Model Checking
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Lecture - 03
Modeling Hardware Circuits

In the last module we saw the concept of transition systems which consist of finitely many states actions and transitions between states. We also saw some examples of transition systems. In this module we will be looking at how one can model hardware circuit as transition systems.

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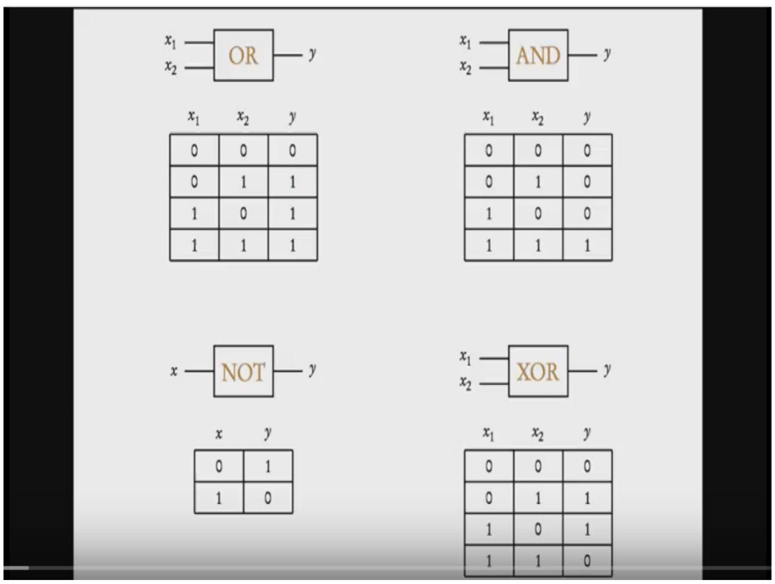

Lets start with some basics of hardware circuits. I think most of you know these gates OR, AND, NOT, XOR. So, OR is a gate which receives two inputs x1 and x2 and gives an output, A gate is nothing but circuit. So, what is the function of the circuit? If the inputs are 0 and 0 the value of the output y is 0 if the inputs are 0 and 1 the value of the output is 1 if the inputs are 1 and 0 the output is 1 again and if the inputs are 1 and 1 it is again 1.

So this computes the OR of these two similarly the AND of these two is 1 when both x1 and x2 are 1 otherwise it is 0. The NOT receives a single input and gives a single output so its just compliments the input. So if the input is 0 the output is 1 if the input is 1 the output is 0 exclusive OR or XOR is 1 if exactly 1 of them is 1 so for instance if x1 and x2 are 0 the output is 0, if x1 is 0 and x2 is 1 the output is 1 because exactly 1 of the inputs is 1.

Similarly if x1 is 1 and x2 is 0 the output is 1 this 1 when x1 and x2 are both 1 the output is 0. This is the only difference with the respect to the OR gate. So these are very basic things about circuits. Now using these gates you can combine them to get more circuits. So these are the Boolean functions representations of these gates.

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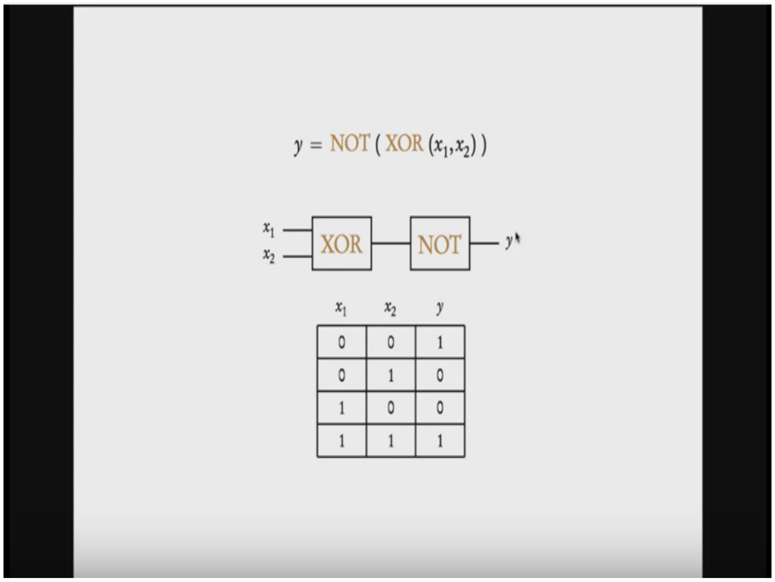

