Discrete Mathematical Structures
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Lecture - 28
Permutations and Combinations

The last lecture we saw about the Pigeonhole principle. Today we shall see Permutations and Combinations.

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What do you mean by Combination? It is the way of selecting some objects out of some objects. Suppose you are having n objects you want to select r objects out of it then it is a selection this is called Combination. And the number of ways of arranging r objects out of n objects that is called arrangement or Permutation. And we shall see some formulae and we shall also workout a few examples regarding this.

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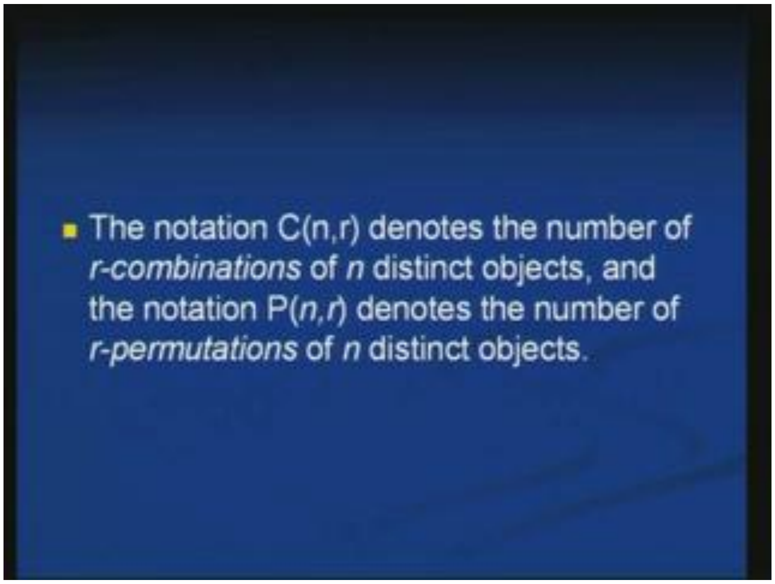

Generally the notation C(n, r) represents the number of r Combinations of n distinct objects and the notation P(n, r) denotes the number of r Permutations of n objects. That is C(n, r) this is the number of ways of selecting r objects out of n objects sometimes it is also denoted as nCr. Similarly, P(n, r) and this is also denoted as nPr sometimes and sometimes in some notations they also represent like this nPr these are other notations. The number of Permutations of r objects out of n objects. That is you are having n objects and you are taking all of them and arranging them that is the number of Permutations of r objects out of n objects.

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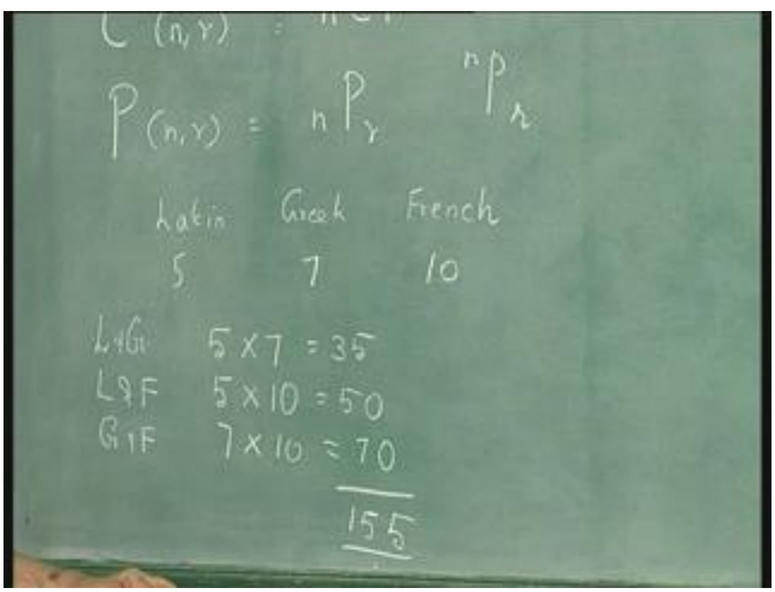

Number of selecting r objects out of n objects is known as the Combination and it is denoted by C(n, r). Other notations are given here. Generally we have two rules.

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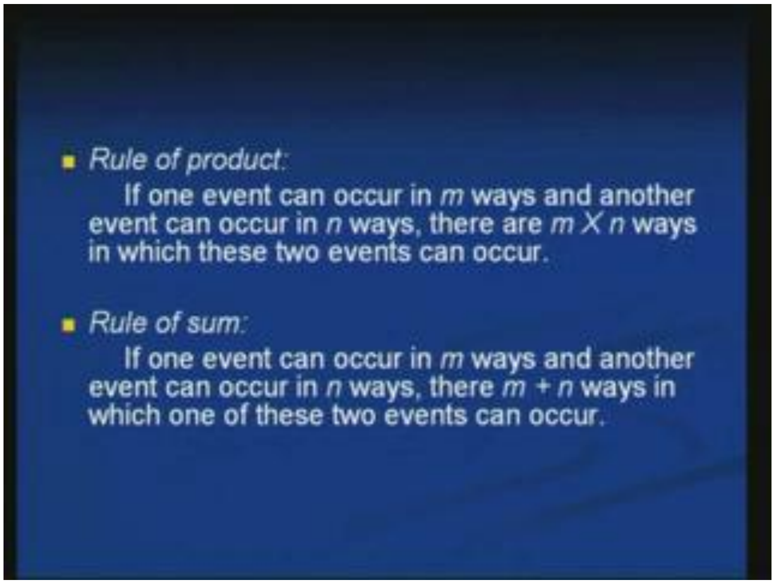

One is the rule of product and another is the rule to attack problems based on Permutations and Combinations. The rule of products is that if one event can occur in m ways and another event can occur in n ways then there are m cross n ways in which these two events can occur. This is known as the rule of product and we will be using both these rules frequently. The rule of sum says: if one event can occur in m ways and another event can occur in n ways there are m plus n ways in which one of these two events can occur. So, we will mention when we make use of these rules.

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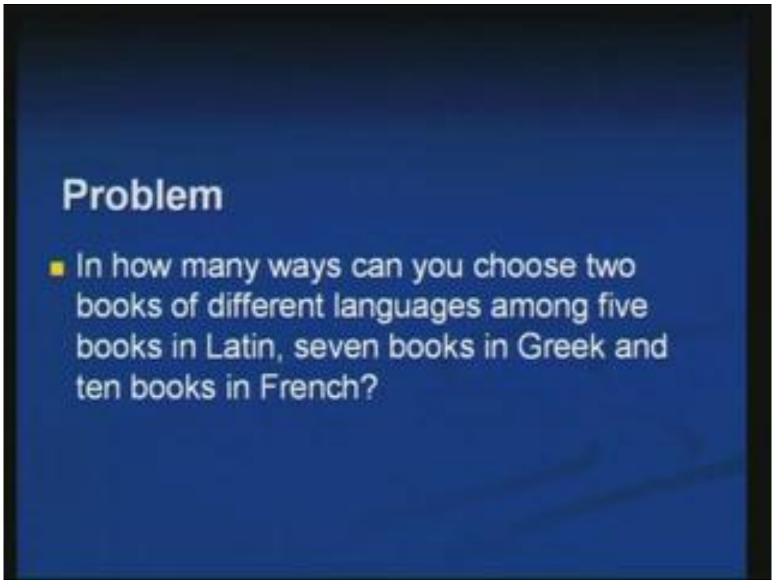

Let us take a simple problem and see how we tackle that problem. The problem is, in how many ways you can choose two books of different languages among 5 books in Latin, 7 booms in Greek and 10 books in French. So you are having Latin, Greek, and French. How many books do you have in Latin? You have 5 books in Latin, 7 books in Greek and 10 books in French. And you are asked to select 2 books of different languages among 5 books in Latin. So you can choose this way, you can choose one book from Latin one book from Greek or you can choose one book from Latin one book from French or you can choose one book from Greek and one book from French.

Suppose you choose from Latin and Greek one book from this and one book from this, in how many ways can you choose a book from Latin? Out of the 5 books you can choose one in five ways and in Greek out of the 7 books you can choose one book in 7 ways so 5 cross 7 there are 35 ways of selecting one book in Latin and one book in Greek. This is the rule of product you are making use of. Then in how many ways can we choose a book from Latin and one book from French? You can choose one book from Latin in 5 ways and you can choose one book from the set of French books in 10 different ways so totally in 50 different ways you can select a book from Latin and the book in French.

Again you are making use of the rule of product. Now, suppose you are choosing a book in Greek and you are choosing a book in French in how many ways can you choose a book in Greek? You can choose a book in 7 different ways and you can choose a book in French in 10 different ways so in 70 different ways you can choose a book from Greek and a book from French. So we are making use of the rule of product. Now we will make use of the rule of sum. So you can either choose a book from Latin and Greek, a book from Greek, a book from Latin and book from French, or a book from Greek and a book from French. So in how many ways can you choose two books from different languages? That means we have to add this, this is the rule of sum so you will get 155.

In 155 different ways you can select two books of different languages among 5 books in Latin 7 books in Greek and 10 books in French. So we have to see how to make use of the rule of product and rule of sum.

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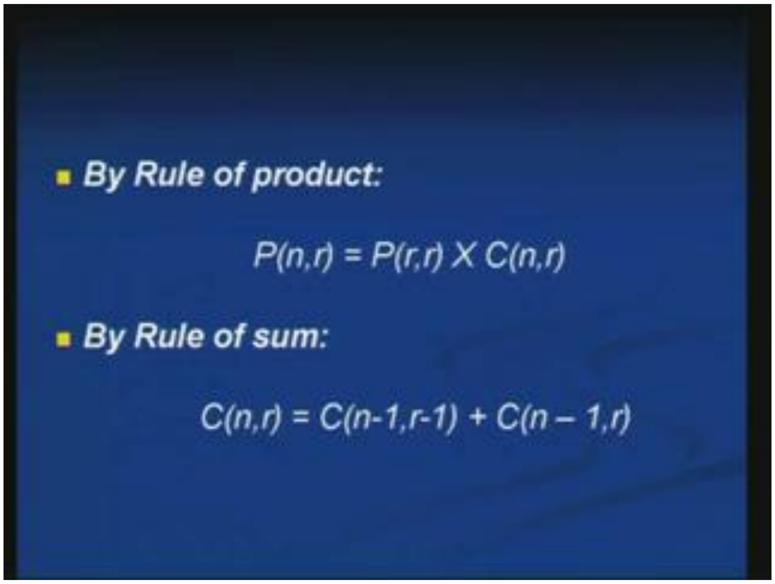

Now, what is the value of P(n, r) and C(n, r)? By the rule of product P(n, r) is equal to P(r, r) cross C(n, r). What do you mean by this?

