# Discrete Mathematical Structures

**Dr. Kamala Krithivasan**
**Department of Computer Science and Engineering**
**Indian Institute of Technology, Madras**

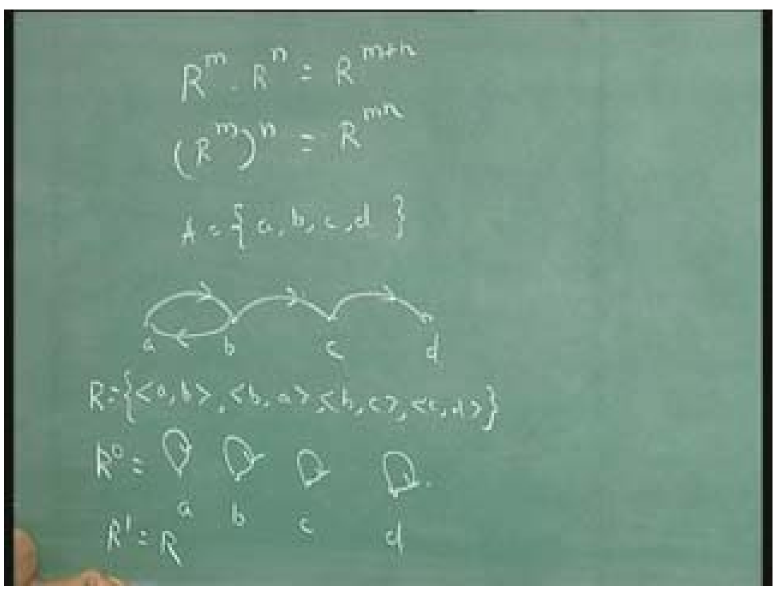
**Lecture - 19**

## Closure of Relation

So in the last lecture we have seen about composition of relations. We have also seen in what is a power of a relation. So if a R power 1 is a relation on A cross B and R power 2 is a relation on B cross C then R power 1 R power 2 you can define on A cross C from A to C. Now, if R is a binary relation on A that is A cross A then you can talk about the power of R. R power 0 is the equality relation, R power 1 is R and R power 2 is R cross R because of the associative property of composition of relation we can say without any ambiguity R power n plus 1 is R power n cross R. We have also seen that R power m cross R power n is R power m plus n and R power mn is equal to R power mn, these things also we have seen.

Let us consider one more example; take a relation on a set A consisting of four elements a, b, c, d and R is written like this by a directed graph a b c d this is the relation. So, R consists of the pairs a, bb, ab, c and c, d. Now what is R power 0? R power 0 is the equality relation on the set so that can be represented by the diagram like this. It consists of the pairs aa, bb, cc and dd. and R power 1 is R.

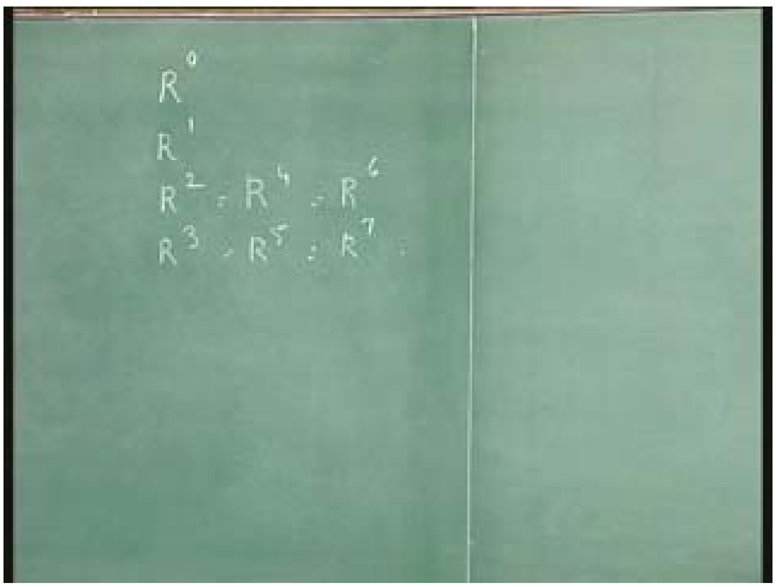
(Refer Slide Time 3:48)



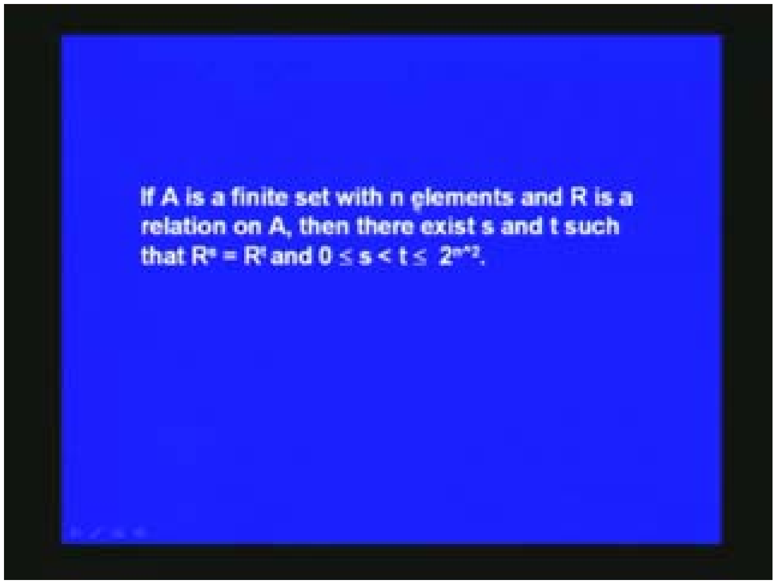

Now what can you say about R squared and R cubed? R square can be represented by the diagram like this. If there is a path of length two that will be replaced by a single arc in R squared. So a to b, b to a you have so there will be a self loop here, b to a, a to b is there so there will be a self loop here and a to b, b to c is there so a to c there will be an arc, b to c, c to d there is an arc so b to d there will be an arc this is R squared. Then R cubed will consist of all paths of length three replaced by a single arc. So what is R cubed? If you write like this, a path of length three will be replaced by a single arc. So R cubed will consist of an arc representing a path of length three in R, R is this, so a to b, b to a, a to b there is a path of length three here so it is like this, then b to a, a to b, b to a there is a path of length three so there will be an arc like this, b to a, a to b, b to c there is a path of length three here so it will be like this, then a to b, b to c, c to d there is a path of length three so there will be an arc like this, this is R cubed.