x_1	x_2	y
0	0	1
0	1	0
1	0	0
1	1	1

Now when you combine them what is the output. The output y is going to be NOT of XOR of x1 x2. So given the inputs you have to first compute the exclusive OR and then take the NOT. So this can be represented by this Boolean function. So if the inputs are 0 and 0 XOR of 0 and 0 is 0 and NOT of 0 is 1. If inputs are 0 and 1 exclusive OR of 0 and 1 is 1 and NOT of 1 is 0. So you can also check the other two entries in the truth table representation of the Boolean function.

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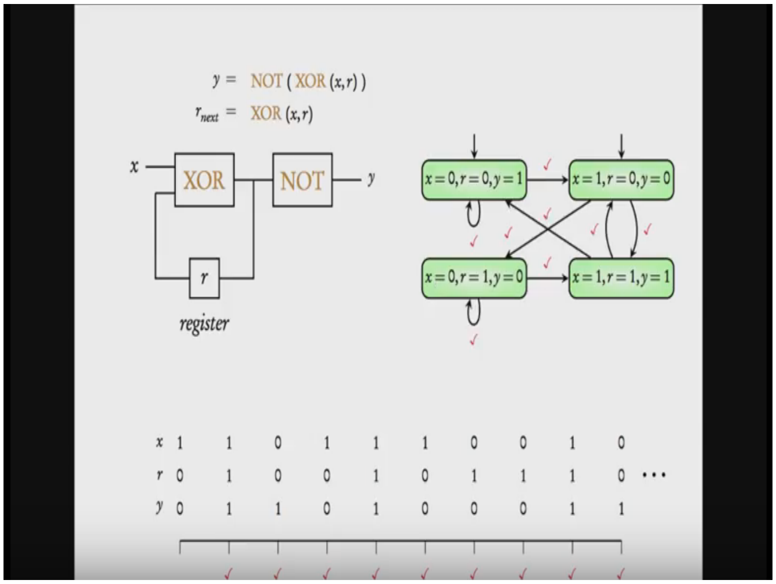

Let us now look at more complicated circuit. Now the inputs to this gate are a bit different there is one standard input and the other input comes from a registered which is part of this circuit. So the output y is going to be NOT of XOR of x and r where r is the value of the registered. In addition to this the value of the register keeps changing because of this connection. So the value of r changes to XOR of x and the previous value of the register.

So such a circuit with this kind of a feedback loop is called a Sequential Hardware Circuit. Now let us look at a sample execution of the circuit. Assume that initially the value of r is 0 and the value of x is 1. Now the value of y is going to be NOT of XOR of 1 0, XOR of 1 0 is going to be 1, NOT of this going to be 0 fine. Now what is the next value of the register it is going to be XOR of 1 and 0 which is going to be 1.

So XOR of 1 and 0 gives me 1 this is fed back to the register. So the register value changes to 1 and now the input here changes to 1. By that time the value of x which is an external input could either be 0 or 1 we do not know. Suppose the value of x is 1 now what is y XOR of 1 and 1 is 0, NOT of 0 is 1. Now what does the value of r change to it is changes to XOR of 1 and 1 which is going to be 0. Now the next input will have r equal to 0. Now by the time the value of x could be anything.

Suppose it was 0 now you have 0 and 0 here XOR of 0 and 0 is 0 NOT of 0 is 1 and now what is the value of r change to it changes to XOR of 0 0 which is going to be 0. The value of x again could be either 0 or 1. Assuming this is 1. XOR of 1 and 0 is going to be 1 NOT of 1 is 0 and you can keep doing this. So this is giving me an execution of the circuit. There are many more executions we started with 1 0 0 and we arbitrarily choose the values of x is here.

So how do we compactly represent the set of an execution of this circuit. In other words, how do we module the circuit. Now we are going to module the circuit as a transition system with some states and transitions. So what would the states of the circuit be?. The state of circuit are determined by the values of x, r and y. The value of x and r could be 0 or 1. Depending on the value of x and r the y gets determined.