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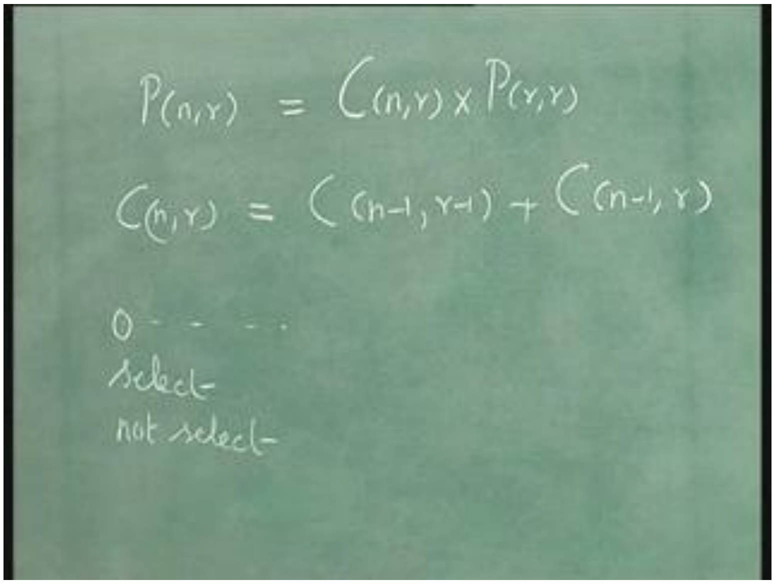

P(n, r) is the number of r Combinations of r objects out of n objects and that is equivalent to C(n, r) cross P(r, r). It amounts to saying that this left hand side represents the number of ways of selecting r objects out of n objects and arranging them. This is the number of Permutations of r objects out of n objects and this is equivalent to saying that you select r objects out of n objects and after selecting you arrange them that is P(r, r) so you can represent in this way.

Now, by the rule of sum you will get this C(n, r) is equal to C(n minus 1, r minus 1) plus C(n minus 1) how we get this? C(n, r) is the number of selecting r object out of n objects. Now, among the n objects you mark a particular object there are two possibilities you can select this object or not select this object.

Suppose you select this object there are two possibilities select or not select. If you select then from the remaining (n minus 1) objects you have to choose r minus 1 objects that is equivalent to saying C(n minus 1, r minus 1). If you do not select this object in the remaining r minus 1 objects (n minus 1) objects you have to select r objects and that is C(n minus 1, r minus 1) you get this and obviously you are applying the rule of sum here. Let us see what is the value of P(n, r)? That is the number of r Permutations out of n objects. It is equal to n factorial by (n minus r) factorial. This is the formula for P(n, r), now let us see how to derive it.

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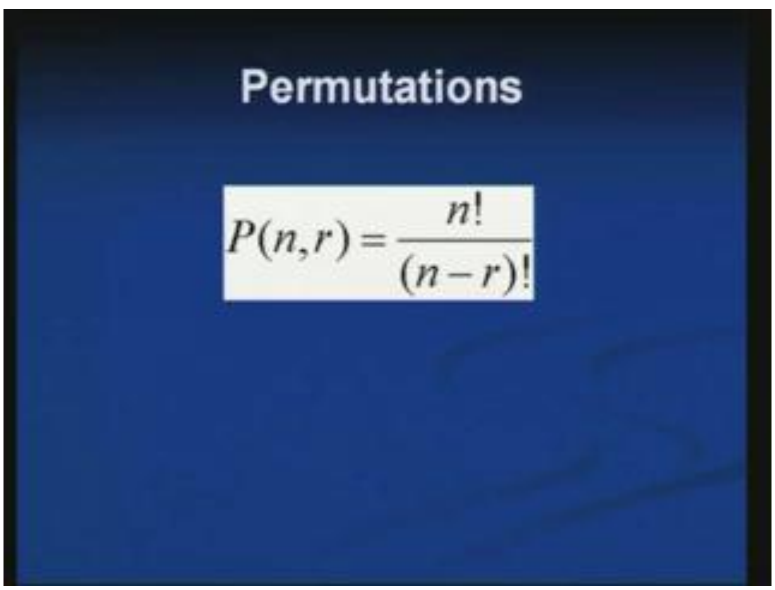

We are having n objects and we are asked to arrange r of them in r places now how will you arrange? So in the first place you can pick one of the n objects and put it here that can be done in n ways. Then after selecting one object and putting it here you will be left with (n minus 1) objects here. So the second place you can fill in (n minus 1) and the first place you can fill in n different ways and after doing that the second place you can fill in (n minus 1) ways.

So the first two places can be filled in n cross (n minus 1) so after filling the first two places you can fill with (n minus 2) objects here, pick one of them and put it in third place and that can be done in (n minus 2) ways. And similarly proceeding the rth place can be filled in (n minus r plus 1) ways after filling the (r minus 1) places the rth places can be filled by selecting one object out of the (n minus r plus 1). So this is the number of ways; this into this into this into this is the number of ways we can arrange r objects from n objects. so P(n, r) is equal to this and this is nothing but n cross (n minus r plus 1) we can multiply by (n minus r) (n minus r minus 1 up to 1) and below also the same number (n minus r) cross (n minus r minus 1) and so on. So the numerator is n factorial the denominator is (n minus r) factorial.

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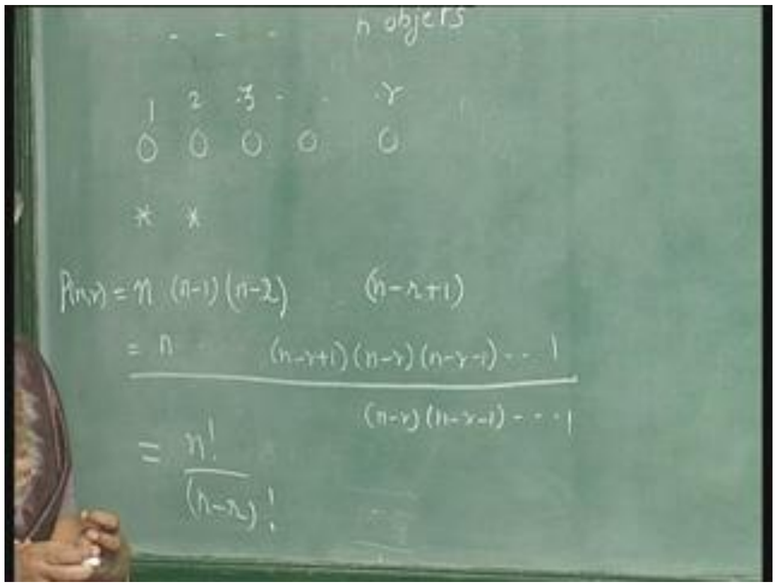

So P(n, r) is equal to n factorial by (n minus r) factorial. You can also derive it in a slightly different manner if that is like this.

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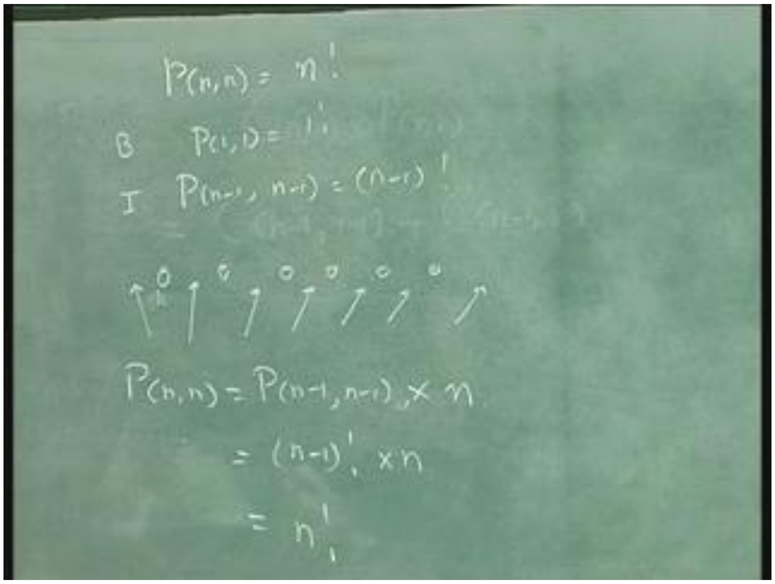

First of all you prove that P(n, n) is n factorial. The number of ways of arranging n objects is n factorial. You can prove this by induction. First of all P(1, 1) is 1 factorial obviously that is true. And suppose P(n minus 1), (n minus 1) is (n minus 1) factorial by induction this is the basic class of the induction this is the induction class. Then from this how do you prove (n minus n) p n is n factorial. After arranging (n minus 1) objects you are having n objects, (n minus 1) of them you have arranged in (n minus 1) factorial ways. Then the last object you can put here in n ways so P(n, n) is P(n minus 1) (n minus 1) cross n. And by induction this is (n minus 1) factorial cross n so this is equal to n factorial. So by induction we can prove that the number of ways by arranging n objects is n factorial. Having proved this how we get P(n, r) is equal to n factorial by (n minus r) factorial.

