Discrete Mathematical Structures
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Lecture # 25
Functions (contd…)

We were considering functions. A function is a map from a set A to B where by a rule you associate an element of B with every element of A. And we have also seen what is meant by a surjective function or onto function and injective function, one-to-one function and bijective function. In a surjective function every element of B will be an image of some element of A. In an injective function different elements of A will be mapped onto different elements. And if a function is surjective as well as injective it is called a bijective function. So we have already seen this definition. Anyway let us recall this.

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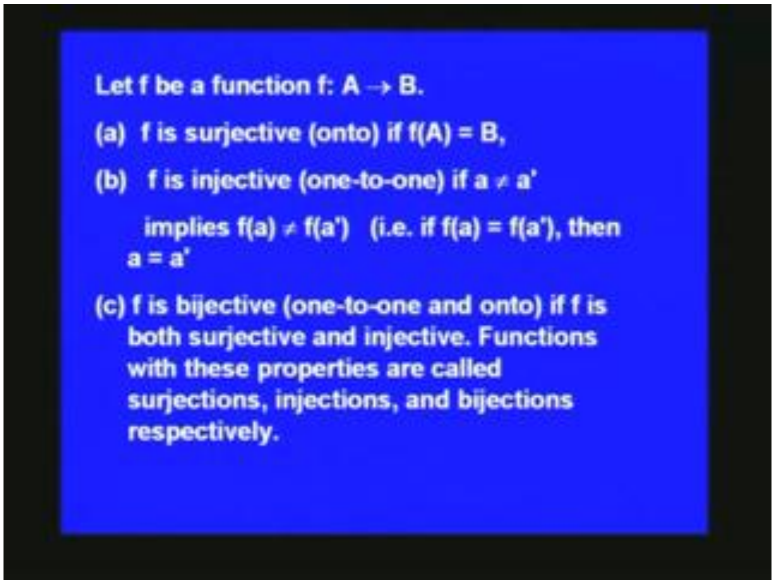

Let f be a function from A to B. Then f is surjective if f(A) is equal to B that is every element of B is an image of some element of A. And f is injective if a not equal to a dash implies f(a) not equal to f(a) dash. That is different elements of a will be mapped onto different elements of B. And if it is both injective and surjective it is bijective.

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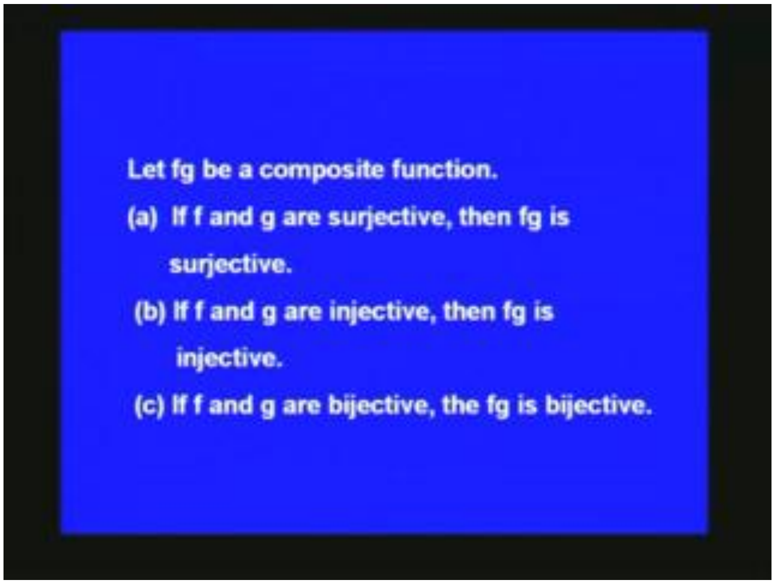

Now, we were considering the use of these functions. For example, the idea will be useful when you consider a hash function.

What is a hash function?
You will have some records and these records will have to be stored in some computer in consecutive memories or addresses so you have locations, suppose you have 400 records of a company employee record or something like that then they have to be stored in addresses between this 499 you have 500 places where it can be stored and these 400 records have to be stored there.

Now how will you identify which one has to be stored where? So it may not be advisable to take the first record here, second record because sometimes you may want to retrieve the record in a very quick manner. So for example this is the employee's record of a company then the social security number could be the key, it is unique, social security number is unique for the person it is a 9 digit number.

Suppose I have say 421, 77, 3824 something like that as a social security number of a person, then you may say that you may use a function h such that this will reduce to 421 and then you can store it in the address 421. Now if it is between 500 and suppose 1000 that is suppose I have something like 678 like this then I can define this mod 500 that is 678 minus 500 is 178 like that you can try to define. But in many places what will happen is people belong to the same area or same locality their social security number may begin with the same 3 digits. If it so happens that in the 400 records you are having 300 of them begin with 421 you have to store three hundred of them here that is not possible.

As far as possible you would like to have each one mapped onto a different number. That is you would like to have an injective function. But we cannot go on defining a new function every time. So we will try to define in such a way that more or less it happens to be an injective function. It may not be exactly an injective function where different elements will be mapped onto different elements but it is almost like an injective function.

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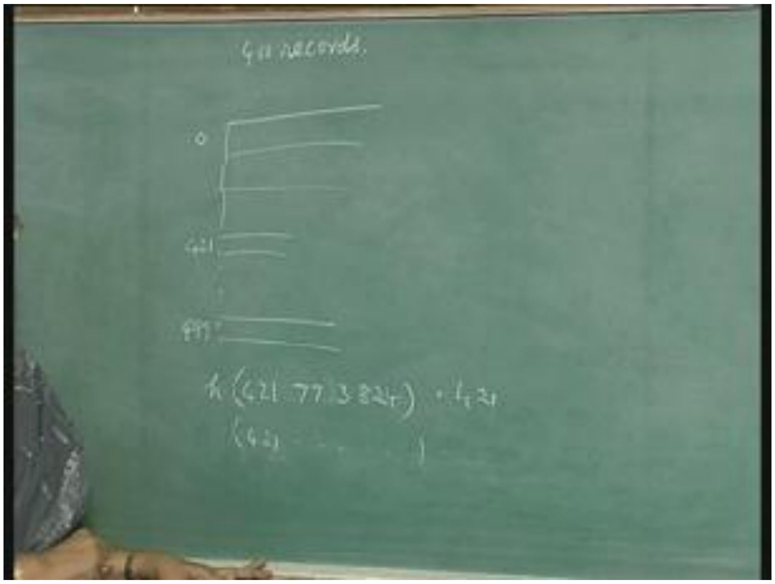

For example, you may define something like this.

Suppose I have a 9 digit number like this something like this, then this has to be reduced to a number with 3 digits where the first digit can be 0 to 4, second digit can be 0 to 9 and the third digit can be 0 to 9.

So for the first digit you can use something like this; this if I denote as x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8 x_9 then you may define like this, first one is, this has to be reduced to a number y_1 y_2 y_3 where y_1 will be between 0 and 4, y_2 and y_3 can be between 0 and 9. So you can define like this something like x_1 plus x_4 plus 7, y_1 is this mod 5 in that case it will reduce to a number between 0 and 4 and y_2 is a number x_2 plus x_5 plus x_8 mod 10 then this will reduce to a number between 0 and 9 and y_3 will be x_3 plus x_6 plus x_9 mod 10. So in this case what will be y_1, y_2, y_3? It is 4 plus 7 is 11, 11 plus 4 is 15, 15 mod 5 is 0.

Then the second number y_2 will be 2 plus 7 is 9, 9 plus 2 is 11, 11 mod 10 is 1. Then third number is 1 plus 3 is 4, 4 plus 8 is 12, 12 mod 10 is 2 so this will reduce to 0, 1, 2 and this record will be stored in the place corresponding to 12. Here also it is possible that different elements will be mapped onto different elements but you cannot ensure this. It may so happen that two of them go to the same address or two of them get mapped onto the same element. In that case you say that a collision has occurred.

Suppose one record you have already stored here another record also you get the same number then a collision occurs. Then the second record you have to store somewhere else may be below that or some other place and then you must have a pointer pointing to that element. This is the way usually it is done in compilers and in several other fields in Computer Science these hash functions. This is not the unique way of defining a hash function there are several ways of defining hash functions and as far as possible you would like them to be injective functions but you cannot ensure that.

So most probably you will select some functions which are almost like injective. And in some cases that condition may be violated and collisions may occur in which case you have to make some adjustments and store the second element which hashes to the same address in a different place and have a pointer to that element. This is how it is done to store elements in a table inside the computer.

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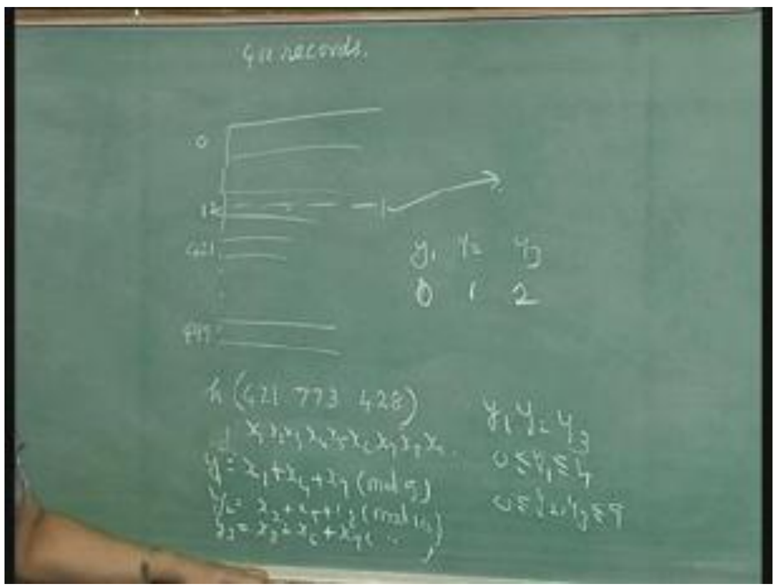

Now we have two sets A B and a function is from A to B, there can be several functions.

