Artificial Intelligence
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Lecture - 26
Game Playing

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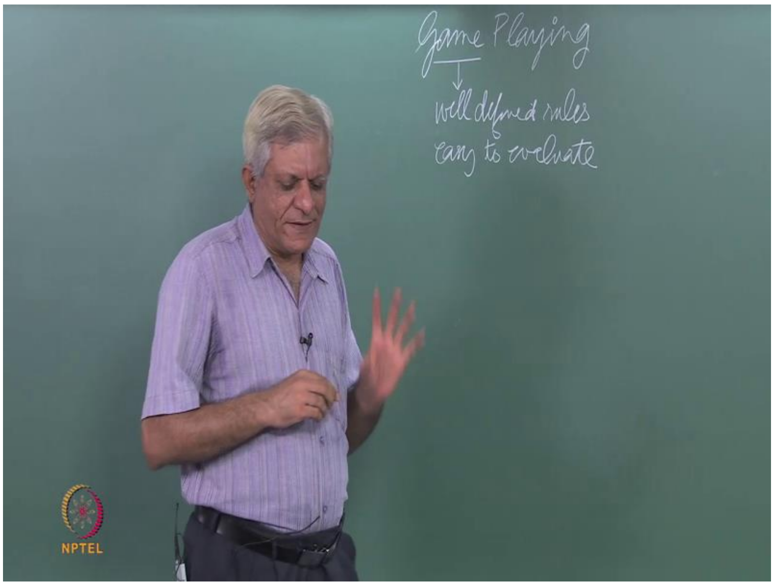

Today, we want to move on to a different topic, which is, games. Now game playing has fascinated computer scientist ever since, the beginning of computing time. Von Newman was the first person to talk about chess playing programs, and we have discussed in the introduction that Alex Bernstein wrote a program in the early 50's, and Samuel wrote a program to play checkers, and so on and so forth. Essentially why do we like games? You can say games, we like, because they have well defined rules. If you solve real problems in the real world, if you want solve computer vision and natural language processing, or let us say, TSP, and things like that; some of the problems, TSP is well defined, but some of the other problems are not so well defined, essentially; the real world problem if you are trying to solve. Whereas, games have clear cut rules; it tells you these are the moves you can make and so on. Essentially easy to evaluate, it is very easy to decide; the rules tell you when a player wins when a player loses.

So, it is easy to judge. Incidentally, the kinds of games we are talking about, these are not the games which are popular amongst students where, you are shooting somebody or killing somebody. We are talking of simpler games, which are like chess and checkers, and so on and so forth, essentially, but the fact is that games, because of these two features; they were well defined and easy to evaluate, or always the good medium for evaluating your reasoning algorithms or search algorithms. Whatever is the technique that you are using, I mean, not only for us, but even in the real world, you can imagine that if you look at our country, you will find the politicians there children, are politicians, that is their only merit.

A film star has children, who become film stars, again, that seems be their only merit. Whereas, if you look at world of sports, children of sports men, do not necessarily, make it begin the world of sports, essentially. Look at Sunil Gavaskar, one of the greatest batsman we had, his son Rohan Gavaskar, could barely, make it to the Indian team, essentially. Whereas, is if he might be a politician, he would have been some big shot by now, essentially. That is because games are easy to evaluate that you do not have bias; you do not have judgment. Somebody saying, no, this is not good and that is good; they have no such things; either, you win or you lose, essentially.

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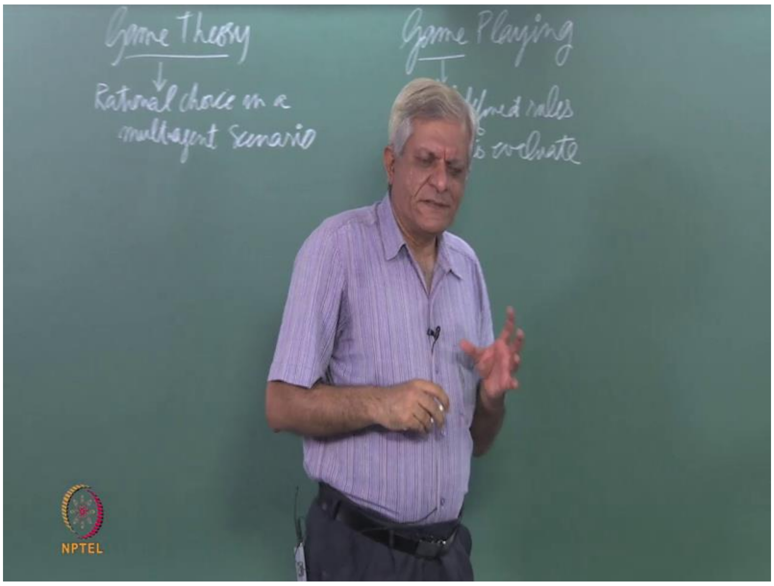

Now, game playing derives some game theory. So, what is the name when associated with game theory? Who is credited with inventing this field of game theory? Not Mash, essentially. Von Newman is the person, we credit with inventing game theory, and what do we mean by game theory? We mean rational choice in a multi agent scenario. So, essentially, game theory is the first place that we have stepped out of a single agent situation. So far, the algorithm that we have looked at, search or whatever else; there was a single agent, trying to solve a problem, and you are looking at what are the approaches solve that problem. In game theory, we first time, introduce more than one player, and therefore, we have to consider the actions of other players, essentially, and that has to be part of listening process that we do, essentially.

