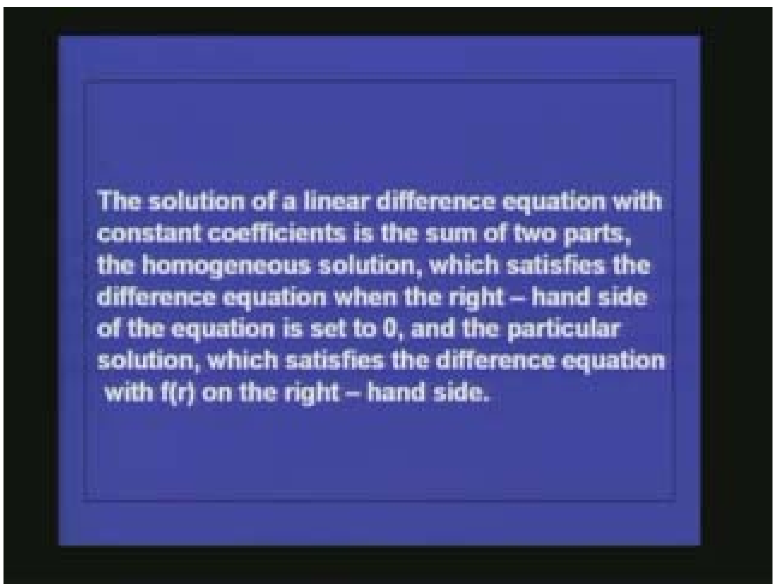
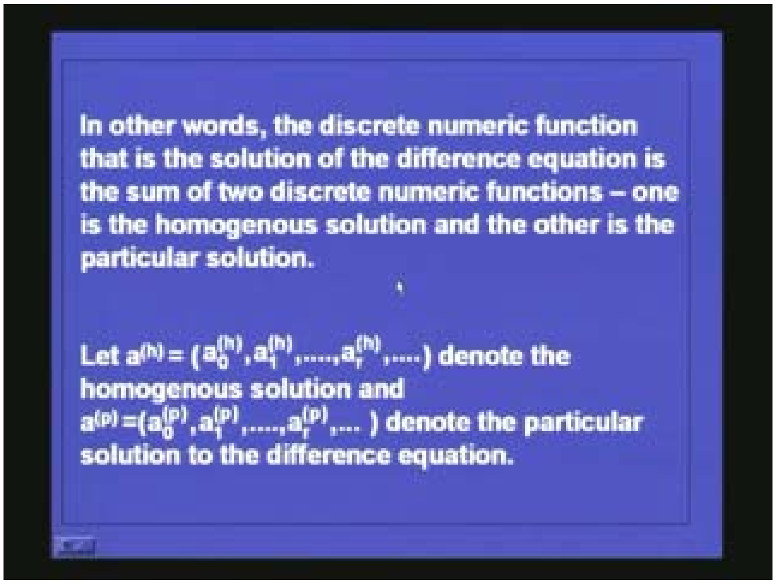
Discrete Mathematical Structures
Dr. Kamala Krithivasan
Department of Computer Science and Engineering
Indian Institute of Technology, Madras
Lecture # 33
Recurrence Relations (Contd....)

In the last lecture we saw about recurrence relations also called as difference equations. We saw what a recurrence relation is and we were also seeing about the solution of a recurrence relation. Actually the solution of a linear difference equation or a recurrence relation with constant coefficients is a sum of two parts. One part is called the homogenous solution and the other part is called the particular solution.

(Refer Slide Time: 01.29)

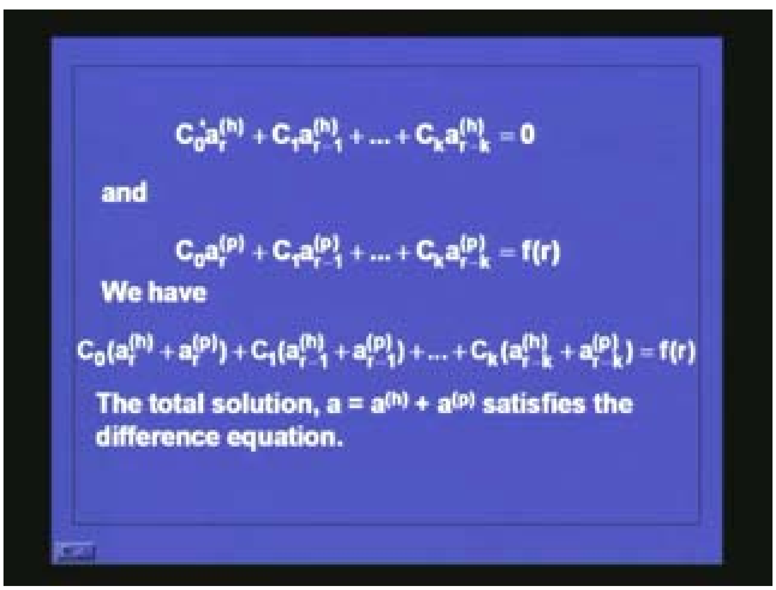


(Refer Slide Time: 1.44)



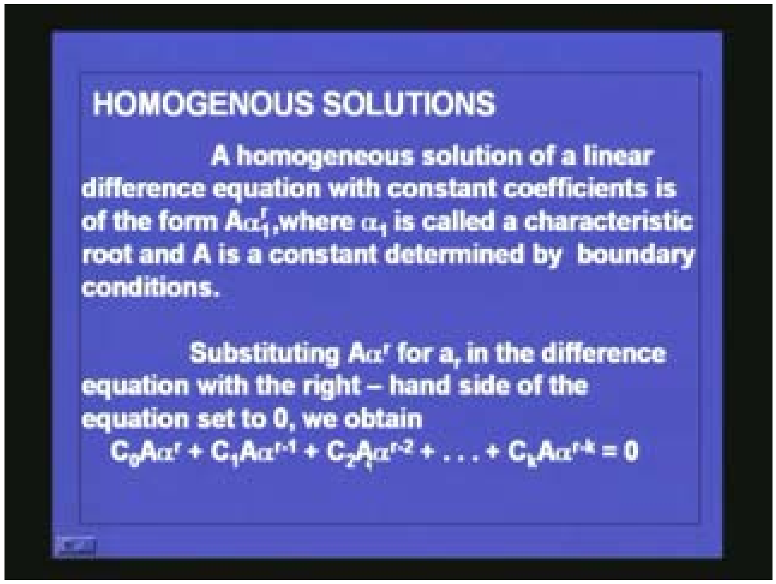
So, if the homogenous solution is given by this and the particular solution is given by this then the total solution is the sum of these two and the homogenous solution satisfies this equation the linear recurrence relation with constant coefficients with right hand side is equal to 0.

(Refer Slide Time: 1.57)



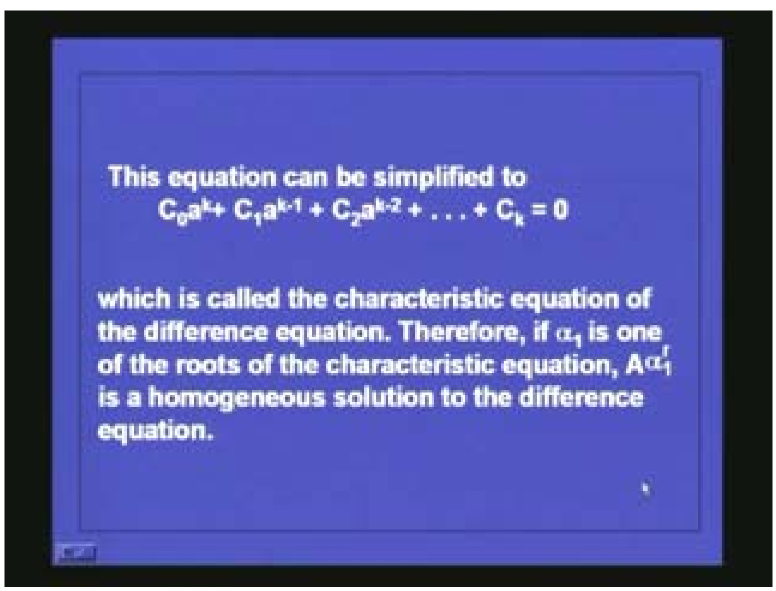
The solution for this is called the homogenous solution. A solution for this with the right hand side given as f(r) is called a particular solution. Now adding these two up you get $C_0 a_r h$ plus a_r to the power (p) etc is f(r) and this is called a total solution while 0 will be satisfied with the particular solution alone because sometimes the boundary conditions have to be satisfied in a proper manner and it is always advisable to have the total solution. The total solution is given by a is equal to ah plus ap it satisfies the difference equation.

(Refer Slide Time: 2.51)



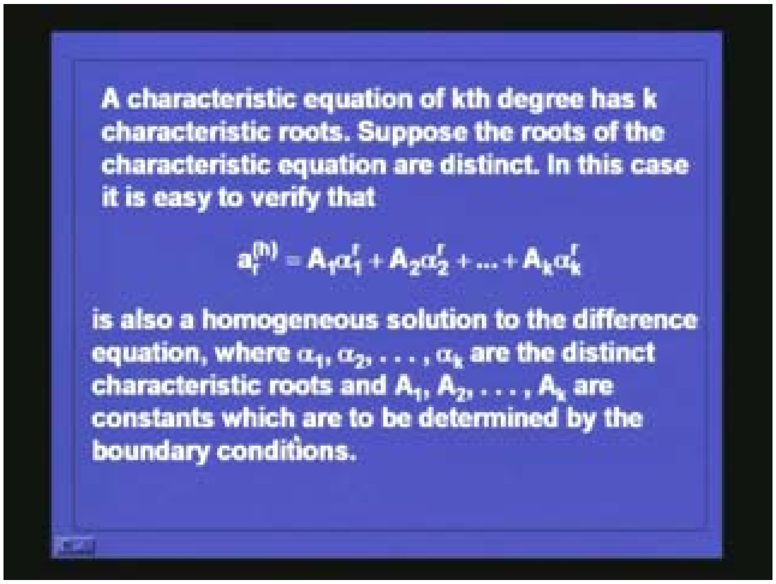
Now we were considering the homogenous equation. If this is the linear recurrence relation then we get an equation like this.

