Discrete Mathematical Structures
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Lecture # 22
Module - 1
Order Relations and Equivalence Relations

We were considering order relation, we saw what is meant by a partial order and a partially ordered set, a quasi order and also linear order and a well order. And, over an alphabet we saw that there can be two types of linear orders; one is the lexicographic ordering and the other is called the standard ordering, it is also called canonical ordering. Let us recall the definitions.

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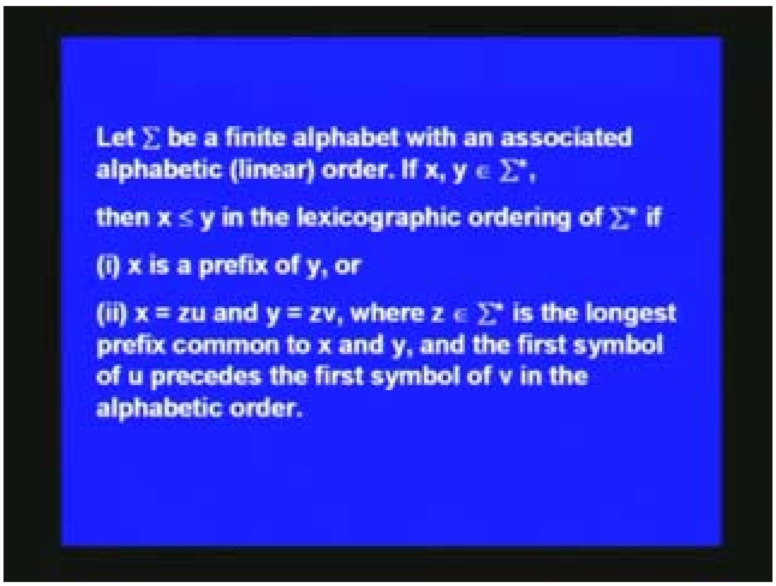

Let sigma be a finite alphabet with an associated alphabetic linear order. If x, y belongs to sigma power star then x is less than or is equal to y in the lexicographic ordering of sigma power star, if x is a prefix of y or x is equal to zu and y is equal to zv where z belongs to sigma power star is the longest prefix common to x and y and the first symbol of u precedes the first symbol of v in the alphabetical order. This is the order which is followed in a dictionary. So, if you take the alphabet say sigma is equal to a, b, c, d then if you have aabd and bacd something like that this will come before this. If you have baac something like that this will come before this but this will come after that in the lexicographic ordering.

If you consider b, ab, aab like that, this will come before this in the lexicographic ordering and this come before this in the lexicographic ordering. It is a linear order if you consider two strings you can say which one will come before that but any subset will not have a least element. For example, if you consider the set b, ab, a squared b like that a power n b this subset does not have a least element so it is not a well order, it is a linear order but not a well order. So in order to introduce a well order we define what is known as a standard ordering on sigma power star. Let us recall this definition.

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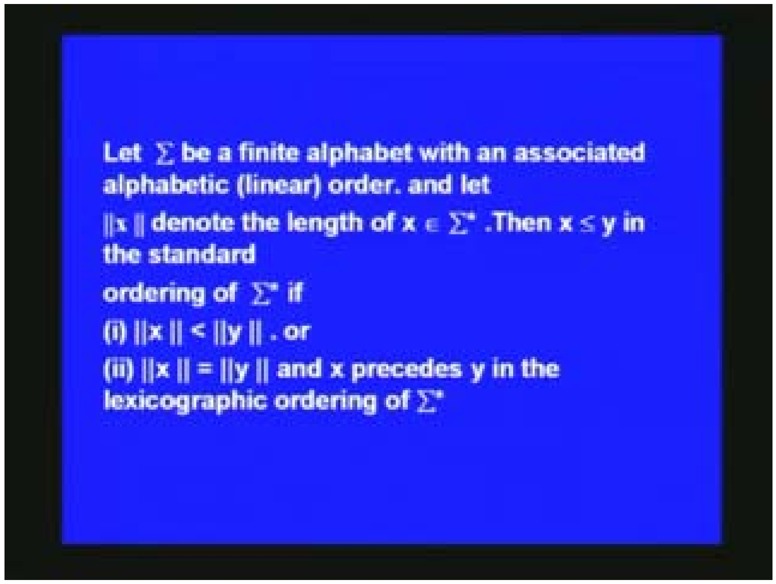

Let sigma be a finite alphabet with an associated alphabetic order and let length of x denote the length of x then x less than or is equal to y in the standard ordering of sigma power star. If the length of x is less than length of y or if the length of x is equal to length of y then x precedes y in the lexicographic ordering of sigma power star.

So, if you start that order that is it is called a standard ordering and in some books they call it as canonical ordering, then the strings will be arranged like this; lambda the empty string then strings of length 1 will occur, they will occur according to the lexicographic ordering that is a, b, c, d then strings of length 2 will occur again they will occur according to the lexicograph ordering.

So next aa will come ab, ac, ad then ba, bb, bc, bd like that then strings of length 3 will come and so on. Here as I told you you can talk about the ith string in the enumeration. If you to want say what is the 27th string in this enumeration you can say what is the 27th string and so on. So this is an enumeration of strings over sigma and this idea is very useful in many places. Now, the set of strings you can talk about lambda is the 0th string, a is the 1st string, b is the 2nd string, c is the 3rd string, d is the 4th string, aa is the 5th string like that you can put them into one to one correspondence with the set of non-negative integers. If in such a case this is a well order you can put the set to one to one correspondence with the set of natural numbers.

In this case you can use the first principle of induction. If you want to prove some property for the set of strings over sigma power star some property you want to prove say you want to prove for all of x p of x where p is some property of the set of strings.

In this case what you do, the first principle is you use p lambda you prove that it holds for the empty string, then you prove that for all of x p of x implies p of successor of x, that is, if some property holds for a string x it will hold for the next string. Then from this you will conclude therefore for all of x p of x this is same as using the first principle of mathematical induction and this can be very well applied.