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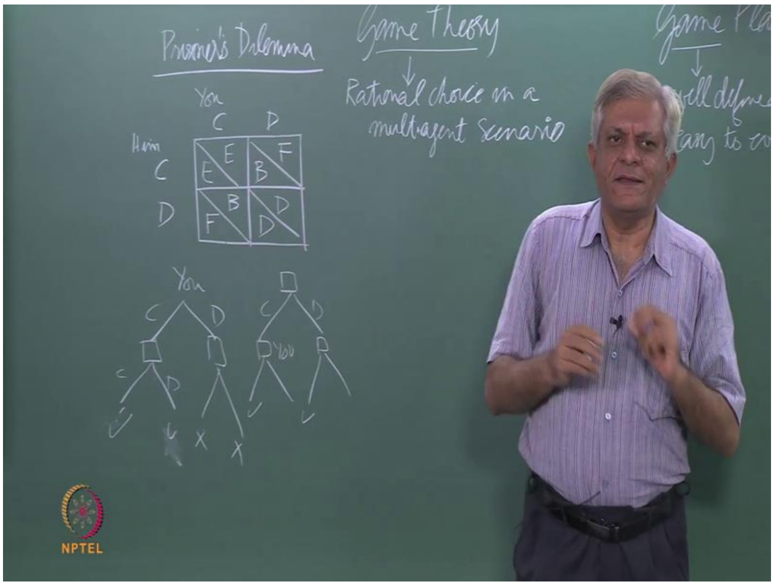

Now, I wonder if we have heard about this game called Prisoner's Dilemma. So, let me introduce this game to you in more familiar setting. Let us say, there are two players as far as this game is concerned, and I am going to write the pay offs here. So, there are two players. Let us say, some hypothetical country, obviously, not our country; there are two students who have been caught cheating, and the Dean has put them into separate rooms, and they are being asked to confess, whether they have helped each other, cheat or not. The payoffs are like this. Let us say, this is you, I mean, you meaning not personally, you, but the player for which, you are designing this game. Let us say, this is him whatever, the other person, essentially. You have two choices; you have to confess or you have to deny, likewise, the other player has two choices; you have to confess or you have to deny.

When the game was originally defined, it was defined in terms of two crooks, who were locked up in two different rooms, and the police were trying to interrogate them. They had to confess or deny, but the principal is the same, essentially. So, what is the payoff here? Let us say, these are your choices; confess or deny. So, if you confess and if the other person confesses, let us say, you get a E grade. This is symmetric for both. So, both people will get E grades. If both of you deny, then let us say, the payoff is this; you both get D grades, because nothing is proved and something like that. Now, if you confess, and the opponent, if you want to call the person, opponent; denies, and then let us say, you get a B grade, and the opponent gets an F grade, likewise, if you deny, and the opponent confesses, then you get an F, and the opponent gets a B grade. So, the top right corner is your grade. The bottom left corner is the other person's grade. These are the four possible situations; you confess, or you denied, opponent confesses, or opponent denied. You can construct that game, three out of this; two represent this choice. Now, remember that this is the game, which is played where, you have to decide sitting alone in the room, and you have been interrogated by somebody. You have to either, confess or you have to deny. You know the payoff matrix. Both players know the payoff matrix. What is the correct strategy to use here? Game theory is concerned with questions like this; that if you know the consequences of your actions, in the context of other people's actions, what are the choices that you will make. So, in this situation, you think it is good to confess, or is it good to deny. Rationally speaking, sorry, you have to speak a bit louder; deny is better; why is it?

See; let me explain this matrix, again. The first, look at the first square here, on the left hand side. It says that you are confessing, and the other person is also confessing. You do not know what is the other person is doing, but if the other person also confesses, then you get an E, and other person gets an E. This square says that if you deny, and the other person confesses, then you get an F; that is your F, stands for fail, and the other person gets a B, essentially. If you confess and he denies, then it is symmetrically, opposite; you get an B and gets an F. If both of you deny, then you get a D each, essentially what should be your strategy?

Student: Confess.

How many people say confess, and how many say, deny? So, others, do not have an opinion.