(Refer Slide Time: 3.09)



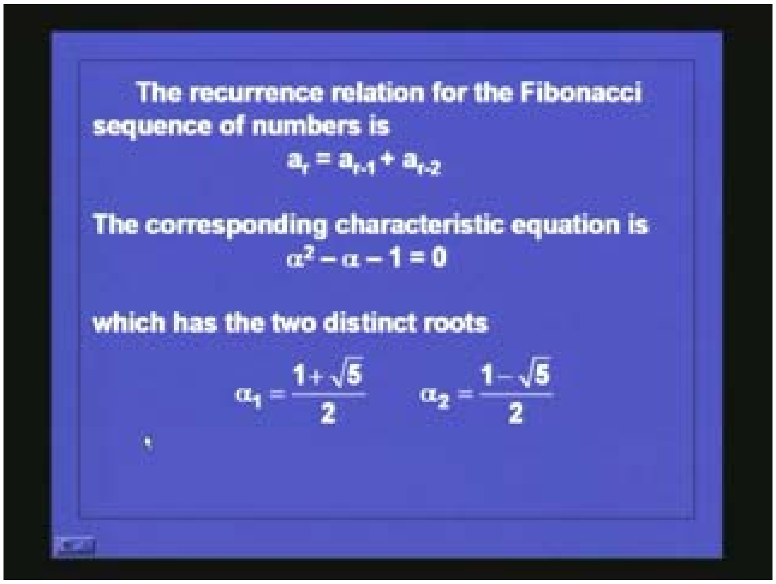
Removing a it is simplified like this. This is called the characteristic equation. And if alpha one is one of the roots of the characteristic equation then a alpha 1 power r is a homogenous solution to the difference equation. We have also seen this in the last class.

(Refer Slide Time: 3.33)



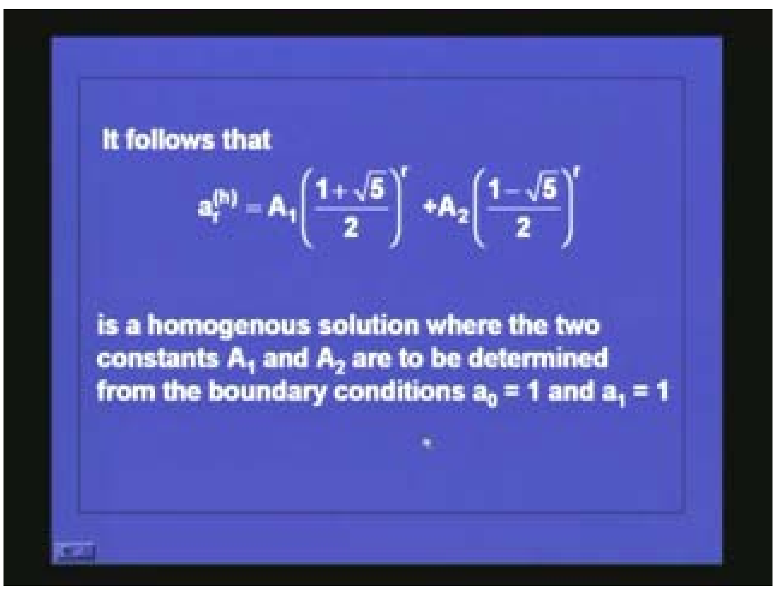
So a characteristic equation of kth degree has got k characteristic roots. Any equation of kth degree will have k roots. If they are all distinct and they are alpha 1 alpha 2 alpha k then the homogenous solution will be of the form A_1 alpha 1 power r, A_2 alpha 2 power r and so on. Suppose the roots of the characteristic equations are all different distinct in this case we can very easily verify that the homogenous solution will be of this form. It is also a homogenous solution to the difference equation. Alpha 1 alpha 2 alpha k here are the distinct characteristic roots and A_1 A_2 A_k are constants which have to be determined by the boundary conditions.

(Refer Slide Time: 04.29)

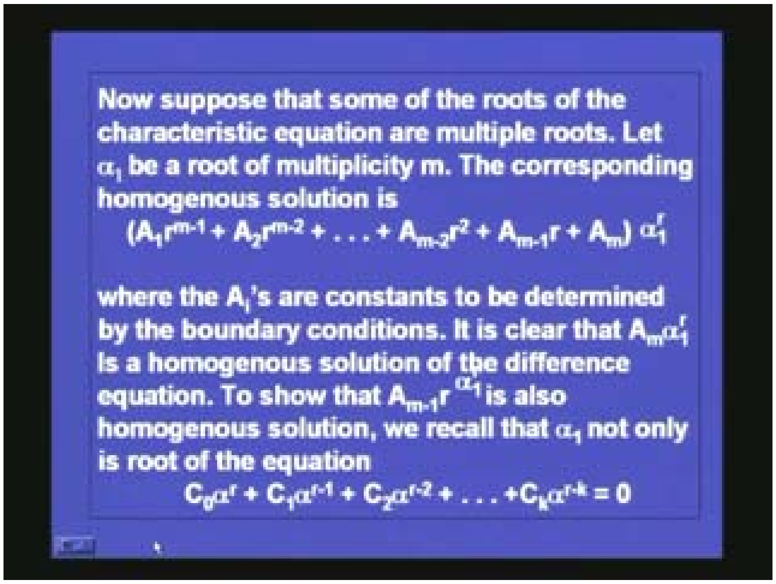


And we consider this example of the Fibonacci sequence, this was the characteristic equation and the two roots were this and so the homogenous solution will be this then A_1 A_2 have to be determined by the boundary conditions which we have seen.

(Refer Slide Time: 04.41)



(Refer Slide Time: 04.51)

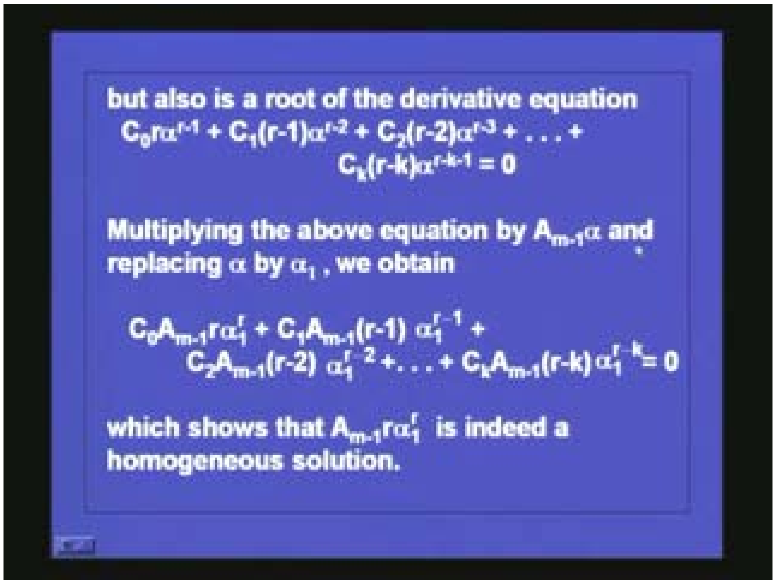


Next we have to consider the case where a characteristic root is repeated or a characteristic root has multiplicity. So what happens in that case? Suppose that some of the roots of the characteristic equation are multiple root they are not distinct but some are multiple roots, let alpha 1 be a root of multiplicity m, then the corresponding homogenous solution is (A_1r power m minus 1 plus A_2r power m minus 2 plus etc $A_{m \text{ minus } 2}$r square plus $A_{m \text{ minus } 1}$r plus A_m) alpha 1 power r where alpha 1 is a multiple root with multiplicity m.

Here the A i's are constants to be determined by the boundary conditions. Now you can very easily see that if you take this portion A_m alpha 1 power r that will satisfy the recurrence relation and so is a homogenous solution of the difference equation. How do you show that $A_{m \text{ minus } 1}$r plus alpha 1r is also a homogenous solution? Similarly, you have to show $A_{m \text{ minus } 2}$r square alpha 1r is also a homogenous solution and so on. Now let us consider the case ($A_{m \text{ minus } 1}$r) alpha 1r.

To show that $A_{m \text{ minus } 1}$r alpha 1 to the power of r there is a slight deviation, it has just gone slightly above. We have to show that $A_{m \text{ minus } 1}$r alpha 1 power r minus alpha 1 power r is also a homogenous solution. How do you get this? We recall that alpha 1 is a multiple root so it does not satisfy the characteristic equation but it also satisfies the derivative of that. So this is not only a root of the characteristic equation which is this but the derivative of that also.

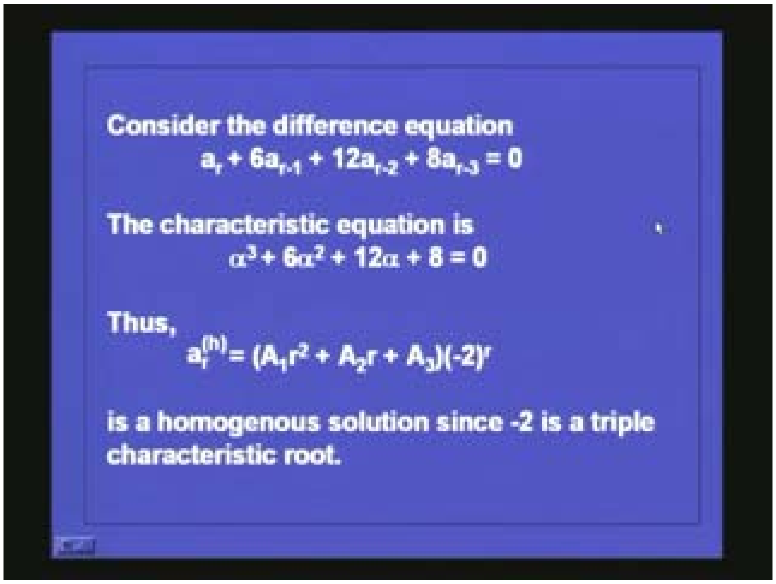
(Refer Slide Time: 07.35)



So the derivative of the equation is C_0r alpha power r minus 1 plus C_1(r minus 1) alpha power r minus 2 plus C_2(r minus 2) alpha power r minus 3 and so on. So, being a multiple root it has to satisfy this equation also. So multiplying the above equation by $A_{m \text{ minus } 1}$ alpha multiply this and replace alpha by alpha 1 you will get this equation $C_0A_{m \text{ minus } 1}$r alpha 1r etc. That is, you are taking the derivative of the characteristic equation and multiplying it by $A_{m \text{ minus } 1}$ alpha and replacing alpha by alpha 1. Now this shows that $A_{m \text{ minus } 1}$r alpha 1r is also a homogenous solution. So this shows that $A_{m \text{ minus } 1}$r alpha 1r is indeed a homogenous solution. If alpha 1 is a double root you can use this, if it is a triple root then it has to satisfy the second derivative also.