Now what about R power 4? So I shall write here, R power 4, let us see what are the arcs representing R power 4 may be this. R power 4 if you take any path of length four should be replaced by a single arc here one two three four so there will be a self loop here, 1, 2, 3, 4 so there will be a self loop here, one two three four so there will be an arc like this, one two three four so there will be an arc from here to here. So you will find that R power 4 is this same as R squared so R power 5 will be the same as R cubed. So in this case you find that R power 0 represents the equality relation, R power 1 represents the given relation, R squared you have found R cubed and R power 4 is the same as R squared and R power 5 is same as R cubed so R power 6 will be the same as this and R power 7 will be the same as this. So after sometime the pattern repeats you do not have anymore different relations but the powers will represent the already existing relations. So we have this result.

(Refer Slide Time: 7:48)



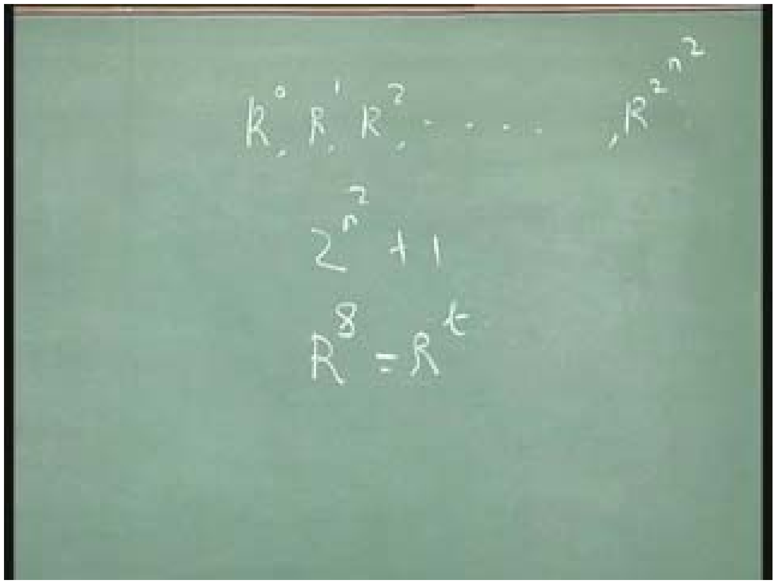
(Refer Slide Time: 8:34)




If A is a finite set with n elements and R is a relation on A then there exist s and t such that R power s is equal to R power t for some s and t between 0 and 2 power n squared, for two values s and t R power s is equal to R power t if the underlined set A consists of n elements. R is a relation on A and A has n elements where n is a finite number why this? It is because if A has n elements what is the maximum number of ordered pair you can have on A cross A n squared elements you can have on A cross A, A cross A will have n squared elements. And any relation on A or rather A cross A can include each one of them or can exclude each one of them. So possibly you can have 2 power n squared distinct relation on A because if you look at it as a graph each one of the arc may be present in the relation or not present in the relation.

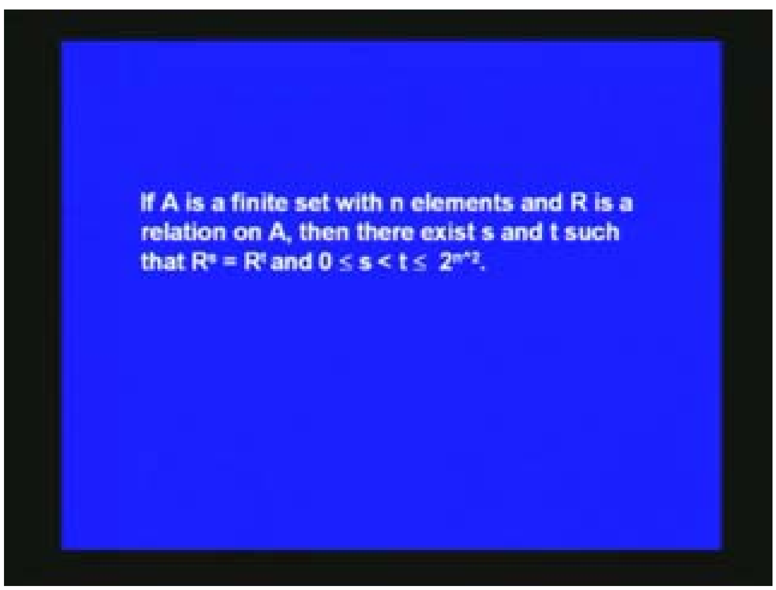
And maximum you are having n squared arcs so possibly totally you can have 2 power n squared distinct relations on A cross A. This includes the universal relation where you have all the elements present all the ordered pairs present and the empty relation where you do not have the any of the ordered pairs present. So looking at that way you see that if you take R power 0, R power 1, R power 2, like that up to R power 2 power n squared in this sequence you are having 2 power n squared plus one element but maximum you can have 2 power n squared distinct relation on A cross A so some of them should be equal, at least for some values of s and t they should be equal. So there will be some s and there will be some t such that R power s is equal to R power t between these values that is what is meant by this result.

(Refer Slide Time 11:33)



A is a finite set with n elements and R is a relation on A then there exist s and t such that R power s is equal to R power t and s and t lies between 0 and 2 power n square.

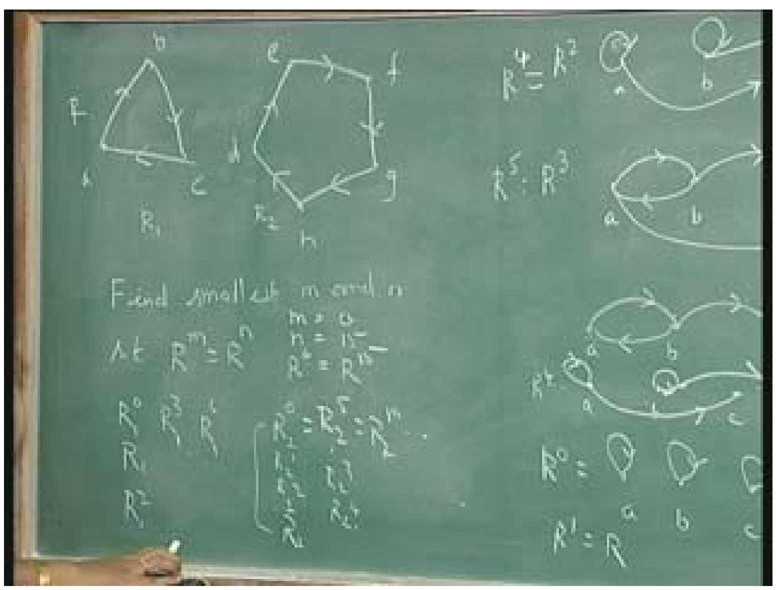
(Refer Slide Time 11:49min)