Student: If you confess, the other deny it should be oppose it.

Sorry.

Student: If you confess and the other deny, it should be oppose it. If you confess and the other deny, the other person should get a B, because in this case, deny is paired in both the cases, irrespective of whatever he says.

Yes.

If you look at the other scenario, let us say, two people have robbed a bank or something like that. I do not know which is the worst; the cheating in a class, or robbing in a bank; anyway, let us say, they have robbed the bank, and then, the police is interrogating that two people separately. They say, if you confess, we will let you off lightly, but the other guy will get punishment. So, it is in that sense that you help them, establish that there was a crime, essentially. So, it is in that sense. That is why, if you confess and the other person denies, you are let off with a B, but other person gets an F, essentially. Otherwise, if you use deny, and the other person confesses, it is the other way alone, essentially. So, many people said confess; why is confess, good? We should give a rational argument. Game theory is about rational behavior that what is logically, the correct thing to do, essentially. You want to maximize your grade in this case, or in the case of those robbers; they want to minimum their punishment, but the problem are equivalent, essentially.

Student: Irrespective of what the other guy does, it is better for you to confess.

Yes, but either that, let us say, your answer is correct, but I just do not want the answer; I want an explanation.

Student: (( )), but if you confess, you will get a B, and which is better than F. If other person is denying, you will get a better grade. if other person is confessing also.

I will encourage you to draw a small tree out of it. So, for example, you confess, deny, and then, the other person, confesses and denies, or you can construct it like the other person confesses and denies, and then, you confess and deny. You can construct the three both ways and then, explore that, essentially.

Student: Either case, if you take the situation where, whatever, regardless of what the other person does, if you say corresponding row; your choice is between confessing and denying, because it is between E and F, and in the other case, it is between B and D. So, it is always better to take the choice of confessing.

Let us argue. Let us just expand up on the argument and say, let us say, if you confess what will happen? That means, you are looking at this row, essentially. You are looking at the first column. If this person confesses, then you get an E. If this person denies, then you get a B, which is, this, so, looking at this thing is, looking at these three structure; first, you look at your decision and then, you look at other two decisions and then, and see which one is better. You will see that in this case, this is better. In this case, this is better and this is bad. If you deny, if the other person confesses, you get an F, and the other person denies, you get a D. So, maybe, this is actually, may be, let us look at the other side of the argument. If the other person confesses, what should you do? If the other person confesses; so, you looking at row, now; we are looking at this tree. You can argue this tree also, but let us look at this, may be, this is simpler. If the other person confesses, you are looking at this row. If you confess, you get an E. If you deny, you will get an F. If the other person confesses, it is better for you to confess, correct. You are looking only at the row. If the other person denies, and if you confess, you will get a B. If you deny, you will get a D.

So, even in this case, it is better for you to confess. So, it is better for you to confess in both cases, essentially. If you look at, you constructed tree like this where, you consider the other person's choice as first, then your choice, and then, you will see that your choices; this is better, and this is better. In this case, these two are better, and these two are the worst, essentially, but this way, it is easy to evaluate. Yes, the actual choice is to confess, and assuming that the other person is also rational; other person also confess. So, your expected grade would be an E, essentially. This E is called the Nash equilibrium in this game, which means that if you divide from this, rather the other person cannot divide from it, without spoiling the game from him, essentially. Such an equilibrium is called the Nash equilibrium, but is this the best outcome? It is not the best outcome in this particular matrix, because if both of them are die hard people, I am not saying that students can be die hard criminals, but we are talking about those robbers now. If both of them are die hard criminals, like you know, for example, it is eulogolized in many films essentially.

So, they are these two criminals, who were really good guys, but you know, they will never confess for anything. Now, if you have two such people, and they have trust in each other in some sense; then, they will both deny. In this matrix, they will both end up in this lower right hand corner, which is the better situation than that situation. So, this of course, is called a Pareto Optimal, but it is not a stable optimal with this problem situation. If I want to exchange these grades for these two corners, this E, E and D, D; then the Pareto Optimal coincide with the Nash equilibrium, and then, it is really the best solution, essentially. Why is it called the Nash equilibrium?

Because in some sense, if you confess, and the opponent is forced to confess, if he does not confess, he will go off, likewise, if he confesses, then you are forced to confess, essentially. Now, obviously, you cannot get into complicate or you try to get into complicated, kind of pleasing that I can figure out that it is real. So, this is the basic idea of game theory, puts you in a situation like this. It is a basic mechanism for studying economics, when people try to evaluate things like, what should be the pricing mechanism; you know, how much will you advertise and things like that; people use game theoretical concept to reason there, essentially. So, rational choice, in the case of multi agent scenario, essentially. So, you can think of Price Wars, for example, as the game between two players. So, let us first characterize the games a little bit. The kind of games that we are going to be interested in, are much simpler in nature.