So taking this and taking derivative once more and so on you will show that the previous term m minus 2r squared alpha 1r will also be a homogenous solution. If alpha 1 is a root of multiplicity m you can take the derivative m(m minus 1) times rather and prove that each one of them is a homogenous solution. Like that we can prove and hence we have this result which we have already seen. If alpha 1 is the root of multiplicity m then the homogenous solution will be of this form A1r power m minus 1 a polynomial of order (m minus 1) A_2r m minus 2 and so on multiplied by alpha 1 power r.

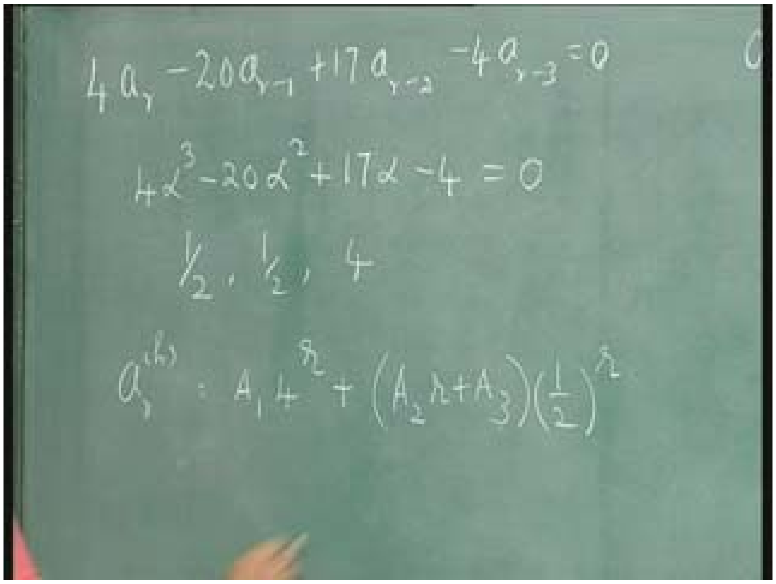
(Refer Slide Time: 9.49)



Consider this example, consider the difference equation a_r plus $6a_{r\text{ minus }1}$ plus $12a_{r\text{ minus }2}$ plus $8a_{r\text{ minus }3}$ is equal to 0. Then the characteristic equation will be alpha cubed plus 6 alpha square plus 12 alpha plus 8 is equal to 0. That is, this expression is alpha plus 2 the whole cube, it is a cube, alpha plus 2 the whole cube, so alpha is equal to minus 2 is a triple root, it is a root of multiple multiplicity 3 so the homogenous solution you will have minus 2 to the power of r but it will be preceded by a polynomial of degree 2 because multiplicity is 3. So you have taken this polynomial as A_1r square plus A_2r plus A_3 and the value A_1 A_2 A_3 are all constants and they have to be determined by using the boundary condition. So, when minus 2 is a triple root the homogenous solution will be of this form.

Now, let us take one or two more examples, let us consider these two examples; one is $4a_r$ minus $20a_{r\text{ minus }1}$ plus $17a_{r\text{ minus }2}$ minus $4a_{r\text{ minus }3}$ is equal to 0. The characteristic equation will be of the form 4 alpha cube minus 20 alpha square plus 17 alpha minus 4 is equal to 0. You can solve this equation and then the roots you will find as 1 by 2 1 by 2 and 4. There are three roots and they are 1 by 2 1 by 2 and 4, 4 is of multiplicity 1, 1 by 2 is of multiplicity 2. So the homogenous equation a_rh will be of the form A_1 4 power r plus this is multiplicity 2 so that will be A_2r plus A_3 1 by 2 power r. Now, here A_1 A_2 A_3 are constants and they have to be determined from the boundary conditions and here all the roots are real.

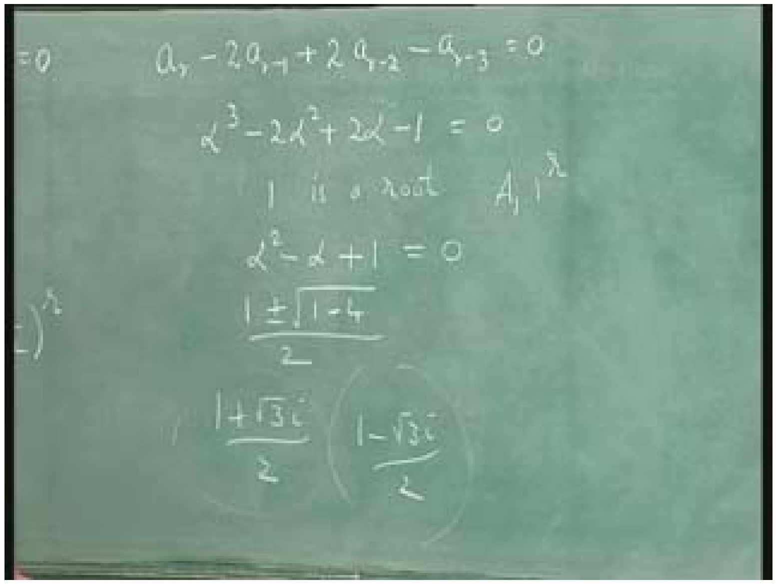
(Refer Slide Time: 12.25)



Now let us take this example, here you can very easily see that the characteristic equation will be alpha cube minus 2 alpha square plus 2 alpha minus 1 is equal to 0 so 1 is a root, alpha is equal to 1 is a root, after dividing by alpha minus 1 you will get alpha square minus alpha plus 1 is equal to 0. and if you solve this the roots will be 1 plus or minus square root of 1 minus 4 by 2 that is the roots will be 1 plus root 3 i by 2 and 1 minus root 3 i by 2. Here you have complex roots.

Of course the homogenous solution corresponding to this will be something of the form A_1 1 power r this is just A_1. You can take as a constant A_1. Now what happens when you take for these two? You can take it as A_1 and this power r and A_1 you have already used, A_2 this power r A_3 this power r. But instead of that because they are complex numbers you generally use the results and take it as B_1 rho cos n theta plus B_2 rho sin n theta.

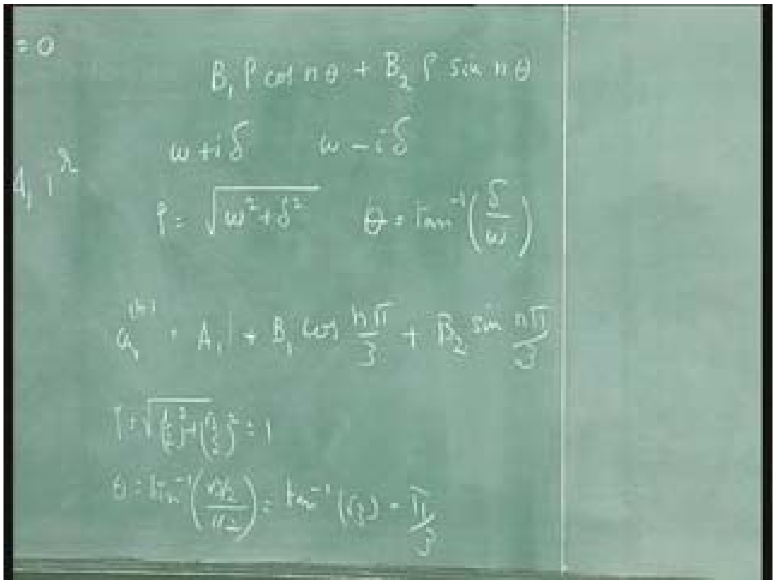
(Refer Slide Time: 13.39)



What is rho and theta?

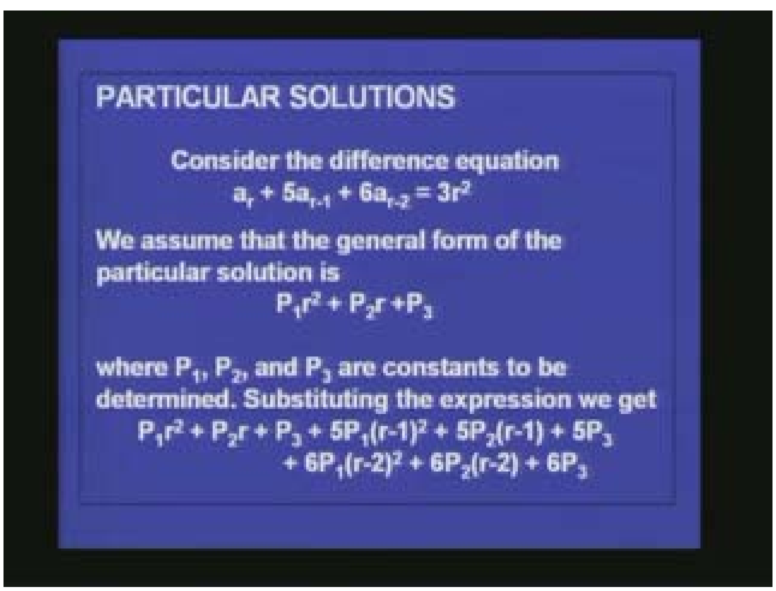
The complex roots occur as pairs, they will occur as w plus i delta w minus i delta. In that case rho is taken to be square root of w square plus delta square and theta is taken to be tan inverse delta by w which is the usual way we take. So the homogenous solution in this case a_rh should be taken in the form A_1 1 power r you can take or leave it out 1 power r is just like that plus B_1, what will be rho in this example? You have rho is equal to square root of 1 by 2 square plus root 3 by 2 the whole square which will be 1 and theta is tan inverse root 3 by 2 by 1 by 2 which will be tan inverse root 3 which is pi by 3. So this will be taken as B_1 rho is 1 so you need not write that, cos n pi by 3 plus B_2 rho is again 1 so you need not worry about that sin n pi by 3. And A_1 B_1 B_2 are constants which have to be determined by the boundary conditions.