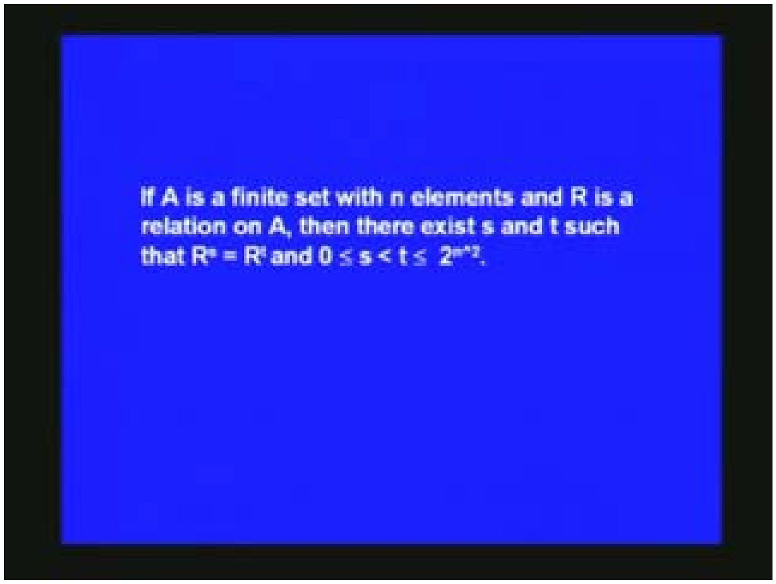

We can also look at another example where this can be made of use of. Look at this relation with eight elements a b c d e f g h, a relation is represented by this. Now find smallest m and n such that R power m is equal to R power n, m less than n. How will you tackle this problem? In the last class we have seen that if you take R like this R power 0 is the equality relation R is this R square is this then R cubed will again be the equality relation because each path of length will be replaced by a loop if you take first component alone. The relation consists of two components. The first components repeats after three steps R power 0 will be is equal to R cubed I will call this as R power 1 and R power 2 so R power 1 will repeat after three steps. Now the same argument you can use for the second component also, $R_2$ power 0 will be the equality relation and $R_2$ power 1 will be this $R_2$ power 2 will be every path of length two will be replaced by an arc, R power 3 will be every path of length three will be replaced by an arc, R power 4 will be every path of length four will be replaced by an arc, R power 5 will be every path of length five will be replaced by an arc.

If you follow the same argument as the earlier case you will find as $R_2$ power 1 $R_2$ power 2 $R_2$ power 3 $R_2$ power 4 will be different but when you consider $R_2$ power 5 $R_2$ power 0 will be is equal to $R_2$ power 5 is equal to $R_2$ power 10 and so on. So the first component will repeat after every three steps, the second component will repeat after every five steps. for the relation to be the same both the components have to repeat in the same manner so you will find that after fifteen steps this will be the same and this will also repeat after fifteen steps, three and five this repeat every three steps this repeat after every five steps so taking the LCM of 3 and 5 after fifteen steps the whole thing will get repeated so the smallest m and n such that R power m is equal to R power n where m is less than n the answer will be m is equal to 0 the equality relation n is equal to 15 this is also will be the equality relation on this so you get R power 0 is equal to R power 15.

(Refer Slide Time 15: 30min)

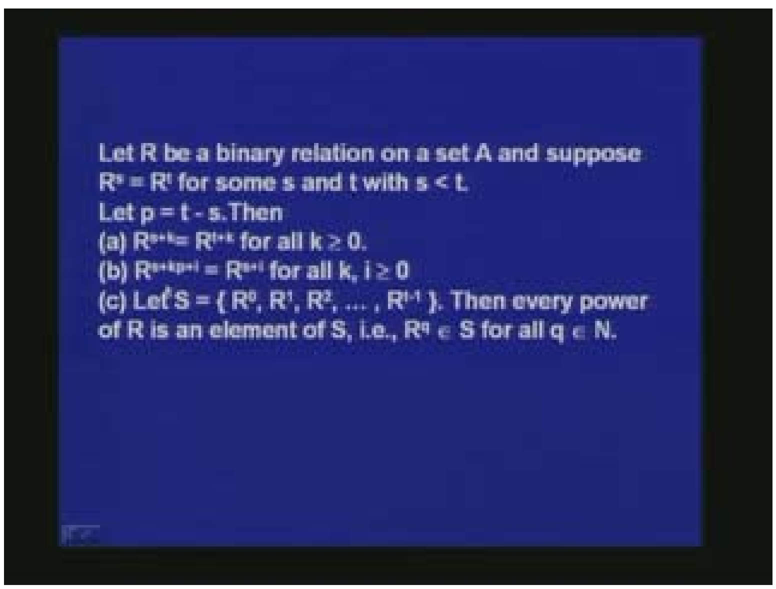


(Refer Slide Time 15: 35)



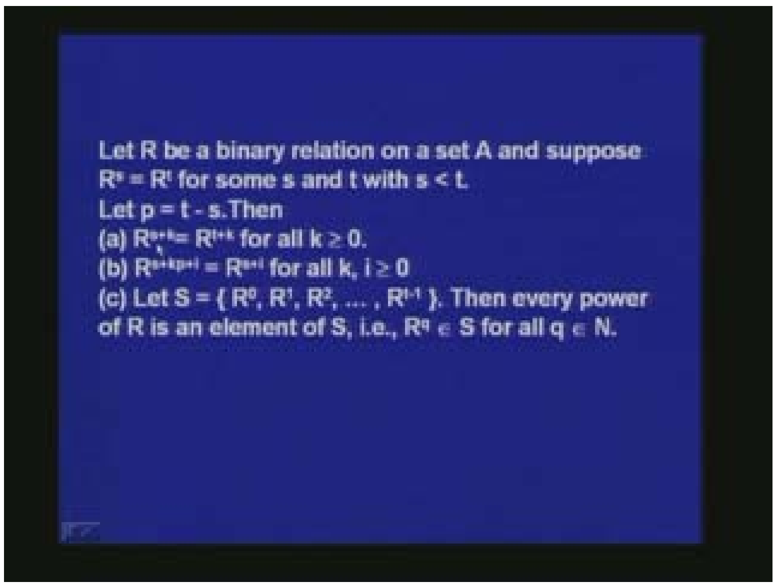

Let us look at the small result on this power of R.

(Refer Slide Time 15: 42)




Let R be a binary relation on a set A and suppose that R power s is equal to R power t for some s and t with s less than t. Before proceeding to that I want to mention one more point that R power s becomes is equal to R power t only if A is a finite set with n elements. If A is an infinite set this may not true at all. For example, let us consider the relation on the set of non negative integers, the underlined set is the set of non negative integers 0, 1, 2 etc and the relation is R is represented like this, x is related to y y is equal to x plus 1. So what is R squared? R squared will represent R squared if y is equal to x plus 2 and any R power s will be x R power s, y if y is equal to x plus s. let us consider this situation in a graphical manner. So the set of natural numbers you can represent like this; R will be represented by this, A is related to B if B is equal to A plus 1. So R power s will be denoted by, 0 will be related by s and 1 will be related to s plus 1 and so on. So if a is related by R power s to b then b will be a plus s. In this case you find that none of the Rs are equivalent R power 0 R power 1 R power 2 etc they are all different. So the result which you had for finite set like this will not hold when A is an infinite set. When A is a finite set we had this result this will not hold when A is an infinite set that is what we have considered now. Let us consider one more result about the power of a relation.