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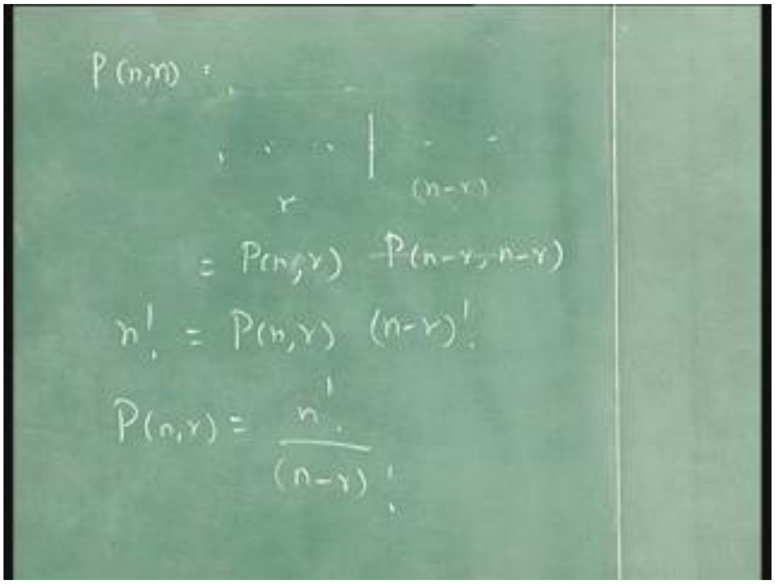

Now P(n, r) is the number of ways of arranging r objects out of n objects. Now, suppose I have n objects and I want to arrange P(n, n) this I can do by arranging them in n places. First I can take r places and fill them then the remaining (n minus r) places I can arrange. So P n, n is equal to first I pick r objects out of n objects and arrange them that is P(n, r) then the remaining (n minus r) objects can be arranged in the (n minus r) places and that is P(n minus r, n minus r). And we know that this is n factorial and P(n, r) is this. P(n minus r) (n minus r) is (n minus r) factorial. So, from this also you get P(n, r) is equal to n factorial by (n minus r) factorial. Of course we have seen that P(n, n) is n factorial.

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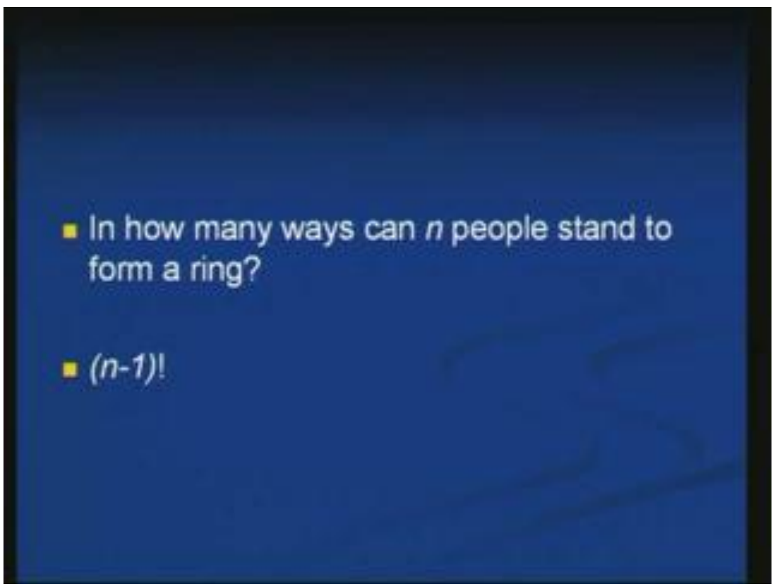

So we can make use of this in falling problems. But instead of having a linear array you want to arrange n objects as a linear array and that is why we have filled the first place in n ways then the second place in (n minus 1) ways and so on.

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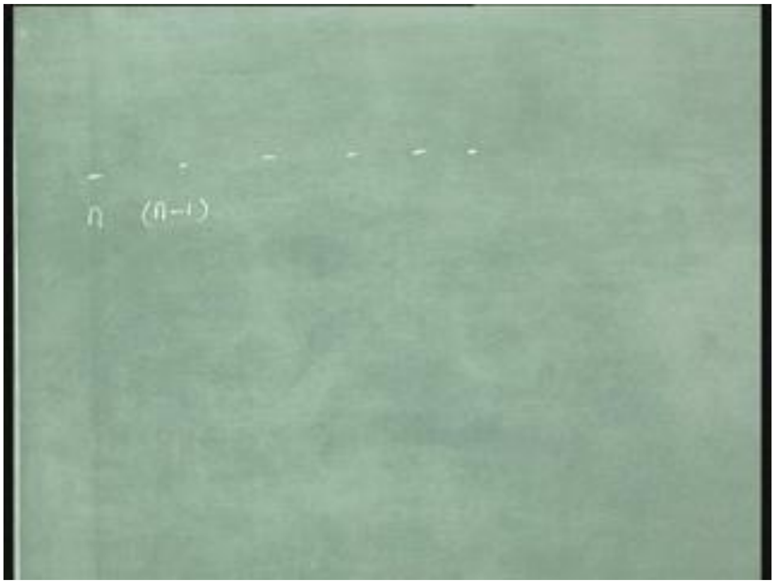

Suppose instead of having linear array you want to arrange n objects in a circular way, that is n objects you have to arrange along a circle {1, 2 to n}, in this case suppose I have one arrangement a_1, a_2, a_n and another arrangement an, a_1, a_2, $a_{n \text{ minus } 1}$.

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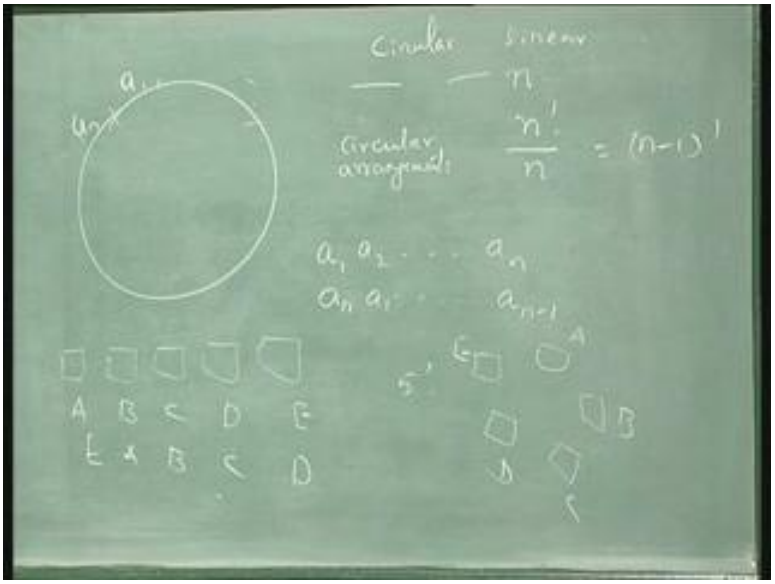

When you look at a linear array suppose you are having chairs you are having 5 people seated in 5 chairs, the 5 persons names may be A B C D E so you can arrange them in something like this in A B C D E this can be done in 5 factorial ways we know that. But when you arrange them in a circle like this 5 chairs are arranged in the form of a circle A B C D E then the arrangement A B C D E and E A B C D are the same as far as the circular arrangement is concerned. But when you consider it as a linear arrangement they are different. So A B C D E, E A B C D, D E A B C, C D E A B, B C D E A are all the same. So in a circular arrangement one arrangement corresponds to five arrangements in a linear manner as shown in this example.

Like that if you have n objects one arrangement here will correspond to n arrangements when you look at it as a linear array. This is circular and this is linear so one arrangement here will correspond to n arrangement and the number of linear arrangement is n factorial this we know so the number of circular arrangements will be equal to, each one corresponds to n so this will be divided by n gives the number of circular arrangements and that is equal to (n minus 1) factorial. So the number of ways in which n people can be arranged in a circular way or n objects can be arranged in a circular way is given by (n minus 1) factorial.

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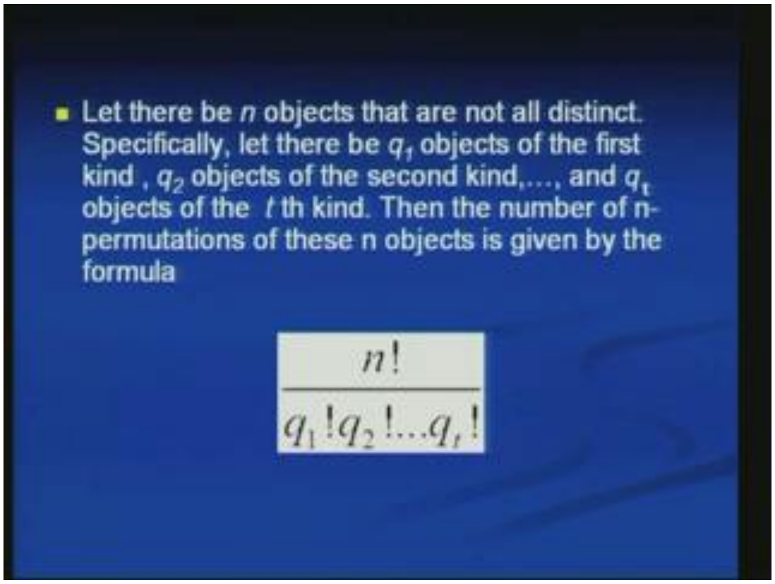

Let us consider some more results. Here whenever I said arranging n objects arranging r objects the objects are considered to be distinct objects. So far we have been considering arranging of distinct objects, P(n, r) denotes the number of ways of arranging r objects out of n distinct objects where the objects are not the same.

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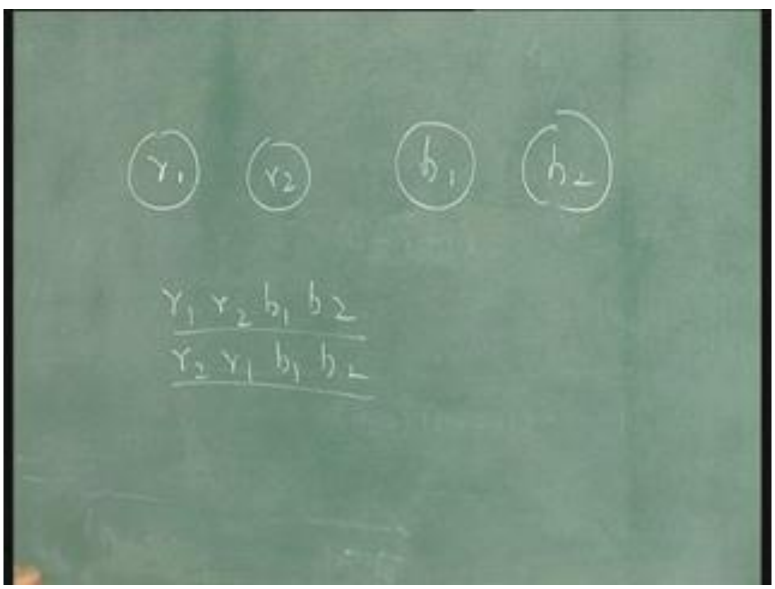

Suppose the objects are not distinct some of them are similar or same then how do you calculate? For example, you may have red balls and blue balls then how do you arrange them. I have red balls r_1 and r_2 they are identical and I have blue balls two blue balls b_1 and b_2, then r_1 r_2 b_1 b_2 is different from r_2 r_1 b_1 b_2 if you look at them as distinct objects.

but when these two red balls are identical these two arrangements are the same red, red and blue, blue that is what you are going to get. So in such a case what is the formula for arranging n objects? Now we have n objects but they are all not distinct, let there be n objects that are not all distinct. Specifically there are q_1 objects of first kind and q_2 objects of the second kind and q_t objects of the tth kind then the number of n Permutations of these n objects is given by the formulae n factorial by q_1 factorial q_2 factorial q_t factorial. So you are having n objects, not all of them are distinct q_1 objects are of the first kind, q_2 objects are of the second kind and so on and q_t objects are of the tth kind. In how many ways can you arrange them?