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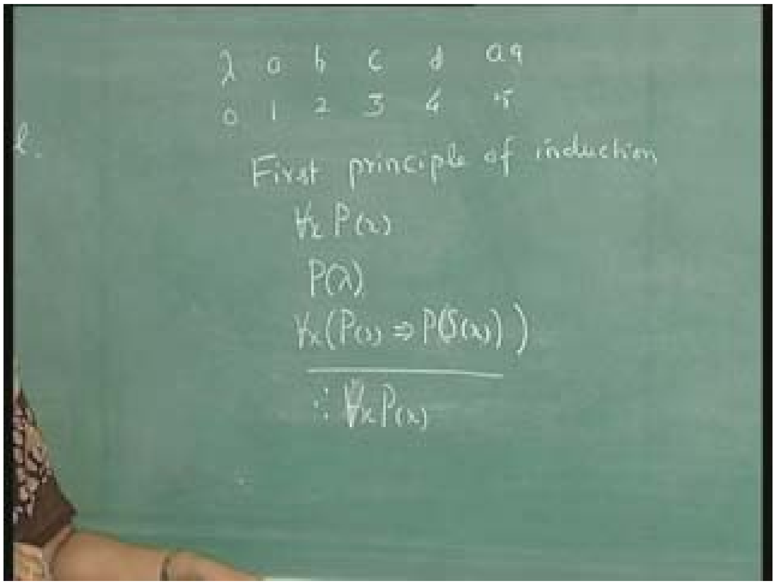

But you may have well orders where you cannot put the elements into one to one correspondence with the set of natural numbers. They may be well orders but you cannot put them into one to one correspondence with the set of natural numbers. In that case you may not be able to use the first principle of mathematical induction you may be able to use the second principle.

Let us consider this, consider the set N cross N set of ordered pairs over N, then you define a, b is less than or is equal to c, d if a is less than or is equal to c if a is equal to c then b should be less than or is equal to d. Or if you look at them like this as the set of grid points in the first quadrant integer integer grid points this is 0 0, this is 0 1, this is 1 0, this is 2 0 and so on. This is 0 1, 0 2, this is 1 1, 1 2 like that. Now, this you can arrange them in this order the linear order 1 2 3 like this and all the elements in this column will come first, then all the elements in this column, then all the elements in this column, then all the elements in this column and so on. So it is a linear order if you take two elements you can very easily say which one will come before which.

And it is also a well order because if you take any subset over here there will be a least element so it is a well order also. But you cannot put this well order into one to one correspondence with the set of natural numbers why? See all these elements have successors, if you have i j, i j plus 1 will be the successor of that. But there are so many elements which do not have proper predecessor.

When you take any element you can say what the successor is. If you take this element the predecessor of that will be this, if you take this element the predecessor of this is this, but the predecessor of this element you cannot specify because this goes up to infinity so you cannot specify the predecessor of this element, so you cannot compare this with the set of natural numbers. In such a case you cannot apply the first principle of induction you have to use the second principle.

Usually you use this to compare two elements in a partially order set. Let us use a this b if a is less than or is equal to b AND a not is equal to b that is you are removing the equality elements from this set because a linear order is a partial order, a well order is a linear order in which every subset has a least element. And linear order is a partial order and a partial order has the reflexive property. But if you remove that you can see a is less than b instead of saying less than or is equal to b, if a is less than b if a is less than or is equal to b but a is not is equal to b that is you are removing the elements from the equality set.

In this case the principle of induction you can say like this; for all of x for all of y y less than x implies p y implies p x, from this we can conclude for all of x P(x). what does that mean? If for all elements less than x the property holds then you can conclude that it will hold for the x also. So from this you can conclude that for all of x P(x). For example, here if you want to use that, suppose I want to prove that initially we can prove for induction, basis class is automatically included in this type of second principle. But if I want to prove the property for this what I assume is I assume that it holds for all this then assuming that the property holds for all these elements I prove that it holds for this.

If you are able to do that then you can conclude that for all of x P(x). That is this is the premise and this is the conclusion and if we are able to prove the premise then you can prove the conclusion, the conclusion follows. How is that, suppose you are considering a set S which is a well order and you are using the second principle so you are able to prove the premise how does the conclusion becomes true? Now suppose T is a subset of S for which p does not hold suppose it does not hold then because if T is not is equal to phi we will show that T will be phi T will be the empty set there is no subset for which p does not hold.

If T is not equal to phi there exists a least element m therefore from this for all of y if y is less than m the property holds that is p of y holds. And m is the least element for which the property does not hold so for all of y if y is less than m the property holds and because this is true therefore this will imply p of m will be true. So T is the nonempty set is not correct T will be empty. So if you are able to prove this premise the conclusion follows.

In the case of well order you can use this again principle of induction. You can use the first principle when you can put it in the form of a set of natural numbers or if it is in one to one correspondence with the set of natural numbers if not you can use the second principle. But if you do not have a well order you may not be able to use the principle of induction at all.

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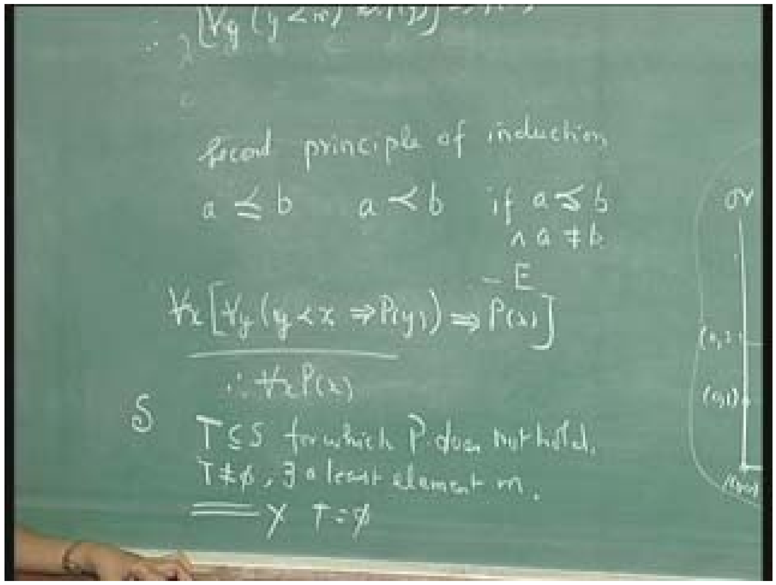

Now, while defining the well order we have seen what is meant by a greatest element and what is meant by a least element. There are some more properties which we will see as to what they are.