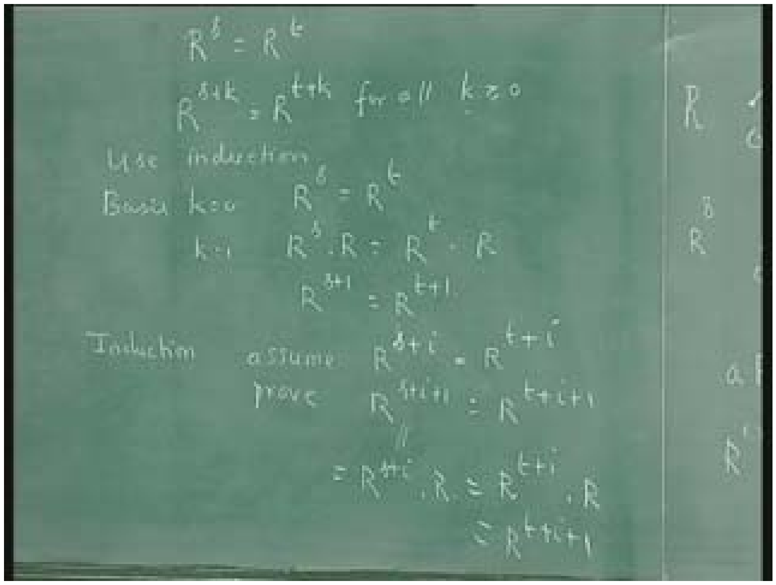
(Refer Slide Time 18:29)




Let R be a binary relation on a set A and suppose R power s is equal to R power t for some s and t where s and t are integers and s is less than t. let p be is equal to t minus s then R power s plus k is equal to R power t plus k for all k greater than or is equal to 0. R power s plus kp plus i is equal to R power s plus i for all k i greater than or is equal to 0 and there is a third result let S be is equal to R power 0 R power etc up to R power t minus 1 then every power of R is an element of the set S. That is, if you have any R power q it will belong to S for all q belonging to N. Let us prove by one by one.

First let us consider the first part R power s plus k is equal to R power t plus k for all k greater than or is equal to 0. So this is what we want to prove. We know that R power s is equal to R power t, we have to show R power s plus k is equal to R power t plus k for all k greater than or is equal to 0. Use induction basis clause k is equal to 0 you know that R power s is equal to R power t, k is equal to 1 you know that R power s cross R is equal to R power t cross R. See you are combining with R you are concatenating with R so this will be R power s plus 1 is equal to R power t plus 1. Then the induction portion is like this; assume R power s plus i is equal to R power t plus i to prove R power s plus i plus 1 is equal to R power t plus i plus 1. You know that this equal to R power s plus i cross R and because R power s plus i is equal to R power t plus i you can write it like this and this is nothing but R power t plus i plus 1. So R power s plus k is equal to R power t plus k for all k greater than or is equal to 1.

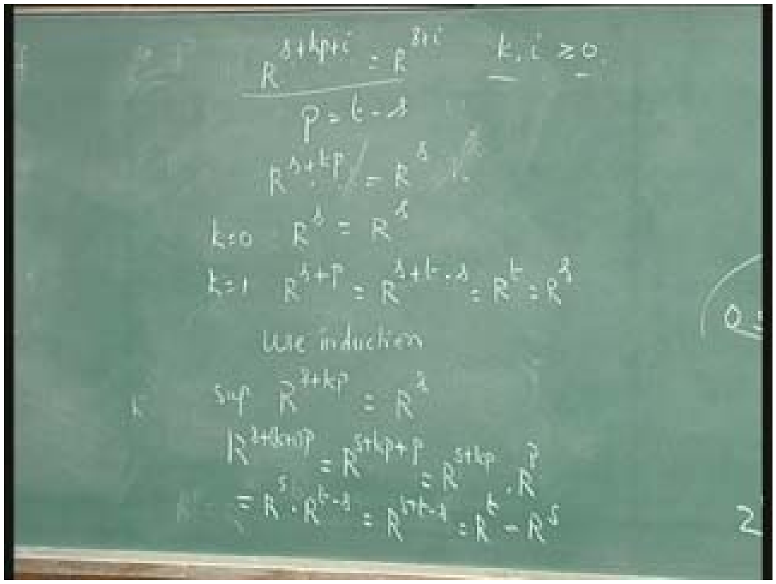
(Refer Slide Time 21:19)




Next we have to take the second portion that is R power s plus kp plus i is equal to R power s plus i for all k, i greater than or is equal to 0. R power s plus kp plus i is equal to R power s plus i for all k, i greater than or is equal to 0. Here again make use of induction where P is t minus s, P is defined as t minus s. Now, from the previous result if you prove R plus kp is equal to R power s then i can vary by adding i on both sides you can say that they are equal, there is no problem. So what you to show that R power s plus kp is equal to R power s. If k is equal to 0 R power s is equal to R power s you get. k is equal to 1 R power s plus p that is equal to R power s p is t minus s is equal to R power t. But we know that R power t is R power s. So this is somewhat like a basis clause of induction and you can show that again use induction portion.

Suppose R power s plus kp is equal to R power s suppose, then R power s plus k plus 1 p is equal to R power s plus kp plus p is equal to, what is R power k plus? This is R power s plus kp R power p but what is p? R power s plus kp is R power s, R power p is t minus s so this equal to R power s plus t minus s is equal to R power t and we know that by assumption R power t is R power s. So we can prove in this manner that for any k and i again if you multiply by this R power i and multiply this by R power i that plus i you will get. So for any k and i greater than or is equal to you can show that R power s plus kp plus i is equal to R power s plus i.