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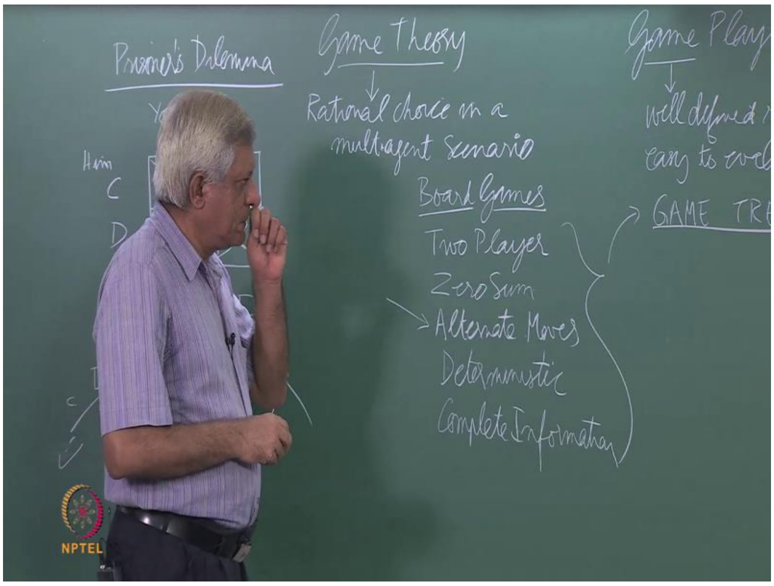

These are called Board games, and as the name suggests, they are played on a board, like chess, for example. In particular, the games that will be interested in, have this following characteristic; they are two player games, which means that there are only two players in this game, but there can be multi player games, may be, will give an example, Zero Sum.

So, we would be interested in games, which satisfy these (( )); two player, zero sum, alternate moves, deterministic games. I might even say, discreet games, in the sense that you know things, like chess and so on, where, you have to choose between the set of discreet, set of choices, essentially. So, two player basically, says that there are two players, but you can have multi player game, essentially. I was just talking about Price Wars. If you think of Price Wars as a two player game, which means, let us say, Pepsi and Coke are getting into a price war, about to decreasing prices to get more market share. Then, you can think of it as a two player game, and you can also think of it as a negative sum game, which means that both the players as going to lose out on the money that they get, essentially.

So, a zero sum game is the game, in which, the sum of the payoffs of different players is 0. So, chess for example, one person wins, and the other person losses. So, if 0 stands for a draw, and minus 1 stands for a loss, and plus 1 stands for a win, then the total sum is always 0, essentially. One person’s gain is another person’s loss. So, these are the game offers where, you have opponents, essentially, but there can be games, when there is negative sum. So, for example, this game is a negative sum game, essentially, assuming that E and D are negative things, which you do not particularly, like.

The Price War game is a negative sum game, because the revenue of both the companies is going to suffer a loss, which is seen as negative, essentially, but you can also have positive sum games. For example, cooperation between people, if two people sit down and study together for an exam, then both of them gains from it, essentially. So, you can have different degrees of games. So, Price War, as I said, can be seen as a two player negative sum game, but if you include the third person, which is the consumer, then it becomes the three player game. Let us say, there is one figurative consumer and then, it becomes the zero sum game, because whatever the company loses, the consumer gains, in terms of price we are talking here, essentially. I missed out one very important characteristic; which is complete information.

In complete information games, each player knows; what are the moves available to the other players. Essentially what are the choices available to the other players, essentially. So, obviously, the game that we have been talking about, like chess and checkers and Cross and Knots, or Go; all these game called Othello in which, you are going to do an assignment. These are board games on which, you can see the board. Both sides can see the board, and therefore, they are complete information games, essentially, but there are many incomplete information games. So, games, like car games, for example, games like Poker; are incomplete information game, but there are also games, which are deterministic versus nondeterministic.

For example, if you have to throw a dies in a game, game like Backgammon or Ludo, or something like that, and that is not a deterministic game where, you cannot predict the outcome of the move, essentially. Backgammon, for example, is a two player, 0 sum, alternate move, and stochastic, incomplete information game. The moment the game becomes stochastic, it becomes incomplete information, because we do not know what happens, when you roll a die or when the opponent rolls a die, essentially. Of course, in the real world, many situations where, you have to do rational decision making. You are mostly operating in an incomplete information world, essentially. An extreme example is two generals, who are fighting a war, essentially. So, you do not really know what the other's army is doing; which side they were moving. Of course, we try to have spies on the other side, and you try to have intelligence, may be, you take satellite images and all kinds of stuffs, but that is an effort to move towards the complete information, essentially.