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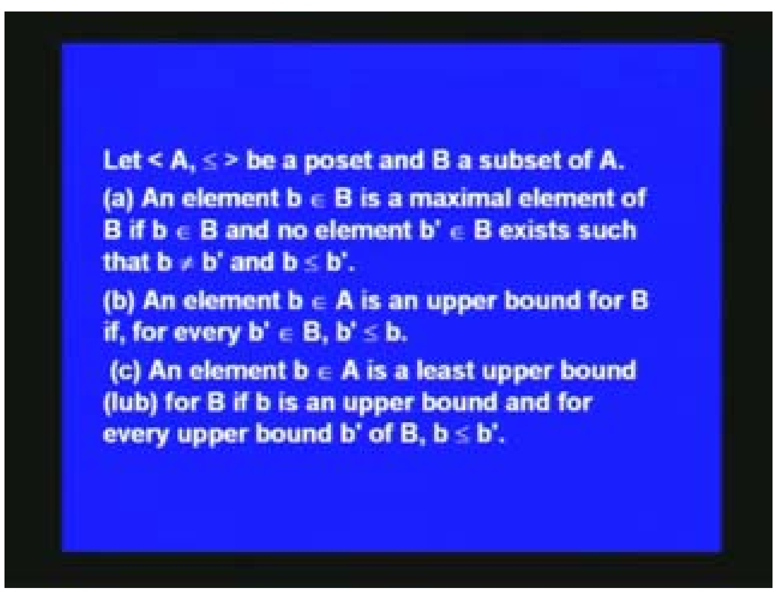

So we have seen what is meant by a greatest element and a least element for a poset for a partially ordered set A less than or equal to we have seen what is meant by a greatest element and what is meant by a least element. If b is a greatest element any b dash will be less than or is equal to b. And if b is the least element any b dash will be greater than or is equal to b. The greatest element and the least element will belong to the set they need not exist if they exist they are unique.

Let A less than or equal to be a poset and b a subset of A. an element b belonging to B is a maximal element of B if b belongs to B and no element b dash belongs to B exists such that b is not is equal to b dash and b is less than or is equal to b dash. An element b belonging to A is an upper bound for B if for every b dash belonging to B b dash less than or is equal to b, an element b belonging to A is a least upper bound for B if b is an upper bound and for every upper bound b dash of B b is less than or is equal to b dash.
Let us take an example instead of looking into the definition blindly like this let us take an example that will make things more clear.

Take the set of natural numbers integers from 1 to 8 and the relation a R b if a divides b. Then the poset diagram will be like this: 1 2 4 3, 1 2 3 4 5 7 and 6 will be like this and 8 the poset diagram will be like this. This particular set does not have a greatest element. You cannot find an element such that every element is less than that. So there is no greatest element here. But by the definition of maximal elements 8 is a maximal element, 6 is a maximal element, 5 is a maximal element and 7 is a maximal element.

Let us look into the definition of maximal element: an element b belonging to B is a maximal element of B if b belongs to B and no element b dash belongs to B exists such that b is not is equal to b and b is less than or is equal to b dash. So there is no other element such like this and this is a maximal element here. There may be several maximal elements here 8 6 5 7 they are all maximal elements there is no greatest element here.

What is a least upper bound and what is an upper bound?
An upper bound is such that, I will use another diagram here, a poset diagram, in this case there is no least upper bound and so on, these things may exist or may not exist. Let me take another example of poset diagram a b c d e h something like that. In this case you will find that the greatest element is e all the other elements are less than e so the greatest element is e the least element is h, this is the set, the set A consisting of a b c d e h. now consider what are the maximal elements here? Here e is the maximal element and h is the minimal element.

A minimal element you can define in a similar manner. How will you define a minimal element? An element b belonging to B is a minimal element if b belongs to B and there is no element b dash belongs to B exist such that b is not equal to b dash and b dash less than or is equal to b. So if you look at this example you will see that this is a minimal element, there is no other element less than this, this happens to be the least element and this happens to be the minimal element. There is only one minimal element in this case.

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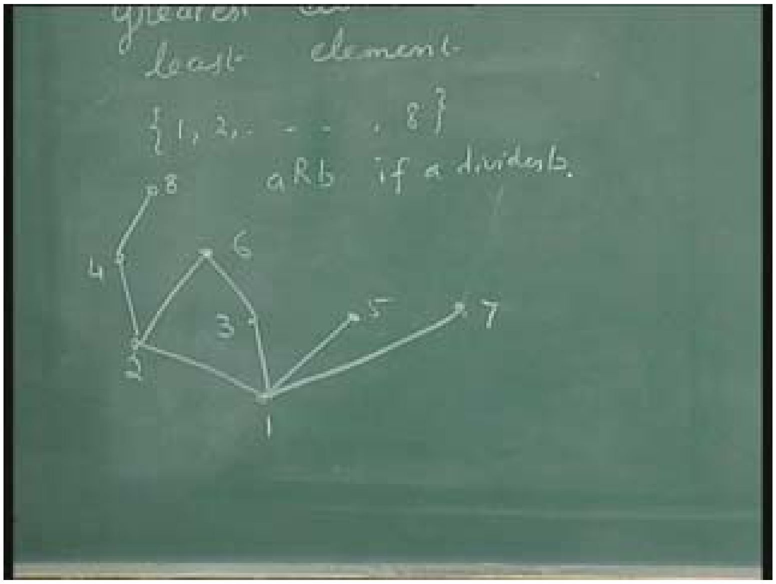

And look at this poset diagram, if you take all the set this is the minimal element, this is the least element, this is the maximal element and that also happens to be the greatest element. Now you take a subset of a, b, c or may be will take a, b, d then among these three elements d is the greatest element, it also happens to be the maximal element. There is no least element here but a and b are minimal elements.

Now what is an upper bound? We are considering the subset b consisting of a, b, d. An element b of A is an upper bound for B if every b dash belonging to B b dash is less than or is equal to b. So, if you look at this, if you take e which is an element of A but it is not an element of B all these elements are less than this. So e is a upper bound, e is an upper bound. if you take a, b, d then a, b, d everything is less than or is equal to d so d is also an upper bound, e is also an upper bound, d is also an upper bound. But d is the least upper bound that is the next definition. An element b belonging to A is a least upper bound for B if b is an upper bound and for every upper bound b dash of B b is less than or is equal to b dash.