Here are the states x could be 0 and r could be 0 if both x and r 0 the value of y is XOR of 0 0 which is 0 and NOT of 0 is 1. First of all there are four states depending on the value of x and r. x could be 0 or 1 here these are the two states with x being 0. One of them has r equal to 0 the other of the r equal to 1. These are the two states with x equal to 1 one of them has r equal 0 and the other has r equal to 1.

So these are the four states of this circuit. Now what are the initial states initially let us assume that the register r is 0. The initial value of the input is unknown it could be either 0 or it could be 1. Under this assumption there are two initial states the once that have r equal to 0. We have determined the states, we have determined the initial states what are the transitions of this transition systems. Lets start from this state, now at x is equal to 0 r equal to 0 y equal to 1. What could the next value of r be?

The next value of r is going to be XOR of 0 and 0 which is going to be 0. Now, what is the next value of x it could be either 0 or 1. So the two transitions are from 0 0 1 it could either stay in 0 0 1 or you can go to 1 0 0. Let me repeat if x is equal to 0 r equal to 0 the value of register changes to XOR of 0 comma 0 which is going to be 0. So the next state will have r equal to 0. Since we do not know what x would be it can be either 0 or it can be 1 depending on what it is we have two transitions.

For example when we were here we computed the value of r next to be 0 and then we arbitrarily choose the value of x to be 1 that means we went to this state. We could have even chosen 0 here in which case it would have depicted staying in this state. Now let us continue drawing the transitions of this transition system. What about this state when you have x equal to 1 r equal to 0 the value of r next is going to be XOR of 1 and 0 which is going to be 1 and the value of x is going to be unknown so it can be 0 or 1.

Hence you have two transitions one going to r equal to 1 x equal to 1 and the other going to r equal to 1 x equal to 0. With similar arguments you can complete the picture and this would be the final transition system. Executions like this correspond to paths in this transition system. For example this is 1 0 0 so you started from here you went to 1 1 1 then you went 0 0 1 then you went to 1 0 0 then we went to 1 1 1 sorry 1 0 0 0 1 0 0 1 0 1 1 1 0 0 1 and so on.

This transition system is giving you a compact representation of the behavior of the circuit. So I hope you understand the modeling of the circuit as a transition systems.

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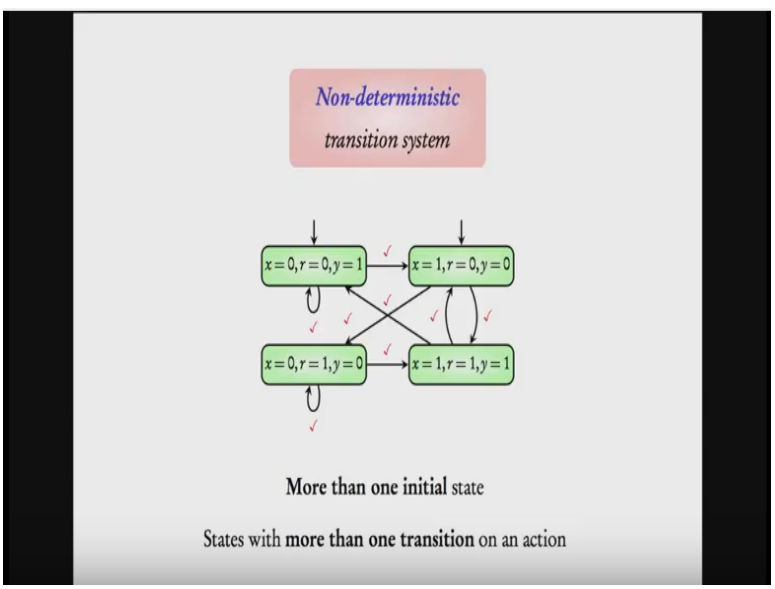

I would like to point out two observations in this transition system. Firstly, notice that there are two initial states there are more than one initial states. Secondly, there are states on which there are more than one transition on an action. For example look at this on a tick you can either go here or you can go here. Look at this on a tick you can either go here or you can go here. So there is some kind of arbitrariness in the choice. This kind of a transition system is set to be Non-Deterministic.

