Artificial Intelligence
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Lecture - 44
Resolution Refutation for PL

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So, we have been looking at reasoning in proportional logic. And today we want to look at recent method known as the resolution method. This is given by Alan Robinson in nineteen sixty five. So, it is a fairly recent method if you compare it with for example, free gaze axiomatic system which is more than 2 years old. This is by recent and it is motivated by the fact that you want to write a computer program to generate proofs which means you have to find proofs automatically. Now, if you look at the high level algorithm for finding proof it basically says picks some rule of inference, pick some applicable data and produce some new data add it to the system. So, it is basically a search algorithm which is searching in this phase of sentences which can be produce and trying to produce the sentences that you are interested in.

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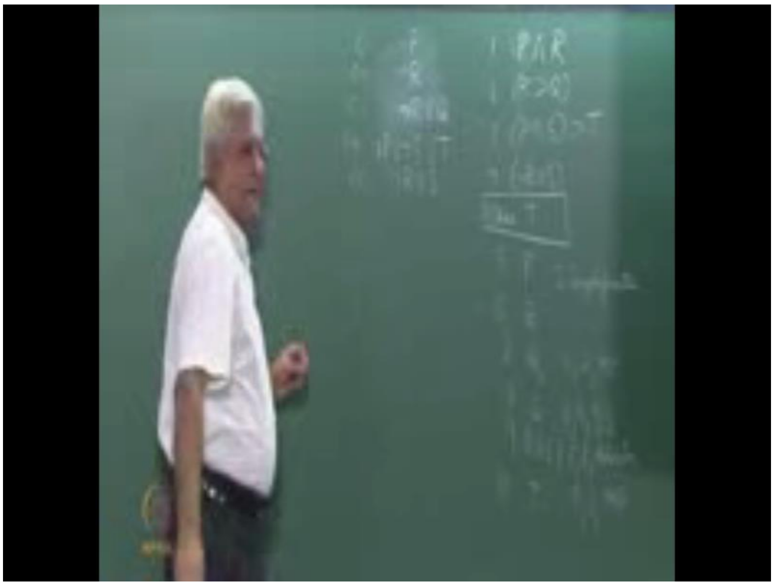

So, to motivate this resolution method let us go back to the problem that we have started with the example that we looking at which said that P and R or something like that R replies Q. P and S implies T and not using the same symbols, but anyway it is a same problem and from this we have to show T. Now, if you if you go back over the proof we did for this and this was to be shown. So, this is 1 2 3 4 then we derive it for example, P from this thing and there was a rule called 1 called simplification. So, we use 1 rule for inference called simplification. And then we derived R by the same rule then from R and R and Q with a derived Q. So, 6 2 the second rule of inherency use there and then from R and not R or S we derived S. So, this is from 6 and 4 and the rule called differential syllogism. So, I just used ds there then from this we get from 5 and 8 we get P and S. And then finally, we get T from 9 3 and so this proof required us to use 3 rules of inference simplification 4 rules of differential syllogism and the addition. And then of course, we have to choose which ruled to pick and what to apply.

So, and so it look nice and simple when you look at just these 4 sentences and the conclusion that you want to draw. But in practice the set of facts that available to a reasoning system would be hundreds of may be 1000s of facts and you may want to ask whether certain formula is true or not. So, it is it is not that only relevant things are given to you there may be many irrelevant other formulas and there may be 100s of rule which may be applicable. So, if you look at the power safety search example like we are looking at in planning. It generates a huge launching factor and the number of possible things we can do is very large essentially. We have not use any axioms here so by and large we try to systems were axioms are not needed. It essentially they just add put enough rules of inference which will work, but as a motivator for this method say some other example we have given a simple formula S and you are asked to derive P or not T we asked to derive.

