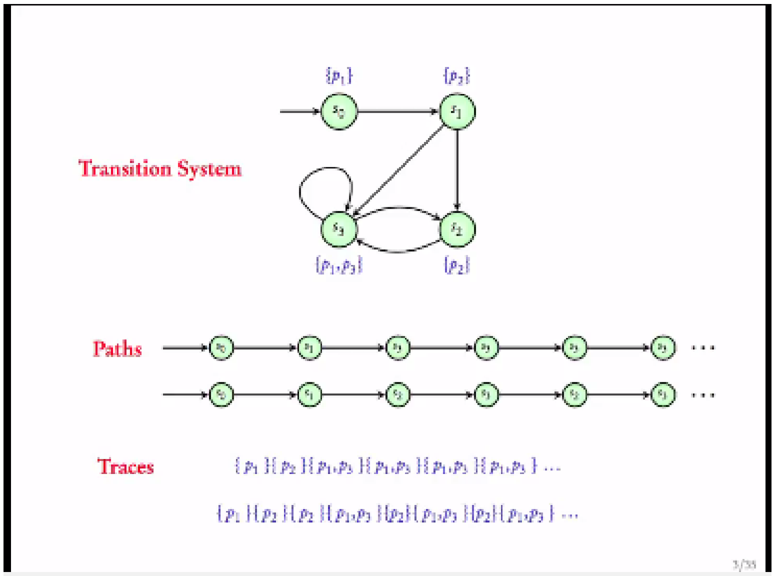

Consider this transitions system. These are the states and these are the atomic propositions which are labeled in each state. We have so far been looking at path in this transition system. For example, say this is S0, S1 S3, S3 and soon and then there is this part S0, S1, S2, S3, S2, S3, S2 and so on. So we have been looking at infinite paths. Once we have paths we can look at traces by restricting to the atomic proposition labels in state.

So S0 is labeled S1 is labeled by P1, S3 by P1, P3 and so on. So each path gives rise to a traces and this Traces is an infinite so we have been looking at ways of expressing sets of infinite words. This is what we had seen so far.

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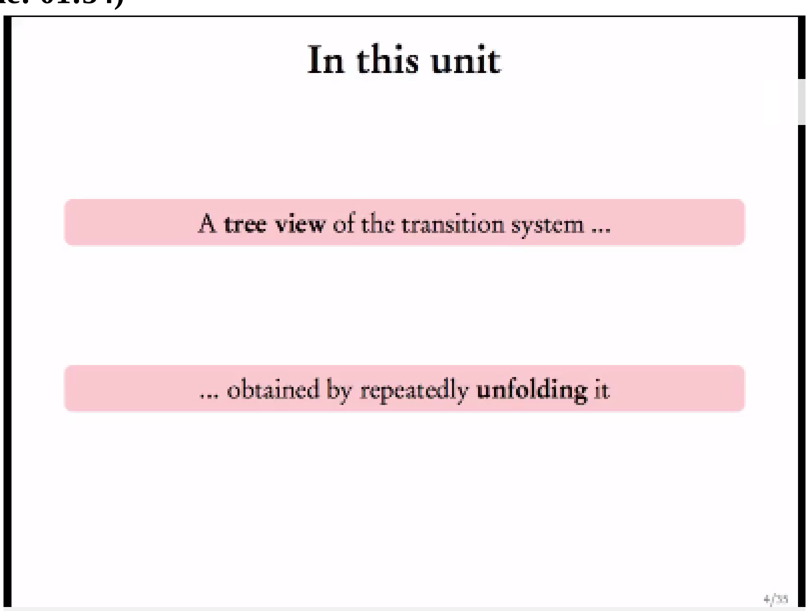

In this unit, we will view the transition system as a tree obtained by repeatedly unfolding the transition system. Let us see what we need.

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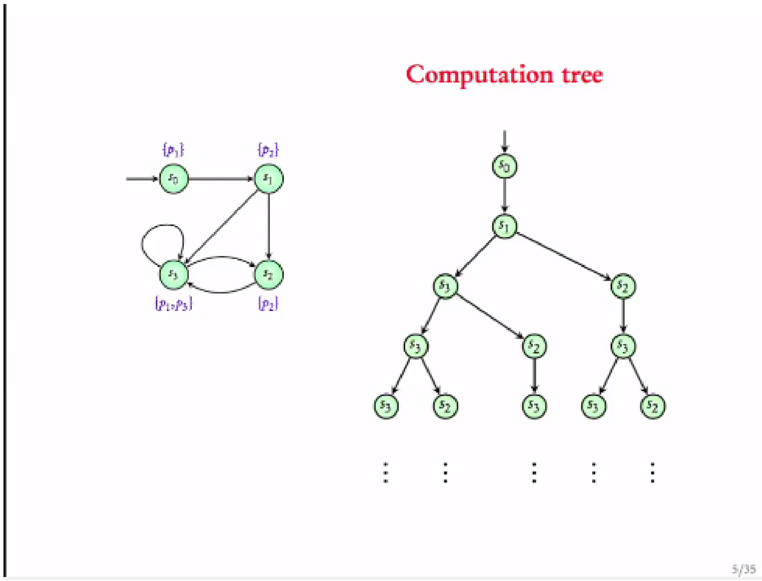

There is a same transition system let us start from this initial state and try to unfold the transition system. So at S0 there is only one transition which is S1. At S1 there are two transitions S3 and S2. So you have these two transitions going to S3 and S2. Now at S3, there are again two transitions going back to S3 and to S2. So instead of giving this as a loop we give it as a tree. So this is the S3 obtained after doing S0, S1, S3.

This is the S3 obtained after doing just S0 and so these two are differentiated. Similarly, at S2 there is only one transition going to S3 and so on. So this is given us the tree at any point there is a node and there are children corresponding to the exists in this transition system. This is an infinite tree, you can keep continuing this path is S3, S3, S3, S3 and so on and then you have S2, S3, S3 and so on. So you know how this transition system unfolds.

This tree is called the Computation tree of the transition system. It is telling us the process of execution of this transition system.

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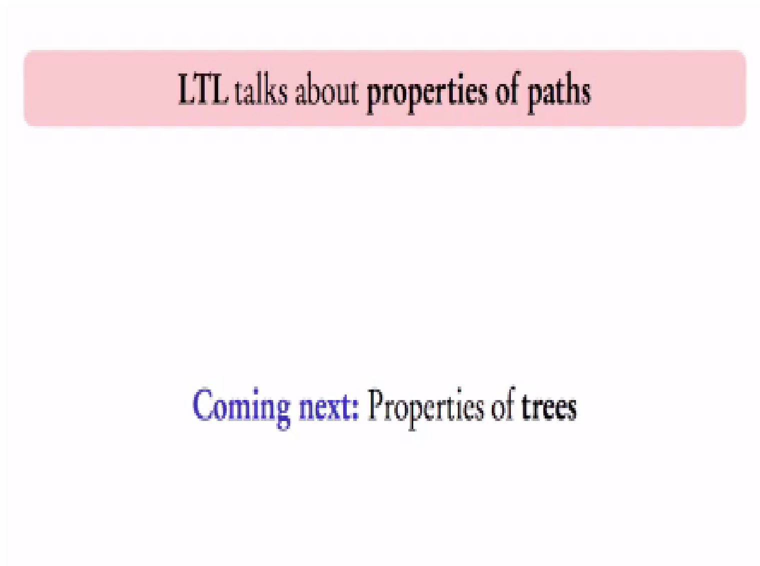

We know that LTL talks about properties of paths. We will now talk about properties of trees.