(Refer Slide Time: 15.49)



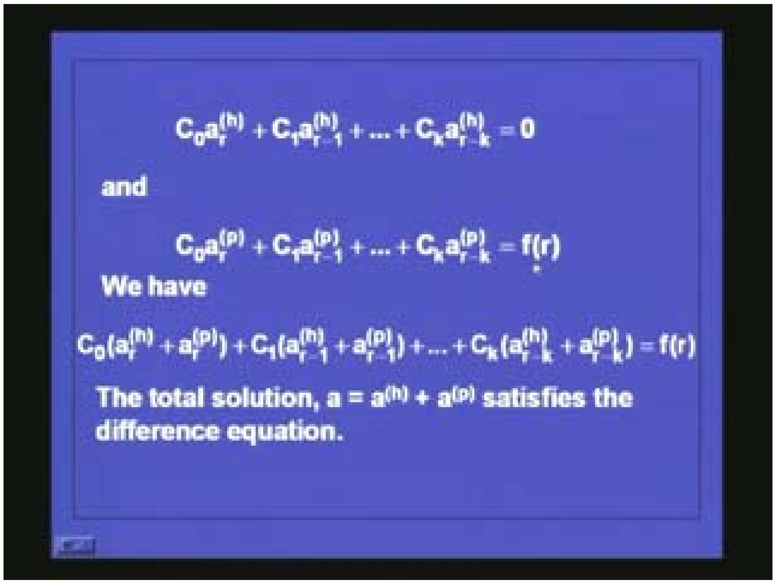
So this is how we go about writing the homogenous solution. Now, as we saw earlier the total solution consists of two parts the homogenous solution and the particular solution. Now what will be the particular solution in this in general? In general there is no set of a strict ways of finding the particular solution. But because of some examples we will see how we have to consider the particular solution and solve the equation for the particular solution.

(Refer Slide Time: 16.34)



So let us take different cases of f(r). We know that the particular solution has to satisfy this equation.

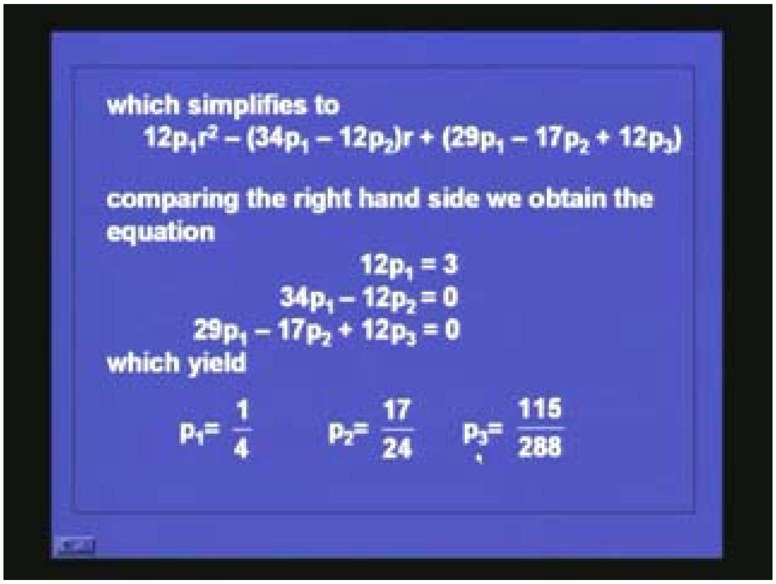
(Refer Slide Time: 16.53)



C_0a_r to the power (p) plus C_1a_r to the power (p) minus 1 and on the right hand side you have the f(r). So depending on the form of f(r) you take the particular solution and find it out. So consider the difference equation a_r plus $5a_{r \text{ minus } 1}$ plus $6a_{r \text{ minus } 2}$ is equal to 3r square. So, on the right hand side you have f(r) which is 3r square. In general you could have a polynomial of degree k. So here it is 3r square polynomial of degree 2.

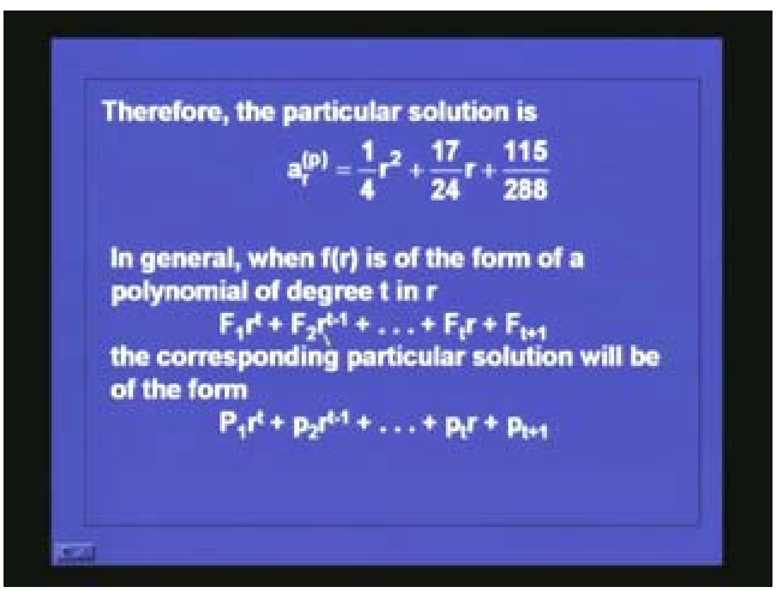
In that case you take the particular solution to be P_1r square plus P_2r P_3 a polynomial of degree 3. Here the values of P_1 P_2 P_3 have to be determined by substituting in this expression. So the general form of the particular solution is this, therefore use this in this equation. Here P_1 P_2 P_3 are constants to be determined. Substituting the expression we get P_1r square for a_r so for a_r we substitute this whole thing P_1r square plus P_2r plus P_3. Then you have $5a_{r \text{ minus } 1}$ for that you substitute 5 (P_1r minus 1) the whole square $5P_2$r minus 1 plus $5P_3$. So, for $a_{r \text{ minus } 1}$ you substitute P_1r minus 1 the whole square P_2r minus 1 plus P_3. Then you have 6ar minus 2 so you substitute for $a_{r \text{ minus } 2}$ P_1r minus 2 the whole square plus P_2r minus 2 plus P_3 so you get this.

(Refer Slide Time: 19.17)



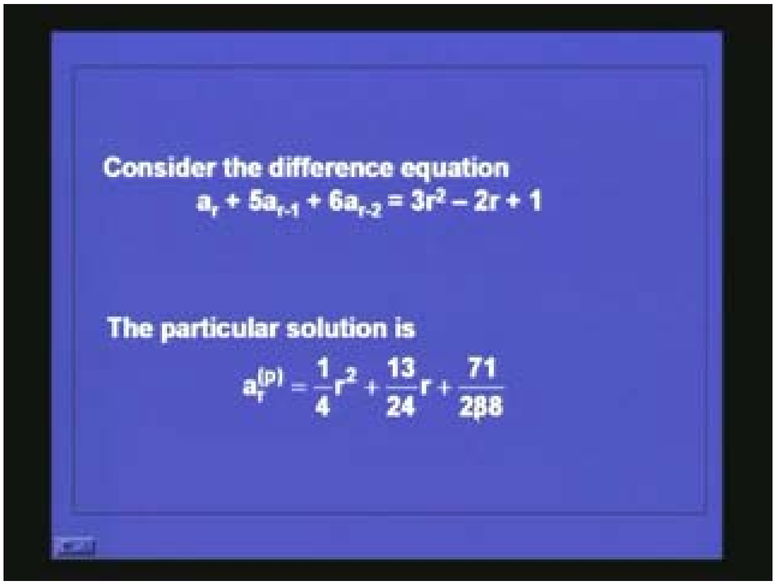
This simplifies to this expression. Equating the coefficients on the right hand side what do you have? you have 3r square, so equating the coefficients you have $12p_1$ is equal to 3 which will give you the value of p_1 as 1 by 4 equating the coefficients of r you will get $34p_1$ minus $12p_2$ is 0 then using the value of p1 as 1 by 4 here you will get p_2 is equal to 17 by 24. Then equating the constants you will get this is equal to 0, using the value of p_2 as 17 by 24 and p_1 as 1 by 4 you will get p_3 as 115 by 288.

(Refer Slide Time: 20.26)



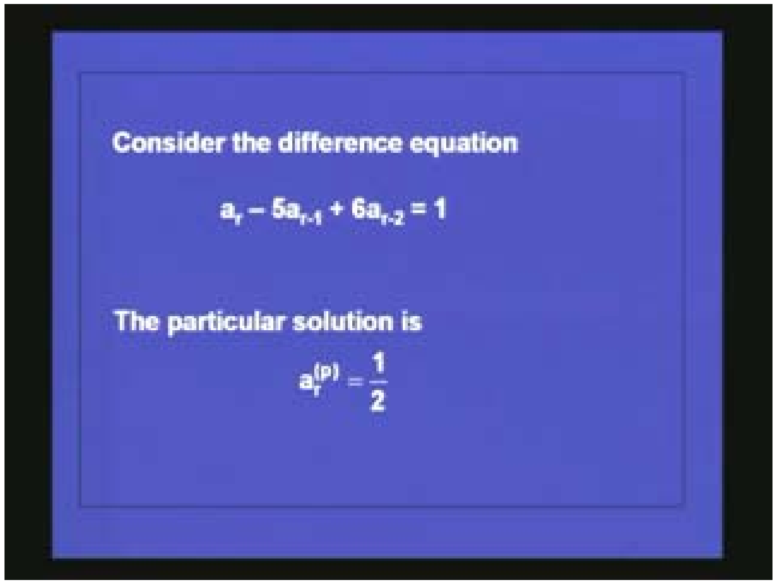
Like that we can determine the value of p_1 p_2 and p_3 and the particular solution is given by a_r to the power (p) is equal to 1 by 4 r square plus 17 by 24 r plus 115 by 288. In general whenever the right hand side f(r) is a polynomial it is of the form of a polynomial of degree t in r it is of the form F_1r power t plus F_2r power t minus 1 plus etc F_tr plus $F_{t \text{ plus } 1}$. That is, the right hand side is a polynomial of degree t then the corresponding particular solution will be of the form P_1r power t plus P_2r power t minus 1 plus P_tr plus $P_{t \text{ plus } 1}$ and substituting in the equation you have to determine the value of P_1 P_2 P_t etc.