For example, I will just take two examples 0, 1 a, b or 1, 2 probably 1, 2 and a, b then I can have a function where these two are mapped onto a, I can have a function where this is mapped onto this and this is mapped onto this like that. Generally the set of all functions from A to B is denoted by B power A, this is the notation used. Now, let us see some properties about this surjective, injective and bijective functions.

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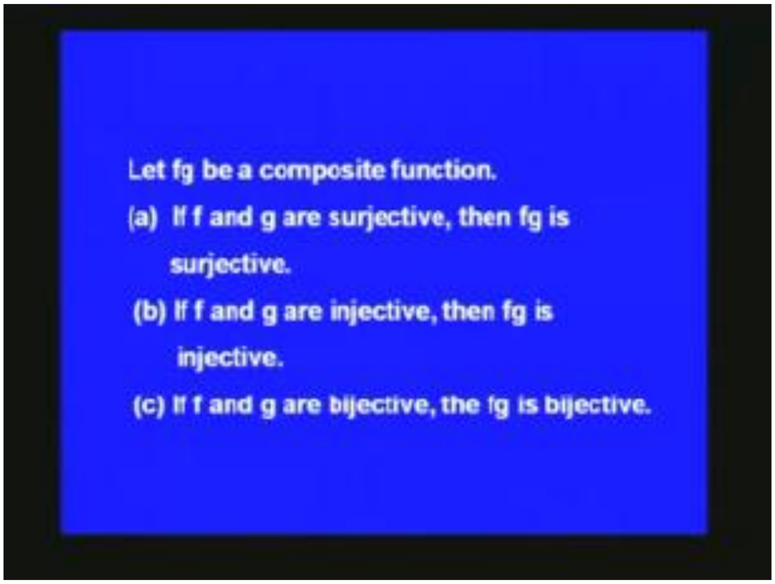

Now if you have a composite function, let fg be a composite function.

- If f and g are surjective then fg is surjective.
- If f and g are injective then fg is injective.
- If f and g are bijective then fg is bijective.

Let us prove this result.

So, g is from A to B, f is from B to C, then f. g is from the composite function is from A to C. It is like this; g you have like this, the composite function will be defined in this manner. So if you have an element a fg(a)n element a will be f of g of a this is how you define. So it maps an element of the belonging to the set A to an element belonging to the set C. Now we want to show that if f and g are surjective onto then fg is surjective how you can prove that. Suppose f and g are surjective that means you want to show that fg is surjective, f and g are surjective because f is surjective take some c belonging to C.

Take some element c belonging to C then there exists b because f is surjective there exists b belonging to B such that f(b) is equal to c then only the surjective condition is satisfied. And because g is surjective there exists a belonging to A such that b is equal to g of a. So what can you say fg(a) is equal to c. So, if you take an element c from the set C there is an element a from the set A such that fg(a) is equal to c that is fg is a surjective function. So if f and g are surjective then fg is a surjective function.

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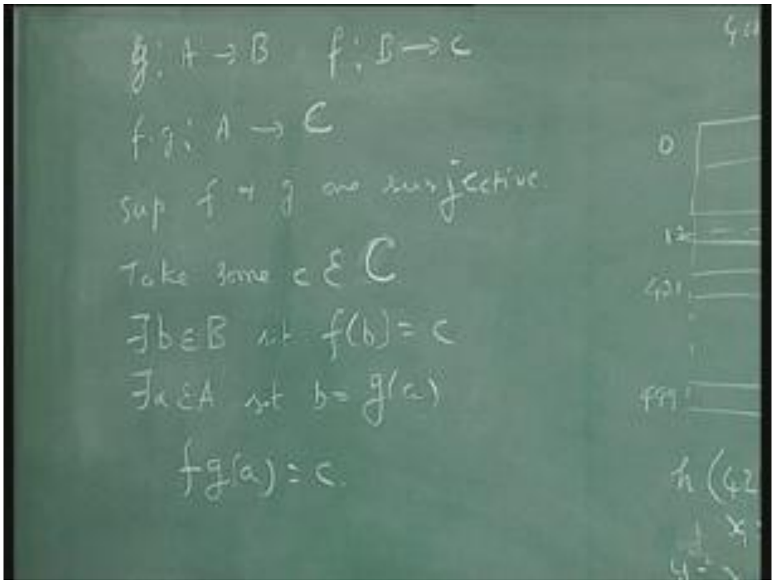

Similarly, if f and g are injective fg will be an injective function. What does that mean? Suppose f and f and g are injective that is take a not equal to a dash then a dash belonging to A then g of a will not be equal to g of a dash why? It is because g is injective, different elements will be mapped onto different elements. So, take f of g of a and f of g of a dash they are different because f is injective this cannot be equal that is fg(a) is not equal to fg(a) dash that is fg maps different elements into different elements so fg is injective that is the second part. If f and g are injective then fg is injective.

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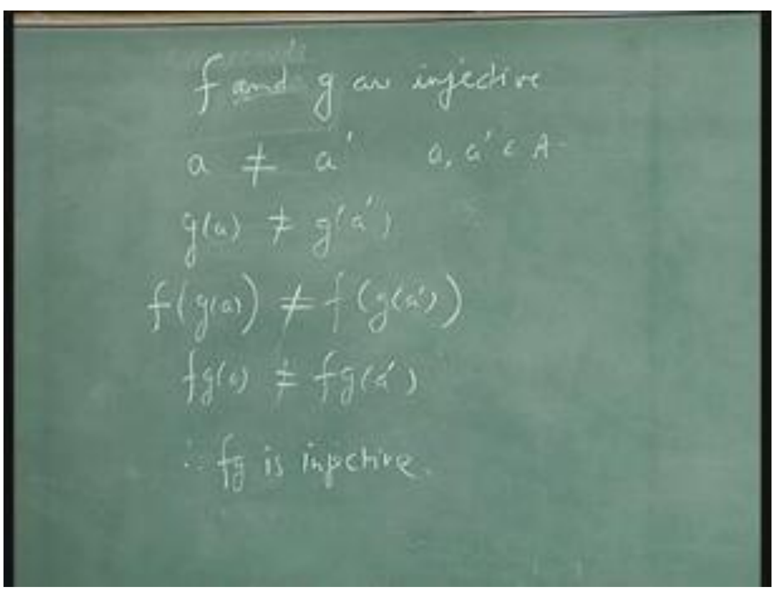

What is a bijective function? If a function is injective and surjective it is bijective.

So from the first two parts the third part will follow. If f and g are bijective then f and g are surjective so fg will be surjective, f and g are injective so fg will be injective because fg is surjective and injective it will be bijective. So the third part follows from the first two parts.

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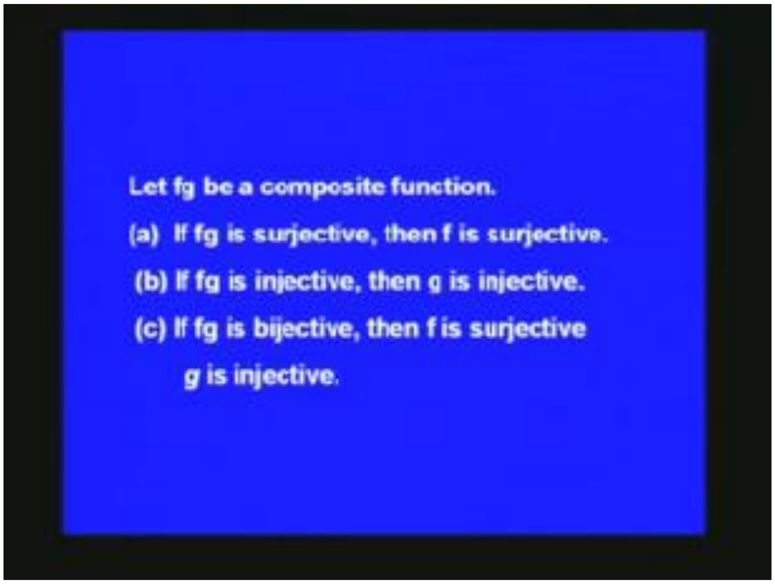

Next consider this, let fg be a composite function. If fg is surjective then f is surjective. If fg is injective then f is injective. If fg is bijective then f is surjective and g is injective.

Now, fg is surjective let us consider this. It will also again follow the similar argument like this. g is from A to B again f is from B to C, fg is surjective, if fg is surjective then f is surjective. Let c belong to C, then what happens here? There exist a in A such that fg a is equal to c. But g of a is some element b so f(b) is equal to c. that is, there is an element b belonging to B such that c is the image of that that means f is surjective. This holds every c belonging to C so f is surjective.

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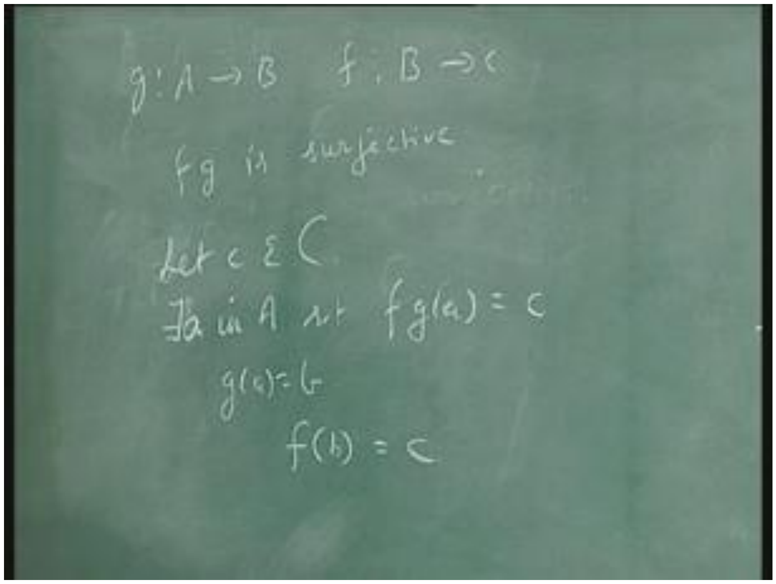

The other way round, if fg is injective g is injective in that case. So we see that if fg is injective then g is injective.