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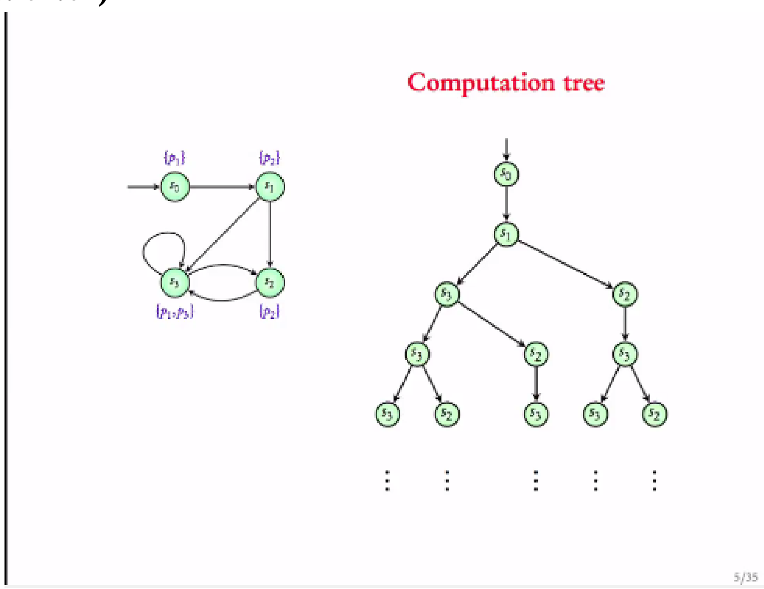

Suppose you have a tree like this we want to ask whether the tree satisfy certain properties not just paths. We want to talk about tree again this entire tree. Does it satisfy a property or not?

We will start off with many examples.

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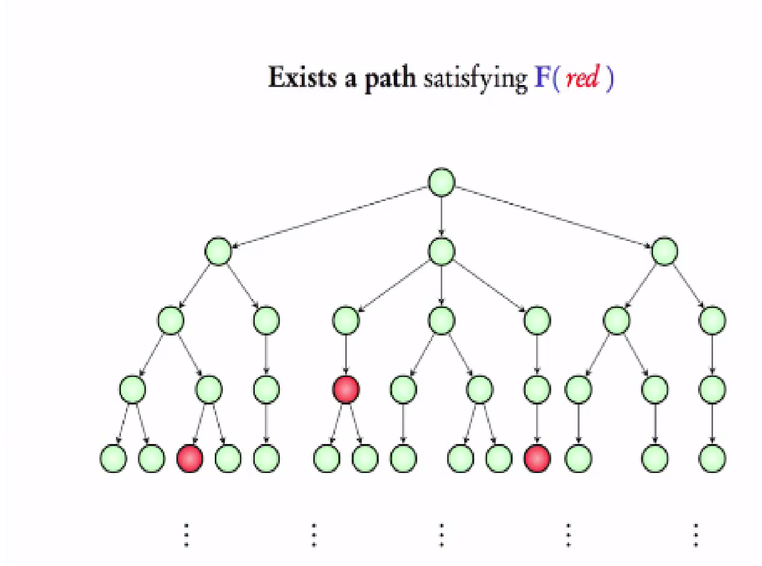

Look at this tree for clarity I have not given the names of the states also I have not given the labeling of atomic proposition on each state. Assume that there is an atomic proposition called red. So these states have been marked red. This tree satisfies the property that there exists a path which satisfies F of red. So look at this path. If you look at this path at some point of time red is true. So this LTL formula F (red) says that at some point of time red is true.

And in this tree there exists a path-- there are many paths where red is true but as long as there one path where F (red) is true this tree satisfies exists a path F (red). I will give these properties in English, later on we will see notations for these properties.

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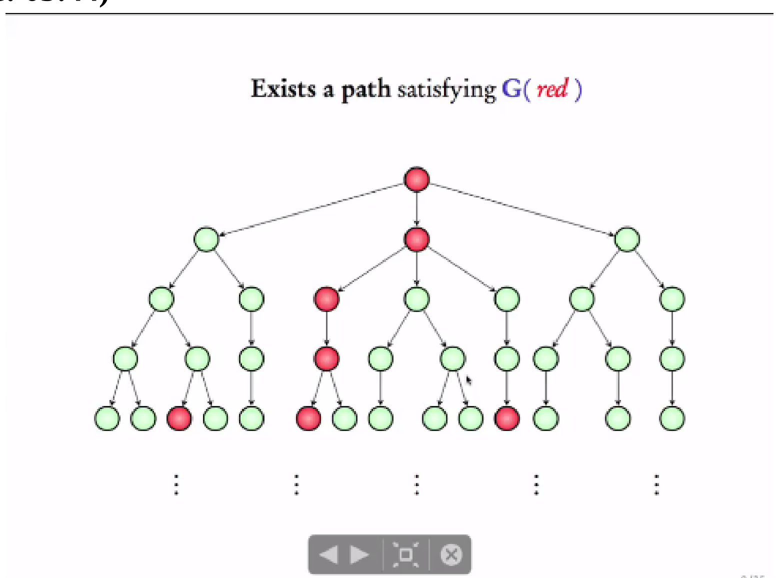

Look at this tree. Look at this path. Assume that this path can be extended to a path where every node is colored red that means every node every state in this path satisfies the atomic proposition red so there exists a path which satisfies G (red), correct? G of red is true on a path if every node in that path satisfies red. So here there exists a path which satisfies G (red).

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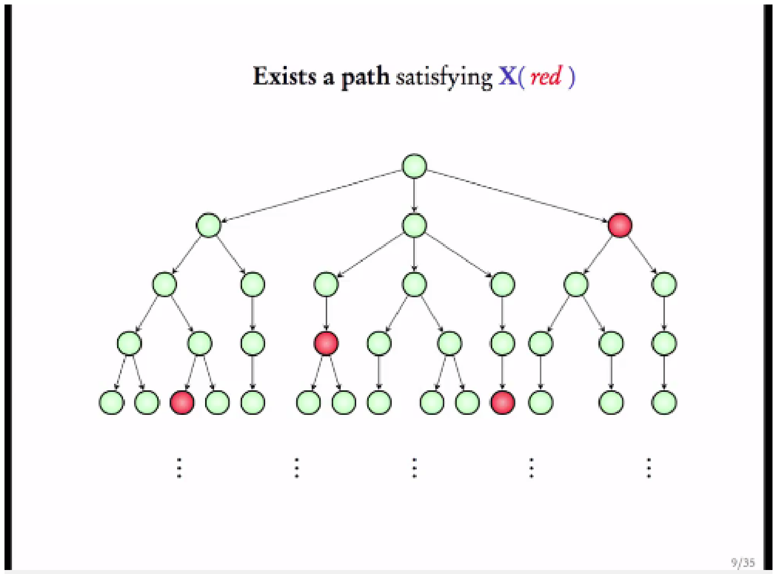

Look at this, this tree satisfies this property. There exists a path satisfying X (red). Look at this path. So some continuation of this path if you look at that path it satisfies X, this is first the initial state of the path and then the next state satisfies red. So X (red) is true on this path and in this tree there exists a path which satisfies X (red).