(Refer Slide Time: 21.16)



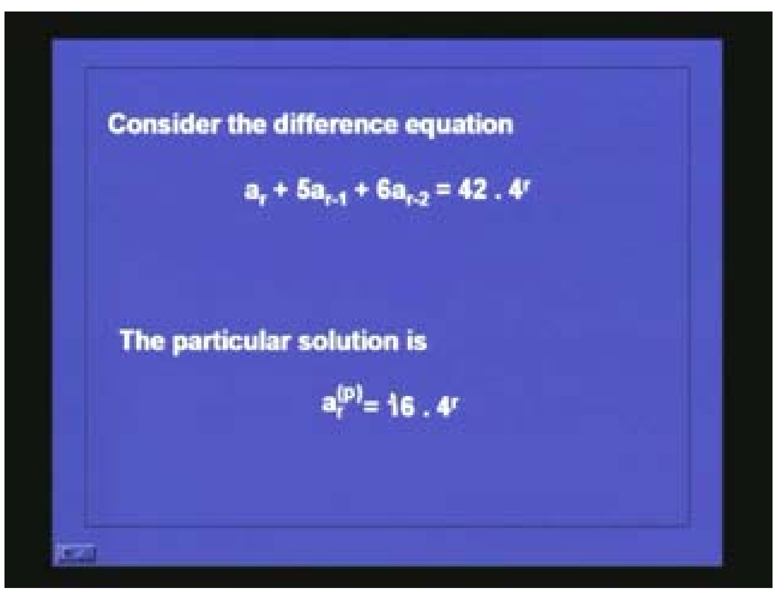
Look at this example, consider this difference equation, the right hand side is a polynomial of degree 2 3r square minus 2r plus 1. In this case again you have to take the particular solution as P_1r square plus P_2r plus P_3 and use this in the equation and find out the value of P_1 P_2 P_3 you will get P_1 as 1 by 4th P_2 as this P_3 as this so the particular solution will be a_r to the power (p) plus is equal to 1 by 4 r square 13 by 24 r plus 71 by 288.

(Refer Slide Time: 22.08)



Next let us consider some more forms of the right hand side f(r). Consider the equation a_r is equal to $5a_{r \text{ minus } 1}$ plus $6a_{r \text{ minus } 2}$ is equal to 1, the right hand side is a constant. So you take the particular solution as just P and a constant and then making use of that you will get P minus 5P plus 6P is equal to 1. That is 2P is equal to 1 or P is equal to 1 by 2. So in this case the particular solution is a constant 1 by 2 and that is what you get if the right hand side is a constant like this. The particular solution a_r to the power (p) is 1 by 2.

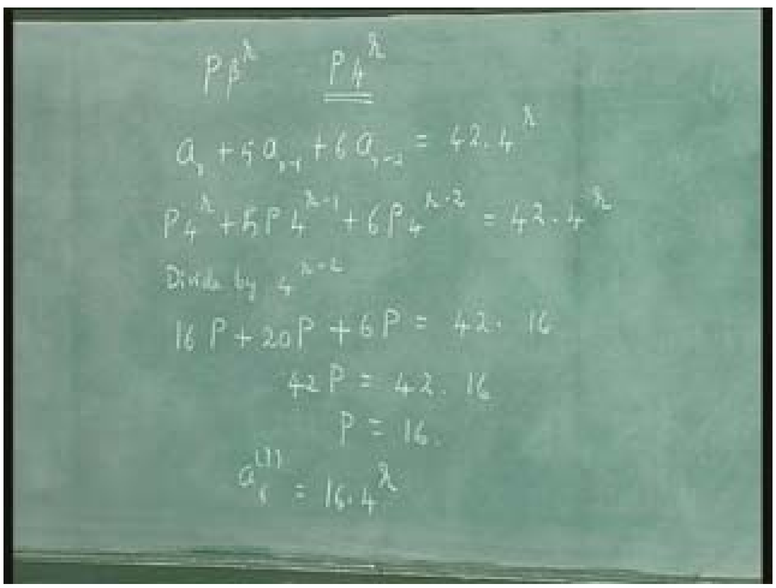
(Refer Slide Time: 23.00)



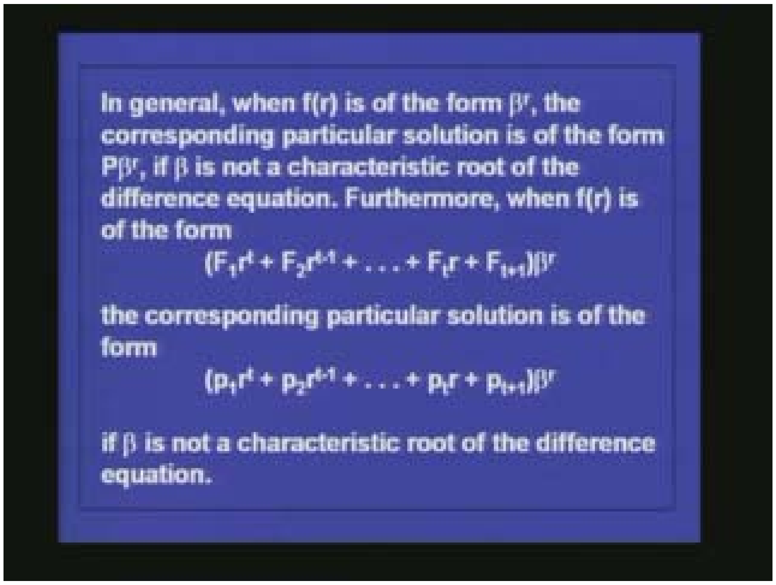
Now let us consider the difference equation a_r plus $5a_{r \text{ minus } 1}$ plus $6a_{r \text{ minus } 2}$ is equal to 42 (4) power r. So the right hand side is of the form some k into beta power r. Now here you have to consider two cases where beta is a characteristic root and where beta is not a characteristic root. Look at this equation, here the characteristic equation will be alpha square plus 5 alpha plus 6 is equal to 0 and the roots will be minus 3 and minus 2 and on the right hand side you are having 42 (4) power r but 4 is not a characteristic root. when this is the case on the right hand side you have some k into beta power r where beta is not a characteristic root you take the particular solution to be p beta power r.

The r in this case it is p is a constant 4 power r and you have to determine the value of p substitute in the equation, the equation is a_r plus $5a_{r \text{ minus } 1}$ plus $6a_{r \text{ minus } 2}$ 42 (4) power r. Use this and substitute here so it is p 4 power r plus 5p 4 power r minus 1 plus 6p 4 power r minus 2 is equal to 42 into 4 power r divide by 4 power r minus 2 so you will get 16p plus 20p 4 r minus 2 is related 4 into 5 is 20 plus 6p is equal to 42 into 6p so this is 36 plus 6 is equal to 42 and 42p is equal to 42 into 16 so p is 16 so the particular solution a_r to the power (p) is 16 into 4 power r and that is what you get. And the particular solution in this case is given by a_r to the power (p) which is 16 into 4 power r.

(Refer Slide Time: 26.00)

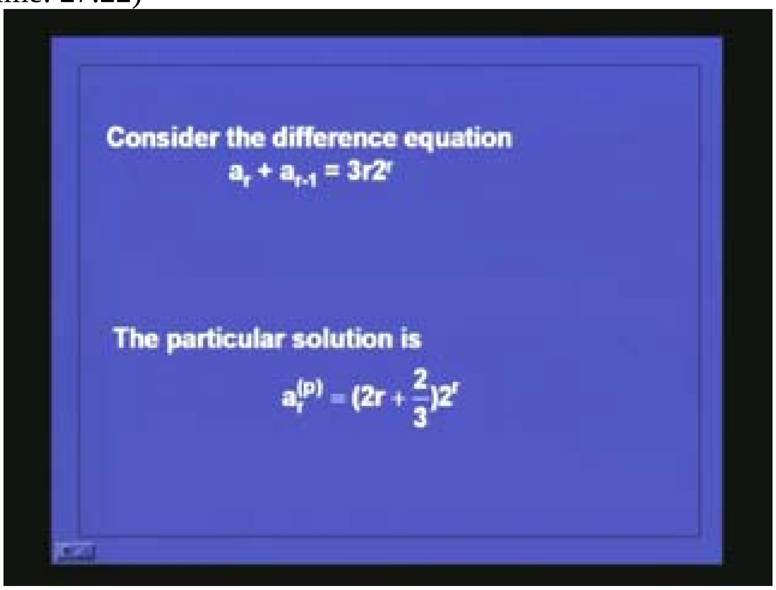


(Refer Slide Time: 26.10)



In general when f(r) is of the form beta power r the corresponding particular solution is of the form P beta power r if beta is not a characteristic root of the equation. Furthermore if f(r) is of the form a polynomial multiplied by beta power r this is a polynomial in r F_1r power t plus F_2r power t minus 1 etc a polynomial of degree t followed by or multiplied by beta power r. Then you have to take the particular solution in the form p_1 r power t plus p_2 r power t minus 1 etc beta power r then substitute this in the equation and determine the value of p_1 p_2 $p_{t \text{ plus } 1}$. This happens when beta is not a characteristic root of the difference equation. We will consider the case in a moment when beta is a characteristic root.