(Refer Slide Time 24:29)

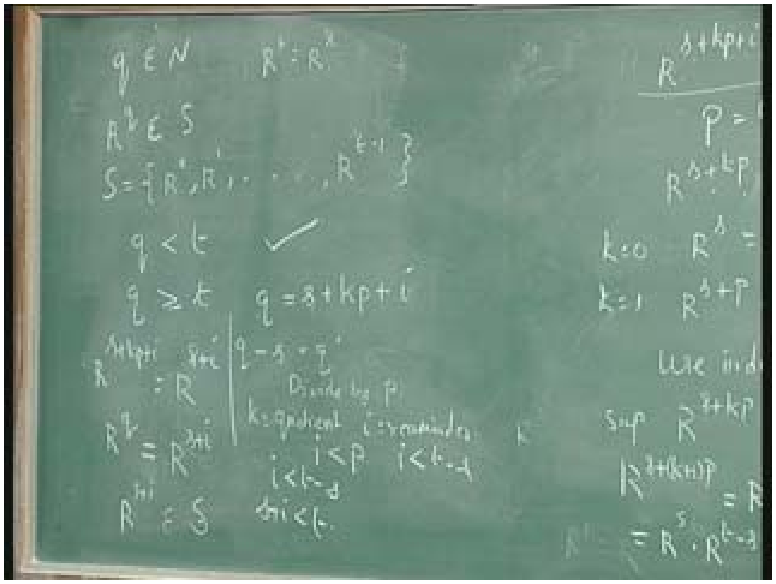



Now let us come to the third part, what is the third part? S is equal to R power 0 R power 1 R power 2 R power t minus s then for any power of R you can say that R power q belongs to S for all q belonging to N. So for any power any q which is a natural number R power q belongs to S where S is defined as R power 0 R power 1 up to R power t minus 1. What we already have is R power t is equal to R power s. And the previous two results we have already proved.

Now, there are two possibilities take q belonging to N, q is any natural number then q is less than t if q is less than t obviously it is in this R power q will be in this side so that is okay. If q is less than t R power q is in this side so the result is proved but if q is greater than t greater than or is equal to t what happens? Then you write q as s plus kp plus i you can write q as s plus kp plus i how it is possible? First from q you subtract s then whatever is remaining divide by p as much as possible where k is the quotient and i is the remainder. So what can you say about i? i will be less than p it is the remainder you are dividing by p so i is less than p, what is p? That is i is less than t minus s. So any q if it is greater than or is equal to t you can write it in this form s plus kp plus I, how do you get this? From q subtract s you get q dash and divide this q dash by p you get k as the quotient and i as the remainder.

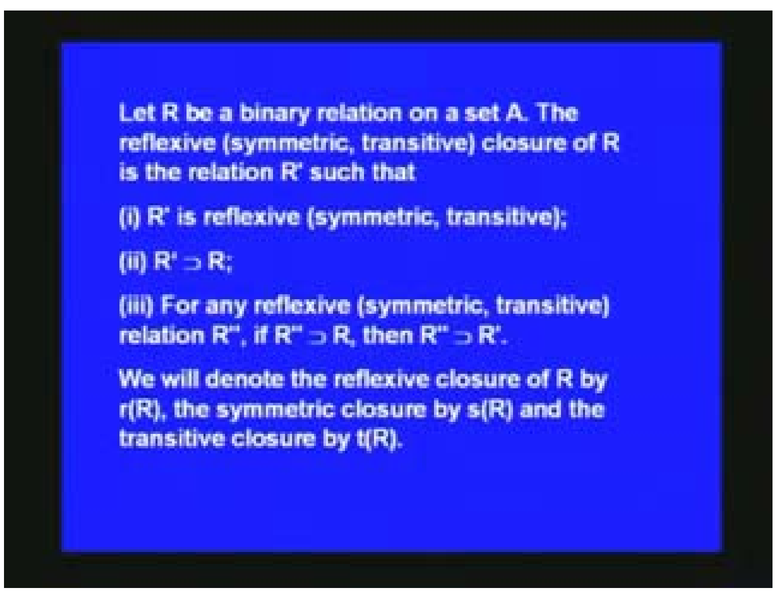
Now, by the second result we know that R power s plus kp plus i is equal to R power s plus i for all k and I this we know from the previous result. So R power q if you write it as R power s plus kp plus i it will be is equal to R power s plus I in this case when q is greater than or is equal to t you can write it as R power s plus i where i is less than t minus i now i is less than t minus s so s plus i will be less than t. So this s plus i is less than t so R power s plus i because s plus i is less than t it will be one of these elements so R power s plus i that will belongs to S. So we have considered both the cases when q is less than t and q is greater than or is equal to t. When q is less than t by definition itself it will belong to this set but when q is greater than or is equal to t you express it in the form express q in the form s plus kp plus i so that R power q becomes is equal to R power s plus I and so s plus i is less than t so this will be one of the elements here and belong to S. So this proves the result. So these are some results about powers of R.

(Refer Slide Time 29:07)



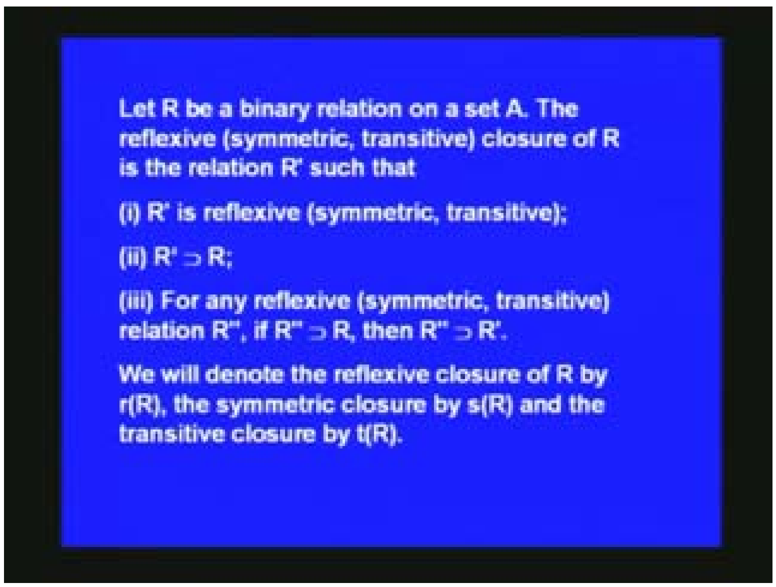
The point to remember is, if you represented it as directed graph, if you represent R by a directed graph then R power N will consist of a directed graph where every path of length N in R is replaced by a single R, this is what you have to remember.

(Refer Slide Time 29: 33)



Next we shall study about some closure. We have seen what is meant by a relation to be reflexive, to be symmetric, to be transitive and so on. We shall see what is meant by a reflexive closure, symmetric closure and transitive closure. Why is it necessary to study this? For example, you may have a communication network in which there is a direct link between A B there is a direct link between B and C and there is a direct link between C and D but there is no direct link between A and D or B and D or A and C. Now, I want to find out from which nodes I can direct a message to which node. See, there is a direct link between A and B and also there is a direct link between B and C so I can root a message from A to C, so I want to find out all the pairs of nodes where from the first node I can root a message to the second node. This in essence will be finding the transitive closure of this relation. So let us see what is meant by reflexive closure, symmetric closure and transitive closure.