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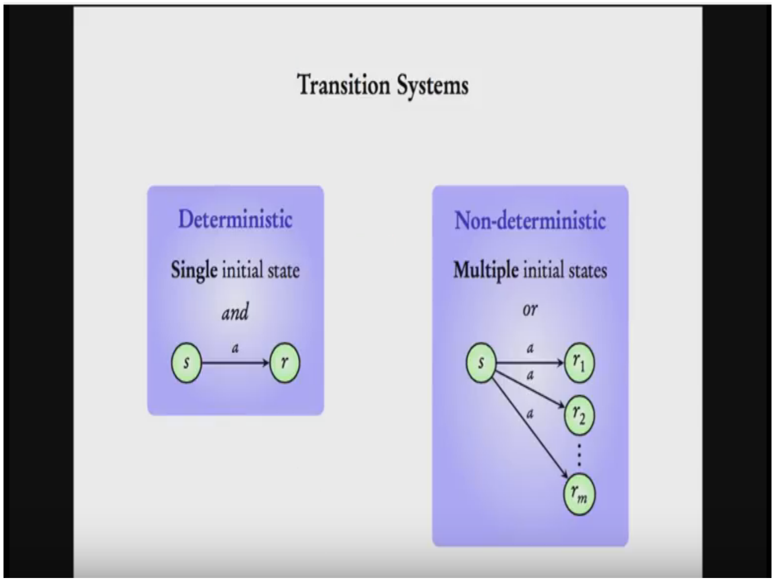

So in the last lecture we saw what transition systems were. Now and this module we just want to classify transition systems into two times one are called Deterministic transition systems and other are called Non-deterministic transition systems. In a Deterministic transition system you have a single initial state and for every state on an action there is a single transition. In a Non-deterministic transition system you could have either have multiple initial state or there can be states with multiple transitions on an action.

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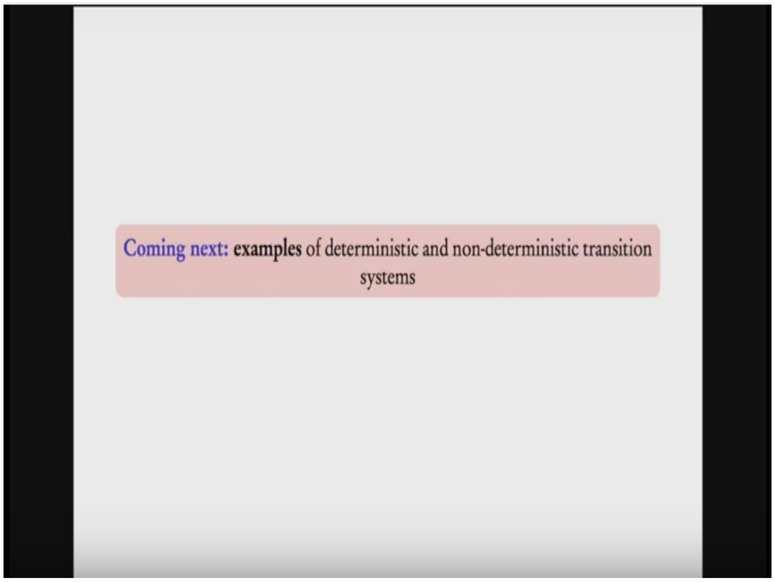

Let me give some examples of Deterministic and Non-Deterministic transition systems.

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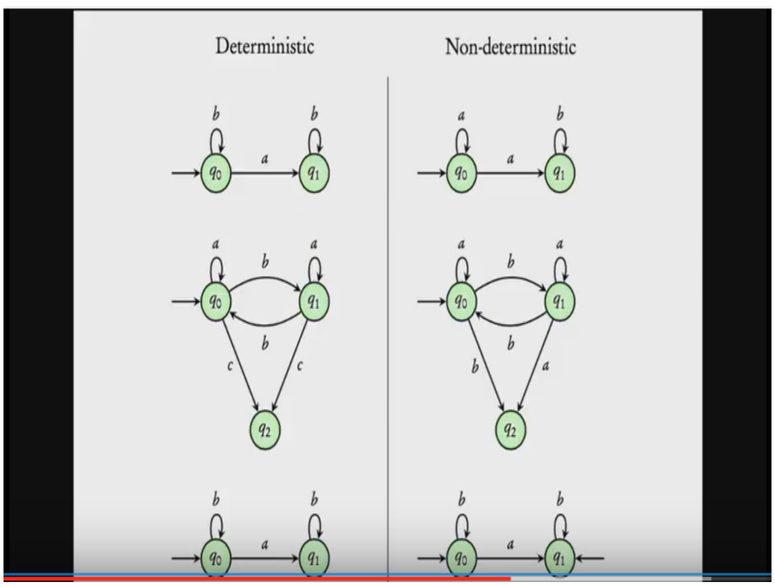