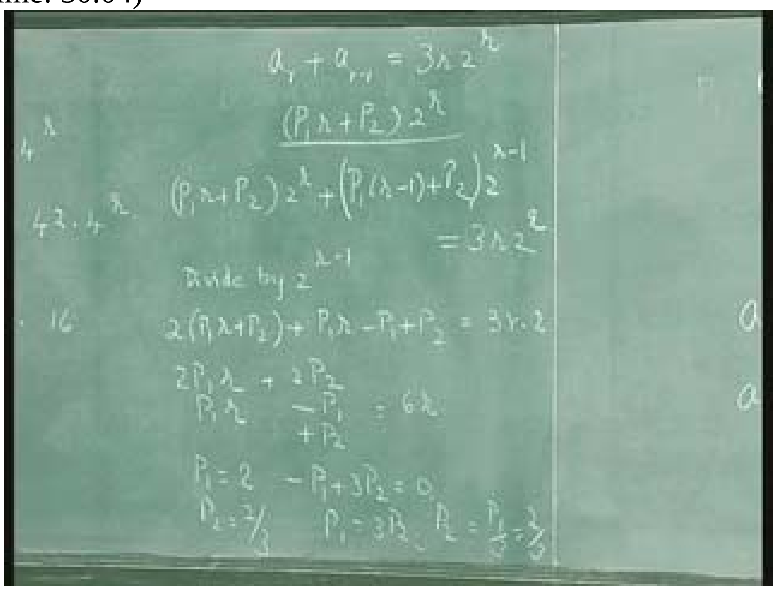
(Refer Slide Time: 27.22)



Look at this equation, consider the difference equation a_r plus $a_{r \text{ minus } 1}$ is equal to 3r 2 power r. Here the root is minus 1 this is different so this is a polynomial followed by 2 power r and polynomial of degree 1 so you have to take the particular solution as (P_1r plus P_2) 2 power r this is a way you have to take the particular solution.

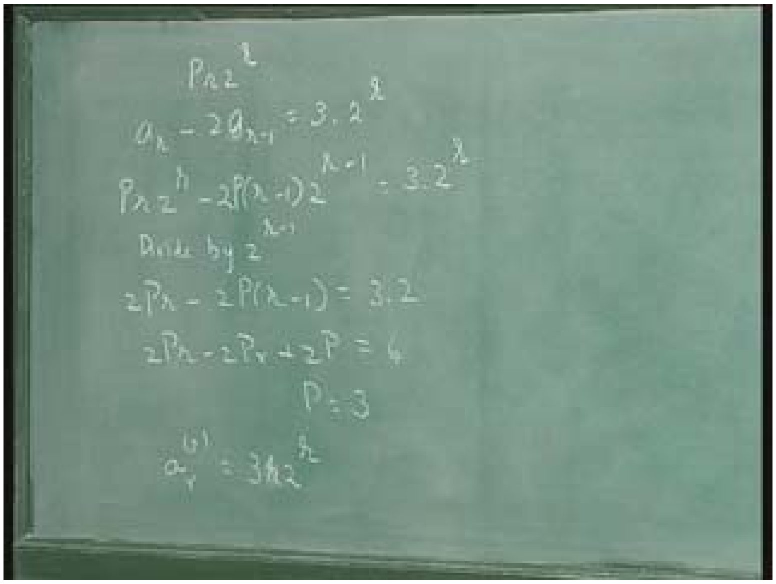
Substitute and find the value of P_1 and P_2 so you get (P_1r plus P_2) 2 power r plus $a_{r \text{ minus } 1}$ will be (P_1r minus 1 plus P_2) 2 power r minus 1 that is equal to 3r 2 power r. So dividing by 2 power r minus 1 you get 2 (P_1r plus P_2) plus (P_1r minus P_1) plus (P_2) is equal to 3r_1 2 that is 6. So 2P_1r P_1r then I am grouping the r terms together and the P_2 terms separately, P_2 minus P_1 plus P_2 is equal to 6r. So P_1 is equal to 2 3P_1 is equal to 6 so P_1 is equal to 2 and minus P_1 plus 3P_2 is 0 so P_1 is 3P_2 or P_2 is P_1 by 3 is equal to 2 by 3 so making use of these values of P_1 P_2 is 2 by 3.

(Refer Slide Time: 30.04)



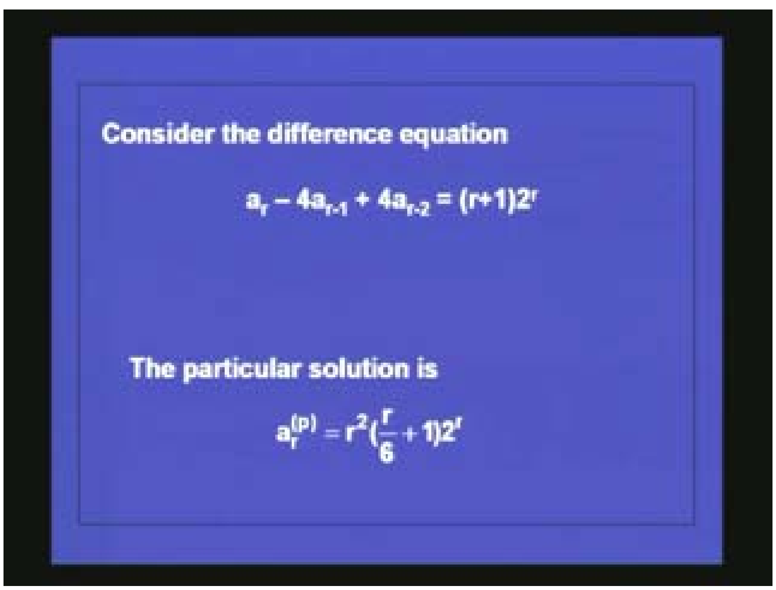
The particular solution is (P_1r P_2) 2 power r which becomes 2r plus 2 by 3 that is P_2 into 2 power r, so the particular solution takes this form. Now what happens when you have something like beta power r where this is a characteristic root? You get this equation a_r minus 2$a_{r \text{ minus } 1}$ is equal to 3 into 2 power r here you have 2 power r but 2 is also a characteristic root. In this case how do you go about considering the particular solution?
The characteristic root is 2 because of this and then you are having 2 power r. In that case you have to take the particular solution as Pr 2 power r where 2 is a characteristic root and you have to take it as Pr 2 power r. So the equation is a_r minus 2$a_{r \text{ minus } 1}$ is equal to 3 into 2 power r so use this as Pr 2r minus 2P into r minus 1 2 power r minus 1 is equal to 3 into 2 power r. So divide by 2 power r minus 1 that will give you 2Pr minus 2P into r minus 1 is equal to 3 into 2 2Pr minus 2Pr plus 2P is equal to 6 so P will be 3. So the particular solution a_r to the power (p) will be 3r 2 power r. So this is how you get the particular solution.

(Refer Slide Time: 32.35)



Let us consider some more examples. Now consider the difference equation a_r minus $4a_{r \text{ minus } 1}$ plus $4a_{r \text{ minus } 2}$ is equal to r plus 1 into 2 power r.

(Refer Slide Time: 32.41)

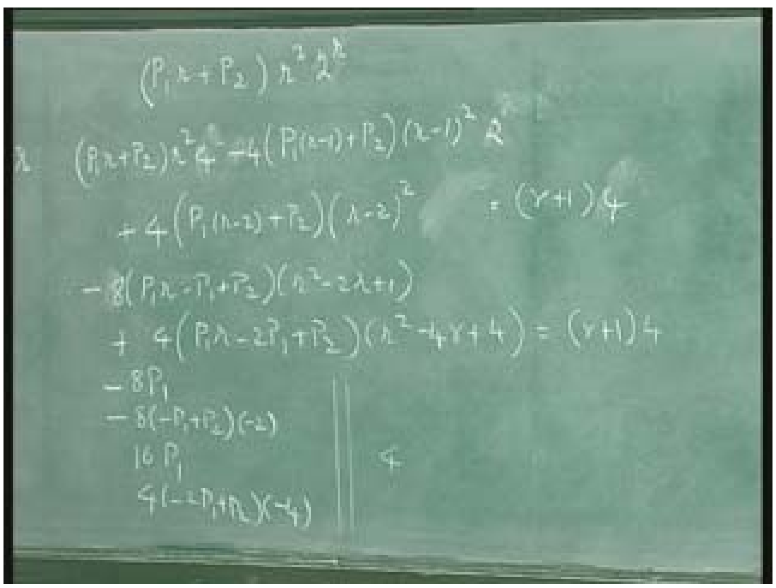


Now you are having a polynomial on the right hand side followed by 2 power r that 2 is also a characteristic 2 and not only that but the 2 is also a multiple root of multiplicity 2 because the characteristic equation for that will be alpha square minus 4 alpha plus 4 so alpha is equal to 2 is a double root. In that case how do you take the particular solution? the particular solution because you are having a polynomial of degree 1 you must take it as (P_1r plus P_2) but then the 2 power r, 2 is a multiple root it is a double root so you will have r and also characteristic root r square 2 power r. You have to take it like this because of the polynomial you have to take it as (P_1 r plus P_2) then 2 is a characteristic root with multiplicity 2 so you have to take as r square into 2 power r and then you substitute in the equation then you will get (P_1r plus P_2) (r square) (2 power r plus minus 4) (a minus 1) so (minus 4) (P_1) (r minus 1 plus P_2) (r minus 1) the whole square 2 power (r minus 1) plus 4 times $a_{r\ minus\ 2}$ that is 4 times P_1 (r minus 2 plus P_2) (r minus 2) the whole square (2) (r minus 2) and that is equal to the right hand side (r plus 1) (2) power (r).

Now expand because you can divide by 2 power r minus 2 and in which case you will get this as 2 square which is 4 and by dividing you will just get 1 2 here not get anything you will get a 4 here if you divide by 2 power r minus 2. Now expand and compare the coefficients. Here there are no terms for r cubed and r squared but there are terms for r and constant term. Look at the constant term and now look at the term for r as to what you will get? There is no r term here but what is the r term here? Let us forget this and expand this minus first portion I am leaving it out because it is r square minus 4 (P_1r minus P_1 plus P_2) (r square minus 2r plus 1) (2) if you bring in and multiply this it will be 8. Here also because of that (r minus 2) term will go, so in this we have to compare the constants and the coefficient of r.

What will be this?
This will be plus 4 (P_1r minus 2P_1 plus P_2) (r square minus 4r plus 4) is equal to (r plus 1) 4. Coefficient of r will be minus 8P_1 into 1 then this r multiplied by the constant term that is minus 8(minus P_1 plus P_2) (minus 2) that will be another term. Then there will be one more term here the coefficient of r into this that is 16P_1 then you have another term 4(minus 2P_1 plus P_2) (minus 4) this should be equated to 4.