(Refer Slide Time 31:16)



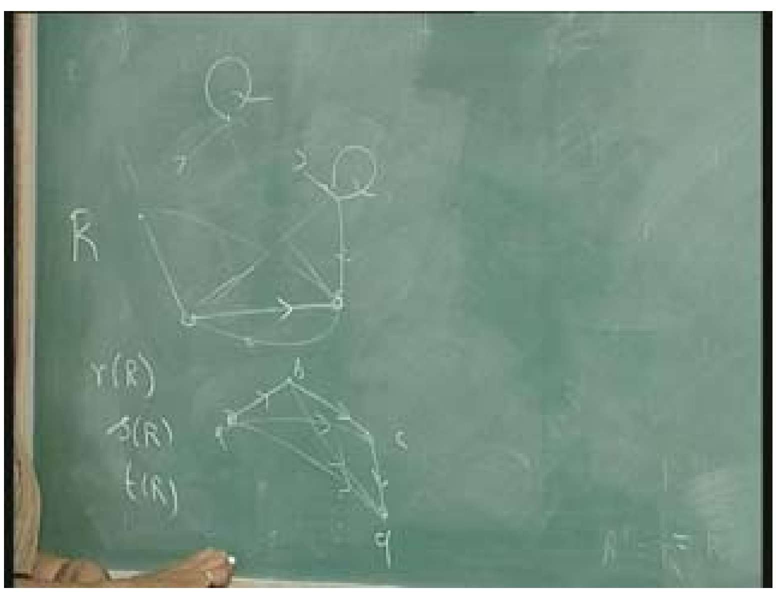

Let R be a binary relation on a set A. The reflexive, similarly we will study symmetric transitive but first we will see reflexive. The reflexive closure of R is the Relation R dash such that R dash is reflexive, R dash contains R, for any reflexive relation R double dash if R power 2 dash contains R then 2 dash contains R dash. This is the definition of reflexive closure. Similarly, for symmetric closure you have to read like this; if R is binary relation on A the symmetric closure of R is the relation R dash such that R dash is symmetric R dash contains R for any symmetric relation R power 2 dash if R double dash contains R then R double dash contains R dash.

Similarly, for transitive closure you have to read like this; let R be an binary relation on a set A the transitive closure of R is the relation R dash such that R dash is transitive, R dash contains R, for any transitive relation R double dash if R double dash contains R then R double dash contains R dash. So the reflexive closure of relation R is the smallest reflexive relation containing R. The symmetric closure of R is the smallest symmetric relation containing R. The transitive closure of R is the smallest transitive relation containing R. We will denote the reflexive closure by r, R symmetric closure by s, R and the transitive closure by t, R. So the reflexive closure is denoted by r, R the symmetric closure is denoted by s, R and the transitive closure is denoted by t, R. In a crude way or a very naïve way you can define like this; let R be a binary relation and you can represent by graph a graph a directed graph R can be represented by a directed graph. Now what is that if it may have some self loops? It may not have all self loops but it may have some self loops.

Now, the reflexive closure of R is the smallest reflexive relation containing R that means you should add self loops wherever it is not there so that it becomes reflexive, it amounts to this, finding the reflexive closure amounts to this reflexive, it amounts to this. Now what about the symmetric closure? So if there is an arc in one direction between two nodes the symmetric closure should also have the other arc in this direction. So when you want to find the symmetric closure of R if there are two arcs like this directed in both directions between two nodes is okay, if there are no arcs between two nodes then also it is okay but if between two nodes if you have one arc pointed like this you must add the arcs the direction also to make it symmetric. So try to add minimum number of arcs to make it symmetric and that will give you the symmetric closure.

Now, what about the transitive closure? The transitive closure is, if there is a directed path between two nodes you must also add this arc, there is a path here you must add this arc. So try to add the minimum number of arcs such that you get the transitive closure. The transitive property means that if there is a path between two nodes then there should also be an arc between the two nodes. So that transitive properties if it does not hold you have to make it hold so whenever you have a directed arc between two nodes you must also try to add the arc corresponding to that.

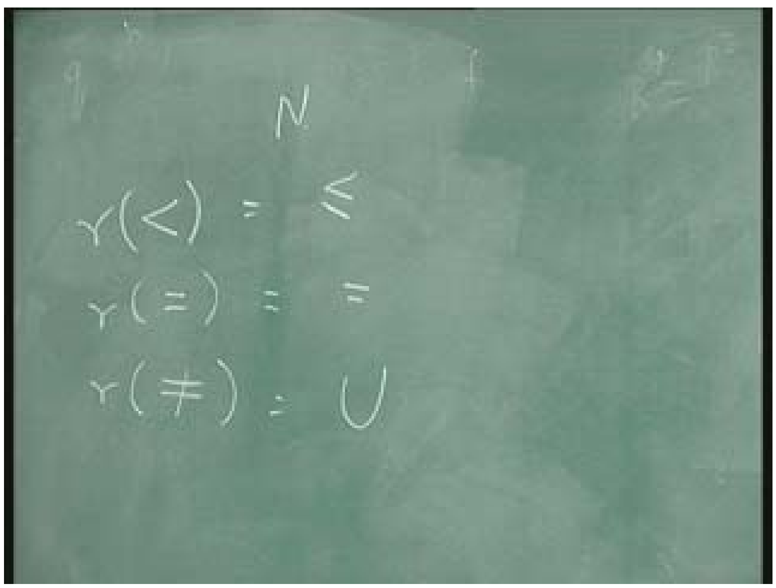
(Refer Slide Time 36:00)



So putting it in a simple manner let us study some properties. Before that let us consider some relation on the set of non negative integers N. Look at the less than relation, what can you say about the reflexive closure of the less than relation? Less than relation is not reflexive, x is not less than x so there will not be any self loops. So, if you look at the natural numbers like this less than relation will have; A is less than B, A is less than 0, 1, 2 like that you have arcs A, B and so on but there will not be any self loop.

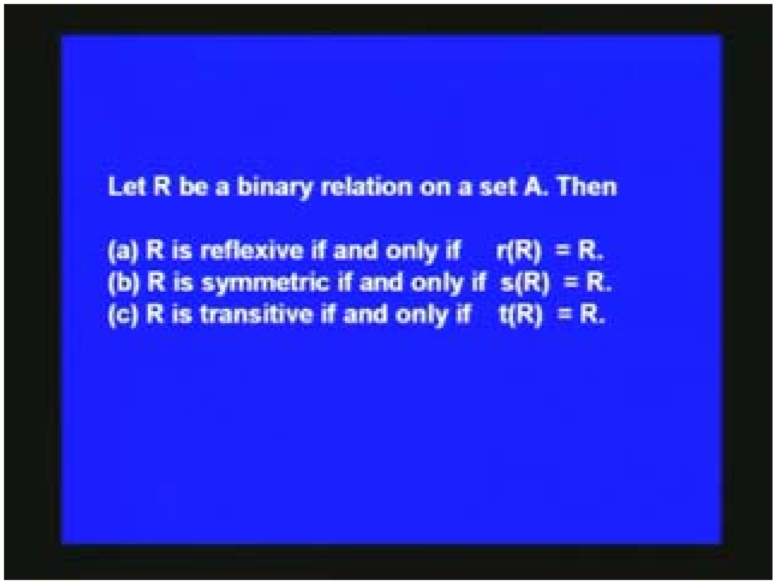
The reflexive closure should have self loop at every node and so the reflexive closure of less than will be the less than or equal to relation including this like this self loop like this. What can you say about the reflexive closure of equal to relation? It is already reflexive so that it will be equal to the reflexive closure of the not equal to relation. Two elements are directed and there is a directed arc between two integers, if they are not the same you may also add self loop at every point you must add the equality relation that will become equal to the universal relation where every number is connected to every other number.