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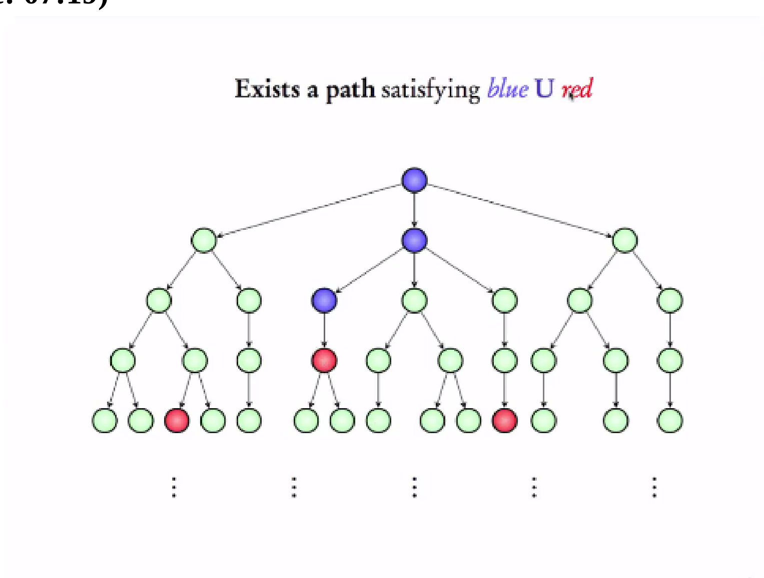

Let us look at this tree. Here there exists a path satisfying blue until red. Correct? This is the path some continuation of this path and in this path blue is true till the point where red becomes true. So-- and this can be expressed using blue until red and the fact that there exists a path satisfying blue until red is expressed here. And there exists a path which satisfies blue until red.

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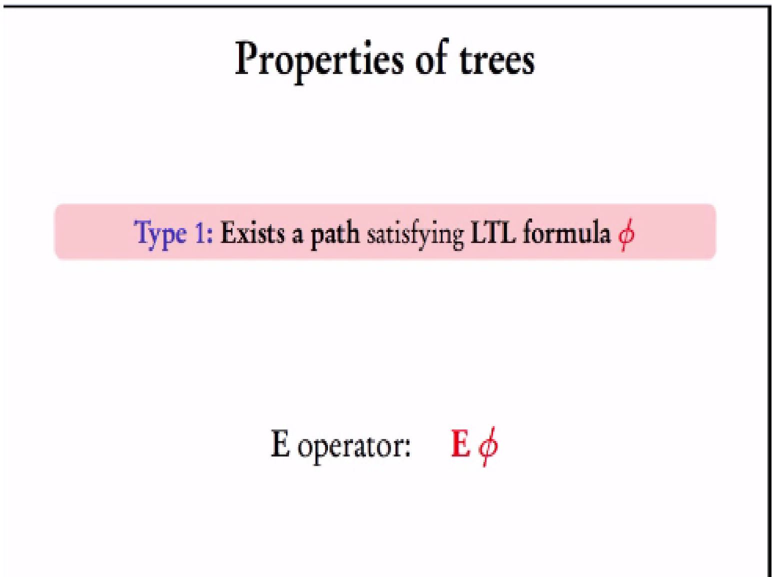

If you had noticed in the last four examples, we were saying that there exists a path satisfying some LTL formula phi. And LTL formula gives you a property or paths and for trees the first type of property that we want to talk about is there exists a path satisfying and LTL formula phi. And the notation for this is E-- E stands for Exists a path and then you can follow it up by saying by using an LTL formula.

E phi tells that there exists a path which satisfies the LTL formula phi. There could be some tree which satisfies this and some trees which do not satisfy it.

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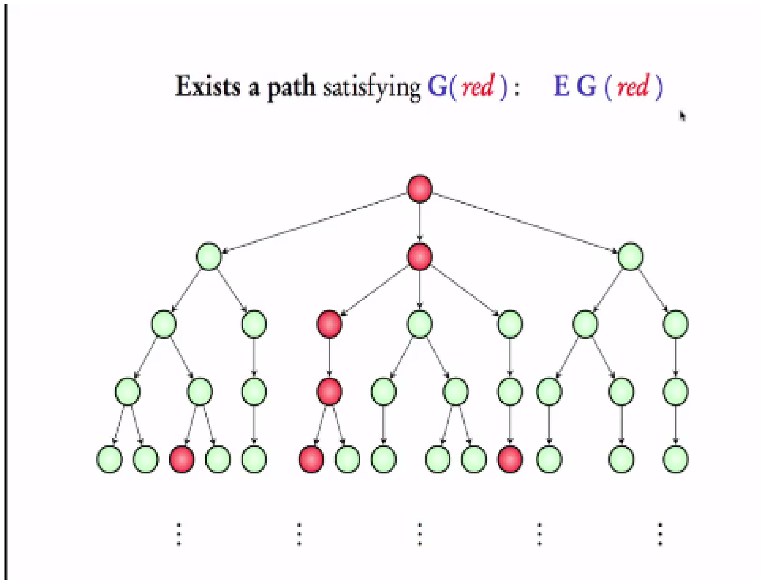

So in this notation exists a path satisfying F (red) will be written as E F (red). You know what F (red) E is the new operator that we have defined in this module. E F (red) is true in this tree because there exists a path satisfying F (red). Similarly, in this tree E G (red) is true. You know what is G (red) and now E stands for exists a path satisfying G (red).

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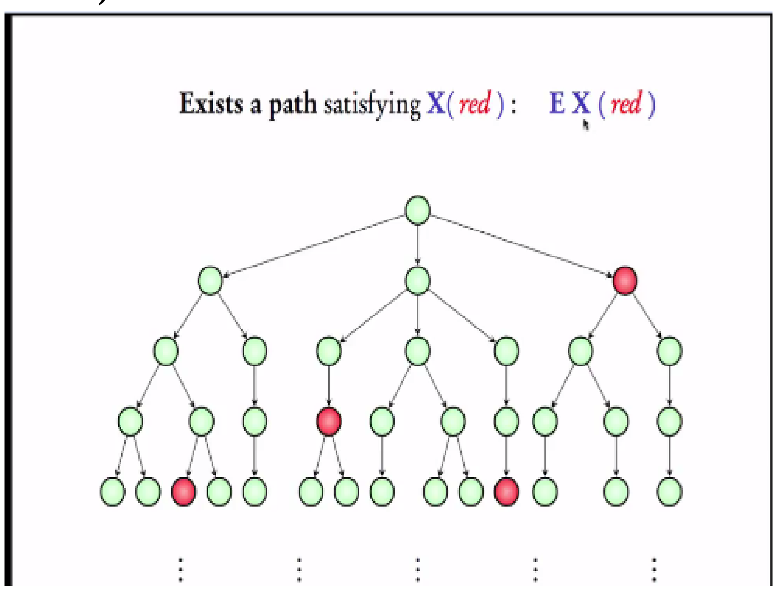

Similarly, this becomes E X (red) because there exists a path satisfying X (red).