But in practice, of course, you have to work when the information is not complete, essentially. Alternate move games are essentially, the simpler kind of games in which, I make a move and then, you make a move, then, I make a move and then, you make a move, which are easier to model for us, essentially. Now, the moment I talk of alternate moves, you can see that we are moving away from these kinds of games where, there is only one decision to be made, and you make one decision. Here, typically, you have a sequence of decisions; I make a decision, then, you make a decision; then, I make a decision, and things like that, essentially. So, such games can be represented by a game tree. A game tree is a layered tree. So, let me draw one, and typically, because we talking about two person games, there are two kinds of nodes; one for one player, and one, the other kind for the other player.

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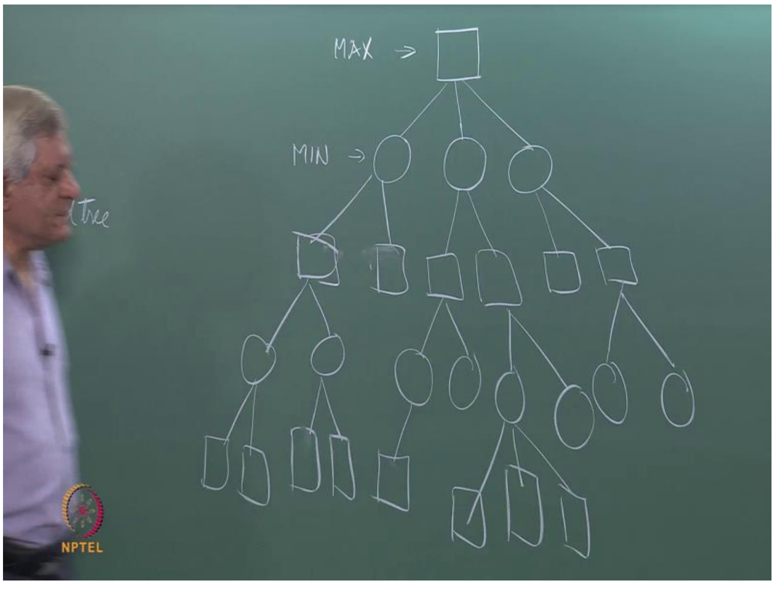

Typically, we draw them differently. So, this is the route node, which the first player has to make a move, and let us say, this is one move that this player makes, out of the many possible moves. Then, the other player makes a move. Then, the first player makes a move, again. Then, the other player makes a move, and so on. So, this is the game, which is a, it is the actual game played, says that one player made this move, and second player made this move, and the first player made this move, and so on. So, square is one player and circle is another player, essentially. Of course, there are many choices that they all have. So, in practice it is a tree or something like this. So, such a tree is called a game tree. It is a layered tree where, each layer represents the choice for one player. This is some tree, I have drawn randomly. Traditionally, this player is called max, and this player is called min, and you can see that the tree, basically, represents a sequence of choices for both players. We are assuming that max plays first. So, max has chosen within these three moves. Then, depending on what max chooses, min has to choose within two moves, and so on, essentially. Such a tree is called a game tree.

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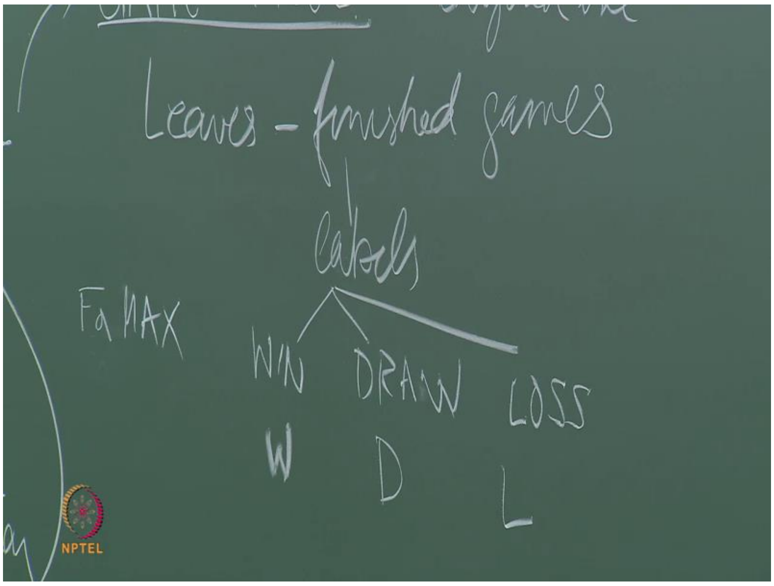

In a game tree, leaves are finished games. So, leaves represent the end of that game, and it has got three kinds of labels. So, a leaf may be labeled by one of these three labels; win, draw or loss; and this word is from max’s perspective. When you say win, it means max has won the game, and if you say loss; that means, max has lost the game, essentially. This is for max. Anything that we do is for max, essentially. So, labels are for max and so on. So, we can complex this to W, D and L, essentially.