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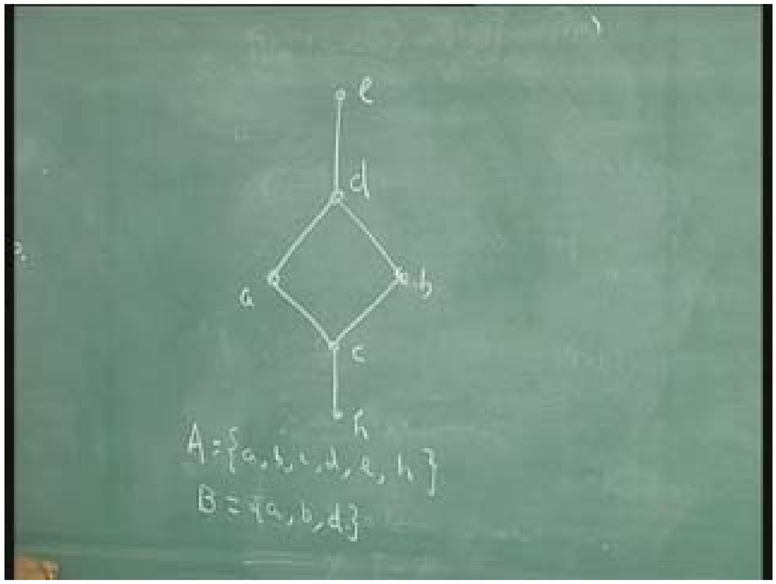

So if you consider this set a, b, d then e is an upper bound, d is also an upper bound but d is the least upper bound. Similarly, you can define lower bound and greatest lower bound.

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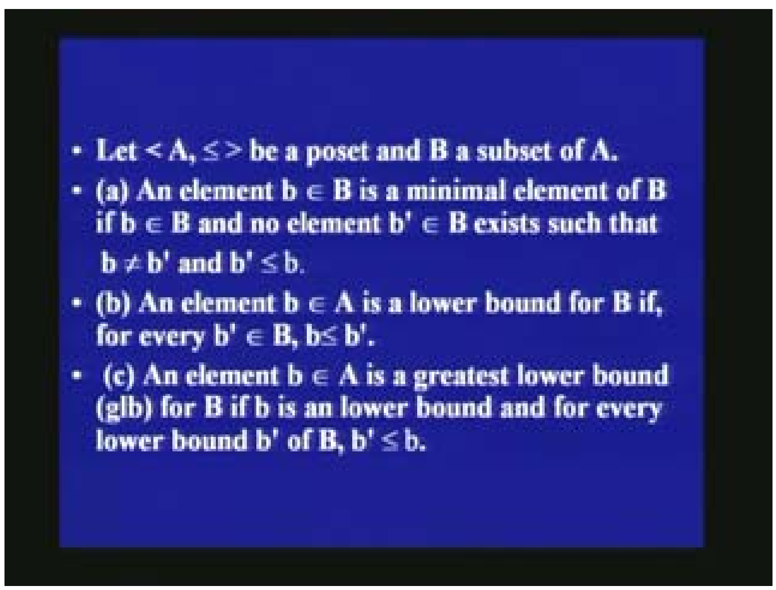

How do you define lower bound?
An element b belonging to A is a lower bound for B if for every b dash belonging to B b is less than or is equal to b dash. So for example if you take this set c is less than or equal to everything so c is the lower bound. Similarly, h if you take h is less than a, d, b

so h is also a lower bound. But among these if you take c is the greatest lower bound if you take the greatest one that is the greatest lower bound.

So you can change a little bit in the definition and get the greatest lower bound. an element b belonging to A is a greatest lower bound which is called glb greatest lower bound, if b is a lower bound and for every lower bound b dash of B this is a lower bound this is the greatest lower bound so what will happen? If b is a lower bound and for every lower bound b dash of B b dash will be less than or is equal to b. |So if you look at this diagram in this case there is no greatest element these are the maximal elements, this is the least element, this is the minimal element, this is the greatest lower bound and only lower bound this is the only lower bound and it also happens to be the greatest lower bound there is no upper bound here.

So these upper bounds, lower bounds may exist or may not exist. And greatest element, maximal elements will belong to the subset. If you are considering a set A and taking a subset B of A, for example here we have taken A to be all the six elements and B a subset having a, b, d around. The greatest element will belong to the set, the least element will belong to the set if it exists. And the maximal elements, minimal elements will belong to the set. But lower bound, greatest lower bound, least upper bound, greatest lower bound need not belong to the set.

For example, here you see that c and h are lower bound they do not belong to this set B. And we have already seen that if the greatest element exists it is unique, if the least element exists it is also unique. Similarly, if the greatest lower bound exists or the least upper bound exists it is unique. These results follow immediately.

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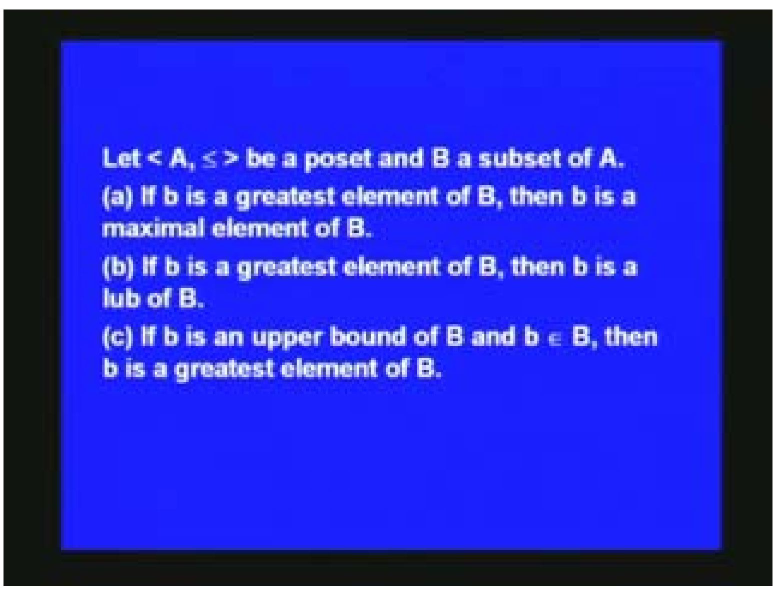

Let A less than or is equal to b a poset and B a subset of A, if b is a greatest element of B then b is a maximal element of B. Obviously, the greatest element has to be a maximal element but not conversely. A maximal element need not be a greatest element, we have seen that. Similarly, least element will be a minimal element. if b is a greatest element of B then b is a least upper bound of B. this also you can very easily see because all elements are less than or equal to the greatest element the same condition holds for least upper bound also. If b is an upper bound of B and b belongs to b then b is a greatest element. If something is an upper bound then everything else is less than that and for the greatest element also the same condition holds.