(Refer Slide Time: 38.09)

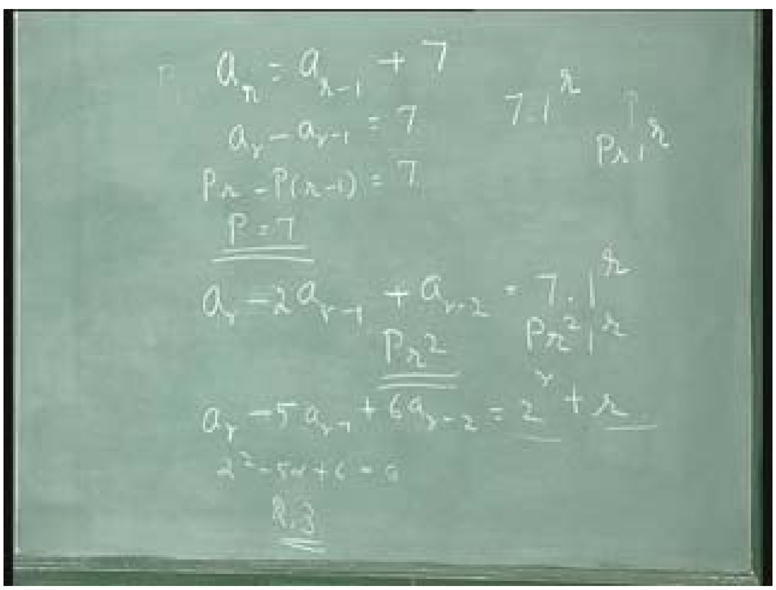


I am not going to solve the complete simultaneous equations, you can go on solving this simultaneous equation and then you will get the value as you have taken the particular solution as (P_1r plus P_2) so here P_1 will get the value 1 by 6 and P_2 will get the value 1. So the solution is (r by 6 plus 1) r squared that r squared is occurring before this, the solution is (r by 6 plus 1) r square into 2 power r. So this is how you have to take the particular solution. Now look at this equation ar is equal to ar minus 1 plus 1 or ar minus ar minus 1 is equal to 7.

The right hand side is a constant so what do you think will be the form of the particular solution?
Initially we realize that in one of the examples when the right hand side was a constant we took it to be a constant. But here note that 1 is a characteristic root. When 1is a characteristic root the right hand side even though is a constant you must look at it as 7 into 1 power r. So you have multiplicity 1for this root so you should not take it as a constant P but you should take it as Pr1 power r or just Pr so make use of that here that is Pr minus P (r minus 1) is equal to 7 that is Pr minus Pr will cancel so P is equal to 7 so the particular solution will be 7 in this case.

Now here look at an extension of that the right hand side is a constant but you have to take it as 7 into 1 power r because 1 is a characteristic root but now it has got multiplicity 2, the characteristic equation for this will be alpha square 2 alpha plus 1 and 1 is a double root here so you have to take the particular solution as Pr squared 1 power r or just Pr squared and then go about solving that. Now you have a combination of all the things.

(Refer Slide Time: 41.47)



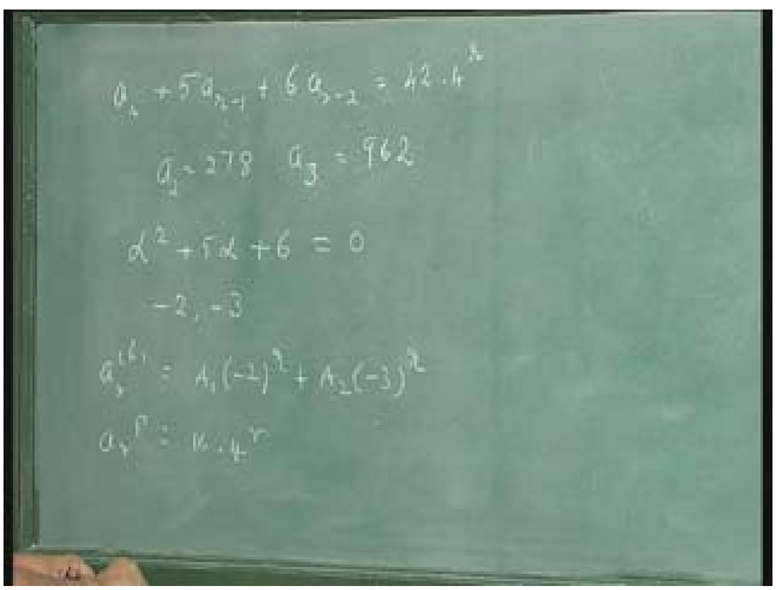
And look at this example; a_r minus $5a_{r \text{ minus } 1}$ plus $6a_{r \text{ minus } 2}$ is equal to 2 power r plus r, you have a polynomial you have an expression of the form beta and r. What are the roots of this equation?
You have to take alpha square minus 5 alpha plus 6 is equal to 0 so the roots are 2 and 3, so you realize that there is a characteristic root here and that power r. So you have to take 1 as P_1, for 2 power r you have to take it as P_1r 2 power r the particular solution has to be

taken in the form P_1r 2 power r. Then there is a second part which is a polynomial of degree 1, r is a polynomial of degree 1 so for that you have to take a polynomial of degree 1 in the general form P_2r plus P_3 which is given by this. You have to make use of this in the given equation, substitute and then try to find the values of P_1 P_2 P_3 which are constants. So these are some of the ways of finding the particular solutions.

So we shall take some examples to see how to find the homogenous solution as well as the particular solution so that we are able to find the total solution. We have already considered this example, look at this example, this is the equation and we have seen the particular solution which is this. So the equation is like this a_r plus $5a_{r\text{ minus }1}$ plus $6a_{r\text{ minus }2}$ is equal to 42 into 4 power r. And to solve the homogenous solution we require the boundary condition.

Therefore, what are the boundary conditions here?
The boundary conditions are given as a_2 is equal to 278 and a_3 is equal to 962 these are the boundary conditions given. Now what is a homogenous solution? The characteristic equation will be alpha squared plus 5 alpha plus 6 is equal to 0 so the roots are minus 2 and minus 3 so the homogenous solution will be of the form A_1 (minus 2) power r plus A_2 minus 3 power r. And we have seen earlier that the particular solution is given by this a_r to the power p is equal to 16 (4r) and this is when you have the right hand side of the form beta power r which is not a characteristic root because beta is not a characteristic root.

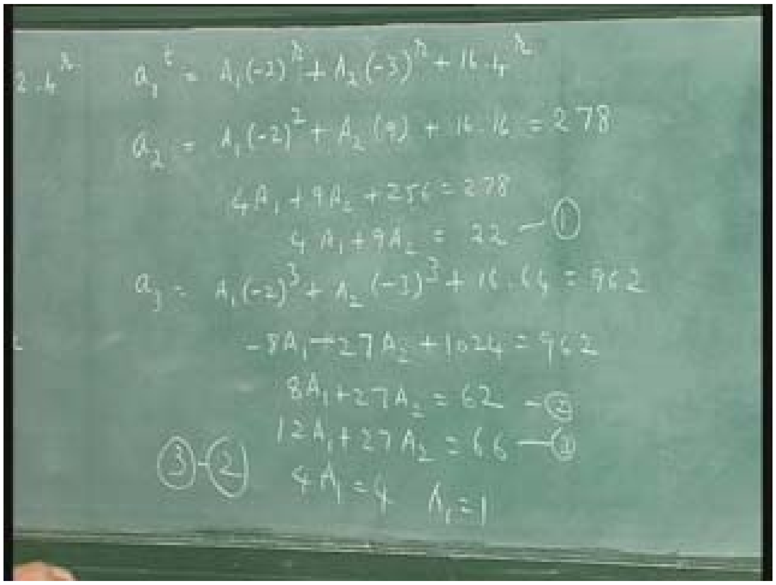
(Refer Slide Time: 44.50)



So the total solution will be art will be A_1 (minus 2) power r plus A_2 (minus 3) power r plus 16 times 4 power r. And we have given the boundary condition as A_2 is equal to 278 and A_3 is equal to 962. So what will be a_2? a_2 is A_1 (minus 2) (2 plus A_2) (9 plus 16) 16 and that is equal to 278. So this will be $4A_1$ plus $9A_2$ plus 256 is equal to 278 which gives you $4A_1$ plus $9A_2$ is equal to 22. Now use the value for 3 r is equal to 3, a_3 is A_1

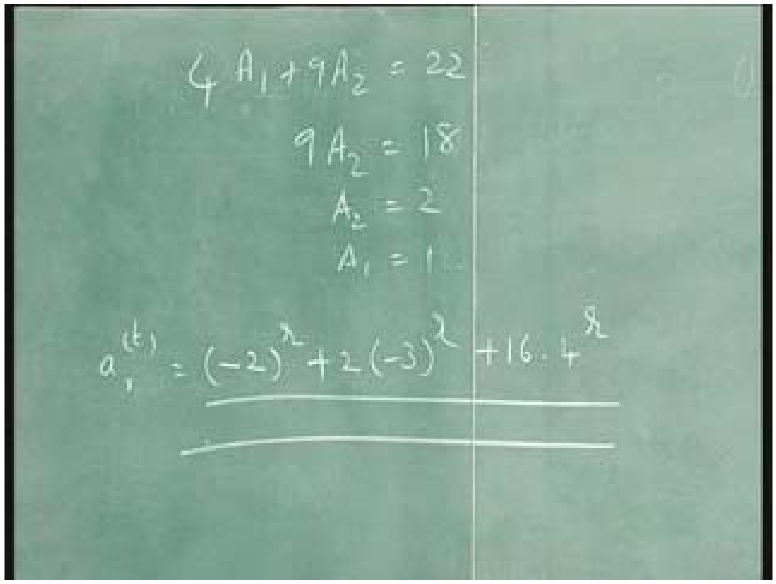
minus 2 power 3 plus A_2 (minus 3) power 3 plus 16 times 64 that is equal to 962. That is minus $8A_1$ minus $27A_2$ plus 1024 is equal to 962, bringing A_1 A_2 the other side and bringing 962 to this side you will get $8A_1$ plus $27A_2$ is 62. Multiply this equation by 3 you will get $12A_1$ plus $27A_2$ is 66 and subtract 2 from 3 you will get $4A_1$ is equal to 4 or A_1 is equal to 1.