So, you must distinguish between this in tactic process of deriving formulas as suppose to the semantic process of looking at the truth value. And saying this formula is true if you look at P or not P you can obviously say it is true it is and it is quite trivial, but the question is can you derive it in the system. So, just say that it is not straight forward to the write P implies P system if not. So, in fact, you cannot derive if you are given only S and you have given all kinds of rule of inference simplification and differential syllogism. There are so many of them it is not going to help you can never derive this from this equation that Robinson's method. Resolution method is a complete method if you use only 1 rule of inference and it does not need any extra axiom system for that is why so very attractive it makes the task of programming. It is simpler to simpler you take that rule all need to decide on which data to apply here that is of true still not a straight forward problem.

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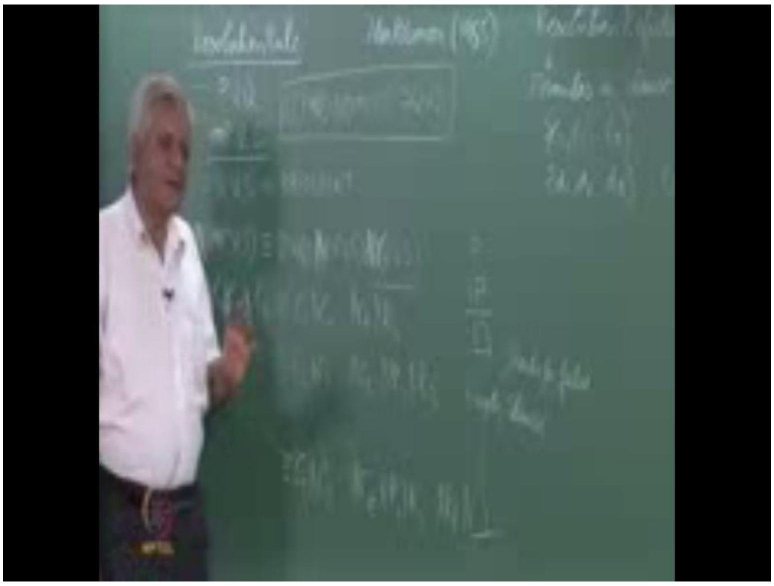

So, what is the resolution rule as it is called? So, will look at the simple form there is a more general form which we will may or may not look at it later if you are given P or Q and if you are given not P or S. We can derive Q or S this is the resolution rule in the simplest form it could be P and Q or R also does not matter. It does not have to be just 2 thing, but in the simplest form it has just 2 proposition here 2 propositions here. So, in the language of resolution method we call each of these elements are literal and resolution effect resolution method works formulas in what is known as clause form. And we are concerned in proposition logic clause form is the same as conjunctive normal form. So, we have to first construct your formulas in clause form or conjunctive normal form which means that you cannot use things like P imply Q. And so if you have a situation like P implies Q you should convert it into not P or Q essentially. So, I will take it for granted that you can convert any given formula into clause form clause form or C n f is basically a formula of the kind C 1 and C 2 and C k where C I is of the form d 1 or d 2 or d r.

Let us say and each d I is either a proposition or negation of proposition. So, it is a very simple structures form at the outer most level it is a conjunction of clauses. So, C 1 and C 2 and C k each clause is a disjunction of literals d 1 d 2 or d R each literal is either a proposition or it is negation. The negation sign is some sense occurs in the inner most package if you imagine brackets all over the places essentially. So, for example, you could start up by converting this problem into clause form saying P is the clause Q is the sorry R is the clause. Then not R or Q is a clause then not P or not S or T is a clause this corresponds to that this is not R or S. So, I converted this 4 sentences into clause form here. So, each one of there is a clause and I can think of it as a P and R and this thing and this thing and this thing have a larger formula. Now, first of all we must ask ourselves is this rule a sound rule or a valid rule of inference and its validity is based on a validity of this equivalence P or Q. Or so I can keep writing this, this should be consistence we need this larger equivalence to argue about what we are doing. But in practice the only think we need this and this implies this which means P or Q and not P or S implies Q or S,