How do you prove that?
fg is injective so if you have a not equal to a dash f(g(a)) is not equal to f(g(a)) dash. But suppose g(a) is equal to g(b), if it were true that g(a) is equal to g(a) dash then if g were not injective you may get a case like this. In that case fg(a) will be is equal to fg(a) dash mapping this on to f you will get this. But you know that fg(a) is not equal to fg(a) dash so this cannot happen so g(a) will not be equal to g(a) dash that is g will be injective.

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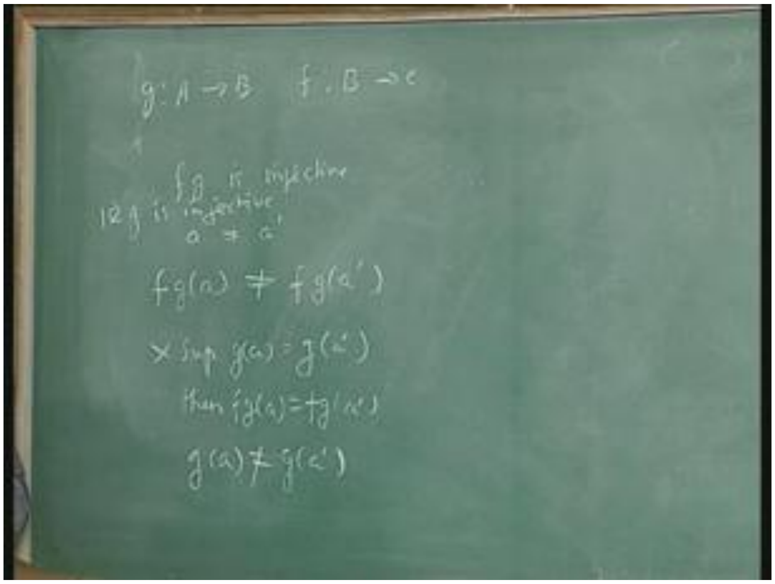

Now, look at the third part if fg is a bijective function it is both surjective and injective. From the first two parts we know that because fg is surjective f is surjective and because fg is injective g is injective. So if fg is a bijective function then you have f is surjective and g is injective. The third part follows from the first two parts. Now we consider different types of functions. You must have already seen these things in your school days but anyway we will consider them separately here.

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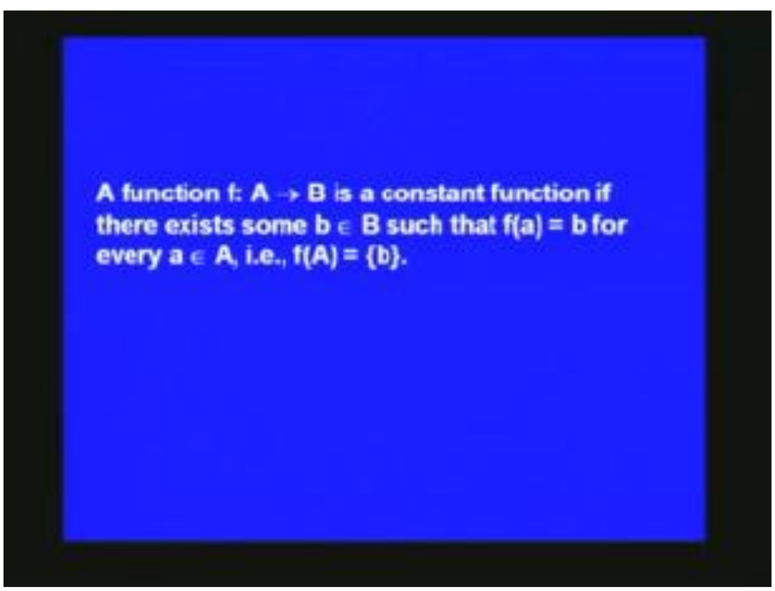

A function f goes A to B is a constant function if there exists some b belonging to B such that f(a) is equal to b for every a belonging A that is f(A) is equal to b.

Suppose I have some 1 2 3 and a b c if 1 is also mapped onto b, 2 is also mapped onto b, 3 is also mapped onto b this is a constant function. Taking the set of real numbers to real numbers a constant function taking the x axis and the y axis, f(x) is equal to c a constant is denoted this is denoted by a line parallel to the x axis, this is c, y is equal to c is a line parallel to the x axis. So this is known as a constant function.

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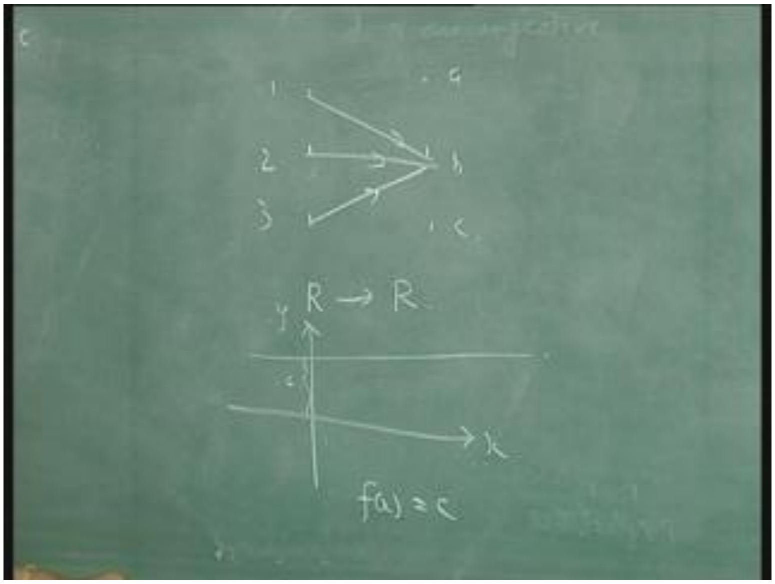

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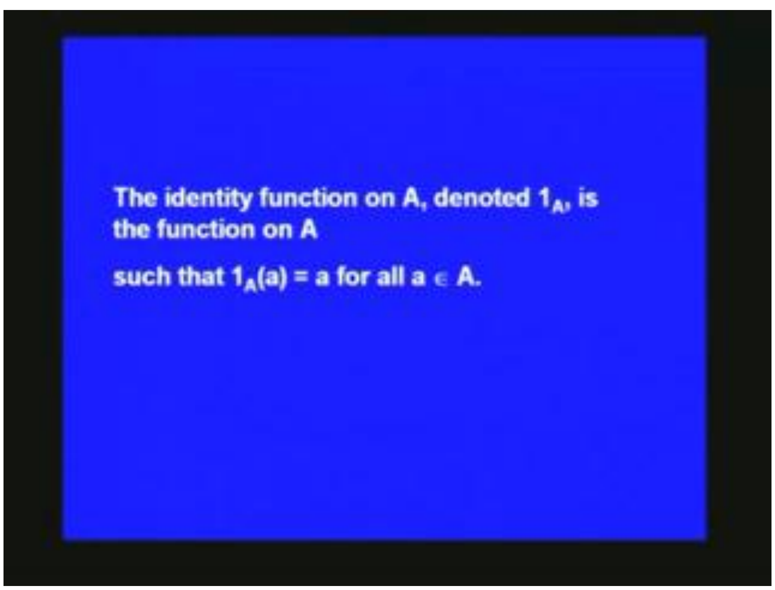

The identity function 1_A is a function on A that is here B is same as A so identity function which is denoted by 1_A is a mapping from A to A. So if you have a b c and a b c a will be mapped onto a, b will be mapped onto b, c will be mapped onto c, this is f of a is equal to a for all a then it is called a identity function and it is denoted by A. Usually the identity function is denoted by this notation but you must specify the set on which it is defined, 1_B is also an identity function but it is defined on the set B, 1_A is a identity function defined on the set A. They are identity functions but they are defined on different sets.

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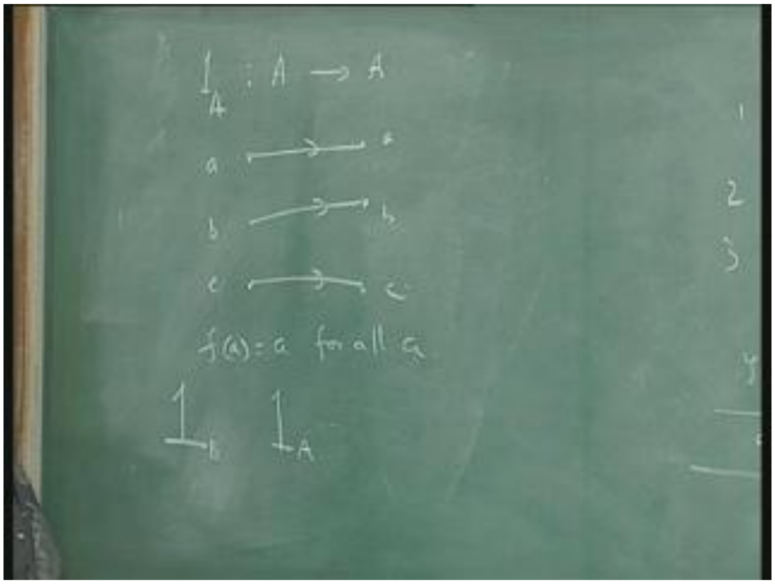

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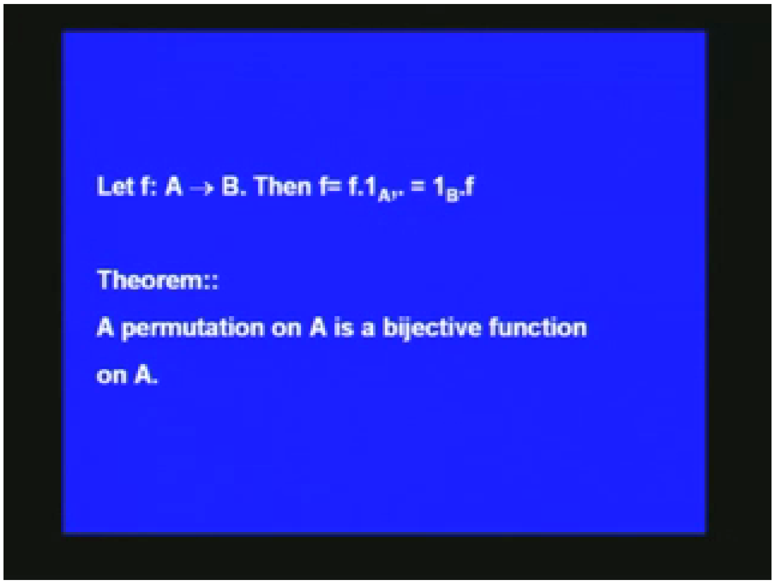

Suppose f is a function from A to B then f is equal to f cross 1_A and is equal to 1_B cross f. Suppose f is a function from A to B then you can say like this, what can you say about f of 1_A that is equal to, take an element a, 1 of a this is like this is equal to f of 1_A is an identity function that is f of a. so for any a this holds so f is equal to the composite function f cross 1_A.