(Refer Slide Time 37:46)



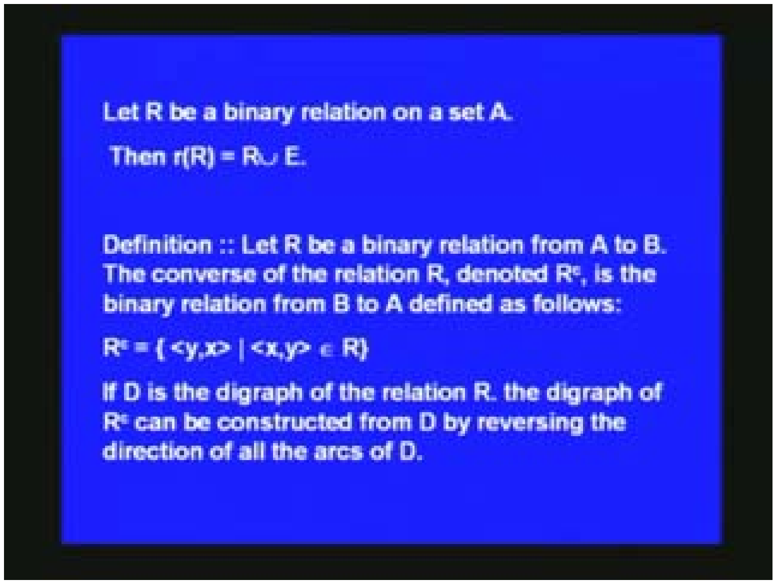

So, this is something about the reflexive relation. Let us see more about reflexive closure.

(Refer Slide Time 38:02)



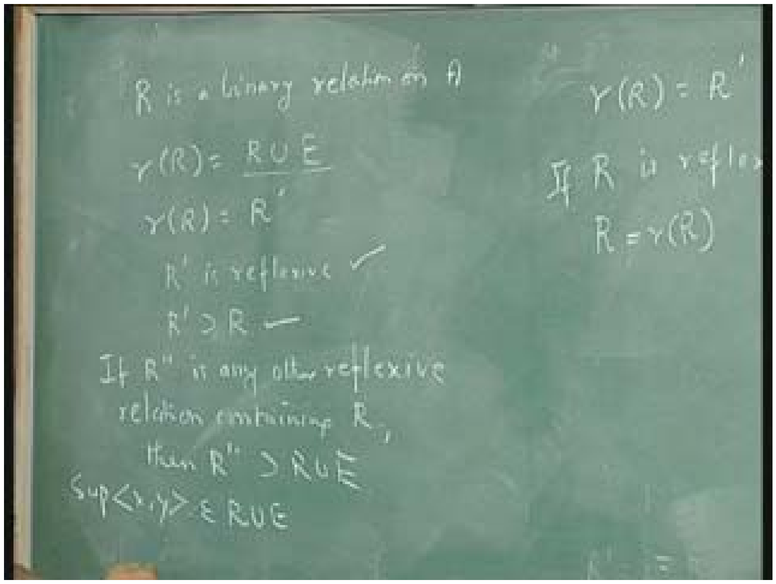

Now, this you can very easily see, let R be a binary relation on the set A then R is reflexive if and only if the reflexive closure of R is equal to R, R is symmetric if and only if the symmetric closure of R is equal to R, R is transitive if and only if the transitive closure of R is equal to R. Let us take the first one and prove, let the reflexive closure of R be R dash, now, if R is already reflexive R satisfies all the conditions mentioned in the definition of the reflexive closure. It is a reflexive, what is a reflexive closure? It is the smallest reflexive relation containing R if R itself is reflexive then it is reflexive it contains R then any other reflexive relation will contain this and all the three conditions are satisfied. So, if R is reflexive the reflexive closure will be R itself. Similarly, if R is symmetric the smallest symmetric relation containing R will be R itself so that is why these results follow, it is not every difficult to see these results.

(Refer Slide Time: 40:00)




Let R be a binary relation on a set A then the reflexive closure of R is R union E. This we already seen, the reflexive closure of less than is less than R is equal to R and so on. So, we want to show that R is a binary relation on A then the reflexive closure of R is R union E that is as graph we are adding self loops wherever we want. How do you prove that? Let the reflexive closure of R be R dash. Then it has to satisfy the three conditions, R dash should contain R dash is reflexive, R dash contains R and if R double dash is any other reflexive relation containing R then R double dash contains R dash this is the definition. Now you can see that when you take R union E it is reflexive because it contains E it is reflexive it contains R that is also satisfied. Now we have to only consider only the third portion; if R double dash is any other reflexive relation containing R then R double dash should contain this R union E. This can be easily proved suppose x, y belongs to R union E there are two possibilities; one is x, y belongs to R or x, y belongs to E and in this case it will actually be of the form x, x E means of the form x, x.

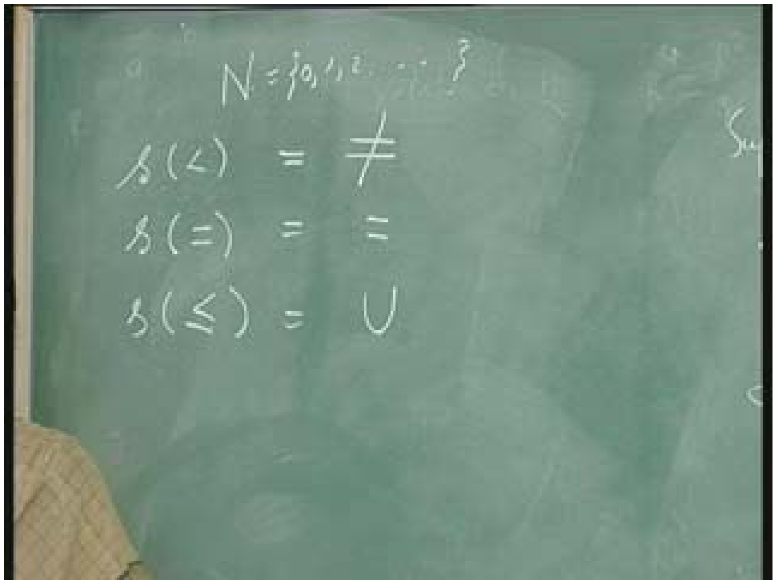
(Refer Slide Time 42:37)




Now, if x, y belongs to R x, y will belong to R double dash the reason is R double dash contains R by the way we have taken R double dash contain R so x, y will belong to R double dash. Now, if it belongs to E it is of this form then also it will belong to R double dash the reason is R double dash is reflexive. R double dash is reflexive so it will contain all pairs of the form x, x. So, whenever x, y belongs to R union E it also belongs to R double dash that means R double dash contains R union E so it satisfies all the three conditions mentioned in the definition of the reflexive closure. And so we find that the reflexive closure of a binary relation R is given by R union E where E is the equality relation R is the given relation and E is the equality relation. Next, we shall go to symmetric closure.