But the upper bound need not belong to the set. If it belongs to the set then it will be the greatest element. So these results follow immediately by a definition. Then we can also see the uniqueness of the least upper bound or greatest lower bound that we can very easily see like this.

Suppose you are having b and b dash as a lub suppose you are having two least upper bounds, then because b is the least upper bound b will be less than or is equal to b dash because the least upper bound will be less than or equal to any other upper bound. And because b dash is the least upper bound b dash will be less than or is equal to b this is an upper bound. So because of these two conditions you get b is equal to b dash or the least upper bound is unique. Similarly, you also see that the greatest lower bound is unique. So, we have considered some thing about partial orders, linear orders, etc. Next we shall consider what is meant by an equivalence relation.

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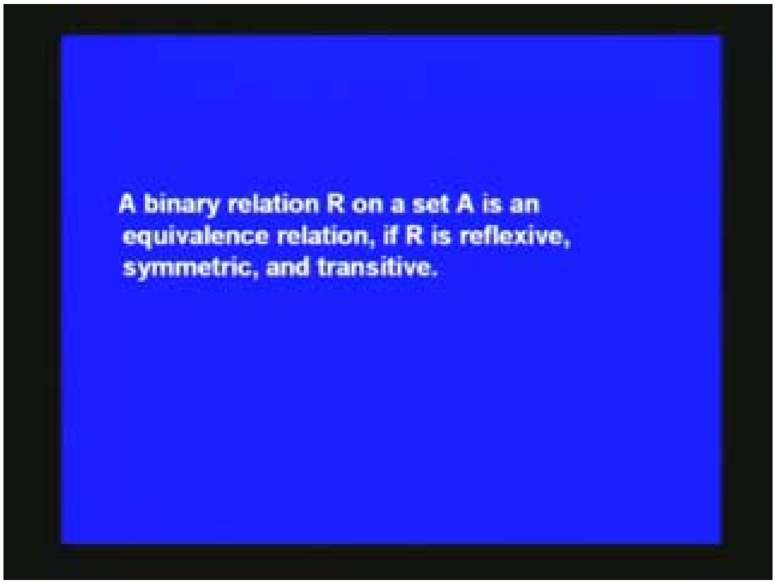

In equivalence relation we consider three properties. We say that you define that relation it is also a binary relation on a set A and a is equivalent to a you always say a is equivalent to itself. And if a is equivalent to b then I can write like this; if a is equivalent to b then we say b is also equivalent to a. this is a symmetric property. Then if a and b are equivalent and b and c are equivalent then a and c will be equivalent this is the transitive property. These three things we considered in equivalence relation. So it can be defined like this a binary relation R on a set A is an equivalence relation if R is reflexive, symmetric and transitive. So, if a binary relation has these three properties it is reflexive, symmetric and transitive then it is called an equivalence relation.

Let us consider some examples of equivalence relation. Take the set of natural numbers, the equality relation is an equivalence relation, it is reflexive, it is symmetric, it is transitive. Now consider a set say A is equal to a, b, c, d and say that it is represented, the relation is represented like this a, b, c, d the relation is represented by the diagraph like this, this is a equivalence relation. It is reflexive, it is symmetric, it is transitive. If these three conditions are satisfied then the relation is called an equivalence relation.

Or if you take something like this, if you take the set of propositional forms that is well formed formula of the propositional logic you say P_1 is equivalent to P_2 if they are equivalent if P_1 is equivalent to P_2 if they have the same truth table then this is reflexive P_1 will be equivalent to itself and P_1 and P_2 are equivalent then that means P_2 and P_1 will be equivalent if P_1 and P2 are equivalent and some P_2 is equivalent to P_3 from that you can conclude P_1 is equivalent to P_3 so the transitive property will hold and this is another example of an equivalence relation. Or if you take the set of parallel lines in a plane all lines parallel to each other are considered equivalent.

So if you have a line l_1 if l_1 is parallel to l_2 they are considered equivalent. In that case you see that l_1 is parallel to itself, l_1 is parallel to l_2 would mean l_2 is parallel to l_1. And if l_1 is parallel to l_2 and l_2 is parallel to l_3 then l_1 would be parallel to l_3 so transitive property also will be proved. So this is another example of an equivalence relation.

Let us also consider some more examples of equivalence relations. The empty relation on the empty set for some reason is considered as an equivalence relation. If you take the empty relation on the empty set it is an equivalence relation. But if you take the empty relation on a finite set or an infinite set that is not an equivalence relation because the reflexive property does not hold there. This is another example of an equivalence relation.

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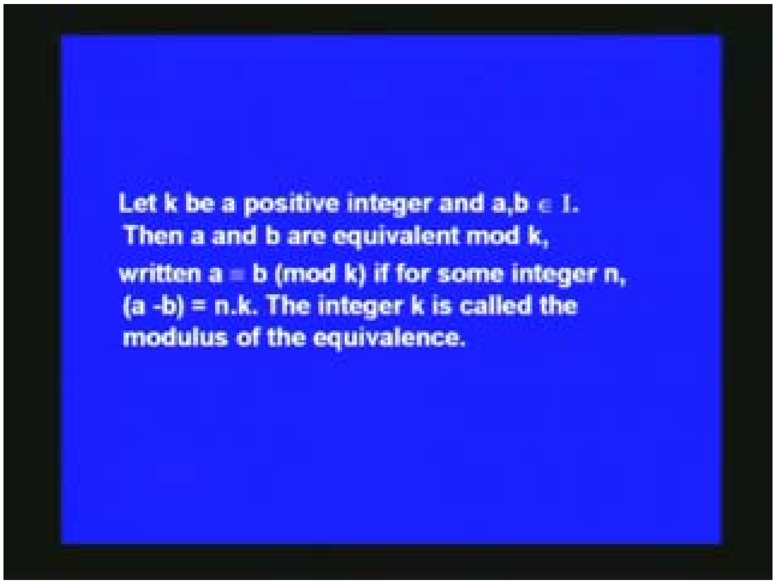

Let k be a positive integer and a, b belonging to integer. And a and b are integers and k is a positive integer then a and b are equivalent mod k you write it as a equivalent to b mod k that is you write it like this; a and b are integers and you say a is equivalent to mod k, k is a positive integer, when a and b leave the same remainder when divided by k that is a minus b is equal to n cross k the integer k is called the modulus of the equivalence. In a sense it means that a is equivalent to b if a and b leave the same remainder when divided by k or a minus b is divisible by k, a minus b is equal to some n cross k.