Now what can you say about 1_B cross f? For 1B cross f the composite function 1_B cross f(a) will be 1_B cross $1_B(f(a))$. But 1_B f(a) is an element in B and any element in B is mapped onto itself by 1_B so this will be f of a. so you can also see that f is equal to 1_B cross f. Please note this difference, f is from A to B, 1A is the identity function on a and 1B is an identity function on B then f is equal to f. 1_A and f is equal to 1_B. f you should not write in the other way round it may not have proper meaning.

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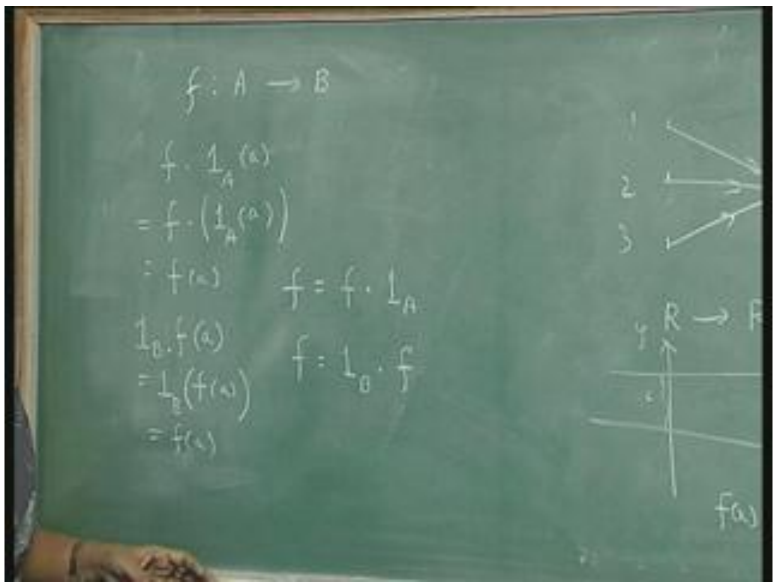

The permutation on a set A is a bijective function on A. consider the elements from 1, 2, 3, 4 a permutation is a rearrangement, a permutation of this is a rearrangement that is 3, 1, 4, 2 is a rearrangement. So 1 is mapped onto 3, 2 is mapped onto 1, 3 is mapped onto 4, 4 is mapped onto 2. You find that different elements are mapped onto different elements so it is injective. And every element here occurs as the image of something or the other so it is also surjective. Since it is both injective and surjective it is bijective. So permutation is a rearrangement of these integers and because it satisfies both the conditions of injectivity and surjectivity it is a bijective function.

Now, you can have compositions of permutations that will also be bijective. Each permutation is a bijection so with the composition of bijective functions will also be bijective.

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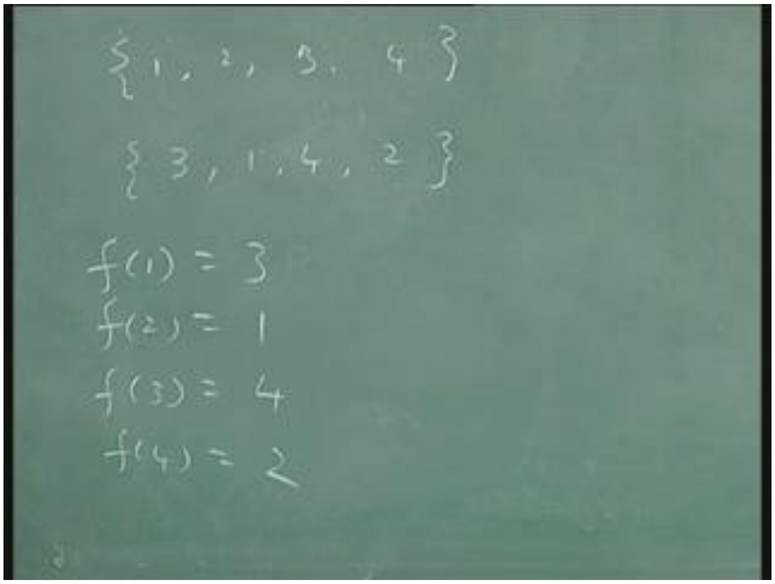

Then these monotonic increasing functions, monotonic decreasing functions these words you might have frequently heard in school or college days. In a formal way let us see what the definition is.

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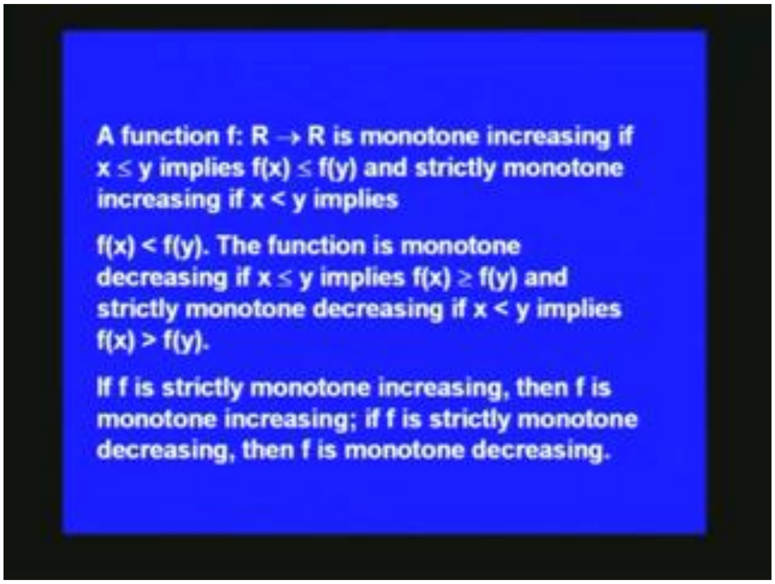

Consider a function from real to real number to real number it is monotone increasing if x less than or equal to y implies f(x) less than or equal to f(y) and strictly monotone increasing if x less than y implies f(x) less than f(y). So, if you draw the graph of a monotone increasing function it will be something like this.

You have two points x and y if x is less than y then f(x) is less than f of y the function will have a graph something like this, this is a monotone increasing function. Nowhere equality holds, it is a strictly monotone increasing function.

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On the other hand, if you have if you have monotonic decreasing function the graph of that will be something like this; x less than or equal to y implies fx greater than or equal to y. The function is monotone decreasing if x less than or equal to y implies fx greater than or equal to fy and strictly monotone decreasing if x less than y implies fx greater than fy.

Therefore, if you have two x values x and y then f(x) is this f(y) is this, this is greater than this the graph will be something like this, this is called a monotone decreasing function. If somewhere they are equal it is monotone, and nowhere any two values are equal it is called strictly monotone decreasing.

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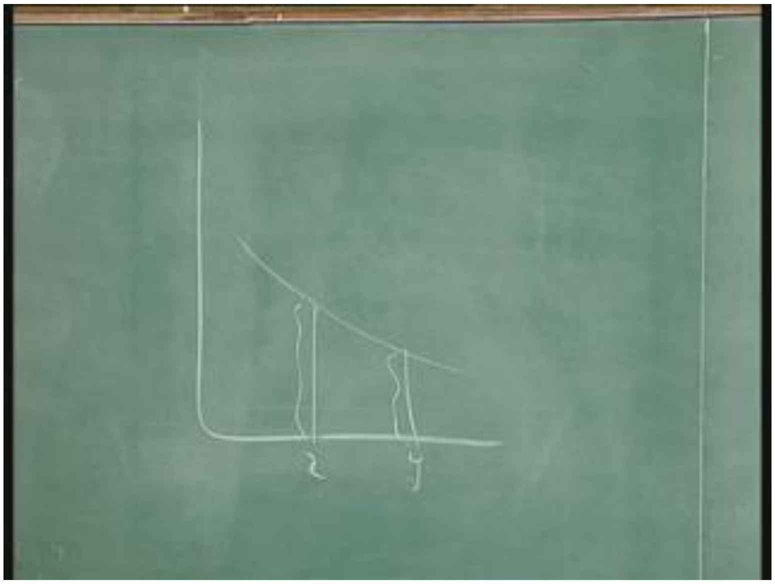

So after this definition the third point follows immediately. If f is strictly monotone increasing then f is monotone increasing. Obviously a strictly monotonic increasing function will be monotone increasing, a strictly monotonic decreasing function will be monotone decreasing. Only thing is in the first one you will allow for less than or equal to, monotone increasing or monotone decreasing you say less than or equal to or greater than or equal to. But in strictly monotone decreasing and monotone increasing you say fx should be less than fy or fx should be greater than fy as the case may be.