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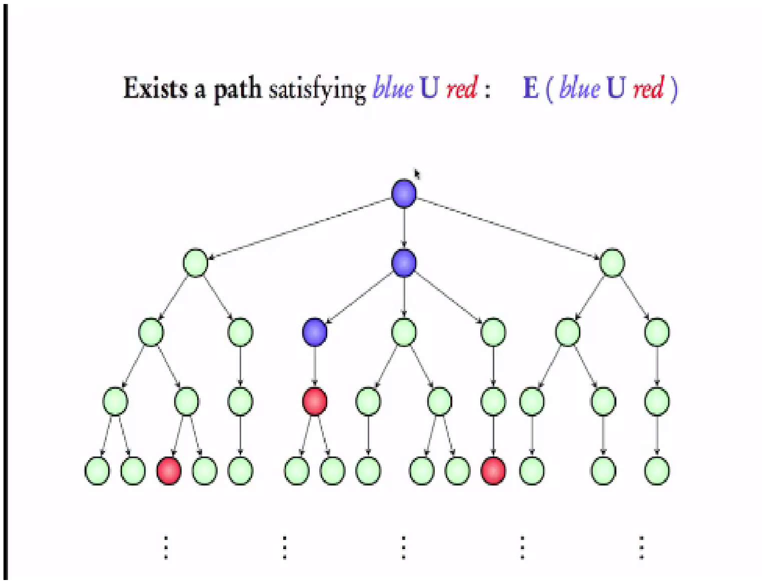

And finally this tree satisfies A of blue until red. So exists a path satisfying blue until red. This blue until red is a property of a path and this E operator will see that there exists a path which satisfies this property.

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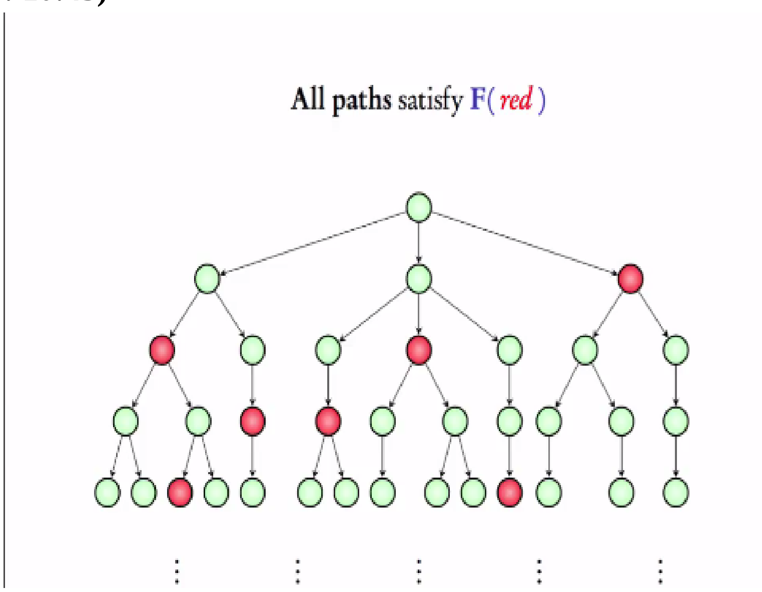

Now we have seen the E operator. Let us see one more type of property on trees. Look at this tree. This tree satisfies the property all paths satisfy F (red). You look at all paths in this tree. For example, look at this path any continuation of a path in this subtree will satisfy F (red). So there are many paths here one that goes left this one or this one the one that comes here and so on.

So any path obtained after this node will satisfy F (red). Similarly look at this path any path after this will satisfy F (red); similarly, any continuation after this will satisfy F (red); similarly, any path from this continuation satisfies F (red) and since here there is already a red any path obtained by continuing this will satisfy F (red). So in particular all paths in this tree will satisfy F (re)d.

So if you just look at some path in this tree and you write it down in a linear way you will see that red will be true at some point. So all paths in this tree satisfy F (red). Previously we were looking at exists a path satisfying F (red) now we say that all paths satisfy F (red).

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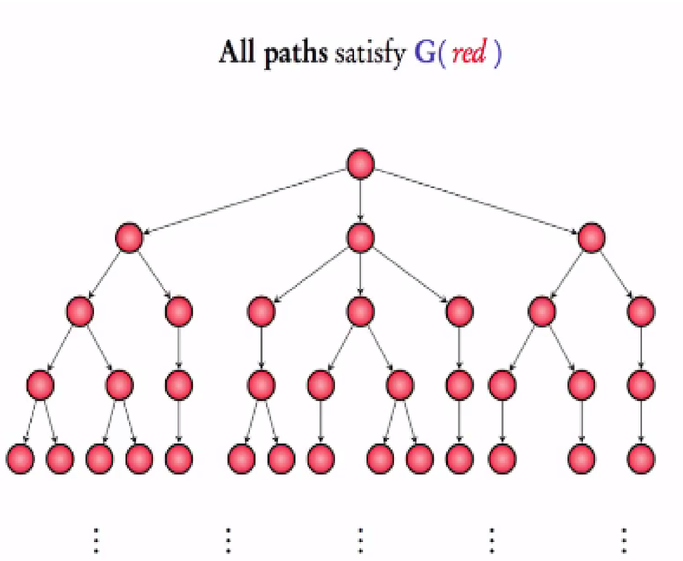

Look at this tree. All paths satisfy G (red). You look at any path and G (red) is true that means red is true all along this infinite path. So this tree satisfies all paths G (red). Okay, so this is a property, all paths satisfy G (red).

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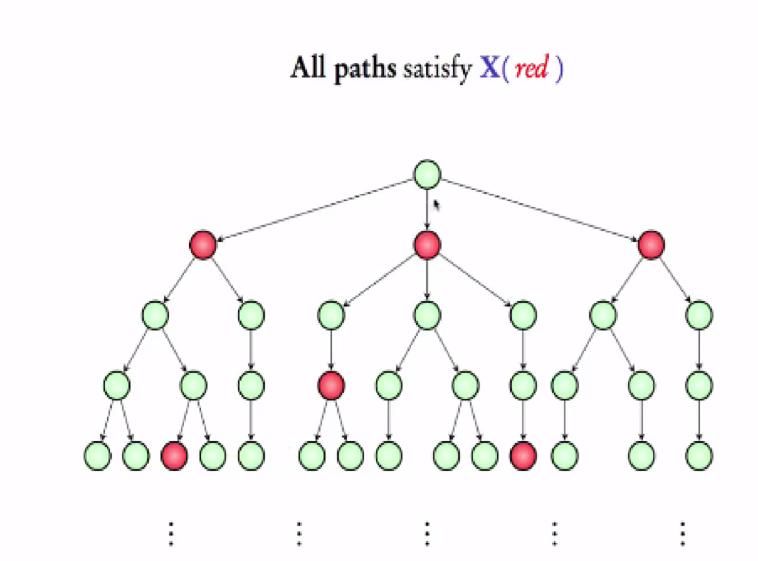

Look at this all paths in this tree satisfy X (red) because the children at the first level satisfy red. So you look at any path X (red) will be true in that path. So you take a path and write it in a linear way X of red will be true.