So, for the rule to be sound so this implication is a total logic it is a rule of inference that we have if you this and if you see this. Then you can produce this and remember that unlike the rule that that we talked about earlier these are not patterns in the sense that you cannot plug in any arbitrary formula here. This is the clause; this is the clause and this is the clause which means the consistence or proposition or negation of propositions it is not a arbitrary formulas but it take any such 2 clauses. So, if you want to describe this rule if you get 1 clause C 1 another clause C 2 and one of them has literal and one of them has a negation of that literal. We can produce a clause to add up all the literals here and remove the, what is cancelling each other in some says. So, we are talked about clause form also instead of writing it C 1 and C 2 we often write it as C 1 comma C 2 comma C k. So, we think as it is a set likewise this 1 d 1 comma d 2 d R where the comma is interpreted appropriately and in this outer level and in this outer level and or in the inner level sorry the joined by the and…

So, the rule of inference required by this implication be true, but we are going to be in trusted in this as you will see in this moment why that is the case. So, this is called the resolvent this is called resolvent I always keep getting let us call this resolvent whenever you add that resolvent to the set of clauses. Then 2 set of clauses are equal in other words what we are saying is that we can keep adding clauses to the set. So, the resolution method works as follows that is start with a set of initial clauses let us called them C 1 C 2 C 3 C k. And then you replace or you add 1 more clause to this which is the first resolvent which means you are saying that this set of clause. I am using a comma here and you should not confuse this comma's equivalent to and here or you can write and that is to be safe I added 1 more clause here then I can add one more. So, what I am saying is there I keep adding this resolvents to my set of clauses and what I get is an equivalent set of clause. This set is equivalent logically equivalent to this, this set is logically equivalent to this.

So, you must prove that this is indeed the cases once you prove this it is easy to prove this, because from the right hand side to the left hand side. It is trivial from the left hand side to the right hand side is based on this that you are adding this extra clause Q or S. And then you can basically show this is the topology and I keep doing this till I get a clause which looks like till I get the formula which looks like this this is called the empty clause till I generate an empty clause. So, what am I doing? I am starting with set of clauses C 1 C 2 up to C k and I added with a first resolvent R 1 then to this added R 2 then R 3 R 4 and so on till R l. And at some point I was able to generate the empty clause and I added the empty clause to this set and that is actually the algorithm terminates. So, what is the algorithm? Algorithm says take any 2 clauses which are which can be resolved which means one of them has a literal. And other one has a negation of that literal resolve them produce a new clause and added to this set. And then pick any 2 clauses apply the resolution rule keep applying the resolution rule keep, adding clauses till at some point you generate the empty clause. How do you get the empty clause? If you see clause like this P if you see clause like not P you can see P or nothing not P or also nothing.

So, when you apply this same rule here there is no Q and there is no S. So, all you left with this empty clause we sometime write with a box sometime we write with this. Basically it is an empty clause empty clause stands for false or it always false. So, I have jumped a little bit ahead from the national deduction of style of proof in the national deduction style of proof. We said that we keep adding new formulas and we terminate when we find the formula that we interested in here I already jumped ahead. And said that the resolution method keeps adding new formulas and it terminates when you generate the empty clause when you find 2 clauses like this. And so this box is also used to denote an empty clause this bottom also can be used to denote an empty clause. Basically an empty clause stands for something which is false in the resolution method terminates with an empty clause. And we will see a moment why or I will tell you in the moment wise, but what the implication of this? What is the implication of the fact that you have added empty clause to your set of clauses?

We have to go back and see what were we are saying here, we are saying that this set of clauses logically equivalent to this set of clauses which is logically equivalent to this set of clauses and so on which is logically equivalent to this clause. Now, what is the truth value of this clause? It is false, because it is something and something and something and something and something and false. So, this whole formula is evaluated to false which means this is false which means this is false and this is false I could have use the resolution method to say terminate for the example. In that example we are doing terminate when generate T essentially, but we are not doing that we are saying that terminate when you generate in an empty clause which means we are trying to show that this formulas are unsatisfiable. That is why this method is called reputation method it is used to show that a formula is unsatisfiable. A formula when I say this is the large formula right this got many clauses inside it and it used to show that the formula is unsatisfiable.