Look at this transition systems first of all these are transition systems I hope that is clear to you. Now let us look at this transition system the initial state is q0 at q0 on a it can either stay here or it can go here this means this transition system is Non-Deterministic. On the other hand look at this transition system there is a single initial state at q0 on a b it has a unique transition, at q0 on an a there is a unique transition, at q1 on a b there is a unique transition.

Now look at this the similar scenario occurs here at q0 on a b it can either go here or you can go here, at q1 on a it can either go here or you can go here. How is this resolved here at q0 on a you can just go here, on b you can just go here, on c you can just go here, at q1 on a you can just go here, on b you just go here, on c you just go here so this is Deterministic. Now what about this transition system at q0 on a b you just go here, at q0 on a you can just go here, at q1 on b you can just go here this is fine but there are two initial states.

So this needs the transition system Non-deterministic. Here there is a single initial state so this is Deterministic. So this is the simple concept I hope you are clear with this.

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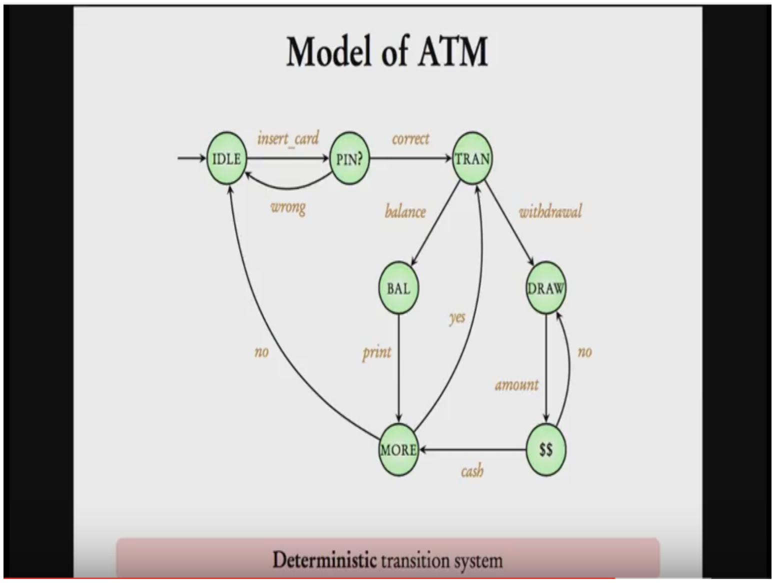

Recall the transition system that modeled an ATM. Is this Deterministic or non-deterministic. How many initial states are there? there is only one initial state. Is there a state which has multiple transition on an action not this one, not this one because from pin on a correct you can just go here on a wrong you can just go here. Neither the state at tran on a balance you can just go here on a withdrawal you can just go here. You can check all the states there is a unique transition on an action.

So this transition system is Deterministic. What about the transition system which modeled the vending machine? This is again deterministic there is a single initial state and every state has a unique transition on an action.

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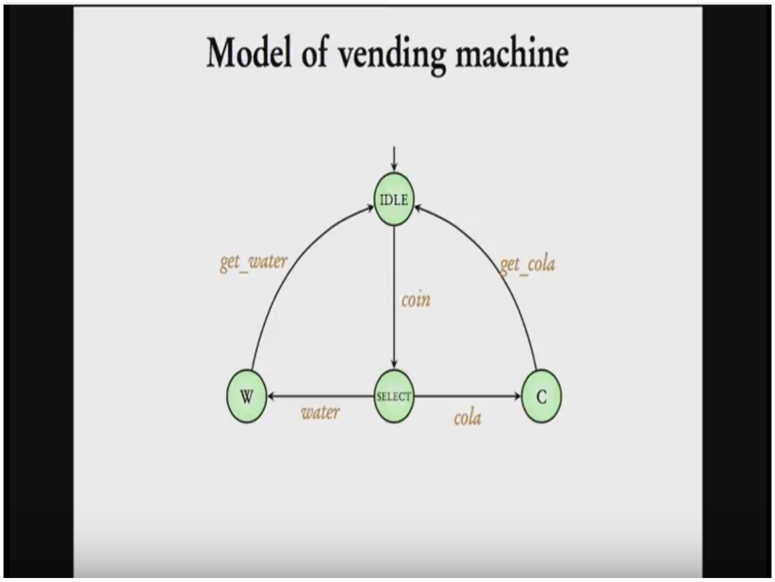