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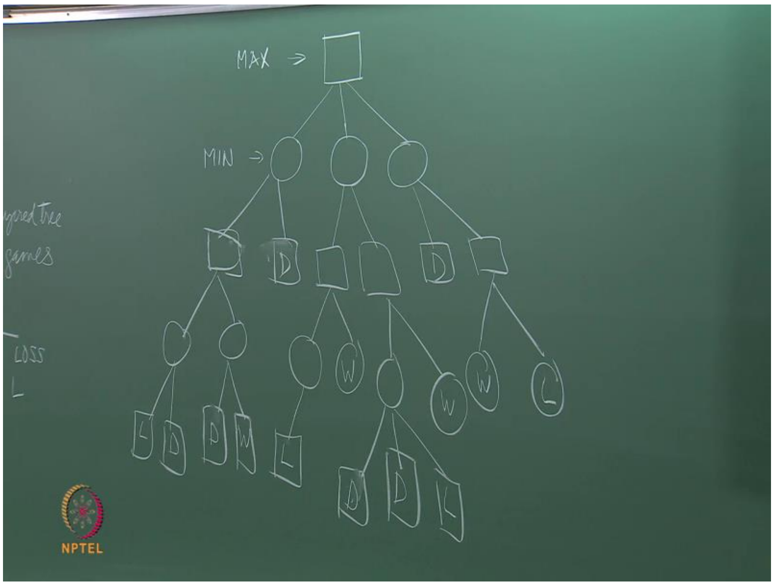

Let us label this arbitrarily. Let us say this is L this is D. This is D and this is W. This is L, W, D, D, L. Every leaf must be labeled. Every leaf has a label. So, basically, the game tree represents all possible games that can be played; given the rules of that game. When we draw the game tree, we do not talk about rules. We just say this, a game tree. You can imagine that if you want draw a game tree for chess and white starts first; white has a set of moves that white can make, which is 8 plus 8, 16 pond moves plus 4 knight moves. So, those of you, who know chess, at the top level, white can make 20 moves, essentially. You could just draw the tree and say, forget about the rules of the game, essentially.

So, a game tree is basically, a representation of the rules of the game. Once we represent the game in the form of a game tree with, we can just reason with the game trees, essentially. Now, every game has an outcome, which will be the outcome, if both players are playing perfectly. We are always interested in rational choice, remember. We do not want to play against an opponent, who can make mistakes, and things like that. We assume the opponent is perfect, and our analysis is based on that, essentially. So, if both the players are perfect, then every game and a game tree is a finite tree, because it has all these leaves, essentially; has on outcome, which can be computed, essentially. So, just like, you can think this as a game tree with only one node, essentially, and its outcome will be determined by what choice you make, essentially. At the same time, this is simultaneous; this is not alternative move, essentially. Both the players are making moves simultaneously, whereas, the games you talking about; first max makes a move; then, min makes a move; then, max makes a move, like chess, for example, and so on.

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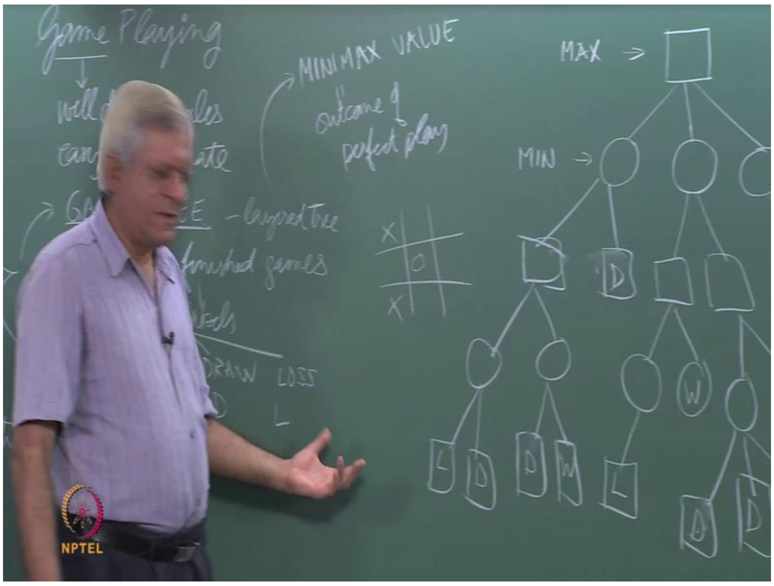