Why is this unequalness relation?
Say for example, in this case if you take a will be equivalent to a itself, the reflexive property will hold because a minus a will be 0 times k so the reflexive property holds. Then what about the symmetric property if a is equivalent to b, a minus b will be some n times k so what can you say about b minus a it will be minus n times k anyway because this holds b is equivalent to a. Or in the sense a and b leave the same remainder b and a leave the same remainder when divided by k. So the symmetric property also holds.

Now what can you say about the transitive property?
If a is equivalent to b you can say a minus b is n1 cross k and if b is equivalent to c then b minus c is equal to n_2 times n_1 times k n_2 times k. So what can you say about a minus c? a minus c add these two you will get a minus c is n_1 plus n2 times which amounts to saying a is equivalent to c mod k here it is mod k, so the transitive property also holds.

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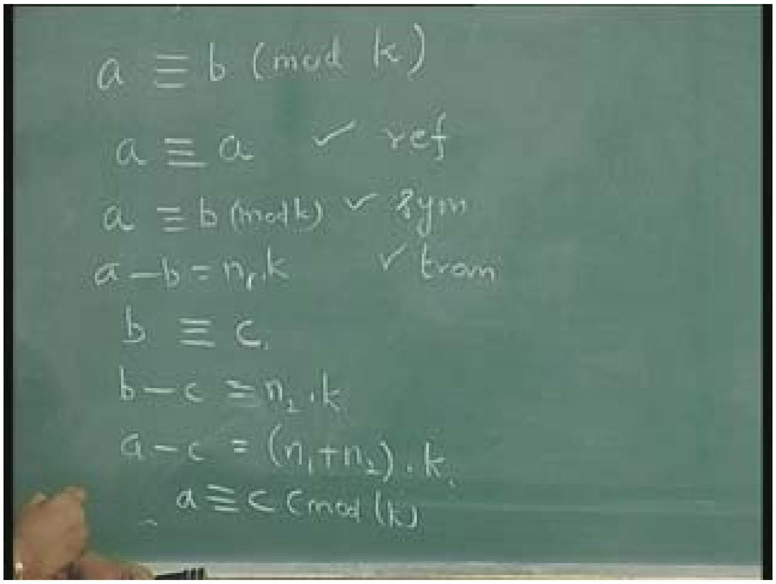

So because all these three properties hold it is an equivalence relation. Now, let us takes k to be 3 and 5 and see what happens and take the set of non negative integers, take underlying set as the set of non negative integers. It is 0, 1, 2, 3, 4 etc and k to be 3. So 0 will be equivalent to 3 because they leave the remainder 0 when divided by 3 will be equivalent to 6 they are all equivalent. So actually 0, 3, 6, 9 will all be equivalent to each other because they leave the remainder 0 when divided by 3.

If you take 1, 4, 7, 10 these are all equivalent to each other because if you take any two of them and subtract one from the other the result will be divisible by 3. Or these elements leave the remainder 1 when divided by 3. And if you take the elements 2, 5, 8, 11, 14 etc these elements are all equivalent to each other because they all leave the remainder 2 when divided by 3. So you see that if you consider mod 3 equivalence if you say mod 3 equivalence you can divide the set of natural numbers into three classes S_1 S_2 S_3 and the elements in any one of them are equivalent to each other. Similarly, you can define mod 5 mod 4 any number you can take and define this modulus equivalence. Here if you take 3 it is called the modulus of equivalence. Equivalence mod k is an equivalence relation over a set A contained in I.

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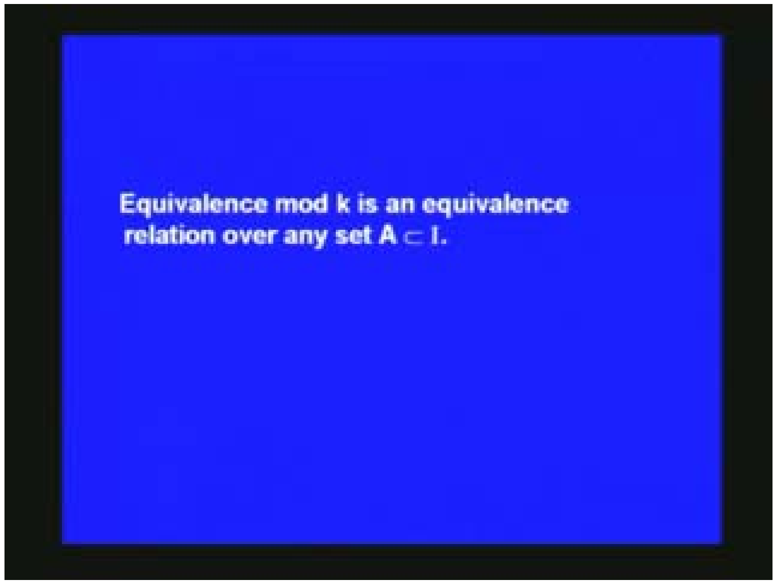

We have taken natural numbers you can take any other set also it will be an equivalence relation. For example, take this set 1, 2, 3, 4, 5, 6, 7, 8 this set and 3 will be equivalent to 6 and in the graph you will represent like this; 1, 4, 7 will be equivalent so the graph representing them the diagraph will be like this and 2, 5, 8 will be again let me remind you that we are considering mod 3 relation. That is 3 and 6 leave the same remainder when divided by 3 and 1, 4, 7 leave the same remainder 2, 5, 8 leave the same remainder, the diagraph will look like this.

Now you can very easily see that this diagraph has got three components and each one is a complete diagraph. Each component is a complete diagraph. So, if you look at it very carefully any equivalence relation if it is over a finite set say then the equivalence relation will be represented by a diagraph which will have some components where each component will be a complete diagraph, why? It is because you have to have reflexive property, you have to have symmetric property and you have to have the transitive property also. So, each diagraph will be a complete diagraph. Now, let us see what an equivalence class is.