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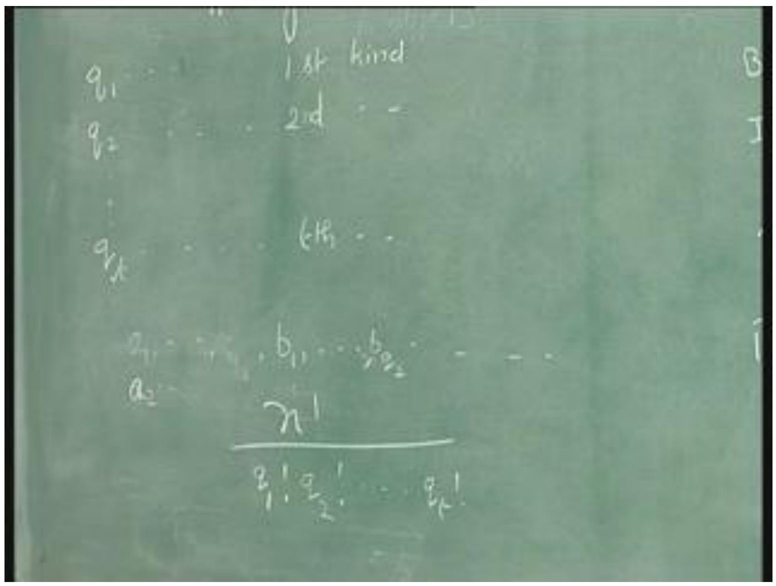

If they are all distinct the answer is n factorial that we know but some of them are similar so the answer will be much less it will be less than n factorial, let us see what it is. Suppose I have q_1 a_1 a_1 aq_1 these are the objects of the first kind q_1 objects. If I mark them distinct then I get different distinct objects. Similarly, the second objects I call them as b_1 b_2 bq_2 make them all distinct and so on. If I do this all objects become distinct. So the number of ways of arranging them is n factorial this we know. But since a_1 a_2 aq_1 are all not distinct here they are all same they are the objects of the same type like red balls blue balls like what I considered earlier.

So if you consider them as same objects then you see that if you take one Permutation here and then in that one Permutation or one arrangement here and within that you can arrange these first objects in any manner you want. See, all these a_1 a_2 aq_1 or a_2 a_2 another arrangement they will all correspond to the same arrangement when we look at them as the same non distinct objects. So if you take a particular Permutation and within that particular Permutation you consider only the objects of the first kind one such arrangement when you look at them as non distinct will give rise to q_1 factorial times the number of arrangements when you look at them as distinct. So when you look at it as non distinct you have to divide it by q1 factorial. This is with respect to the first kind of object then the similar thing holds for the second kind of object.

Look at the second kind of objects; the Permutation here when you look at them as non distinct objects it will give rise to q_2 factorial times the number of arrangements when you look at them as distinct object. So one Permutation gives rise to q_2 factorial Permutations when you look at them as distinct objects. So when you look at them as non distinct objects we have to divide this n factorial by q_2 factorial and so on so similarly q_t factorial. So the number of ways of arranging n objects when the objects are all not distinct and where q_1 of them are first kind and q_2 of them are second kind and qt of them are of the tth kind is given by n factorial by q_1 factorial q_2 factorial and q_t factorial this is the formula so this is given by this.

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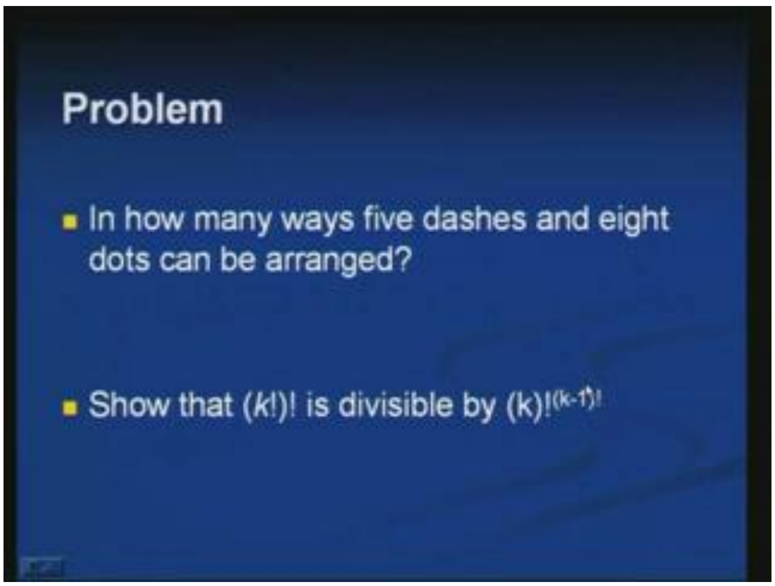

Let us do a problem under this. In how many ways 5 dashes and 8 dots can be arranged. You are having the telegraphic code but earlier we used to have dashes and dots.

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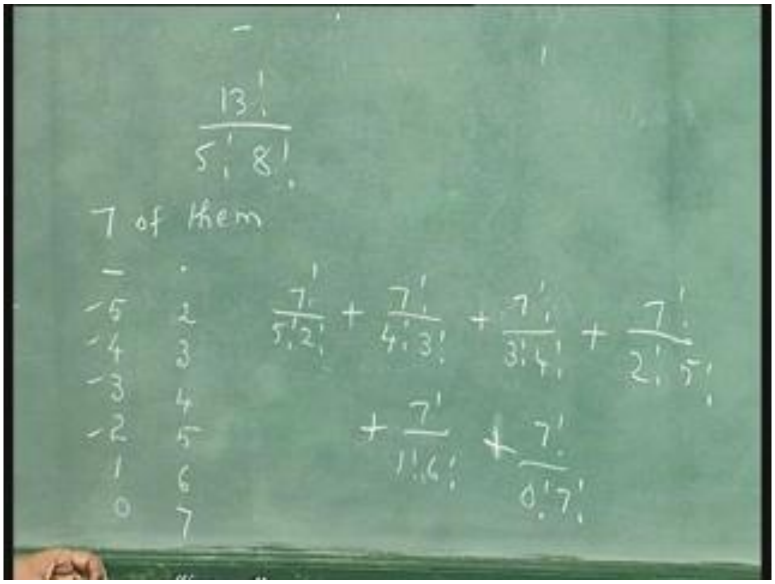

There are 5 dashes and 8 dots. In how many ways can you arrange them? Totally you have 13 factorial but you cannot make a distinction between the dashes and the dots so that will be 5 factorial and 8 factorial. And the total number of ways of arranging 5 dashes and 8 dots is this. Suppose I want to arrange only 7of them and do not want all, this is arranging all the 30. Now I want to arrange only 7 of them pick up 7 of them and arrange them.

In how many ways can you arrange 7 dashes and dots picking them from 5 dashes and 8 dots?

Now at the most there are 5 dashes so you can select 5 dashes and 2 dots for 7 or you can choose 4 dashes and 3 dots or you can choose 3 dashes and 4 dots or 2 dashes and 5 dots or 1 dash and 6 dots or you can select all the 7. So, for each one what will be the formula? If take 2 dots and 5 this will be 7 factorial by 5 factorial cross 2 factorial if you select 4 and 3 that will be 7 factorial by 4 factorial cross 3 factorial. If you select 3 and 4 it will be again 7 factorial by 3 factorial cross 4 factorial. And if you select 2 and 5 it will be again 7 factorial by 2 factorial 5 factorial plus this we have 1 and 6 it will be 7 factorial by 1 factorial and 6 factorial this plus 7 factorial by 0 factorial 7 factorial. The 0 factorial is taken as 1 so this we have to sum up.

Let us take one more problem. This is about numbers but using combinatorial arguments how you prove this?

Show that k factorial is divisible by k factorial to the power of (k minus 1) factorial.

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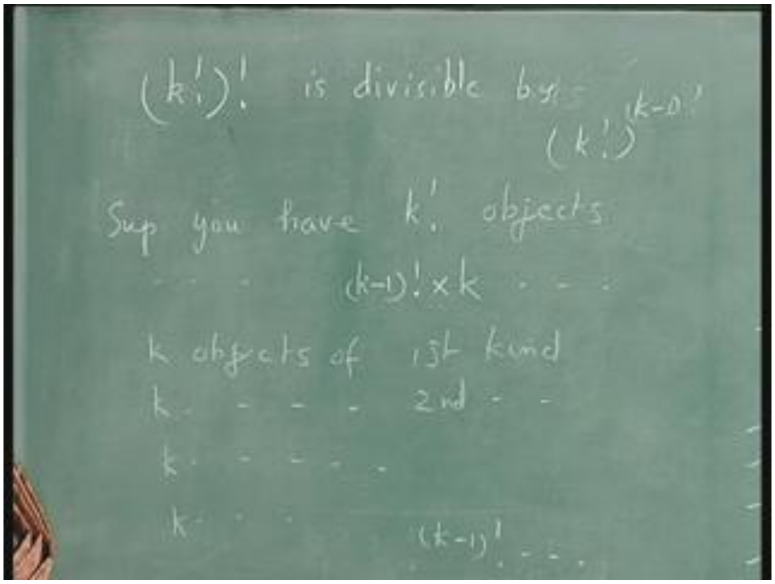

The problem is to show that; K factorial factorial is divisible by k factorial to the power of (k minus 1) factorial. How do you prove this using combinatorial argument?
Suppose you have k factorial objects actually that is equivalent to saying you are having (k minus 1) factorial cross k objects. And suppose they are not distinct there are k objects of first kind, there are k objects of second kind, there are k objects of third kind and so on like that there are k objects of (k minus 1) factorial. So out of the k factorial objects k of them are identical of the first kind and k of them are identical of the second kind, k of them are identical of the third kind and so on.