How do we listening? The outcome is called the minimax value, is equal to outcome of perfect play, one more thing. By perfect play, we mean that each player is making the correct choice, essentially. Let me ask you, for example, you are familiar with Knots and Crosses, right. This game which children play, or Tic Tac also, as it is called. You put one cross, somebody puts a knot and you put another cross, and so on and so forth, right; you are familiar with this game. So, let me ask you; what is the value of this game? What is minimax value of this game? It is wrong, right; which means that we everybody knows that if we played perfectly, then the game is withdrawn. So, how would you compute the value, minimax value of the game? Is that we would have a backup rule, we should back up value from bottom to top, and the backup rule will basically, look at the kind of node that it is; I will not write the rule; I will just state it here. So, for min, it means that if you can back up L, if another choice is L; then back up L. If none other choice is L, then, if you have D; back up D. Otherwise, you back up W. You have forced to back up W, in some sense, because min; the desired outcome for min is the leaf labeled L, because min wants to win the game, which means that max should lose the game. Therefore, min is trying to drive the game towards L. So, you can see what is happening here. The first move is made by max. So, let say max makes this move. Then, it means, it up to min; so, min will try to drive the game towards L. For example, there are some Ls here, but if min drives the game here, unfortunately, I have drawn both of them like this. So, let us say, this is L. If min drives the game here, you can see, max can choose W, and end the game there. So, it is better for min to play this move. If min plays this move, then max will not choose that, of course, you know, and max will choose this, and at this stage, min can win the game. You can see that if min makes this move, then min can win the game, which means, max should not really, make this move, essentially, because perfect win will have been here, essentially. So, this kind of analysis is simplified by this back up rule, which says that you back up from leaf to the routes, and for min, you choose.

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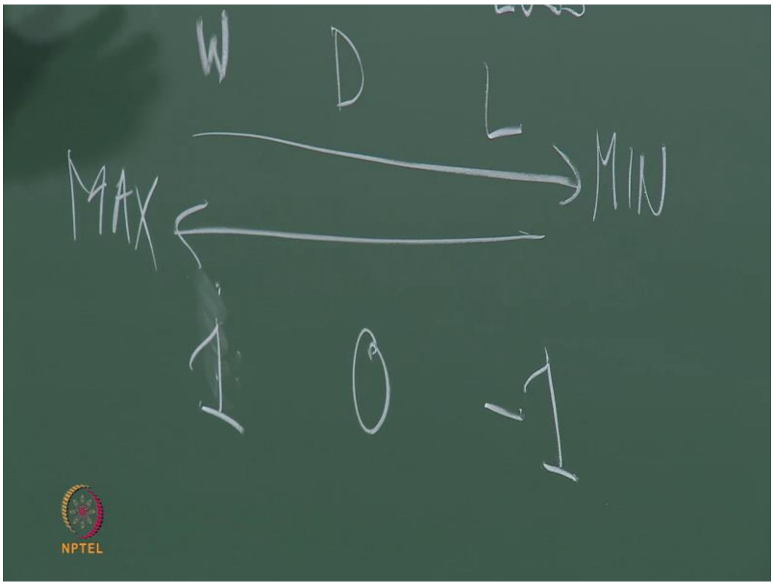

So, for min, the order of preference is this. For max, it is this. It means, min will prefer L to a D; if it can see L and D, it will take L, whatever, it will always choose the right most; max will choose the left most. So, as we will see, it helps to think of this as a supply chain kind of a thing where, max is supplying some values to min, and min will choose. Min’s choice is always, prefer the L and then, D, and then, W. Likewise, min is making offers to max, and max will choose, and so on, and so forth. Before I fill up the tree, let me also say that we can represent this by 1 or 0, or -1 as well; just a numerical way of representing this outcome, which of course, gives us a clue, as to why max is called max, and min is called min; because max is driving it towards 1, trying to maximize the value of the game, and min is trying to drive it towards 0, sorry, towards minus 1, and trying to minimize the value of game. So, that is why the players are called max and min. In terms of numerical value, it is easier to state the backup rules. Max chooses the maximum of the values, available to it, and min chooses a minimum of the values, available to it. What do you mean by available to it? That, at from the next layer, what is being supplied to it, essentially. So, let us fill up this tree. This is L; this is D; so, min will choose L here.

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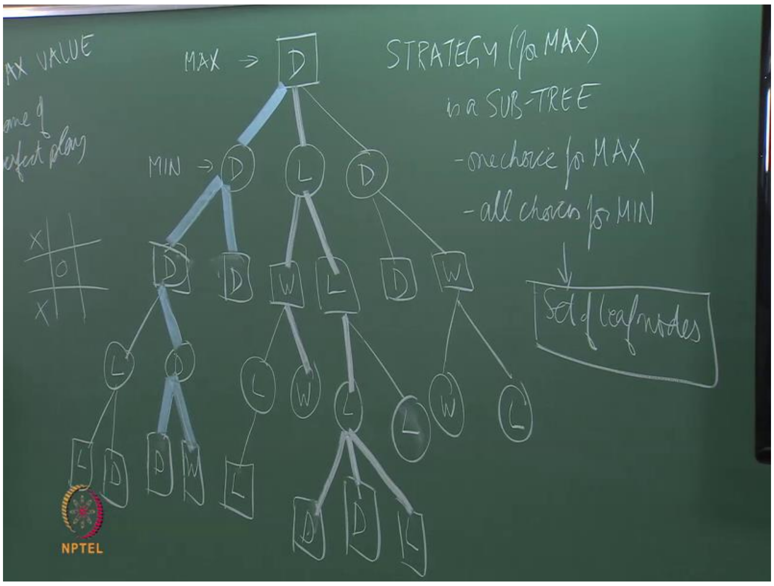