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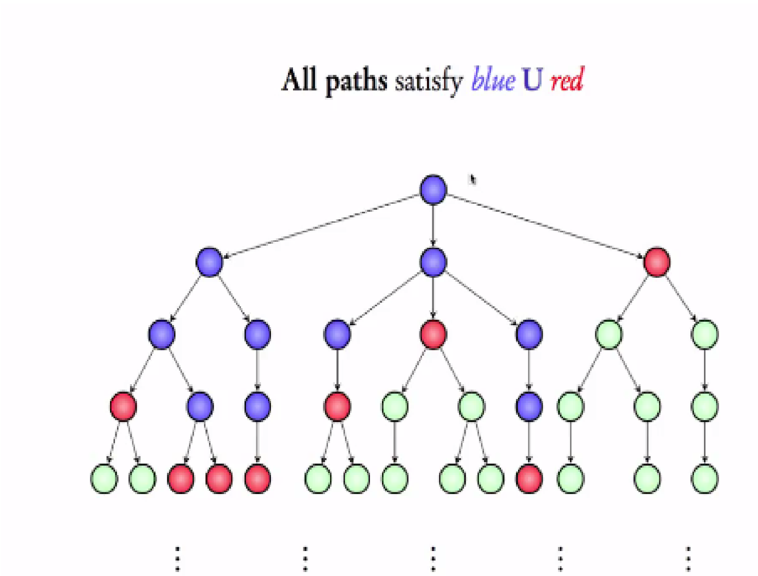

What about this? All paths satisfy blue until red because you look at any path till red is true blue is true. So here all paths that are continued after this state will satisfy blue until red similarly all parts after this continuation after this continuation after this continuation and so on. So all part in this tree-- you try to find some paths in this tree and every part in this tree will satisfy blue until red.

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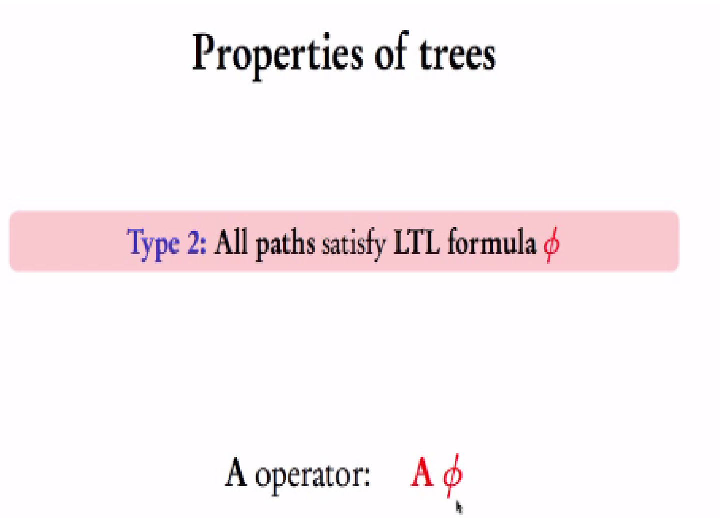

So this is the second type of property over trees. All paths satisfy LTL formula. So an LTL formula is a property over a path and over a tree we have this additional thing which says that every path satisfies this LTL formula. And for this we have the A operator. When you say A phi it means that all paths in this tree satisfy the LTL formula phi.

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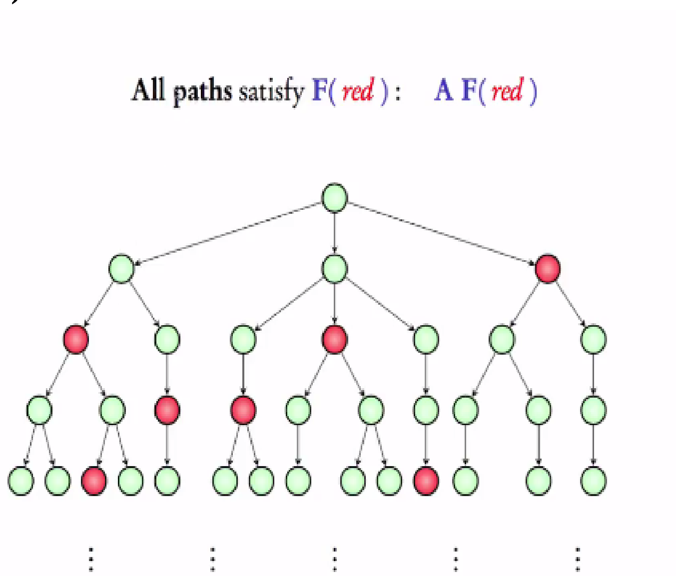

So this translates to A F (red), this tree satisfies A F (red).

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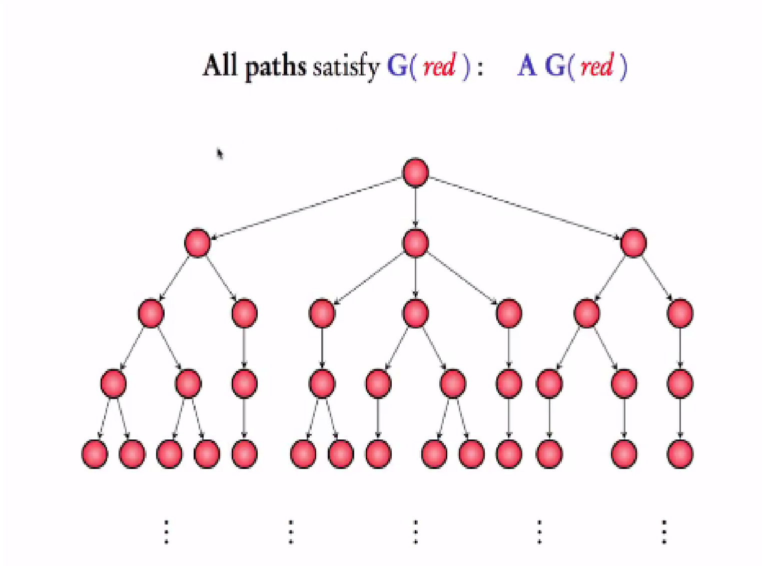

This tree satisfies A G (red) all paths satisfy G red can be written as A G (red). This is the notation.

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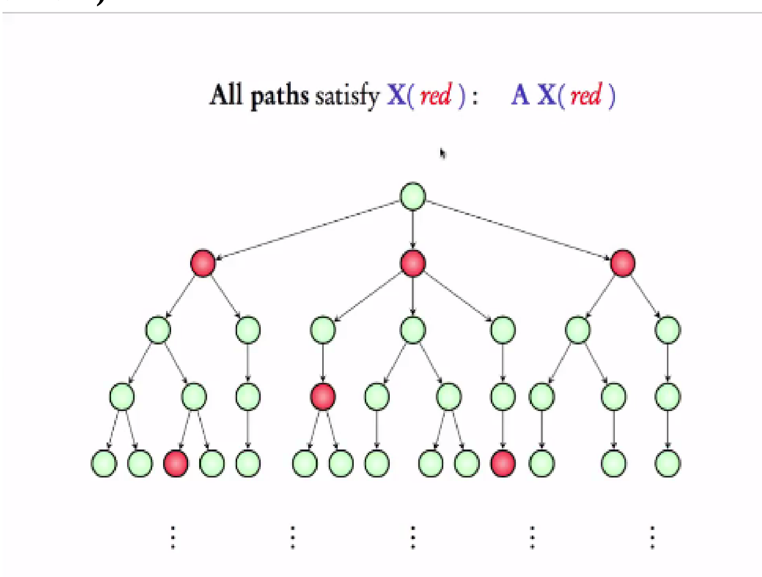

Similarly, this will become A of X (red).