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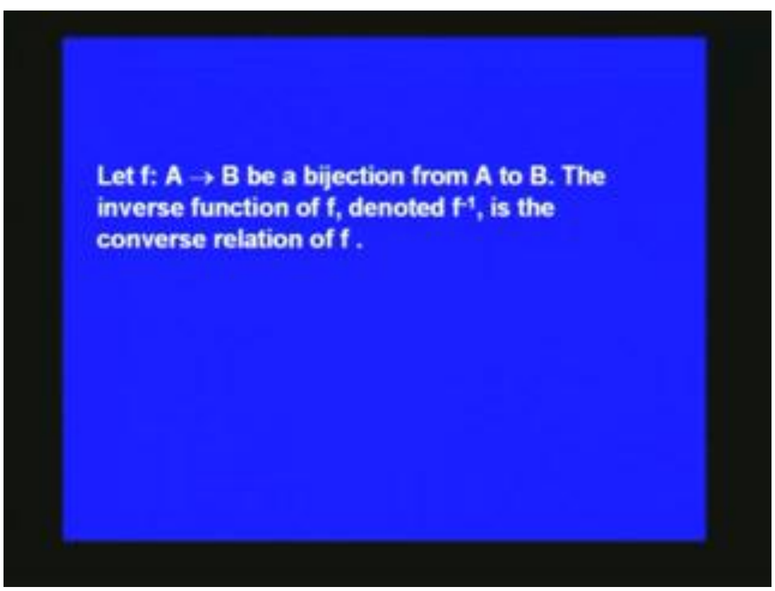

Let A be a bijection from A to B then what is the inverse function f? It is the converse of the relation f. So, you have a bijective function from A to B say 1 2 3 a b c you have a bijective function, 2 is mapped onto c and 3 is mapped onto b. So you say f of 1 is equal to a, f of 2 is equal to c, f of 3 is equal to b then the inverse mapping f inverse is from B to A it is a converse relation you have to change the direction of the arrows. So it is defined from B to A and it is like this; f inverse of a is 1, f inverse of b is 3, f inverse of c is 2 so the converse relation represented by this is called the inverse function f inverse and it is denoted by B to A.

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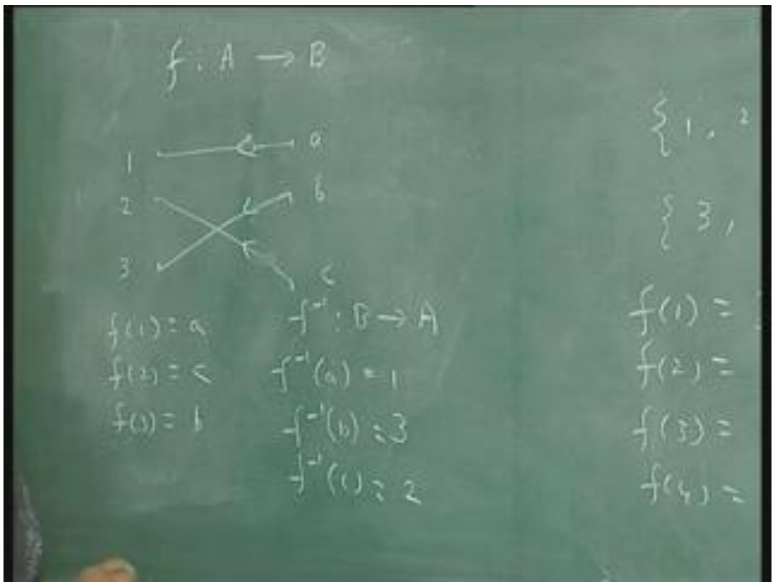

Suppose take the set of integers to integers. Consider the function f from I to I set of integers to set of integers such that f(x) is equal to x plus 1. Then 2 will be mapped onto 3, 3 will be mapped onto 4, 4 will be mapped onto 5 it is a bijective function. You know that different elements will be mapped onto different elements.

What is f inverse of x? It is x minus 1 because if you map 2 onto 3 f(2) is 3 then f inverse of 3 will be 2 so that is x minus 1. You define f inverse in this manner. Such functions are called inverse functions.

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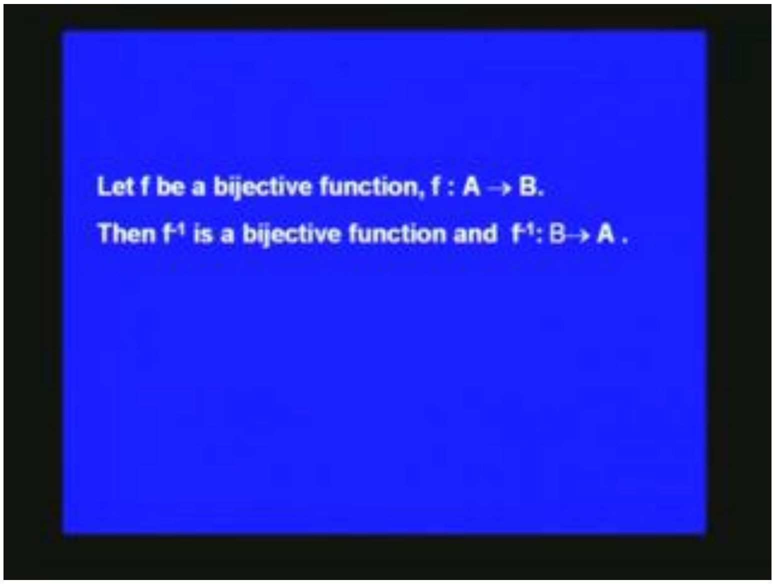

Let f be a bijective function from A to B then f inverse is a bijective function and it is from B to A this is what we have seen now.

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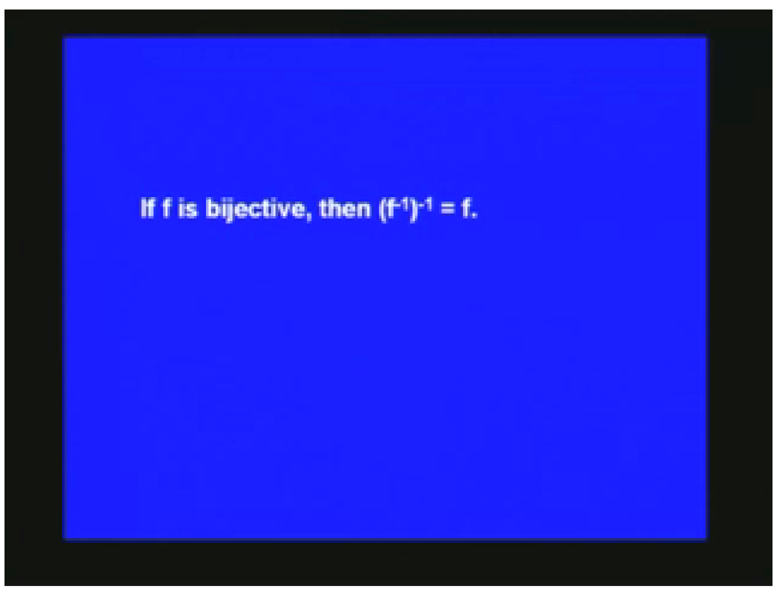

If f is bijective then f inverse that is inverse(f inverse) is f itself, how is that?
Look at this example, originally we had a, this is f then f inverse is you reversed all the arrows f inverse was represented like this, this is f inverse. Now, again take the inverse, again you have to change the arrows, so for f inverse inverse is f itself.

Now how do we represent what is f inverse f? So you know that f inverse f is the identity function but on what it is identity function on what? f is from A to B f inverse is from B to A so f cross f inverse will be, that is this one will be an identity function on A because this is from A to B and this is from B to A so f inverse dot f will be an identity function on A.

But what can you say about f f inverse, this is from B to A and this is from A to B, so f dot f inverse will be the identity function but now it will be on a different set B, f dot f inverse will be on B.

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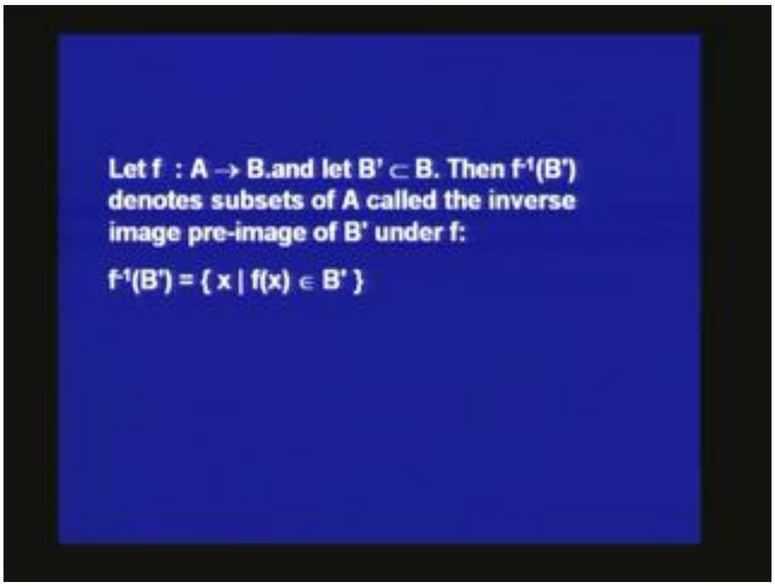

Let f be from A to B and B dash a subset of B. Then f inverse B dash denotes subsets of A called the inverse image or pre-image of B dash under f. That is, if you have something like this f is from A to B this is A this is B.

Now you have a subset B dash, some elements of A dash subset of this are mapped onto this. Now, f inverse of B dash is a set of all elements whose image is in B dash and such a set A dash is called a pre-image.

So it is defined like this: f inverse of B dash is a set of elements x such that f(x) belongs to B dash. That is called a pre-image A dash is called a pre-image of…

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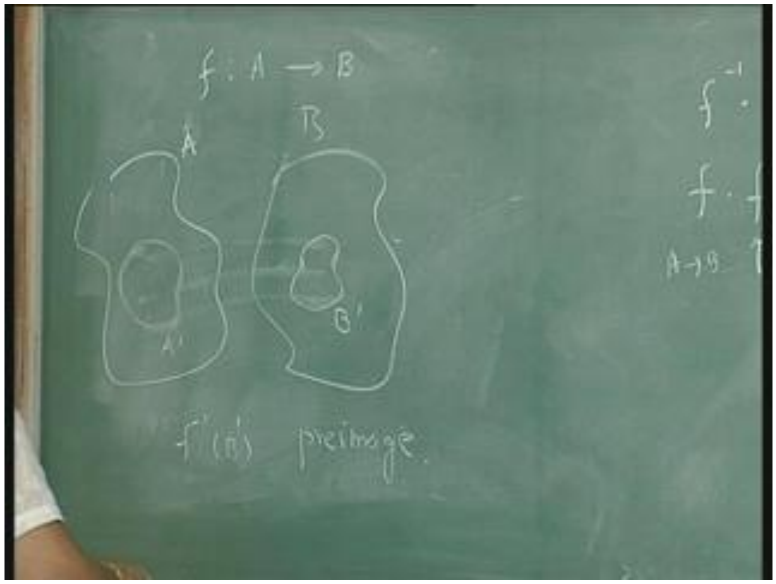

Next, we see that a function can introduce an equivalence relation. We have already seen what is meant by an equivalence relation. An equivalence relation is a relation which is reflexive, symmetric and transitive. A function can introduce an equivalence relation on a set. How can that happen?