This is D; this is W; min will choose a D here. This is L; this is D; the max will choose a D here. This is, both these are Ds; so, min does not have a choice. Here, it is L and so, there is only one node. This gets backed up; L. Max will choose W here, and we analyze this part of the tree, earlier, right; min will choose L here; max does not have a choice; so, max has to choose L here; min will choose L here; here, max will choose W between

W and L; min will choose D from this. So, as a top level, you can see that max has three choices. One of them leads to a node, which will evaluate, eventually, evaluate to a draw. This move will eventually, lead to a loss, and this move will eventually, lead to a draw. So, max can choose one of those other two moves, essentially. Rationally speaking, and the minimax value of the game is therefore, D.

So, this is the game, which has a minimax value of D, essentially. In some sense, solving a game amounts to trying to find the minimax value of the game. It is called minimax, because at alternative layers, you are minimizing and maximizing, essentially. So, in some sense, you are trying to distinct. Another way to put it is that we are trying to find a strategy for max. Obviously, at the first level, we have known that max should make as this move or this move, but on what basis, are you saying that; that basis can be formulas in the notion of the strategy, and a strategy; we are talking about max at this movement, but we can talk similarly, about min; is a sub tree, which is constructed as follows, that one choice for max, and all choices for min. So, by strategy, we mean that max is freezing max’s choices is and saying; this is my strategy; this is how I am going to play this game, essentially.

It means that every plays in the game, max says that this is the choice I will make. So, that is why, the strategy is the sub tree that we construct by making one choice for max, and all choices for min. Why all choices for min, because it is max’s strategy, and max does not know what min will play, essentially. So, max has to account for, or cater to all of min choices. So, the strategy must consider all of min's choices. So, we can draw a strategy for max. One strategy is, let us say this one. Now, we have chosen this, but for this, we must choose both; because all choices for min.

Then, we can have this choice for max, and let us say this choice for min. So, this is, the thick lines represent one strategy for max. Of course, yes, max has occurs to many different strategies, and you can say that the task is to find the best strategy for max, essentially. Let us draw another strategy for max, which is on this left side of the tree. So, max could have chosen this move, and then, of course, you have to choose both these moves. Then, max could have chosen, let us say this move, which is the better move and then, min would chosen. So, that is how, we construct the strategy. At max level, you choose all, sorry, at max level, you choose one choice for max. At min level, you choose all choices for max. What the strategy gives you is the set of leaves. Every strategy has a set of leaf nodes, which tell you about this. This is what will happen, if max chooses this strategy. So, the question I want to ask you is, given a strategy; how do we evaluate a strategy? Let us work with this one; numerical representation.

Let us say the leaves are labeled with 1, 0 and minus 1. Given a strategy, S, and I talk about the value of a strategy; how can I compute the value of a strategy?

Student: Worst possible one.

Worst possible one and by that, you mean the lowest value essentially.

So, if you look at this white strategy, in this white strategy, the leaves have W, D, D, L; four nodes in the leaves. The worst of them is L or minus 1. So, that is the value of this strategy. If max chooses this strategy, he will end up losing the game. If min is perfect, because the choices that are left, are only for min, essentially. If min is a perfect player, then if max chooses this strategy represented by this tree, then max will end up losing the game, and the value of the strategy is the lowest value of the leaf nodes, essentially.

If you look at this strategy, then you can see that there are three leaf nodes here; D, D, and W. The worst of them is D, and therefore, the value of the strategy is D or 0, in this case. If you looking at minus 1, 0 and 1, value of the strategy is 0. Now, none of them is a winning strategy. A winning strategy would be a strategy, whose value was 1, essentially. Ideally, of course, you want to find a winning strategy for a player, or max, in this case, because we are writing the program for max, let say, and somebody else is handling min, or if you cannot find a winning strategy, then at least, you should blue one, essentially. So, I will leave you with this question here.

Can we think of this as an AND OR tree, and can we use A0 star algorithm to find an optimum strategy? Because you can see that this like an OR choice, and this is like an AND choice. This is like an OR choice, and this is like an AND choice. So, can we think of it as an AND OR tree, and can we use A0 algorithms? So, I will stop here and in the next class, we will take it up from here, essentially.