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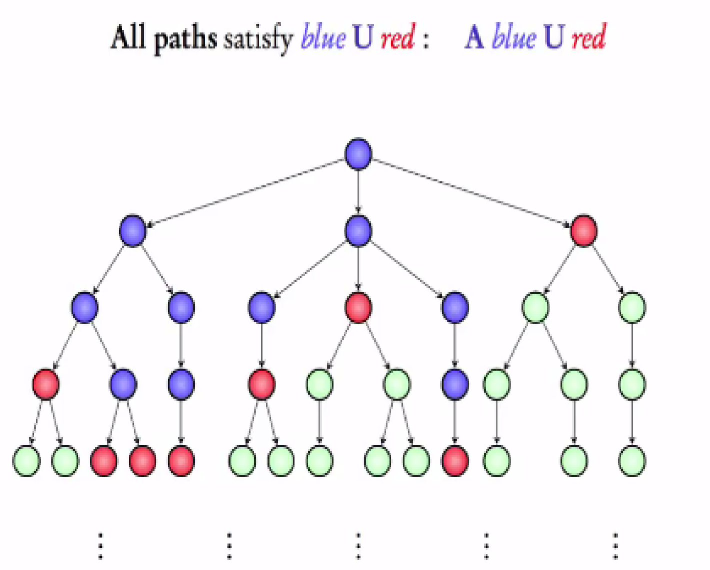

And this becomes A of blue until red. So all paths satisfy blue until red.

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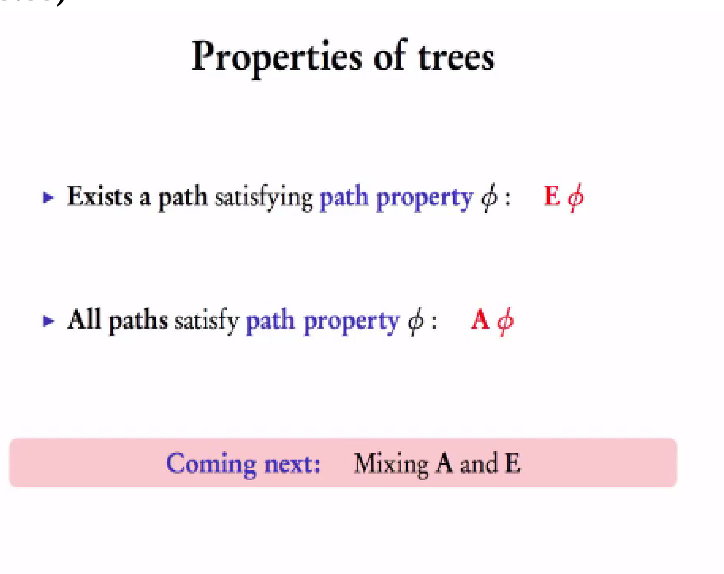

So we have seen two types of properties over trees there two operator A and E. When you say A of an LTL formula note that an LTL formula is a path property that is why I emphasized this. When you say A of some path property it means that all paths in this tree satisfy this path property. When you say A phi it means that there exists a path satisfying the path property. These are the two operators and they are very simple.

What we will now do is we will try to do a mixing of A and E. Let us see what we need.

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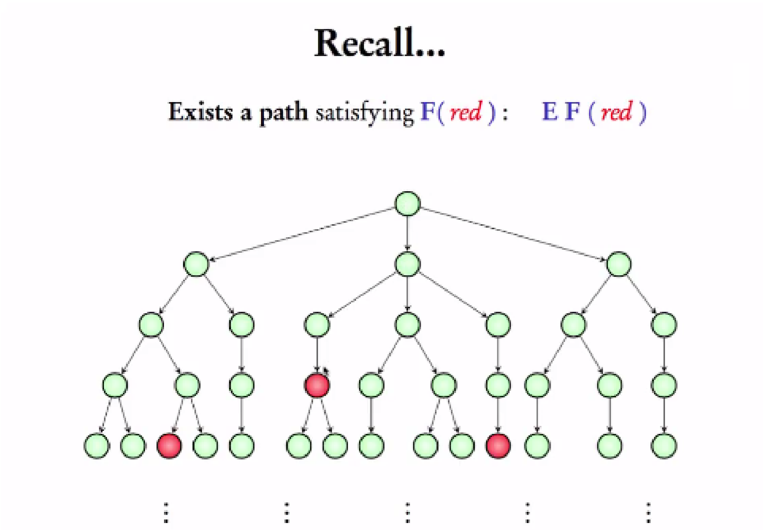

So we call A F (red) is exists a path where F (red) is true.

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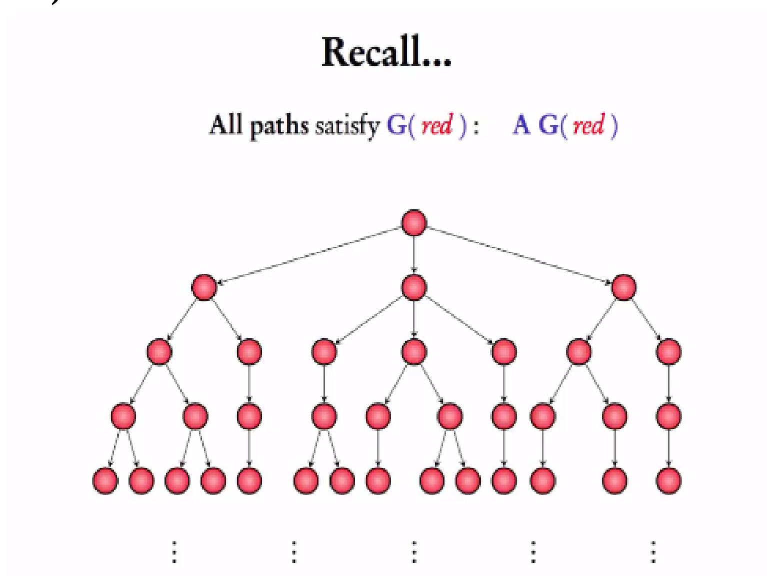

And A G (red) this all paths satisfy G (red).

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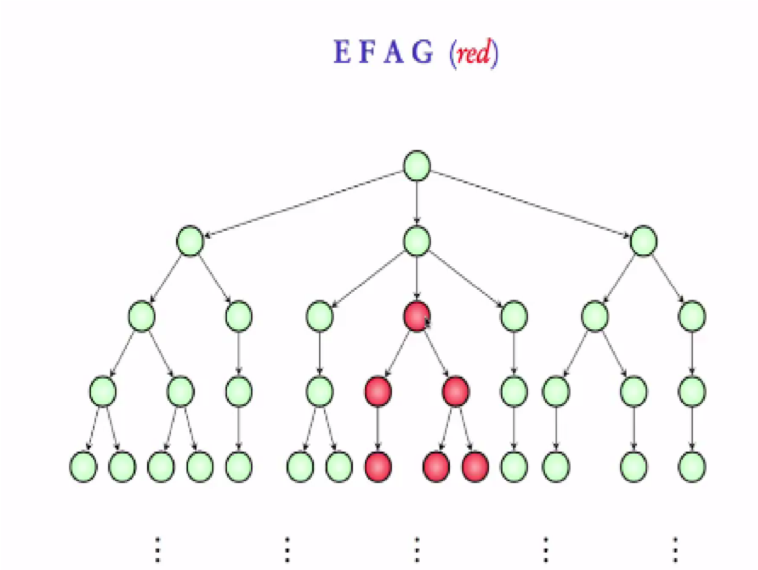

Now look at this tree. This subtree satisfies G (red) and there exists a path to the root of the subtree. This looks like a mix of E and A indeed so there exists a path such that F A G is true there exists a path where at some point of time A G (red) becomes true. So you look at this path at some point of time you reach a state that satisfies A G (red). Look at this state.