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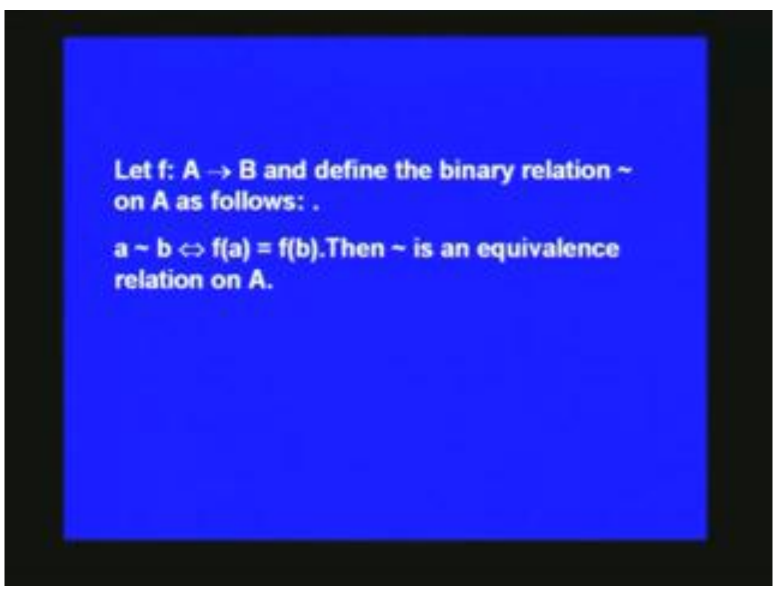

Suppose f is a function from A to B there are some elements like this 1 2 3 4 5 B has three elements a b c, now 1 is mapped onto a, 4 is mapped onto a, 2 is mapped onto c, 3 is also mapped onto c, 5 is mapped onto b. If this is the mapping then 1 and 4 are mapped onto a. so they belong to one class, 2 and 3 are mapped onto c they belong to one class, 5 is mapped onto b that is in one class.

You are partitioning a into subsets like that. And we know that a partition induces an equivalence relation. So you say that two elements a and b are related a related to b if f(a) is equal to f(b). Then this relation you say a related to b if f(a) is equal to f(b). then you say that this relation is an equivalence relation because a will be related to a obviously. And if a is related to b f(a) is equal to f(b) so b also will be related to a. The symmetric property is also satisfied. And if a is related to b f(a) is equal to f(b) and if b is related to c that means f(b) is equal to f(c) so f(a) is equal to f(b) is from this you can conclude a and c are related. So this satisfies the transitive property equal to f(c). So, all these three properties reflexivity, symmetric and transitivity are satisfied so this is an equivalence relation.

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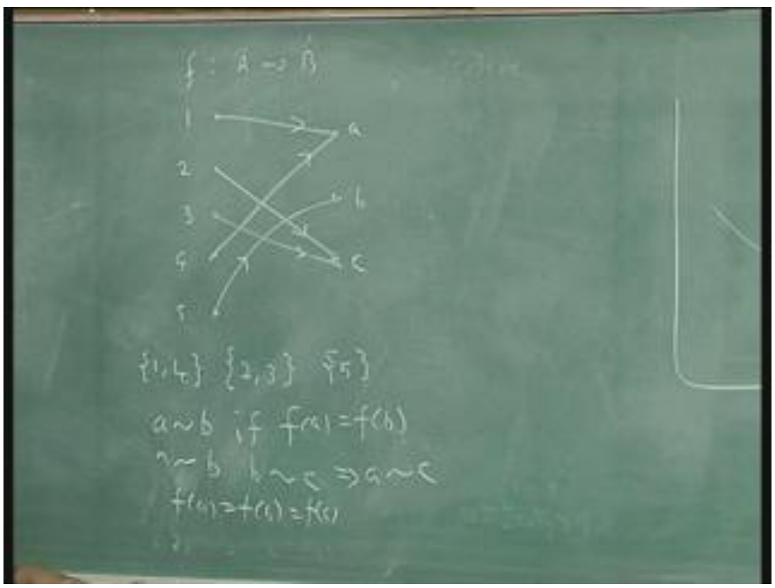

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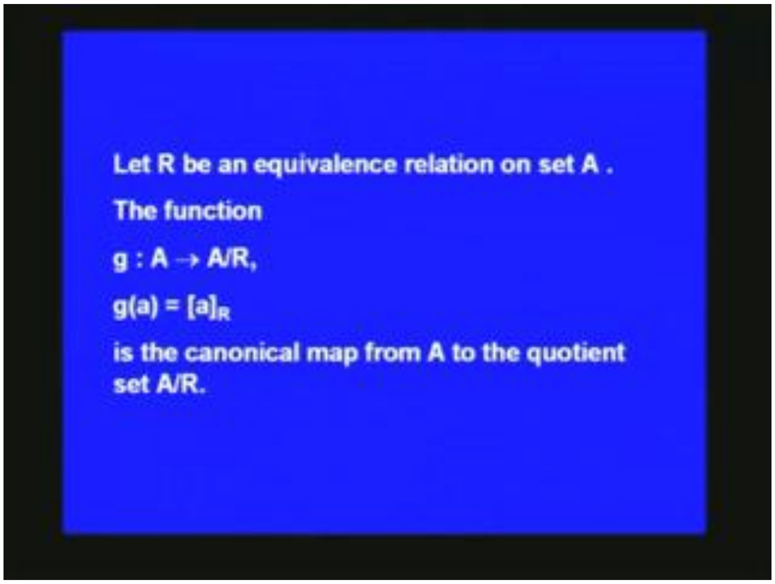

Let R be an equivalence relation on a set A then the function g A cross AR is a canonical map from A to the quotient set A by R. So, an equivalence relation on set A divides it like this, then this is the equivalence class containing 1. So f(a) is the equivalence class containing a. if you define like this here there are three equivalence classes if I call them as E1 E2 E3, 1 and 4 are mapped onto E1 2 and 3 are mapped onto E2, 5 is mapped onto E3 such a map is called a canonical map and it is denoted by A by R.

Now we talk about extension of functions and restriction of functions.

Sometimes we note that you may have a function defined like this, you take the set of integers to set of integers and define like this f(x) is equal to x plus 1. This is a function from integers to integers. But if I restrict a domain and say that non negative integers, I define f dash this is from non-negative integers to non-negative integers but you define the same one you will define like this f dash(x) is equal to x plus 1. This is the same as f(x) as far as the non-negative integers are considered but you are not considering the whole domain you are considering only a subset of the original domain.

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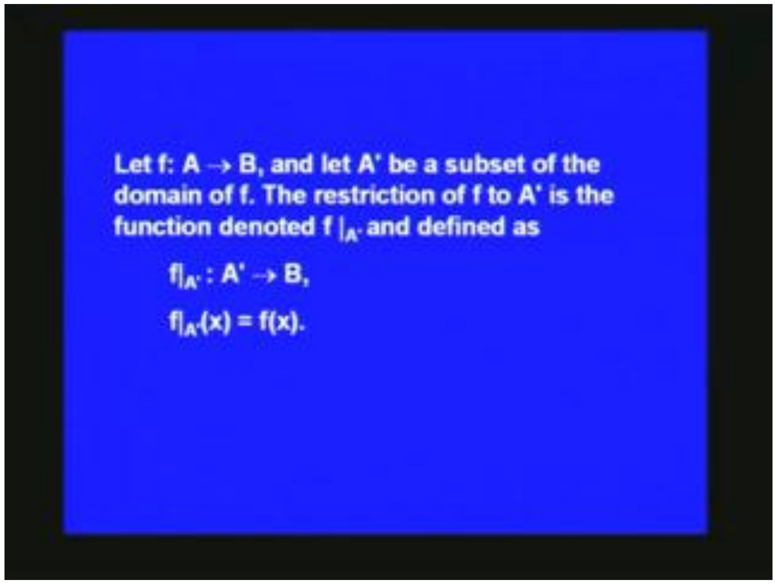

So let A and B be two sets and f is a function from A to B and let A dash be a subset of the domain of f, A dash is a subset of A.

The restriction of f to A dash is the function denoted by f A dash that is it is denoted like this f A dash. It is defined as from A dash to B but as far as the elements of A dash is considered it is the same as f(x). f dash x is also the same as f(x) as in the case instead of defining from integers to integers you can restrict the domain you need not have to restrict the codomain the co domain can still be an integer but it does not matter it will be mapped onto non-negative integers only. So you need not have to change the codomain but you are restricting the domain, N is a subset of I. If you restrict and you define the same map this is called a restriction. You are restricting the domain that is all and it is usually denoted in this manner.

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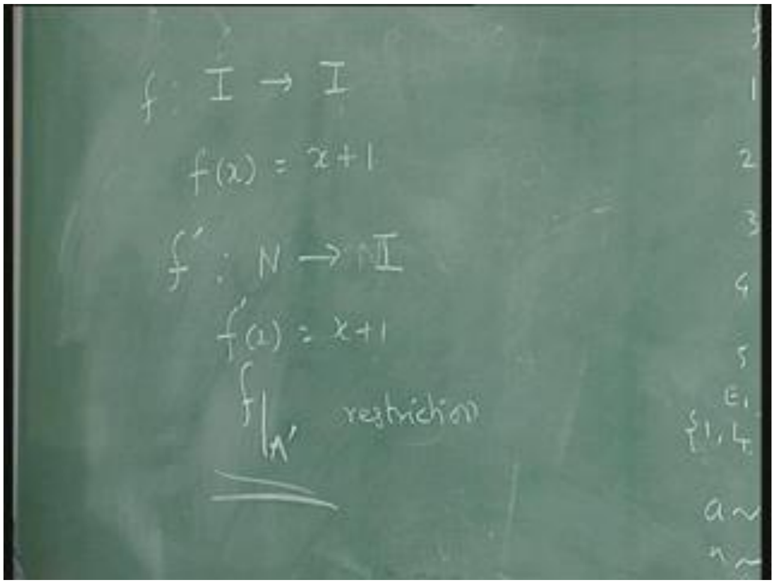

You have a function from A dash to B that is f, then you have function g from A to B where A is a super set of A dash. A is a bigger set than A dash then g is called an extension of f to the domain A if g A dash is equal to f.