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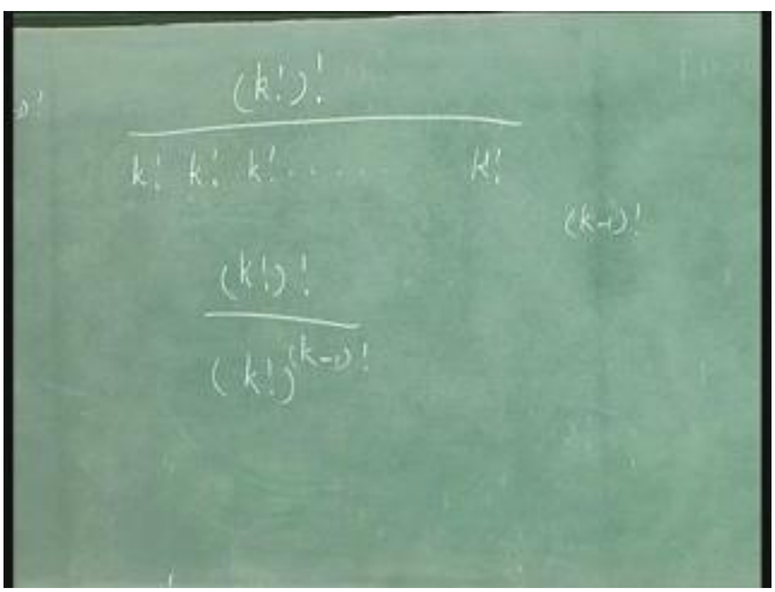

So what is the number of ways of arranging them? The number of ways of arranging k factorial objects is given by k factorial by cap. If they are distinct the number of ways of arranging k factorial object is given by this formula. But we know that k of them are of one kind, k of them are second kind and so on. So using the formulae which we derived earlier you have k factorial because first kind objects are of k type then there are k objects of second type, there are k objects of the third type and so on like that you have, this number is (k minus 1) factorial times. So you are dividing this by (k minus 1) factorial time. So this is equivalent to saying k factorial factorial by k factorial to the power of (k minus 1) factorial.

And what is this? This is the number of ways of arranging k factorial objects of which k of them are first kind k of them are second kind and k of them are (k minus 1) factorial. And this is something, it should be an integer it cannot be a fraction or something it has to be an integer. And so this is an integer, this is an integer which would mean that this number is divisible by this number. Hence, by a combinatorial argument you are proving that this number is divisible by k factorial factorial is divisible by k factorial to the power of (k minus 1) factorial.

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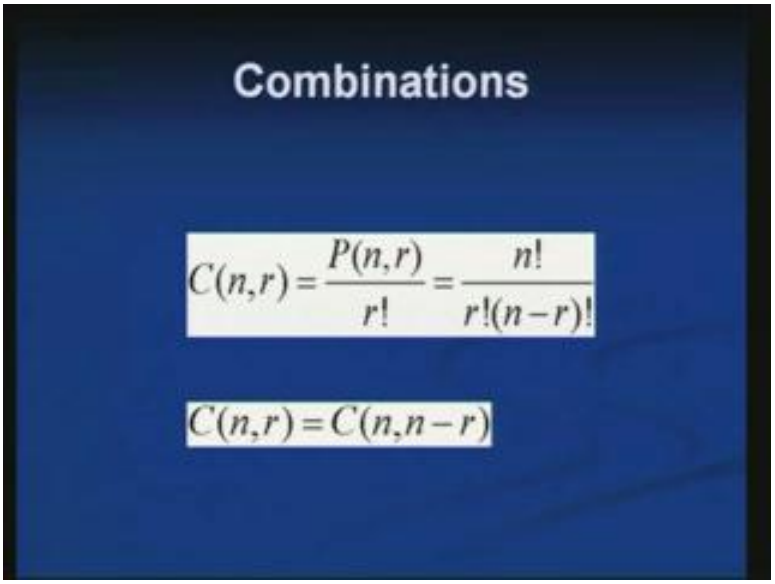

Now, let us consider Combinations or the number of selections. C(n, r) is given by this formula C(n, r) is equal to P(n, r) by r factorial is equal to n factorial by r factorial cross (n minus r) factorial how do we get this.

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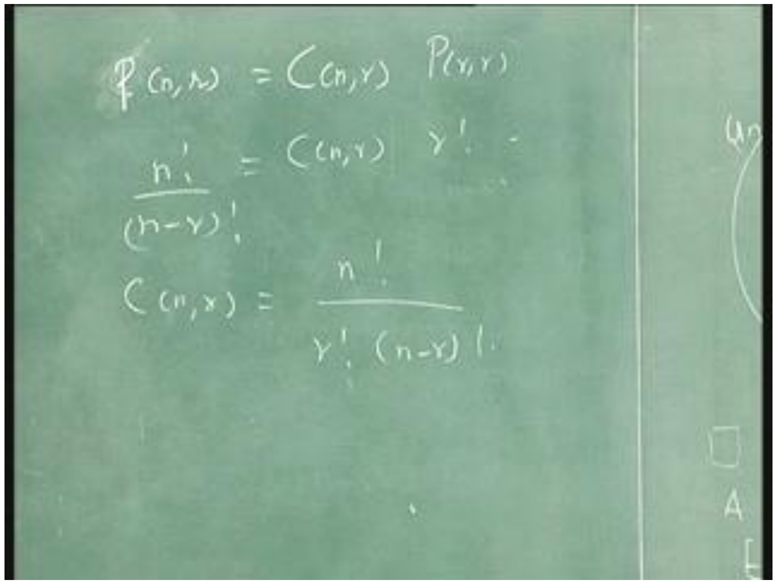

See, C(n, r) is the number of ways of selecting r objects out of n objects. And what is P(n, r)?
P(n, r) is the number of ways of arranging r objects out of n objects. So you get this; when you want to get P(n, r) you have this, first you select r objects out of n objects that is C(n, r) then arrange them that is P(r, r) that is given by C(n, r) cross r factorial. And P(n, r) we know it is n factorial by (n minus r) factorial from this we derive C(n, r) is equal to n factorial by r factorial cross (n minus r) factorial. So we had derived the formulae C(n, r) is equal to n factorial by (n minus r) factorial cross r factorial. And we also see that C(n, r) is equal to C n, n minus r. We can derive at this straight away from the formula or you can also give it a combinatorial explanation.

What is C(n, r)?
C(n, r) is the number of ways of selecting r objects out of n objects. It is also equivalent to saying that you are rejecting (n minus r) objects you are not selecting (n minus r) objects. So the number of ways of rejecting (n minus r) objects is, or you are choosing (n minus r) objects and rejecting them that is given by C n, n minus r. So the number of ways of selecting (n minus r) objects out of n object is the same as the number of ways of rejecting n objects out of n object and so C(n, r) is equal to C(n, n) minus r and this is also derivable from the formula.

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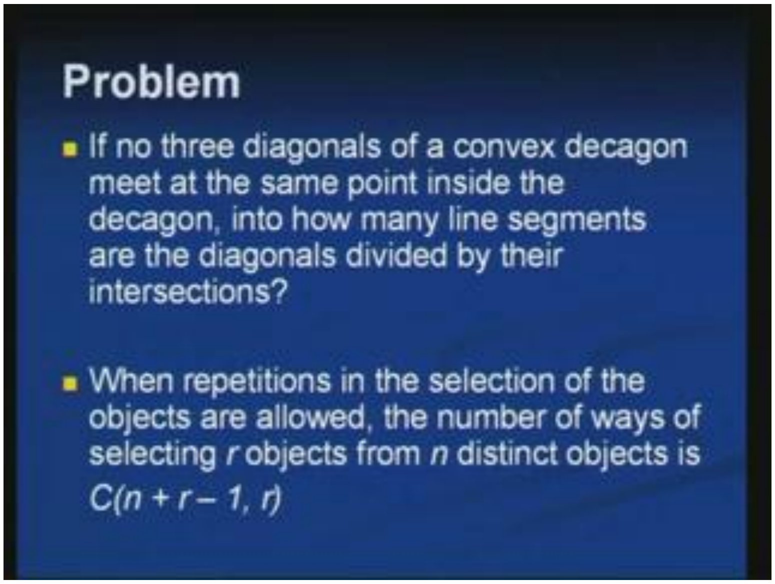

Let us take an example: if no three diagonals of a convex decagon meet at the same point inside the decagon, in how many line segments are the diagonals divided by their intersections. So, simple problems we can consider; in how many ways can you select 2 books out of 5 books is 5 C 2 ways and so on C 2 5 ways and so on those things are simple, let us take a slightly more involved problem, this is the problem we are considering.