All paths starting from this state satisfy G (red). So we say that this state satisfies A G (red) so that is this path of the formula. And then this state is reachable via path from the initial state. So there exists a path which reaches a state where A G (red) is true. So this is the first example where we have mixed A and E and we are getting something interesting. So you reach a state such that subtree below it satisfies A G (red).

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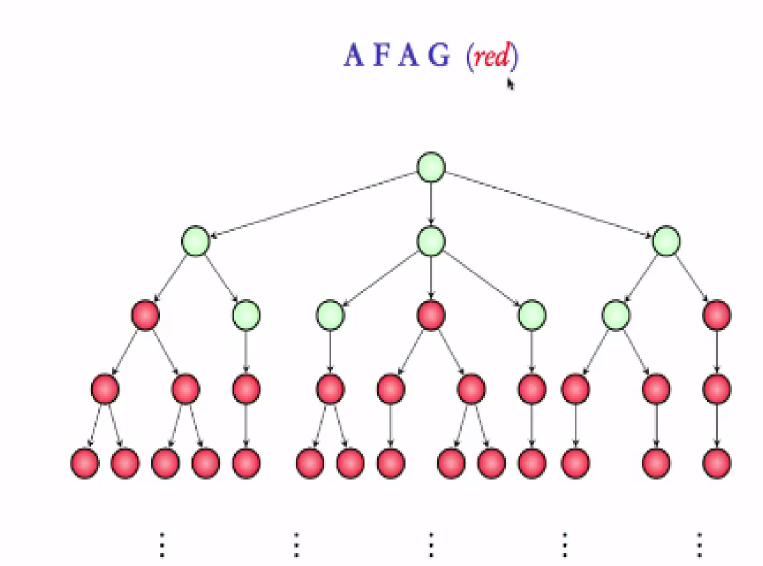

Here look at all the paths. At some point of time you reach a state where A G (red) is true. From this state A G (red) is true. From this state A G (red) is true. From this state A G (red) is true because all paths out of this state G (red). Similarly, all paths out of this state satisfy G (red) so A G (red) is true here, A G (red) is true here similarly A G (red) is true in this state. And hence you have this A F A G (red).

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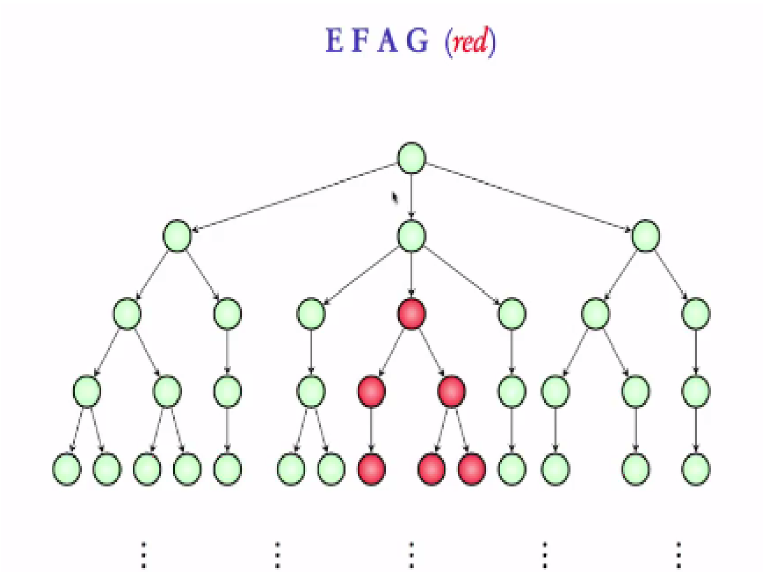

This was E F A G red.

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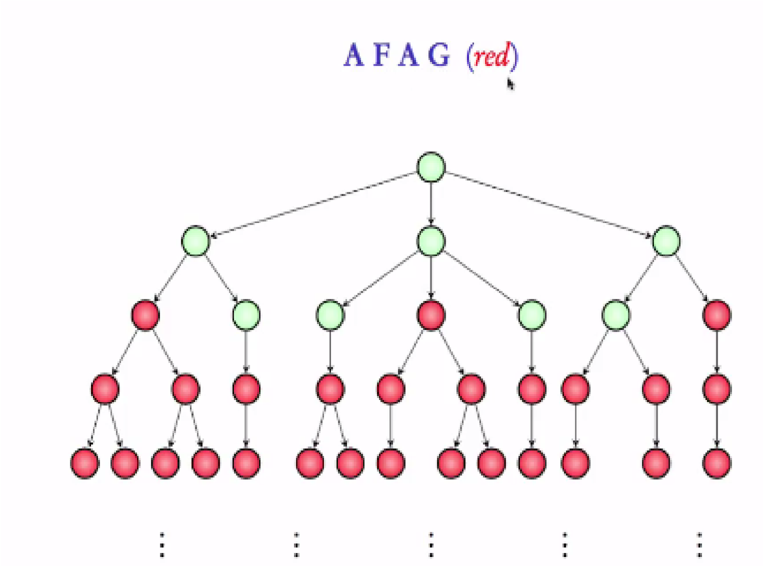

This is A from all paths you reach a state that is given by A F such that A G (red) is true. This is simple. From all paths you reach a state from which A G (red) is true.

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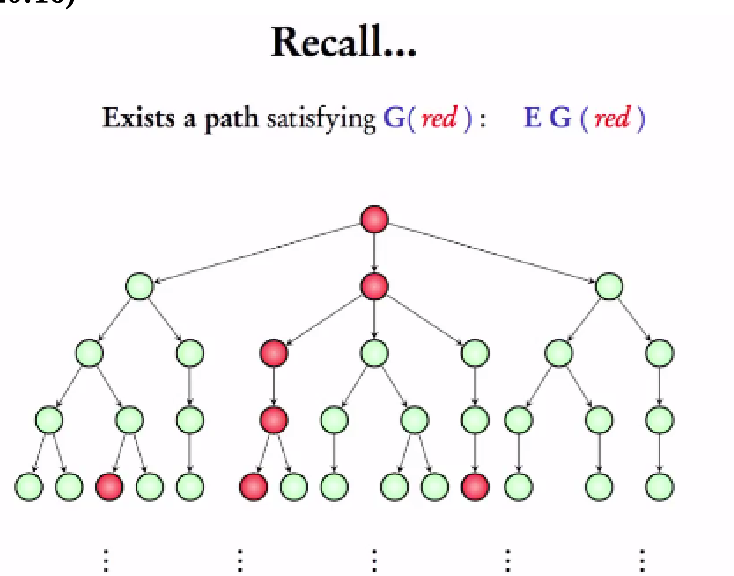

Let us know look at other example but before that recall A G (red) is there exists a path where G of red is true and A X (red) is there exists a path where X (red) is true.