And the reason why the resolution method is implemented in this manner is, because it has been shown that the method is complete. If you want to derive null clause it is not necessarily complete if you want to derive any arbitrary clause which may be an tail by the system. So, remember completeness means whatever can be derived basically resolution method is complete for showing that a formula is unsatisfiable. If you give with an unsatisfiable formula it will there will a derivation which will derive the null clause from the set of unsatisfiable formula is it going to help us. We are not interested in unsatisfiable formulas or are we could we interested in unsatisfiable formula. So, let us a question I asked you earlier in the last class we have 3 kinds of formulas; one is valid formulas which are always true on the all valuations there are satisfiable formula which are true under some valuations. So, for example, P implies Q is true in under certain valuations then there are unsatisfiable formulas which have which are true for any valuation which are false for all valuations.

And essentially what we are saying is that if we have unsatisfiable formulas then you can test it using syntactic procedure of this resolution method which means keep applying the resolution method and you will be able to derive null clause. And so first of all you have to convince that deriving the null clause is indeed showing the unsatisfiable. Because of this equivalence this whole formula is equivalent to the 1 before that which is equivalent to this which is equivalent to this which is equivalent to this which means the original set that started with is shown to be unsatisfiable. And remember that everything is to be expressed in clause form. So, if I could so have that problem to solve if you look at that top 5 lines there I have 4 premises given and 1 confusion to be drawn can I set of produce an unsatisfiable formula from there. So, the process is straight forward. So, let us look at the clause form here. So, I have C 1 C 2 C 3 C 4 C 5. What am I trying to do? If I look at my logical notation I am trying to do this P 1 P 2 P 3.

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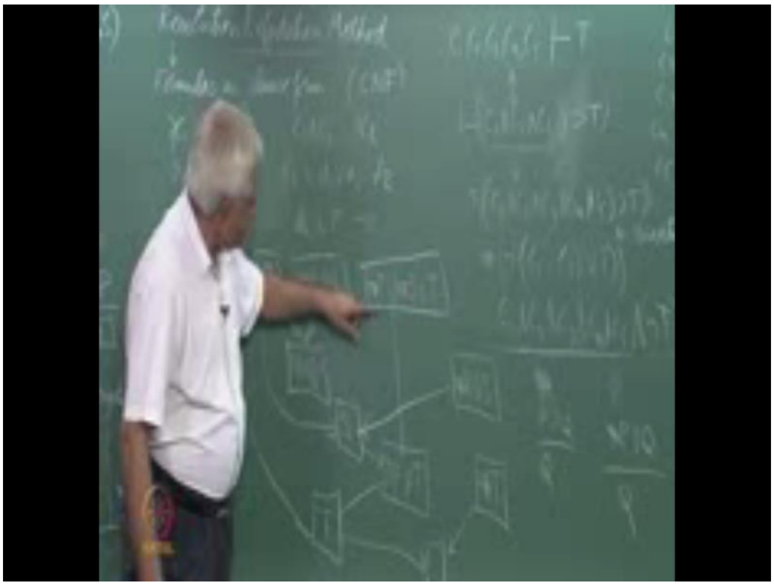

So, I am saying given these 5 clauses all those 4 statements which are equivalent to 5 clauses, because this is the end statement it is broken down into 2 clauses. Because the clause form says that the clause is has only the this junction side inside. So, P becomes the separate clause R becomes the separate clause this becomes R or Q this becomes not P or S or T and this is anyway not R or S. So, what am I really trying to do was this that can I derive T from this clauses. If you go the back to the deductions you know it said that if you can do that then you should be able to derive C 1 and C 2 sorry. So, remember that that is what a deduction said that you could take actually you could do it 1 by 1. But eventually we will come to this formula and I will leave that as a small exercise. Because we are also shown somewhere in between if we have P and Q and the whole thing implies R then P implies Q implies R. So, the 2 formulas are simple. So, eventually we can all the ans come together here. So, this is the set of premises implies the conclusion is what we are trying to show to be true.