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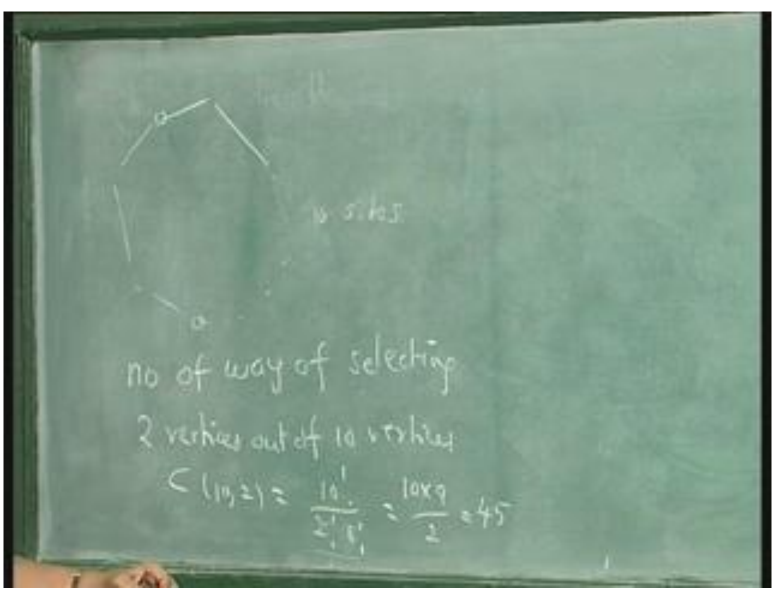

If no three diagonals of a convex decagon meet at the same point inside the decagon in how may line segments are the diagonals divided by their intersections. How do you argue this? So you are having a convex decagon that is 10 sides and 10 vertices of course, how many diagonals it will have? A diagonal is obtained by selecting two vertices out of n vertices 10 here. So the number of ways of selecting 2 vertices out of 10 vertices is given by C 10, 2 is 10 factorial by 2 factorial cross 8 factorial that is 10 cross 9 by 2 is equal to 45. But each line segment joins 2 vertices, out of the 45 here 10 are sides, you want only the diagonals we do not want the sides.

There are 10 sides so the number of diagonals is 45 minus 10 is equal to 35. Now how many intersecting points you can have? If you choose four points then their will be a intersecting point corresponding to the four vertices. If you choose any four you will get an intersecting point. And it is said that all the intersecting points are all distinct. So how many intersecting points you will get?
Number of intersections, the number of intersection is given by C number of ways of selecting 4 vertices out of 10 vertices gives 10 factorial cross 4 factorial cross 6 factorial and that is equal to 10 cross 9 cross 8 cross 7 by 1 cross 2 cross 3 cross 4 is equal to 210. So the number of intersection is 210.

Just consider two diagonals and one intersection, this is one diagonal, suppose there are two intersections on this line, this is the diagonal there are two intersections initially you had one this intersection has added this line segment this intersection has added this line segment so each intersection adds one line segment for our calculation. And this intersection is on two diagonals so it will add one line segment in one diagonal and another line segment in another diagonal. So each intersection will add to two line segments.

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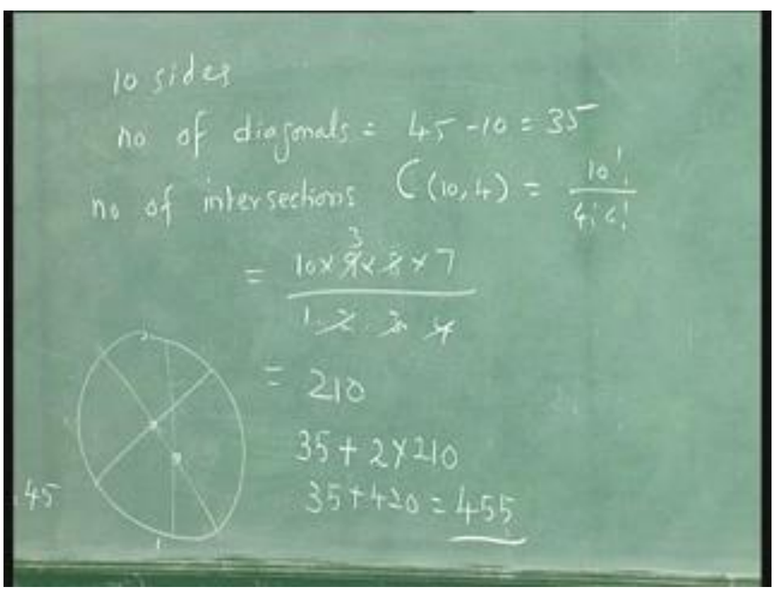

So if you look at it the question is how many line segments are the diagonals divided by their intersections. Initially there are 35 diagonals and each intersecting point adds a line segment and it adds line segment in two diagonals. The number of intersecting points is

210 that is 2 cross 210 times that will give you 35 plus 420 is equal to 455. So in a convex decagon if you assume that that no three diagonals of the convex decagon meet at a same point that is the intersecting point of the diagonals are all distinct in that case the number of line segments into which the diagonal is to be divided is given by number 455 and you get this by arguing out this way.

The next, we will consider one more method: When repetitions in the selections of the objects are allowed what is the number of ways of selecting r objects from n objects n distinct objects. Of course we are not considering non distinct object we are considering distinct objects. When the repetitions in the selections of the object are allowed the number of ways of selecting r object from n distinct object is given by C(n plus r minus 1, r).

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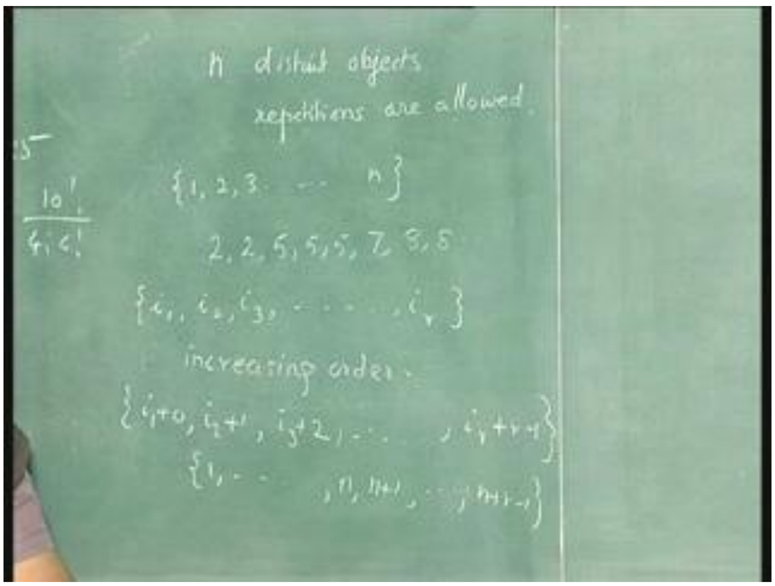

How do you get this? So n distinct objects are there and you are allowing repetitions. Instead of looking at as n distinct objects you look at them as numbers 1, 2, 3 to n. And I want to select r numbers where repetitions are allowed. For example, 2 I can select two times, 5 I can select three times, 7 I can select once, 8 I can select two times and so on that I can select. I want to select r of them out of n repetitions are allowed. Now in that case I select and write them like this I, i_1, i_2, i_3, i_r. Here some of the i's may be the same then they are not distinct repetitions are allowed but I am arranging them in increasing order, they are arranged in the increasing order like this 2 5 7 8 like that. Now, I add 0 to i_1, 1 to i_2, 2 to i_3 and so on and (r minus 1) to i_r. So to i_1 I add 0, to i_2 I add 1, to i_3 I add 2 and so on to i_r I add (r minus 1). Again they are all numbers but they are all distinct numbers they are not the same. So this way you are selecting r distinct integers now the integers are not from 1 to n but they can be up to (n plus r minus 1) because if you add r minus 1 to n you will get (n plus r minus 1). So this is equivalent to saying that you are selecting r distinct objects i_1, i_2 plus 1, i_3 plus 2. You are selecting the integers i_1, i_2 plus 1, i_3 plus 2, i_r plus r minus 1 they are distinct integers from the set {1, 2, 3 up to n plus r minus 1} this is equivalent to the number of ways of selecting r objects out of (n plus r minus 1) distinct objects. And so that is given by C cross C(n plus r minus 1, r) like that we get this result.

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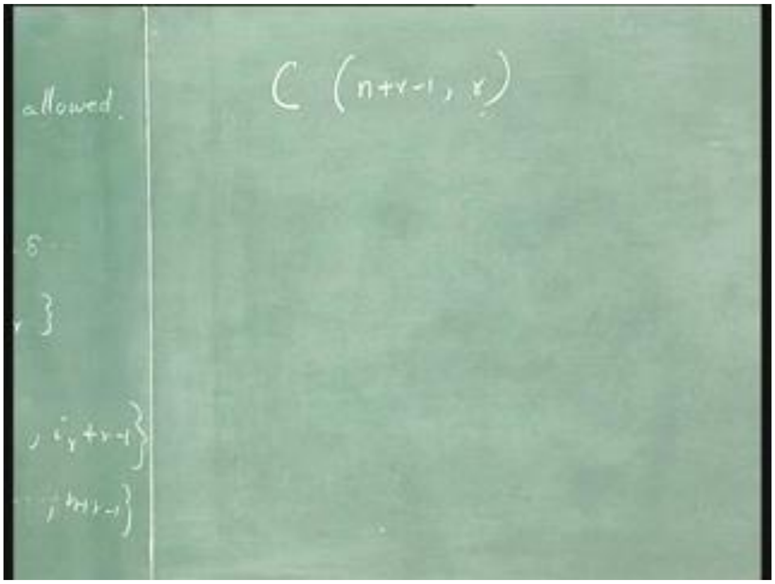

So when repetitions of selection of objects are allowed the number of selecting r objects from n distinct objects is given by C(n plus r minus 1, r).

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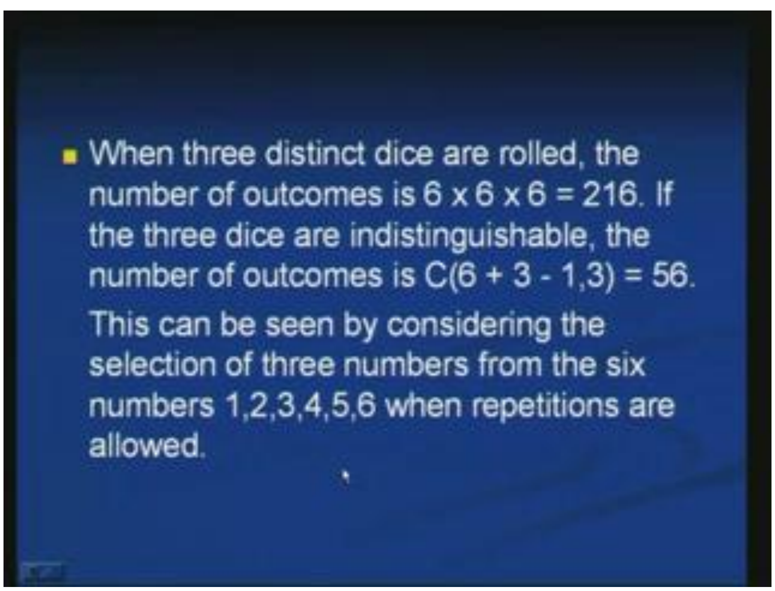

Let us consider a simple problem here; you are having 1 rupee coins, 2 rupee coins and 5 rupee coins you want to select 6 of them. From a collection of 1 rupee coins, 2 rupee coins and 5 rupee coins you want to select 6, in how many ways can you select?