(Refer Slide Time: 47.11)



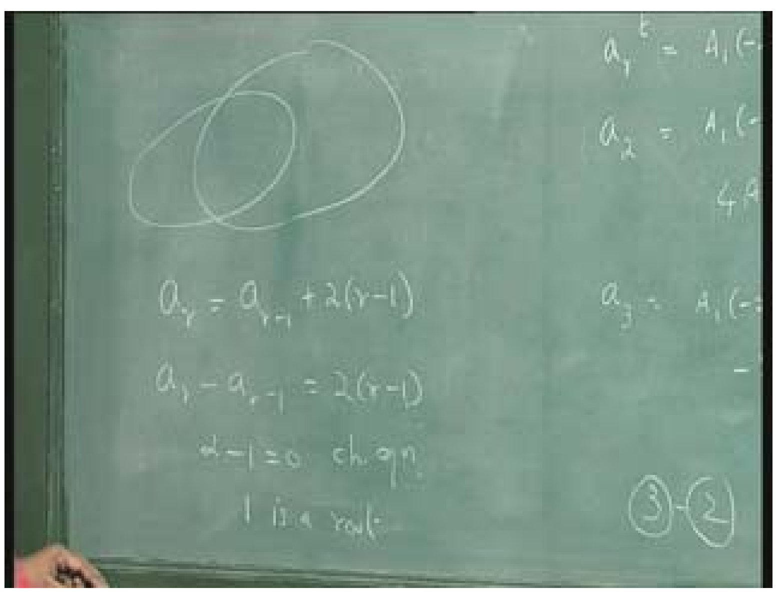
If A_1 is equal to 1 then $4A_1$ plus $9A_2$ is 22 so A_1 is 1 so $9A_2$ will become 18 or A_2 is 2. So solving for A_1 and A_2 you get A_2 is equal to 2 and A_1 is equal to 1 so the total solution art becomes a_1 (minus 2) power r that is 1 (minus 2) power r plus A_2 (minus 3) power r that is 2 (minus 3) power r then the particular solution is 16 (4) power r. So this is the total solution. Like that we can get the total solution. We have also considered another example.

(Refer Slide Time: 48.06)



Let us see how we go about finding the total solution for that. If you remember we considered the ovals in the last lecture. If two ovals intersect in two places the number of regions into which is dividing the plane is 1 2 3 4 and then if you have n ovals in what way the plane can be divided and into how many regions the plane will be divided? And we obtained the recurrence relation a_r is equal to $a_{r \text{ minus } 1}$ plus 2 (r minus 1) this is the recurrence relation we obtained and what will be the solution for this? You can write it as a_r minus $a_{r \text{ minus } 1}$ is equal to 2 (r minus 1) so alpha minus 1 is equal to 0 is the characteristic equation and 1 is the root.

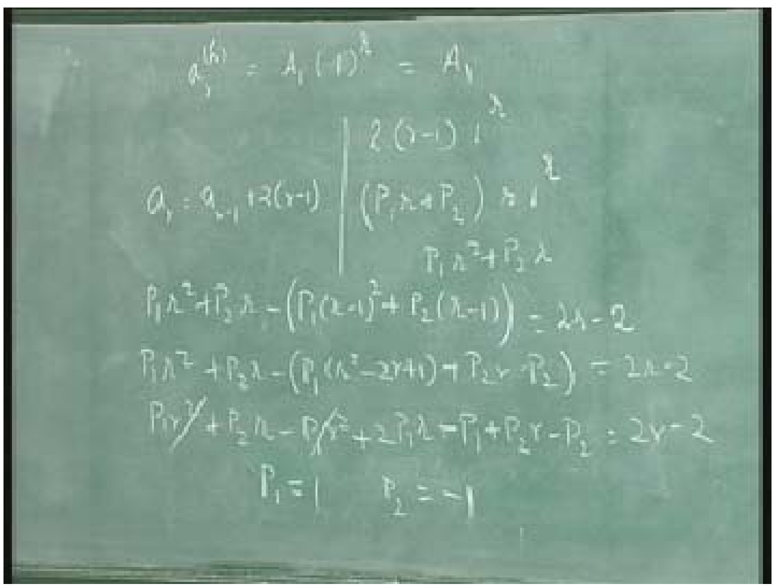
(Refer Slide Time: 49.13)



So the homogenous solution will be of this form. The homogenous solution will be of the form A_1 (1) power r that is just A_1 the homogenous solution will be of this form. What will be the form of the particular solution?

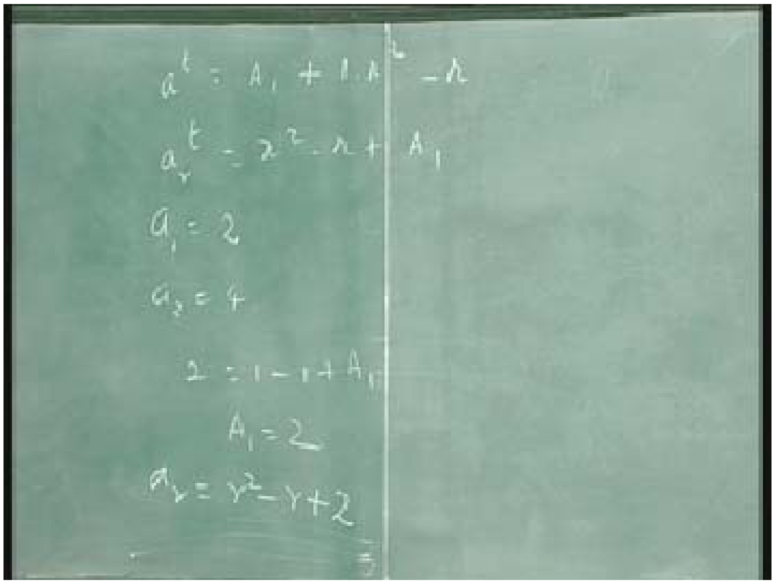
The particular solution should be of the following form: 1 is a characteristic root so you must look at the right hand side as 2 (r minus 1) to the power of 1 power r, you have to take it like this. So the form of the particular solution should be, this is a polynomial so you must have (P_1r plus P_2) then you have 1 power r and 1 is a characteristic root so it should be of the form r (1) power r and now this 1 power r is again 1 so you have to take the particular solution as P_1r squared plus P_2r. The equation is a_r is equal to $a_{r \text{ minus } 1}$ plus 2 (r minus 1). So use this particular solution here and solve for P_1 and P_2 so that will be P_1r squared plus P_2r minus P_1 (r minus 1) the whole squared plus P_2 (r minus 1) is equal to 2r minus 2. So, that will be P_1r squared plus (P_2r minus P_1) r squared minus 2r plus 1 plus P_2r minus P_2 is equal to 2r minus 2. That will be P_1r squared plus (P_2r minus P_1r) squared plus 2P_1r minus P_1 plus P_2r minus P_2 is equal to 2r minus 2. So from this if you solve you will get P_1 equal and P_2 is equal, like this.

(Refer Slide Time: 52.19)



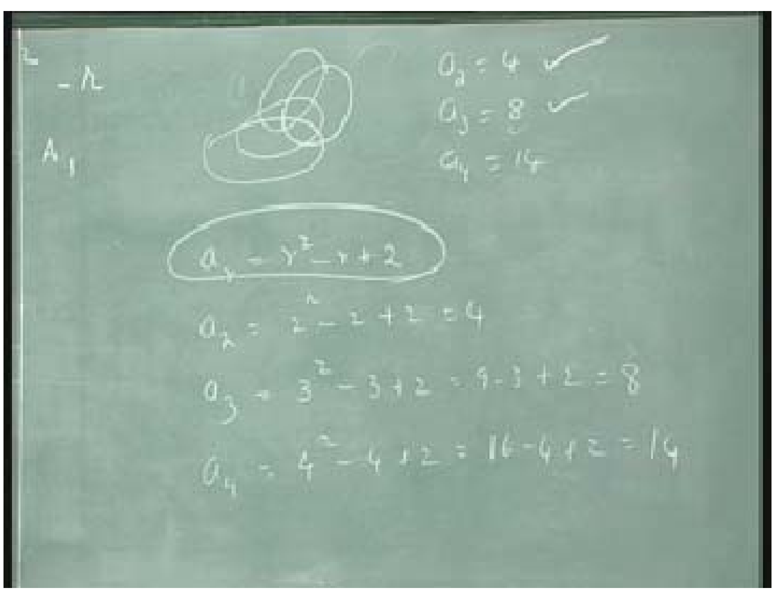
So P_1 will be 1 and P_2 will be minus 1 so the total solution will be of this form and at will be A_1 plus P_1r squared plus P_2r so that is r squared minus r plus A_1 this is the total solution. And you have a_1 is equal to 2 and a_2 is equal to 4 and so on so from that you can solve for a_1 so put r is equal to 1 you will get 2 is equal to 1 minus 1 plus A_1 so A_1 will be 2. So we get the total solution as a_r is equal to r squared minus r plus 2.

(Refer Slide Time: 53.11)



Now let us see whether it verifies what we got in the last lecture. In the last lecture we considered figures like this so that a_2 is 4 and one more oval like this we got a_3 is equal to 8 and then drawing one more oval like this we got a_4 is equal to 14. Let us see weather these values are correct for this equation, a_r is r squared minus r plus 2 so what will be a_2? a_2 will be 2 squared minus 2 plus 2 will be 4 which verifies our value. What will be a_3? a_3 is 3 squared minus 3 plus 2 that is 9 minus 3 plus 2 is equal to 8 which is also correct. a_4 will be 4 squared minus 4 plus 2 will be 16 minus 4 plus 2 is 14. So we see that what we have got is the correct answer.

(Refer Slide Time: 54.25)



So, recurrence relations really help to solve such problems. Looking at this sort of a figure if you think that in how many regions the plane will be divided into all of a sudden you cannot get this expression that easily. But by writing down the recurrence as a_r is equal to $a_{r \text{ minus } 1}$ plus 2 (r minus 1) and then solving it we are able to get an expression for a_r in this manner. So recurrence relations are really useful in many ways, they help us to solve the problems in a very convenient manner. In the next lecture we shall see how to solve the recurrence relations by the means of generating function concept.