So, what are trying to show? You are trying to show that this formula this larger formula is a topology what I have with me is a method which can show that a given formula is unsatisfiable can I brief this gap is not it? So, that is why I ask this question there are 3 kind of statements valid statement, satisfiable statement and unsatisfiable statement. If I take a negation of a formula what where does this how to how do they get related to each other if I take a negation of satisfiable formula for example, if I take the negation of valid formula or take a negation of unsatisfiable formula. So, there is a clear relation and you need to think about little bit about this between the valid formula and the unsatisfiable formulas. A valid formula is true under all valuations which means every row in the truth table. It will end up with true, if take if you take the negation of that formula then every lower ends of become false. So, I have this. So, I am trying to show that this is true which means you show that negation of this C 1 and C 2 and C 3 and C 4 and C 5 implies T is unsatisfiable.

Instead of trying to show that that this is the topology which means you can be derived and assuming a sound and complete system. Instead of trying to show that it is a topology I can say that take it is negation and that there is a unsatisfiable formula. So, you must convenience yourself that this is the sound step that if I take topology and convert and convert its take it is negation it becomes unsatisfiable. So, I produce now what I think is a unsatisfiable formula and of course, to check whether it is unsatisfiable instead of constructing a 2 table. I will use the resolution method to check whether that is the case. Now, just serious of steps if I go to take this not inside so I have to convert this into clause form. Remember that the resolution method applies only to clause form if I take this not inside I will get not of not of C 1 C 5 or T. So, remember this equivalence which said that alpha implies beta is not equivalent to not alpha or beta. So, this whole thing I am taking as alpha. So, I am taking not of this alpha and or this T and then when I have push this not finally, inside what I will get this C 1 and C 2 and C 3.

So, that is the logically equivalent formula now I have the formula that I have been interested in clause form. So, each of these is a clause I we already said that. So, each of them is a clause. So, we end this, this is just a literal from not T is also clause essentially. Now, we have a approach to using the resolution method which is that you take the set of clauses that are given to you which is the optimizes. You take the conclusion and take it is negation not of T and add it as a clause and the claim is that this set is unsatisfiable is it unsatisfiable we can try and show that using the resolution method essentially. So, the resolution proofs all usually shown as directed graph directly recycle graph. So, the resolution they are called. So, let us try and do the resolution method here. So, just write the clauses again now this is P this is 1 clause this is R this is not R or Q not P or not S or T not R or S. These are the 5 given clauses and the negation of this is P I have the 6 clauses. So, C 1 to C 5 when negation of the goal and negation of the conclusion which is the 6 clauses.

So, before we proceed you should be able to see this is a very proof contradiction what we are saying for the given these clauses assume that conclusion is not true. And then show the whole set becomes unsatisfiable which means of course, this is unsatisfiable. The only reason why this could be unsatisfiable if this last clause was not correct which means C 1 C 2 C 3 C 4 C 5 are anyway given to us you cannot assume them to be false. The only reason why this formula becomes false is because of not of T is there so not of T meaning is false. Therefore, not of T must be false itself it means T must be true it is proved by contradiction. So, that with a small task of finding a proof and what is the proof is a derivation which ends in an empty clause or a null clause. So, let us see we can take any 2 clauses and resolve them. So, for example, a h m well in this there are not too many choices, but in general a larger problem will after you many choices, but in this case we do not have too many. So, let us take this and this and you get negation of Q. So, you must see that what I am doing is applying the resolution step from this clause. I am removing R from this clause I am removing not of R.