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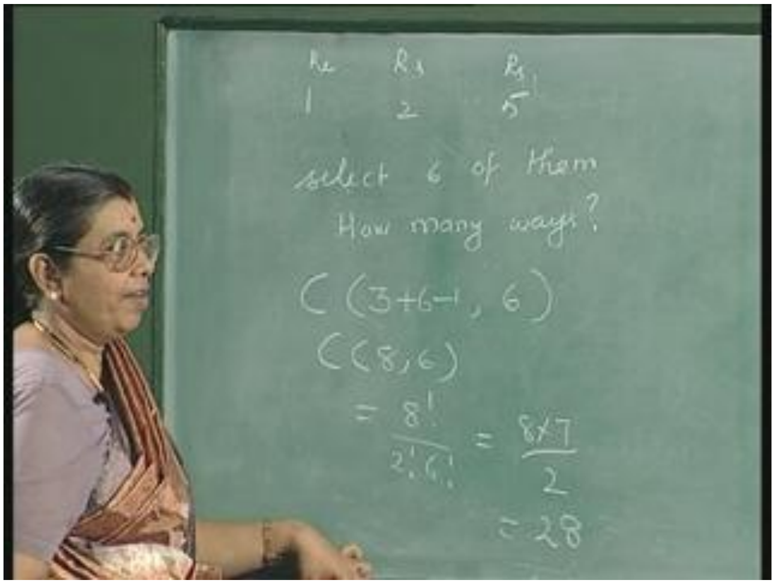

The answer is, there are three types and you have to select six of them by repetition you can use repetition. So the answer using the formula is C(3 plus 6 minus 1, 6) this is the answer. What is this? C(8, 6) and that is given by 8 factorial by 2 factorial cross 6 factorial which is 8 cross 7 by 2 is equal to 28 this is the answer. And, if you have some doubt about this let us check how you get this 28.

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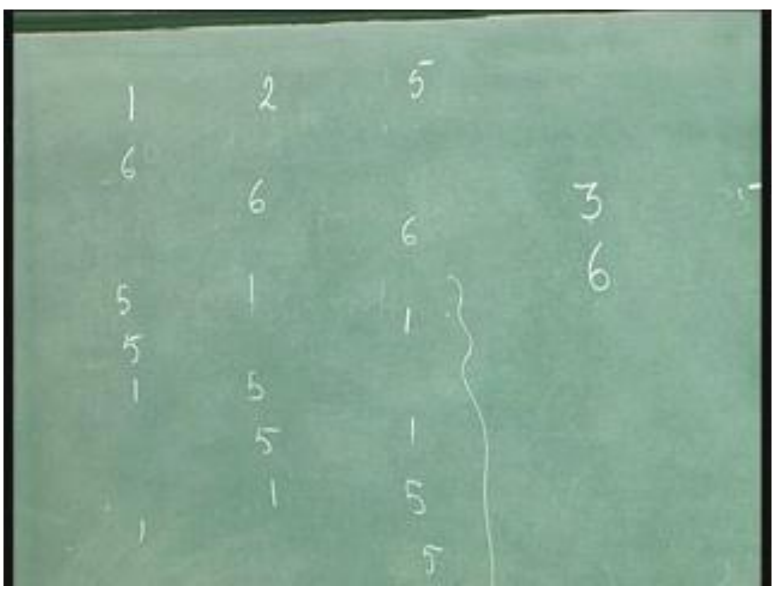

So you have 1 rupee coins, 2 rupee coins and 5 rupee coins you can select all of them to be 1 rupee coins, you have to select 6 coins you can select all of them to be 6 or all of them to be 2 rupee coins and all of them to be 5 rupee coins and that can be done in three different ways. If you select five of them as 1 rupee coins the sixth one you can choose as a 2 rupee coin or you can choose as a 5 rupee coin. Similarly, if you choose 5 here you can choose 1 here or you can choose 1 here. If you choose 5 here you can choose 1 here or you can choose 1 here and this way this accounts to 6 possible ways. And if you select 4 of them and again two of them from this and two of them from this like that you can select 4 from here 2 from this and 2 from this that again can be done in 6 ways.

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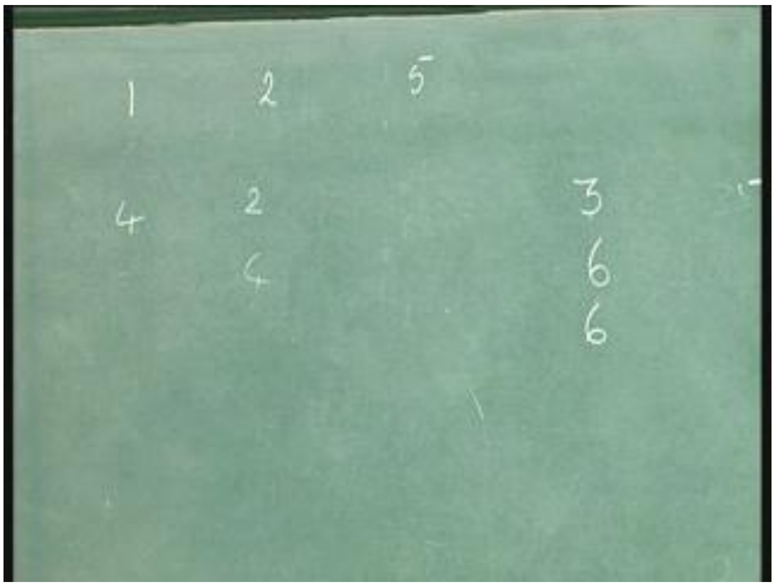

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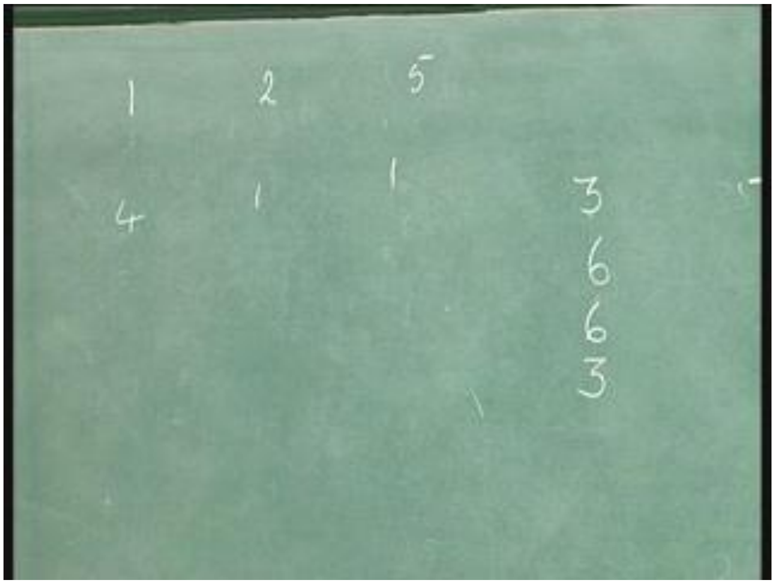

You can select 4 of them from one kind and 1 and 1 from the other two kinds that will give rise to three different ways.

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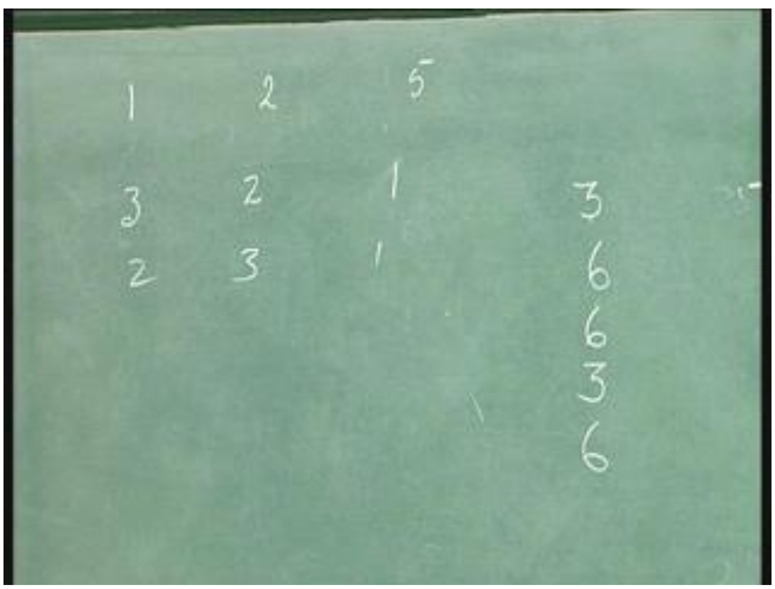

And if you select 3 of them of one kind and 2 of them from the one and 1 from the other one that is you can have 3 2 1 or 3 2 1 like that and this way you can select in 6 possible ways. And you can select 2 2 2 that is in one way.

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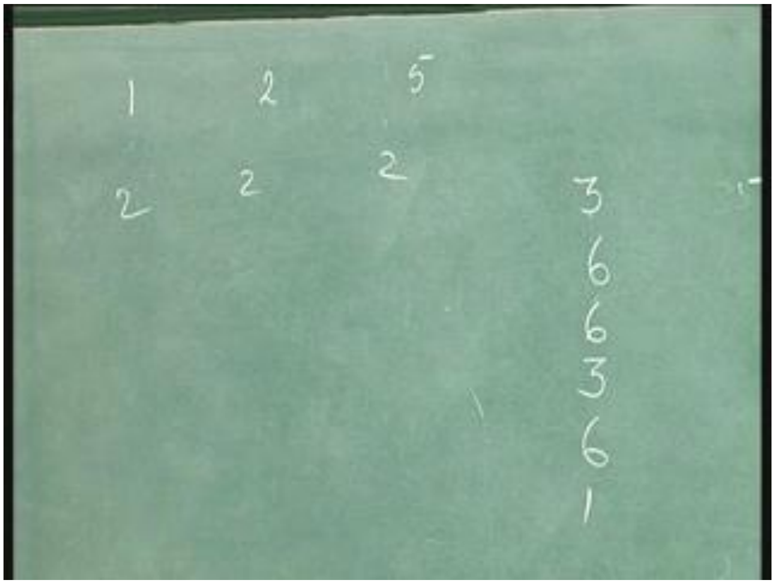

Or you can select 3 and 3 or you can select 3 and 3 here or you can select 3 and 3 here and that is again 6.