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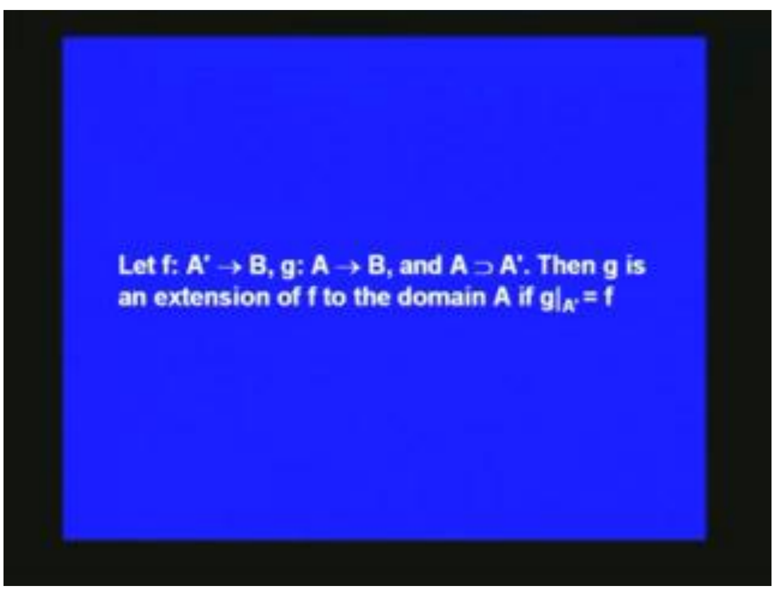

For example, you consider this f is a function say in this example non-negative integers to I that is f(x) is equal to x plus 1 or whatever it may be x minus or x plus 2 or something. Then g you define the same function but the domain is a bigger domain.

Again you define g of x is equal to same way then this g is called an extension of f because when you restrict g to the domain of A dash that is equal to f.

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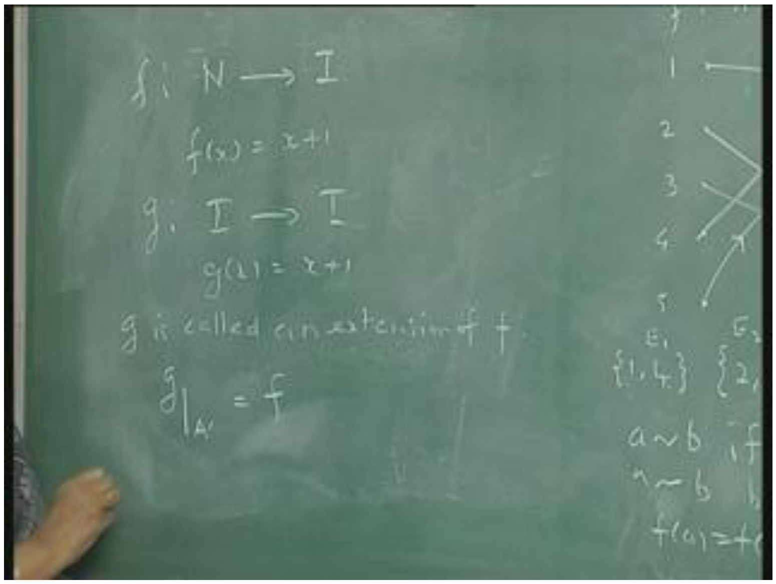

If you look at finite sets this extension can be defined in this manner:
Suppose I have a function f between two sets, a function f like this 1 2 3 4 and a b c d then I want to have some more elements, this function is f, I want to have some more elements 5 and 6 but as far as 1 2 3 and 4 are concerned it is the same map for 5 and 6 I define like this 5 is mapped onto d and 6 is mapped onto c something like this I define. This is an extension of what we defined earlier. You are having some additional things defined and you are making the underlying set as a bigger set.

Now, only for three elements this function is defined then I can remove these things. Now, for subset we are defining the function. But it is the same as the original function but we are making the domain smaller. This is called the restriction of the function. So you can define restriction and extension. Later on you will see that these ideas are very much used in automata theory. You will first define a transition function delta on K cross sigma and then extend it to K cross sigma power star and so on.

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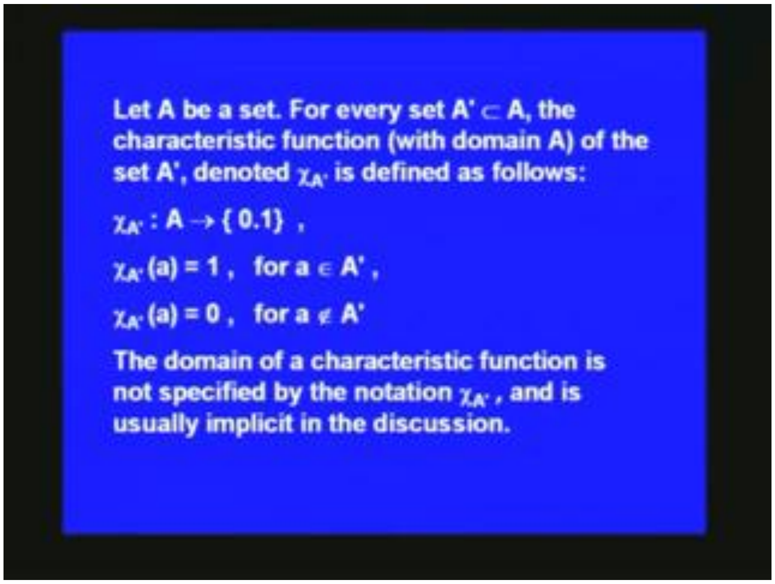

Now, what is a characteristic function?
Let A be a set. For every set A dash contained in A the characteristic function with domain A of the set A dash is denoted by KiA dash is defined as follows:

It is a map from A to the set 0, 1 that is every element in a will be mapped onto 0 or 1. If it is mapped onto 1 when will it be mapped onto 1? If a belongs to A dash and if a does not belong to A dash it will be mapped onto 0. So you define like this KiA is a mapping from a to the element 0, 1 such that KiA dash is equal to KiA dash of a belonging to A dash and KiA dash of a is equal to 0 for a not belonging to A dash, this is how you define.

Take for example a set like this;
A is equal to a, b, c, d it has got four elements. You take a subset A dash is equal to a, c how do you define the characteristic function KiA dash? KiA dash a is equal to 1 because it belongs to the subset, KiA dash b is equal to 0 and kiA dash c is equal to 1 because it belongs to the set, KiA dash d is 0 it is because d does not belong to the subset. So this is known as a characteristic function.

The domain of the characteristic function is not specified by the notation KiA dash. You do not specify A explicitly here it is usually understood.

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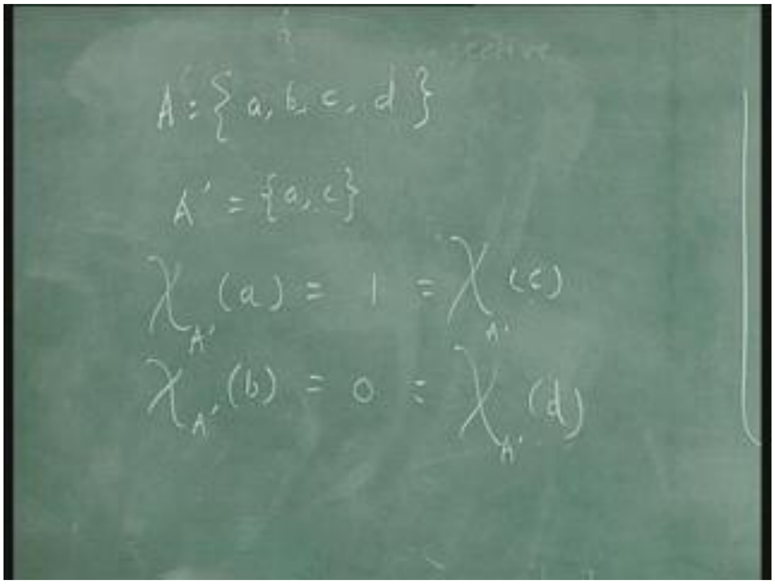

Now, you must have heard the word fuzzy set, what it is.

Characteristic function KiA dash is a map from A to 0, 1. If you take some element in A either it belongs to the subset or it does not belong to the subset, there is a marked distinction. Either it will belong or it does not belong so it is mapped to one of the values 0 and 1. But if A is mapped onto some real value between 0 and 1, it may be mapped onto say 0.5, 0.2, 0.7 such a set is called a fuzzy set. That is, characteristic function will not map elements onto 0 and 1 but it will map something onto a real value between 0 and 1.