So, from this clause I am removing R and from this clause I am removing not of R. So, what I am left with this Q then is that correct this Q is not very useful for me I can take this and this and I can get S. So, remember this is a directed graph I can take any 2 clauses. So, this is just a set. So, I can take now this and this I get not P or T see I can resolve this with this or I can resolve this with this. But just for the sake of the argument let say we will resolve this with this. So, you can see that actually we have derived T we have actually derived T, but off course, as we know from the national deduction method that is quite possible to do that. But our method does not stop here our method says we have must derive the null clause. So, you can see from this and this we can derive the null form. So, this graph represents the proof that the given set of clauses is unsatisfiable and the proof wholes, because of this equivalence that we have seen that we are keep adding new things. And in the end we added the empty clause and in the empty clause basically stands for false you can see that the empty clause is saying T is true. And at the same time you are saying not T is true and obviously that stands for false or contradiction.

If we can derive an empty clause then we have shown that this set of clauses unsatisfiable which means this large formula is unsatisfiable. And it can only be unsatisfiable if not T false nothing else can be false because say that are premises given to us if not T is false then T must be true. So, let us quickly just look at this rule what does says it says P or it says in terms of literals at least P and P implies Q. And you can do Q now you can see that if I want to converted into clause form. I have P and not P or Q and I deriving Q because you know these 2 are equivalent essentially. So, we can see that is just 1 example of the resolution rule been applied or if you look at says P implies Q and not Q implies not P. And you can see this is again not P or Q and not Q and not P. So, again in this form when you replace P implies Q with not P or Q you can see it is a same resolution rule being applied take a literal from here Q take the negation from other 1 and cancel that and whatever remains is that. So, all these rules as we can see are special cases of the resolution rule that we have stated here. If at the resolution rule more general than this as you saw in this example we does not have to be only 2 literals it can be more than 2 literals.

So, we use not P or not S or T and consider it with S. So, removed S from here and what was left was not P or T is not it? So, that is so the resolution method is a sound and complete proof procedure for the case of proportional logic is it to introduced in 1965. And since an lot of theorem proving work is based essentially on the resolution method essentially it is most popular way of proving theorems, but so far we have seen the proportional logic. So, what is the language of proportional logic? We have set of propositions and they could be stand for anything. And we do not know the only thing that is sort of this system is solving out for us is the meaning of the connectives when you have this connectives what do the sentences mean? What do compound sentences mean when they when are they true and then we can set of know extend that truth value of essentially if you go back to the example we started with this the Socrates argument.

All men are mortal Socrates in a man; Socrates is man this is beyond our reach to talk about this argument is beyond the reach of proposition logic. Because we cannot really do anything with this sentences essentially, because what you need here is to somehow understand what you mean by all. And then relation between being man and being mortal and then to say the same relation between being man and being mortal this is applied to Socrates, because Socrates happen to be a man. Now, this is not possible in proportion logic, because if you say this is P this is Q then there is nothing we can be do. There is no logical connective applied here essentially we could say for example, m stands for man and n something else R stands for mortal. And you could say m implies R is, but you can see that motion of all is essentially. To handle this we need to go to first order logic which is a more expressive language which allows us to look inside sentences.

Remember in proportion logic a sentence is atomic and the only thing you can say about a sentence is that in our mind it stands for something. And whether it is true or false, but in first order logic a sentence can be broken down into its consequence. And you can look at the relation between the different constitutes what is the relation between of being between being a man and being a mortal? And then saying that this relation carries a what to Socrates, because Socrates is a man therefore, same relation was applied to him that also mortal that is possible in first certain logic. First certain logic will introduce motion of individuals men Socrates and so on. In proportion logic there is no individuals there are only sentences, the sentences can only be true or false whereas, the individuals are things which can participate in relation essentially. So, in the next class, when you meet on Wednesday we will look at of logic and we will look at all these proof methods in the context of first order logic plus some extra rules. So, we will stop here.