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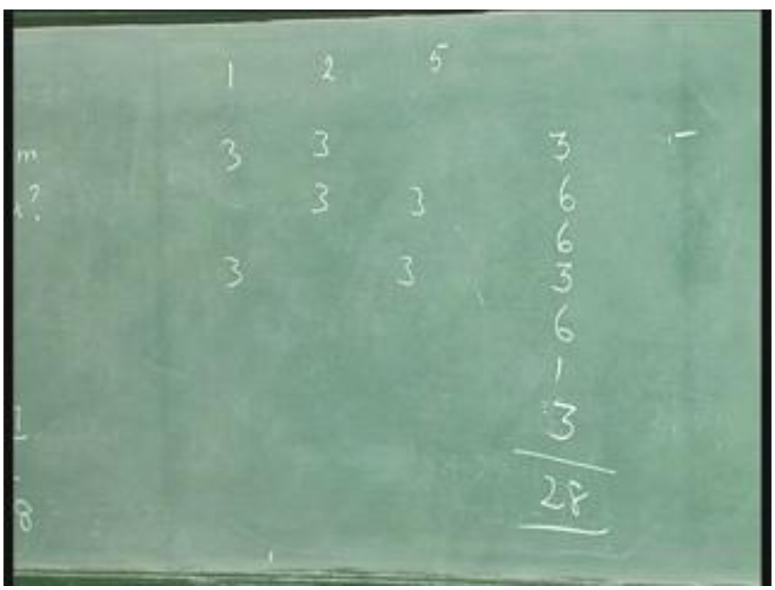

This is 3 so this gives rise to 3 and by adding up all these you get this answer 28. Now another example is this; when three distinct dice are rolled the number of outcomes is 6 cross 6 cross 6 is equal to 216. So you are having three distinct dice and the number of ways you can have the possible outcome is first one it can show any one of the six numbers and the second time you can show any one of the six numbers and so on. So you have totally 6 cross 6 is equal to 216 ways.

If the three dice are indistinguishable that is if you do not want to distinguish between the first, second and third one then the number of outcomes you have to use the formula C. So there are three dice and six outcomes so it will be given by C(6 plus 3 minus 1), 3 which is 56. This can be seen by considering the selection of three numbers from the six numbers when repetitions are allowed so you have to look at it this way and give the answer. So the first one getting 216 is easy but getting this number you have to look at this in this manner, you are selecting the three numbers from the six numbers where repetitions are allowed. You have to look at it to get the answer.

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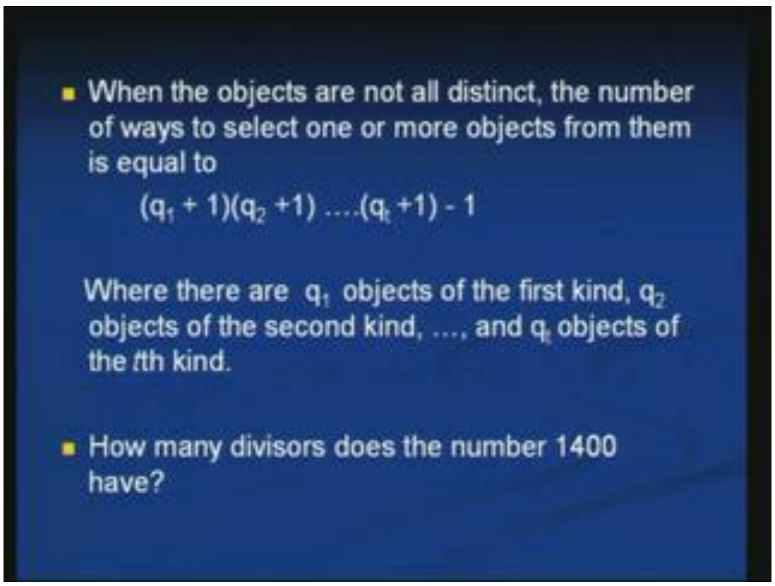

Another problem is; when the objects are not all distinct the number of ways to select one or more objects from them is equal to (q_1 plus 1) cross (q_2 plus 1) cross (q_t plus 1) minus 1 where there are q_1 objects of the first kind q_2 objects of the second kind and q_t objects of the tth kind. There are q_1 objects of the first kind so the number of possibility is you can select one of them, you can select two of them, you can select three of them, you can select q of them you need not select any one of them so the number of possibilities is given by (q_1 plus 1). And there are q_2 objects of the second kind so you can choose one of them, two of them, three of them, q_2 of them or not select any of them that is given by (q_2 plus 1).

Similarly, proceeding with this you get (q_t plus 1) the number of ways of selecting one or more objects of the tth kind or not selecting any of them. But the question is the number of ways to select one or more objects, so not selecting any of them that possibility should be subtracted that is why that minus 1 is there. So the number of ways of selecting 0 or more objects of the first kind is given by (q_1 plus 1). The number of ways of selecting 0 or more objects of the second kind is given by this. The number of selecting 0 or more objects of the tth kind is given by that. So the number of ways of selecting 0 or more object is given by this. But the number of ways of selecting 0 objects is not allowed so that is minus 1. So this formula gives you the number of ways to select one or more object from them when q1 of them are of the first kind and so on. So, making use of this we can solve some problems.

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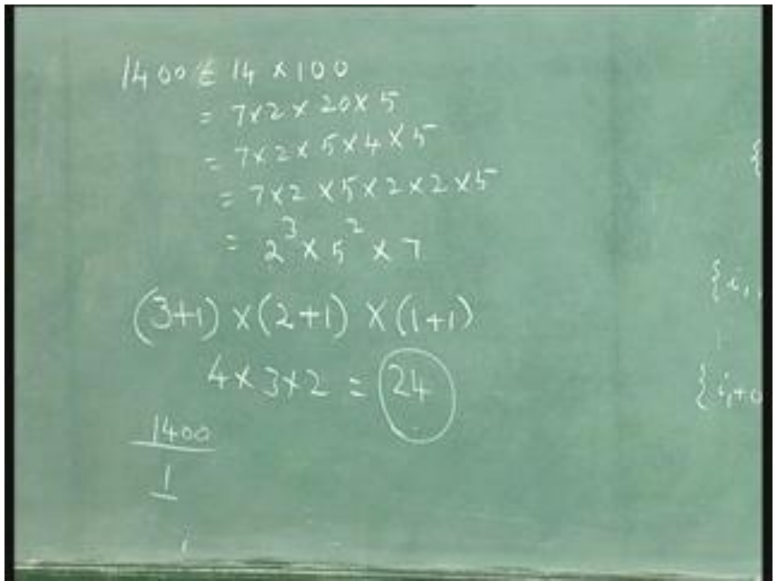

How many divisors does the number 1400 have?
So, 1400 you can write as 14 cross 100 is equal to 7 cross 2 cross 20 cross 5 is equal to 7 cross 2 cross 5 cross 4 cross 5 is equal to 7 cross 2 cross 5 cross 2 cross 2 cross 5 is equal to 2 cubed cross 5 squared cross 7. A divisor of the number is obtained by selecting sum or 0 of them or all of them, again sum or 0 or all of them and by selecting 7 or not selecting 7. So the total number of possibilities is, you can select 1 2s 0r 2 2s or 3 2s or no 2s at all. So that is given by (3 plus 1) ways. So in the earlier formulae if you look at it, it is given by this q_1 plus 1. And similarly, you can select 2 5s or 1 5s or no 5s. So that is given by 2 plus 1 and you can select 7 or not select that is two possibilities that is 4 cross 3 cross 2 is equal to 24 ways. But please remember here that, if we select all of them we get 1400 and if we do not select any of them we get 1. So 1 and 1400 if you look at them also as divisors then the number of divisors of 1400 is given by this.

Here we are taking 1 and 1400 also as divisors. So we have considered some problems based on Permutations and Combinations. The main thing is each problem we have to consider in a different manner and there are some formulae. The main formula is P(n, r) is n factorial by (n minus r) factorial and C(n, r) is n factorial by r factorial cross (n minus r) factorial. And by looking at the problem we have to decide whether there are repetitions or no repetitions whether they are distinct objects or non distinct objects and things like that and then apply a corresponding formula and argue it out.

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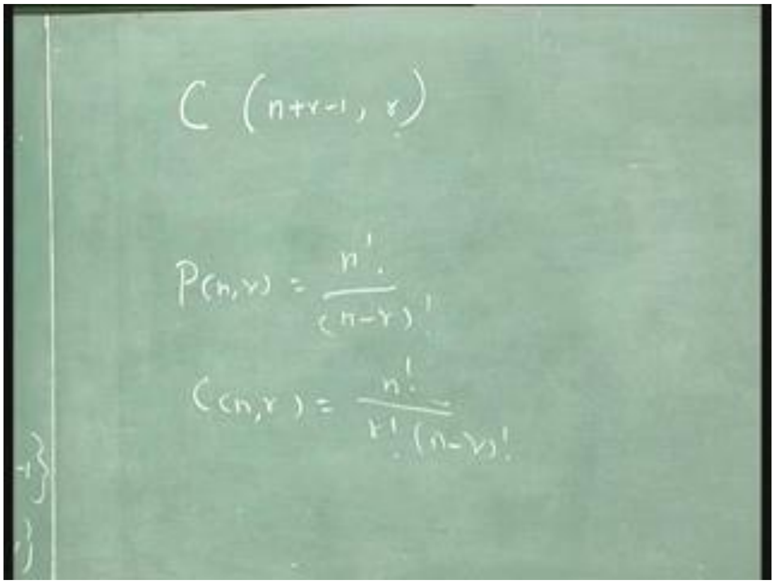

So we shall consider some more Combinations and Permutation by distribution of distinct objects non distinct object into distinct cells and non distinct cells in the next lecture.