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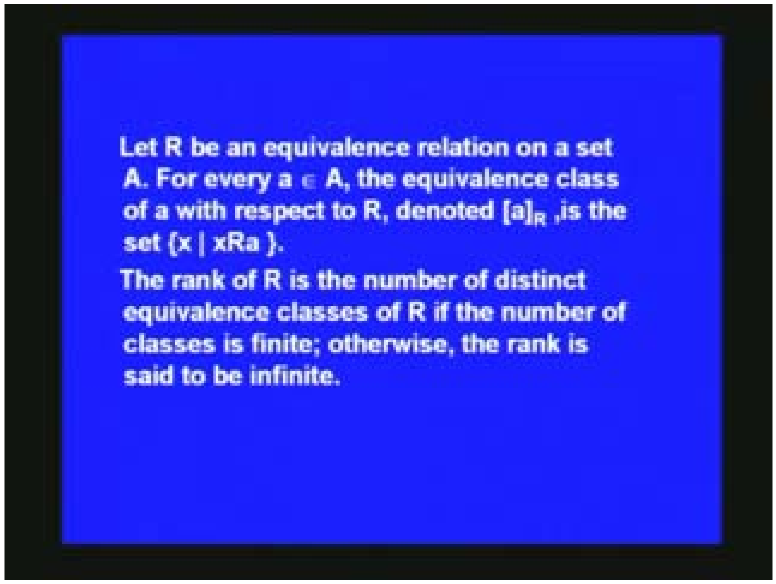

Let R be an equivalence relation on a set A. For every a belonging to A the equivalence class of a with respect to R denoted by aR is the set x x related to a. The set of all elements x such that x is related to a. The rank of R is the number of distinct equivalence classes of R if the number of classes is finite otherwise the rank is said to be infinite.

Now, consider some equivalence relation, we have considered the mod relation. You are having a set A and an equivalence relation R on that. Now, the equivalence a is a particular element of a and this denotes the equivalence class of a. This denotes the set all x such that x is related to a. You denote it like this. This is called the equivalence class of a with respect to R, the set of all elements which are equivalent to a particular element.

Now let us consider this example mod 3. Now, if you take 7 what is the equivalence class of 7? All elements which are equivalent to 7 that would mean by this relation of course if you want to specify the relation you can mention it as R. So, all elements which are related to 7 belong to this equivalence class. So S2 is this class and if you say 5 all elements which are related to 5 belongs to the equivalence class so this is one equivalence class.

Here there are three equivalence classes this is one equivalence class, this is one equivalence class and this is the third equivalence class. The number of equivalence classes is called the rank of the equivalence relation. Here in this example it is 3, it is also called the index of the equivalence relation. In this case also there are three equivalence classes each complete diagraph represents one equivalence class so the rank here is 3.

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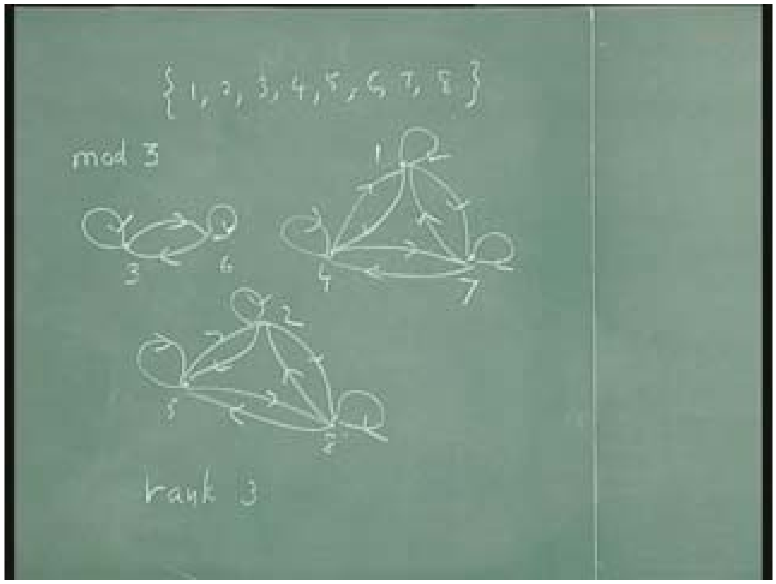

If you consider mod 5 relation then 0, 5, 10 will be in one equivalence class and 1, 6, 11 etc will be in one equivalence class, then 2, 7, 12 will be in one equivalence class and 3, 8, 13 will be in one, 4, 9, 14, 19 will be in one so there are five equivalence classes here. So the rank is 5 or the index of the equivalence relation is 5. You can also have equivalence relation of infinite index. If you consider the set of natural numbers and the equality relation on that then that is an equivalence relation. But each element will belong to one equivalence class and so the number of equivalence classes will be infinite in that case so the rank is infinite.

Another example of an equivalence relation of infinite index is the set of parallel lines. If you take the set of all parallel lines they form an equivalence class. Now all parallel lines with slope 30 degrees form one equivalence class. All parallel lines with the slope 31 degrees will form one class and the degrees can be in fractions also so you can have infinite number of equivalence classes. This is also denoted as rank or index.

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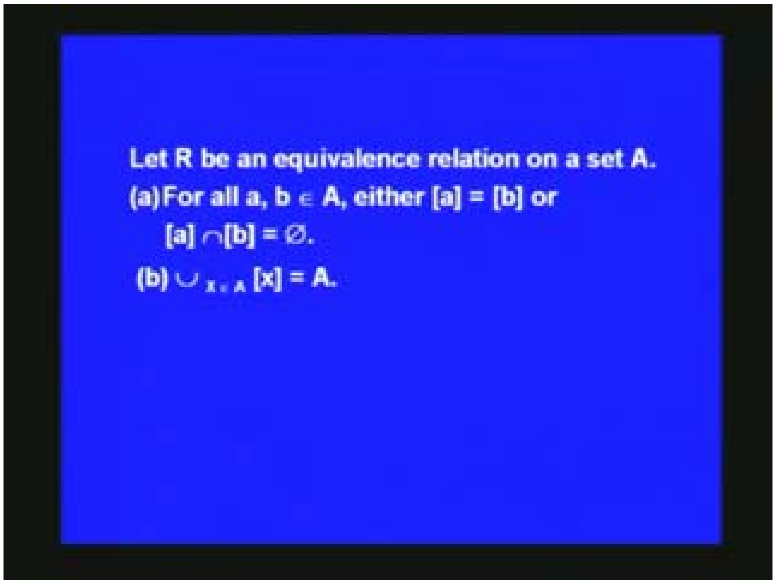

Let R be an equivalence relation on a set A then for all a belonging to A either a is equal to b or the equivalence class of a is equal to the equivalence class of b or the equivalence class of a and b are disjoint. And the union of all the equivalence classes constitute the set A. we have to look into the proof of this. Let A be a set and R be an equivalence relation on a set then if you have two equivalence classes a and b then either they are equal or they are disjoint this is what we want to prove.