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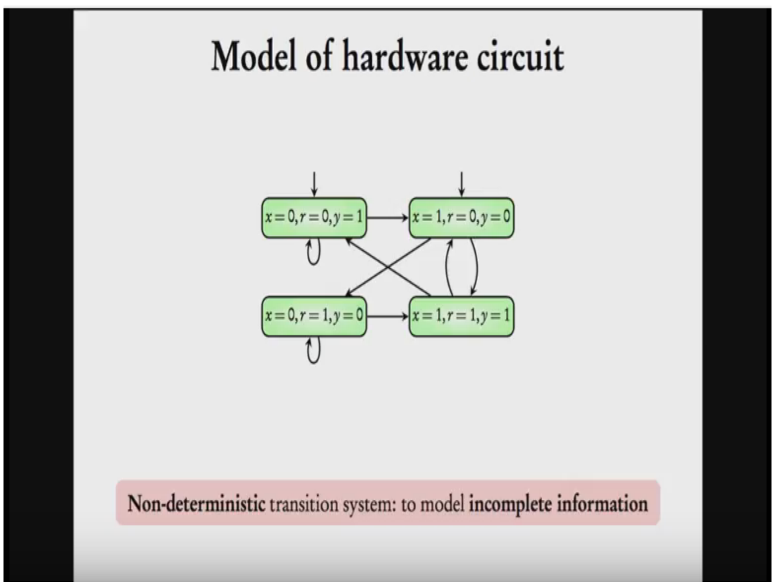

What about the hardware circuit so note that since every action was a tick I have removed the tick from the picture for clarity. So when you have only one action it is irrelevant. Note that here are the multiple initial states and there are multiple transitions from a state on an action. So this is Non-deterministic transition system. Why did we make use of non determinism here? We did not know what the next value of x would be it could either be 0 or it could be 1.

So there was an incomplete information such incomplete information can be modeled using this feature of non determinism.

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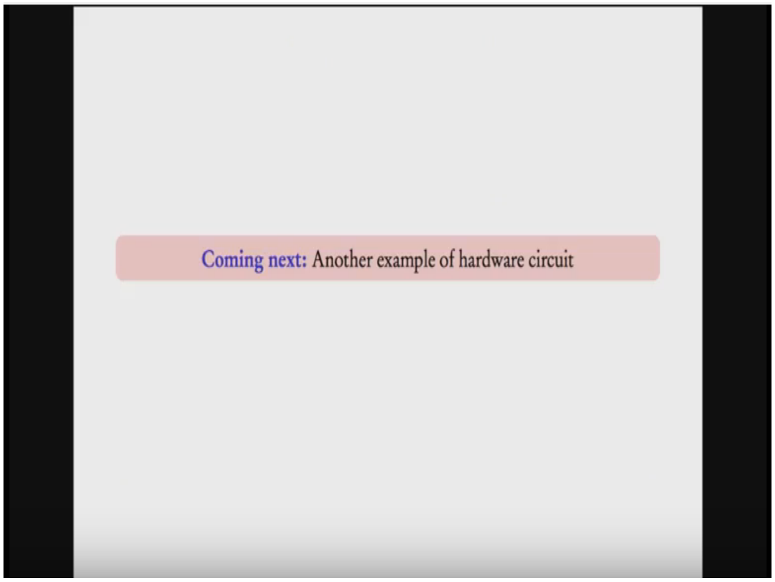

So let us look at another example of hardware circuit.