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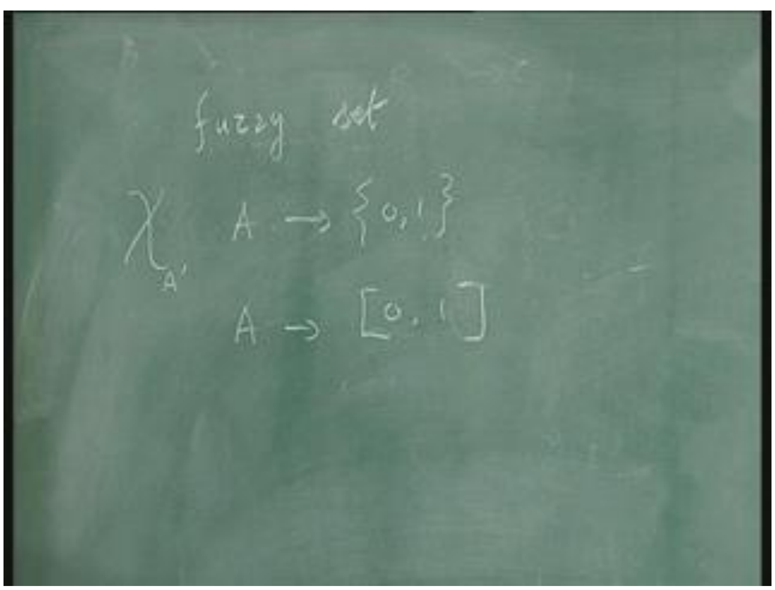

Take for example, the set of tall persons. If I say set of tall persons who is a tall person? Do you call a person tall if he is 6 feet, if his height is 6 feet or if his height is 7 feet or if his height is 5 feet 9 inches? And when do call a person not tall, if he is 3 feet or 4 feet, 5 feet what is the criteria?
Now, somebody may be taller than me and you may not call him as a tall person or I may call a person not tall you may call a person tall so it is slightly subjective to your views.

Now, in this case if you map the element, suppose a person who is 6 feet tall may be a member of the set of tall persons with value 0.8, a person who is 7 feet tall may be the member of the set of tall persons with membership 0.95, a person who is 4 feet tall may be a member of that set with membership value 0.3 so the membership value is a real number between 0 and 1 and such a set is called a fuzzy set.

We have seen several things about functions. We have seen what is meant by a monotone decreasing function, what is meant by a constant function, what is meant by characteristic function and so on. We have also seen what is meant by a recursive function. Now let us try to do one example, define a function f like this, it is from N to N it is a set of non-negative integers to N such that it is defined like this f(x) is equal to x minus 10 if x is greater than 100 f(x) is equal to f(f(x)) plus 11.

Now, calculate f(98), how will you calculate this? If you have f(105) that will be 95 obviously, 101 will be 91, but if you consider 98 how will you calculate this?
F(f(x)) plus 11 98 plus 11 that will be f cross f of 109 but this is greater than 100 so this will be minus 10 so f(99) so what do you get? You get f(98) is equal to f(99). But what is f(99)? f(99) is equal to f(f(99)) plus 11. That will be is equal to f(f(110)) but this is greater than 100 so you will subtract 10 so that will be f of 100. So you find that f of 98 is equal to f(99) is equal to f(100). And what is that? f(100) is equal to f(f(100)) plus 11 that is f(111) this is still greater than 100 so this will be 91. So you find that f(98) is equal to f(99) is equal to f(100) is equal to f(101) is equal to 91. All these take the constant value 91. But f(102) will be 92, 103 will be 93 afterwards it will be just minus 10. I have taken a value 98.

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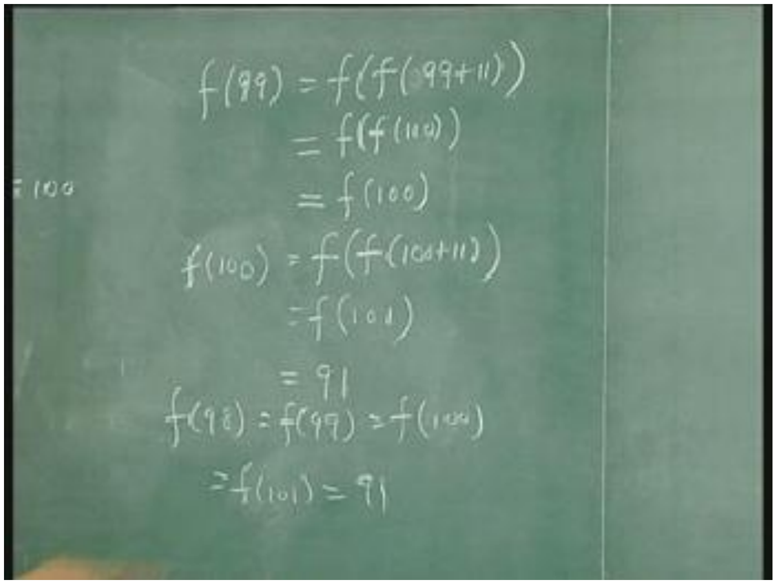

Suppose I take f(89) or something like that still if you calculate the value you will find that that will be only 91. f of any value you take where x is between 1 and 99 then this will be only 91. This is called McCarthy's 91 function. The way you define, it is not very easy to understand or realize the fact that this is that function. That is, all values between for all arguments between 1 and 100, 101 it will take the value 91 and later on it is this value because there are two recursions inside this, it is a recursive function no doubt but in the definition itself two recursions are involved.

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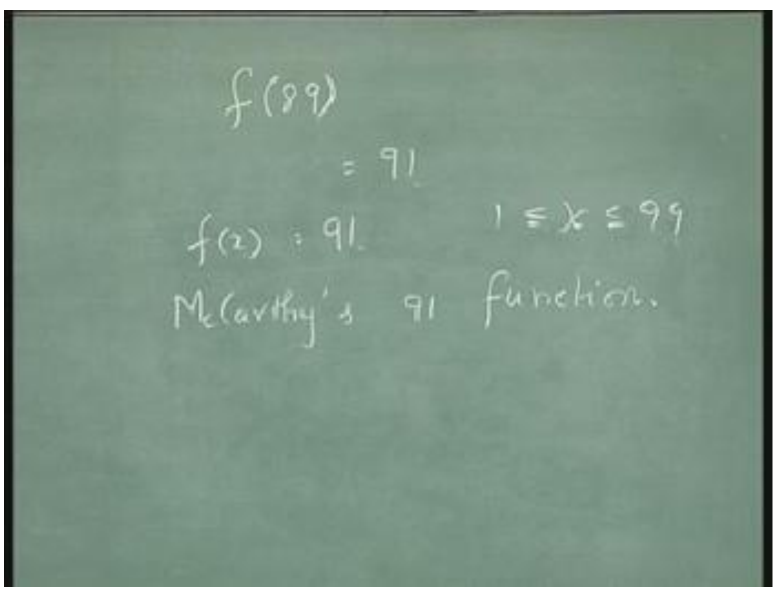

Now, you can talk about left inverse and right inverse of a function and so on. And you can bring out the relationship between them.

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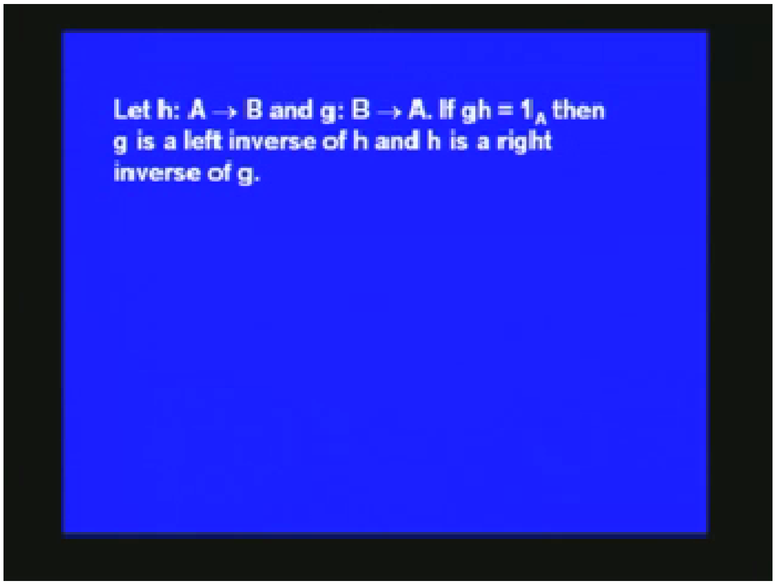

Let h be a function from A to B and g be a function from B to A. And g(h) is equal to 1_A then g is a left inverse of h and h is a right inverse of g. When the function is bijective we have already seen what is meant by an inverse function. When f is bijective we define f inverse. But you may have one sided inverses. You may have left inverses and you may have right inverses. In the bijective case f is from A to B, f inverse will be from B to A. In that case we have seen that f(f inverse) is this is B to A A to B so this will be 1_B and f inverse of f will be 1_A identity function on A.

Now if you have two functions g and h g is from B to A and h is from A to B so g(h) if you say it is A to B B to A so it is an identity function on 1_A. Then you say if g(h) is 1_A then you say g is the left inverse of h g is the left inverse of h and h is the right inverse of g, this is what you define.

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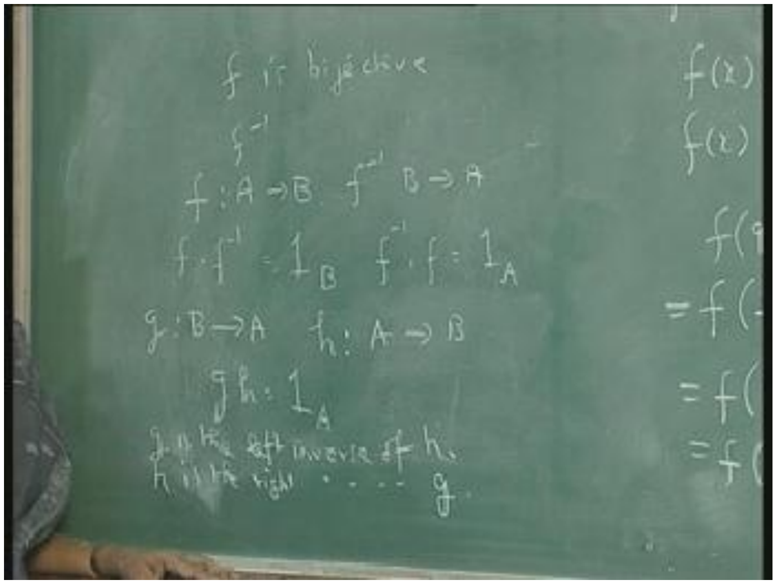

Now these left inverses and right inverses to occur you must have certain properties. If a function has to have a left inverse it should have a certain property. Similarly, if a function has to have a right inverse it has to have a certain property. And also left inverses need not be unique and so on, we shall study about it in the next lecture.