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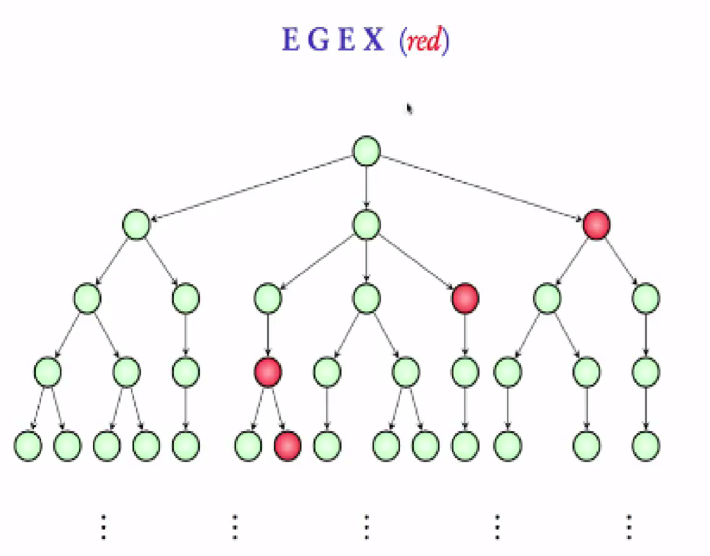

Now E G A X. Look at this path. In this state E X (red) is true because there exists a path out of the state which is this one that satisfies X 9red). From this state A X (red) is true. Look at this state from this state there exists a path where X (red) is true so this state satisfies E X (red). Look at this state there exists a path satisfying X (red). So starting from this state look at this path, if you look at this path next of red is true.

Similarly start from this state look at this path rather start from this state. In this state E X (red) is true because there exists a path out of the state satisfying next of red. Similarly, you can assume that from this state there exists a path satisfying X (red). So if you look at this path every state in this path satisfies E X (red). So this path satisfies G of A X (red) and since this is a path in the tree we write exists G E (red).

This is a slightly complicated example please try to understand it step by step.

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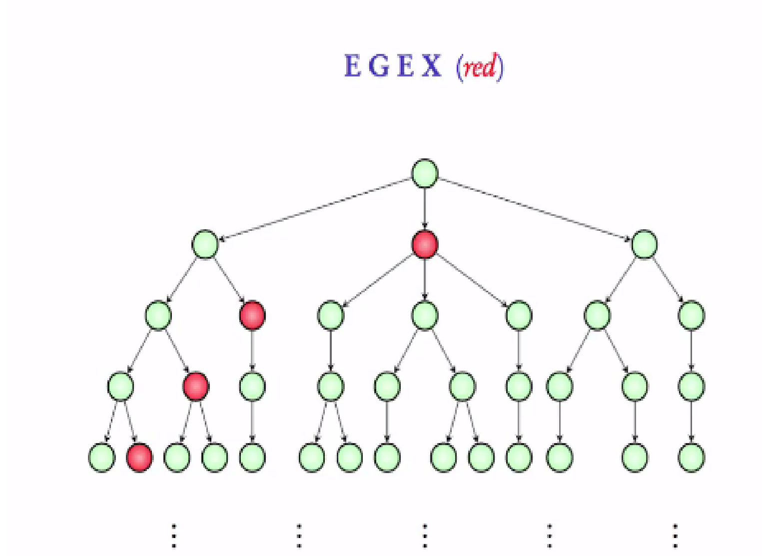

This is another tree which satisfies the same property E G E X (red). Look at this path the left most path. Every state satisfies E X (red). Note that, -- does not mean that that the path that we chose needs to be this one. For example, in this state E X (red) is true because of this path. Similarly, E X (red) is true in this state because of this path and so on. So every state every state along this path satisfies the fact that E X (red).

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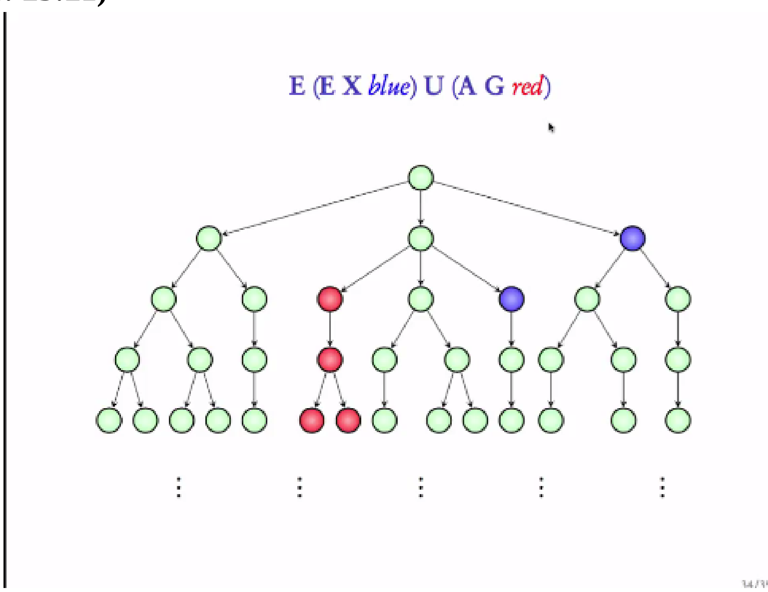

This is a fairly complicated formula this is there exists a path where something until something is true. You know what is E X blue? Look at this state forget about the everything and just look at E X blue. This state satisfies E X blue because there exists a path satisfying its blue. Similarly, this state satisfies E X blue because there exists a path satisfying X blue. Now look at A G red. Forget everything and look at A G red.

You know that this state satisfies A G red because all paths starting from this state satisfying G (red) so this state is set to satisfy A G red. Now we said this satisfies E X blue. This satisfies E X blue and this satisfies A G red. So there exists a path that satisfies E X blue until A G red. Okay. So this A G red is true in this state; E X blue is true in these two states, so we can write this formula which is that (E X blue) U (A G red) is true on this path so E of (E X blue) U (A G red).

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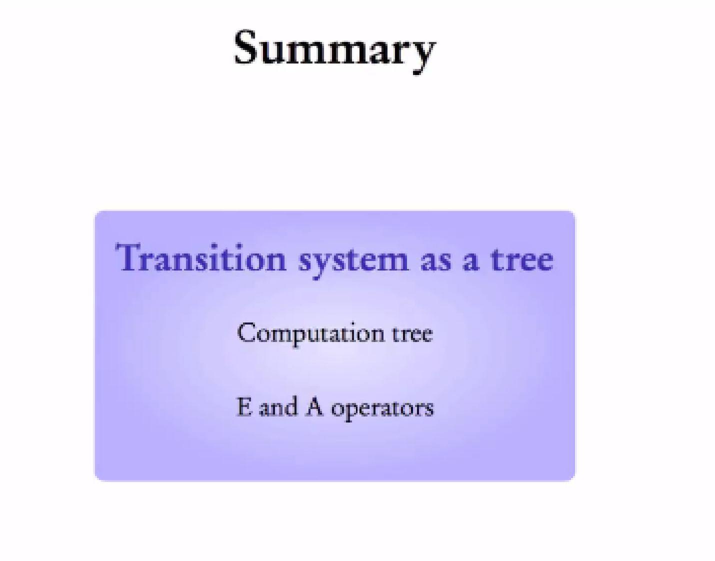

So we have introduced a tree view of the transition system. What is our goal, we want to talk about properties of these computation trees the computation tree of a transition system and obtained by repeatedly unfolding the transitions and to talk about properties on trees we need some notation; we know about properties over paths so that has been extended to trees by using the E and A operators.

E says that exists a path that satisfies a certain properties. A says that every path in this tree satisfies the path property and the last three or four example that we saw had a mix of E and A and this could give certain complicated properties over trees saying that there exists a path which reaches a point from where the entire subtree satisfy certain properties that is an important thing which could be useful in practice.

And we also saw some other property saying that there exists a path where something is true until something else is true and each of the something was had nested E and A. This was the E X blue until A G red. So E of E X blue until A G red. So we have given you an idea of E and A and how to mix E and A to get complicated properties. The next module we will give the formal syntax and semantics of a logic over these computation trees.