Let a and b belong to A and the equivalence class of a is denoted like this the equivalence class of b is denoted like this. Now, we want to show that they are equal or they are disjoint. Suppose they are not disjoint then you get a figure like this. This is the equivalence class corresponding to a and this is the equivalence class corresponding to b so a is in this and b is in this.

All elements which are equivalent to a are in this set and all elements which are equivalent to b are in this set, they are not disjoint so there is an element c here. What we want to show that is if they are not disjoint they will be equal.

How do you go about proving this?
Now because a and c are in this equivalence class a will be related to c and because c and b are in this equivalence class c will be related to b.

By transitive property a will be related to b. Now, take in arbitrary element x belonging to the equivalence class of a then x is related a and we know that a is related to b. So by transitivity x will be related to b and what does that mean? That means x belongs to the equivalence class of b. This means the equivalence class of a is contained in the equivalence class of b. By taking an element y here and taking that corresponding arc b to a you can show that the equivalence class of b is contained in the equivalence class of a. So from this you conclude that the equivalence class of a is equal to the equivalence class of b.

So we find that if the equivalence of a and if the equivalence class of b are not disjoint then every element x belonging to the equivalence class of a will also be an element of the equivalence class of b and every element in the equivalence class of b will also be an element of the equivalence class of a. That is equivalence class of a will be contained in the equivalence class of b and vice versa which leads us to the conclusion that the equivalence classes are equal. So, if you take any two equivalence classes either they will be equal or they will be disjoint.

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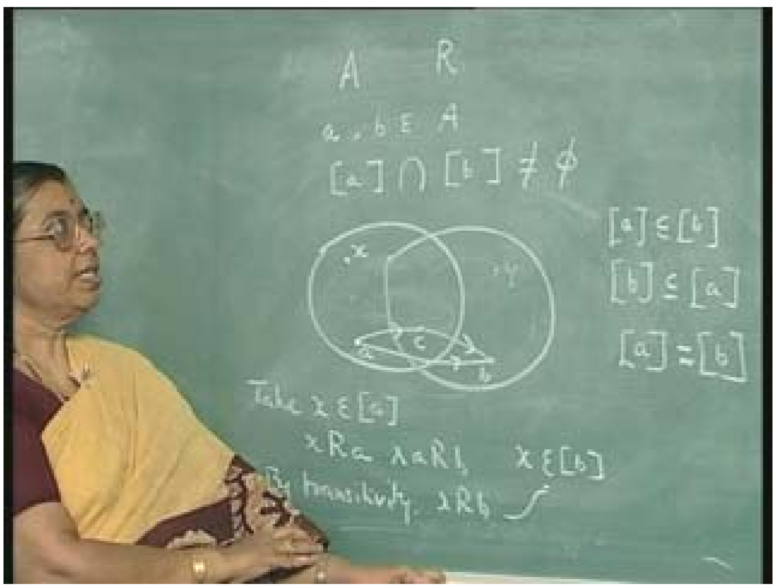

Let us take that example, see mod 3 we had three equivalence classes and 1, 4, 7 etc 2, 5, 8. Now, if you take the equivalence class of 4 that is S_2 if you take the equivalence class of 3 that is S_1 they are disjoint, if you take the equivalence class of 4 and if you take the equivalence class of 7 they are the same. So either they will be the same or they will be disjoint, that is what is meant by the first part of the proof.

Let us go to the second part now, what is the second part?
The second part says that union x belongs to A equivalence class of x is equal to A, the second part says union x belongs to A equivalence class of x is A. Now, again we have to prove it in two parts; union of x belonging to A x contained in A and the other way round. Let us take this part that is the first part, suppose there is an element belonging to union of x belonging to A x in a sense it means that c belongs to some equivalence class a where a is an element of a. But this equivalence relation is completely contained in a but a is contained in A. From this we conclude that union of x belonging to a x is contained in A.

The other way round we can show that A is contained in union of x belonging to A equivalence class of x. Say c belongs to A, take some c belonging to A then c will belong to equivalence class of c and that is one of these classes. So from this you can conclude that A is contained in union of x belonging to A, this is one of these classes so if c belongs to A then c will belong to union of x belongs to A and from this you get this conclusion. So if you take the equivalence classes either they will be disjoint or they will be equal and the union of all of them will make the entire set.

For example, in the mod relation you found there were three equivalence classes they were disjoint equivalence classes and the union of all of them made the whole set the set of natural numbers.

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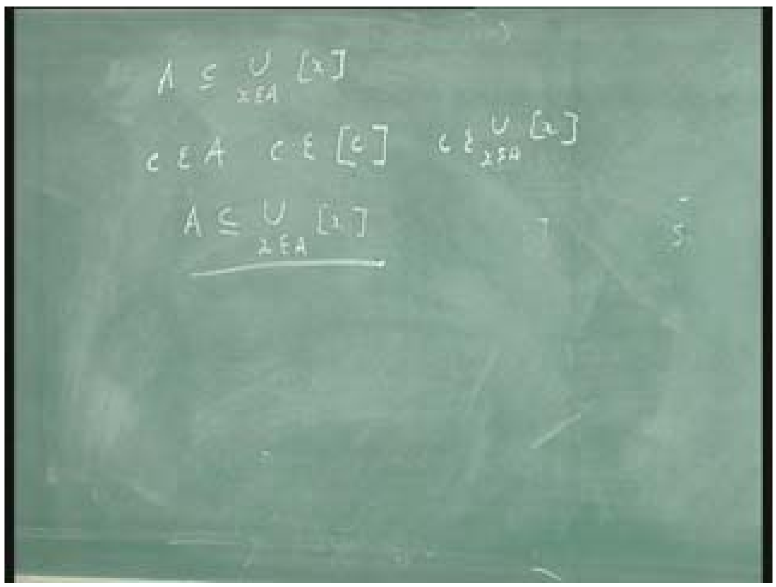

We have seen what is an equivalence relation and what are the equivalence classes induced by equivalence relation, what is the index of the equivalence relation and so on. In the next lecture we shall see more properties about equivalence relations. And we shall also see what is meant by a partition and what is the connection between equivalence relations and partitions. We shall also see some of the properties of partitions and how they are related to some operations on equivalence relations.