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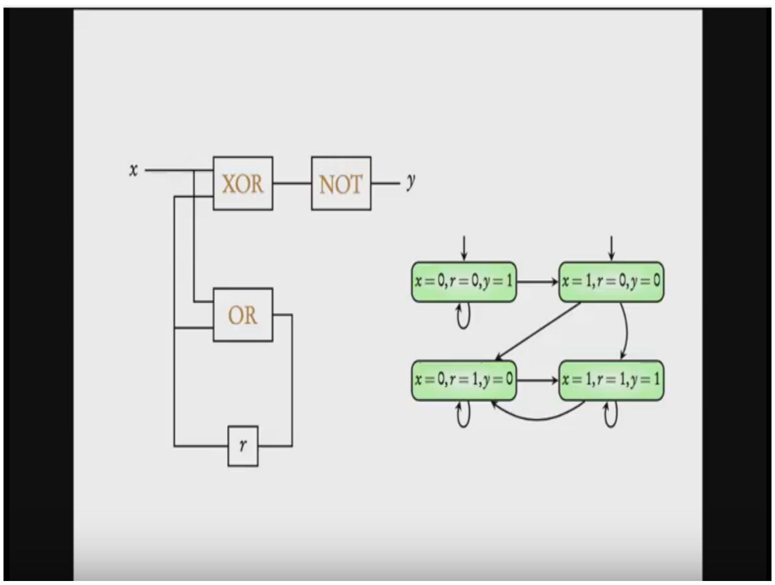

Its a modification of the previous circuit so the inputs come from a register and an external input x. So the register value goes both to this gate and to this gate. Now the output y is jut NOT of XOR of x and r it is going to be the same. Now how does the register value change? the register value changes as OR of x and its previous value based on this is going to be the transition system. The states are again x is equal to 0 0 1 0 1 1 0 1 and the value of y is going to be NOT of XOR of x and r.

So the states are the same as in the previous circuit. Now the transition are different because the next value of r is going to be OR of x and its previous value. So the next value of r is going to be r of 0 and 0 so r is going to be 0 but we do not know what x is so this goes here with x is equal to 0 r is equal to 0 at here with x being 1 and r is equal to 0. Now once you are here r of 1 and 0 is going to be 1. So the two transitions from here are going to go x equal to 1 r equal to 1 and x equal to 0 r equal to 1.

So once the value of r becomes 1 it will maintain that value because r of 1 and anything is going to be 1. So that will give you this transition and this transition because you do not know again what x is going to be. So I hope this transition system is also clear. This way given some hardware circuit like this, this could be other gates NAND, AND, OR okay. You can come up with the finite state machine corresponding to that. Note that you could also have more inputs, more registers, more outputs and based of the number of inputs registers and outputs you can create your transition system.

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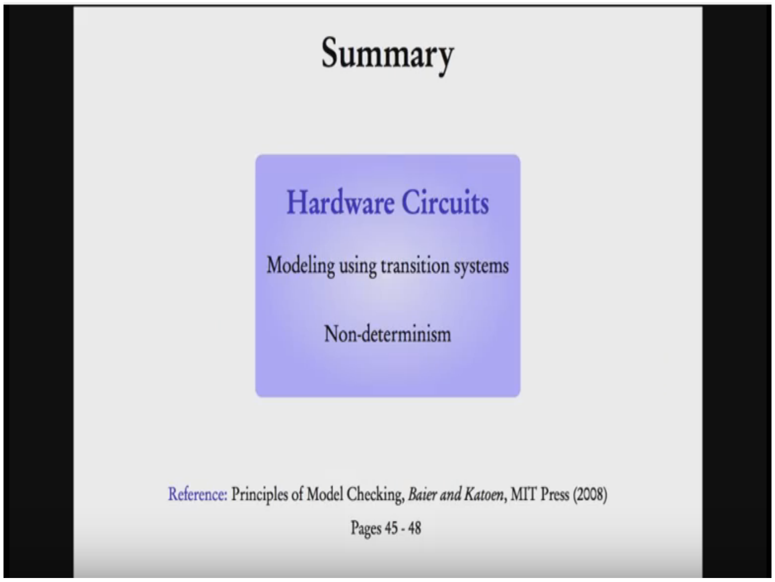

Let me summarize this module, so in this module we looked at sequential hardware circuits. We saw how to model them using transition systems while doing this we made use of non determinism. So we give a classification as to what Deterministic transition systems are what Non-deterministic transition systems are. So I have given the reference yet again there are more details in the book. If you understand it is well and good but you do not have worry if you do not understand all the details. For this part of the course as long as this is clear it is fine.