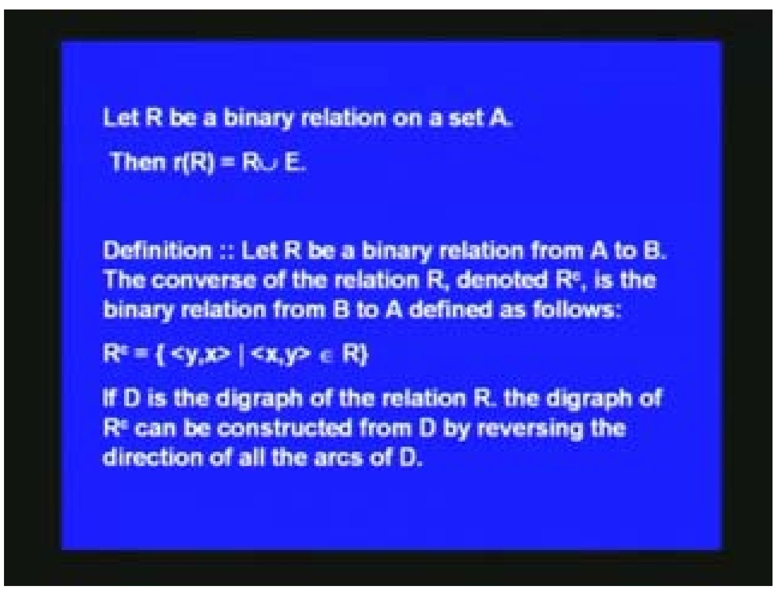
Again let us consider some examples. As I mentioned earlier in a symmetric closure if you represent it as a graph if you have an arc in one direction you must also add the arc in the other direction. So let us consider the set of natural numbers or non negative integers 0, 1, 2, 3, 4 etc. What is the symmetric closure of the less than relation? A is less than B then you must also add the arc the other way round so the symmetric closure of less than relation will be the not equal to relation. And the symmetric closure of equal to relation is equal to and the symmetric closure of less than or equal to relation is the universal relation because equal to means no arcs between any two nodes you will not add anymore arcs. In the less than or equal to again if there are self loops at every node and also when there is an arc like this you will also add the other way around so that will give you the universal relation on the set of natural numbers.

(Refer Slide Time 45:57)



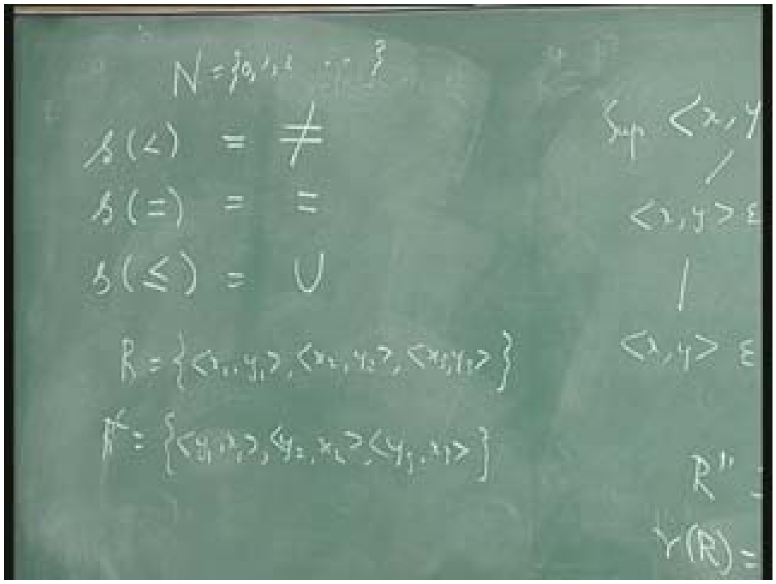

Now, how we go about the finding the symmetric closure of a binary relation R. for that we have to know that what is meant by the converse of a relation.

(Refer Slide Time 46:09)



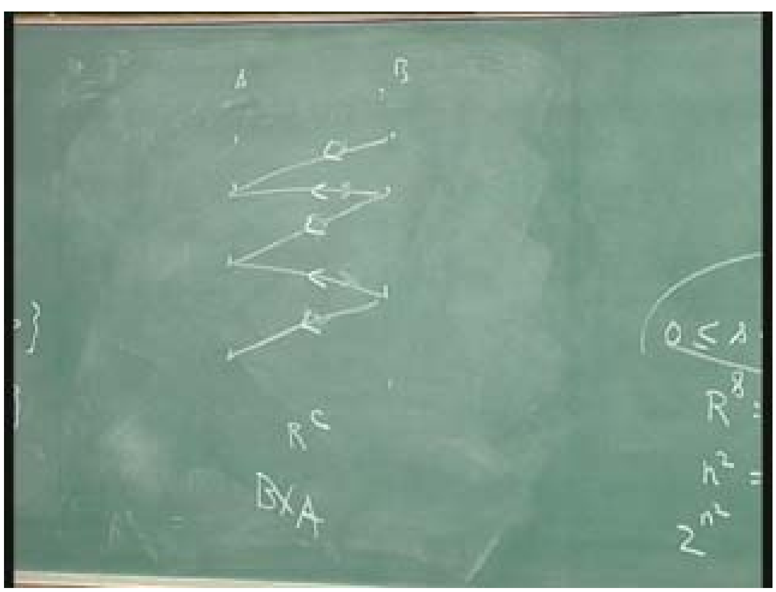

Let R be a binary relation from A to B. the converse of the relation R denoted by R, c is the binary relation from B to A defined as follows: R, c is y, c while R, c consists of ordered pairs y, x such that x, y belongs to R. So, suppose I have a relation R consists of three pairs $x_1$, $y_2$ $x_2$, $y_2$ $x_3$, $y_3$ R, c will consist of $y_1$, $x_1$ $y_2$, $x_2$ and $y_3$, $x_3$ all the order reversed.

(Refer Slide Time 47:04)



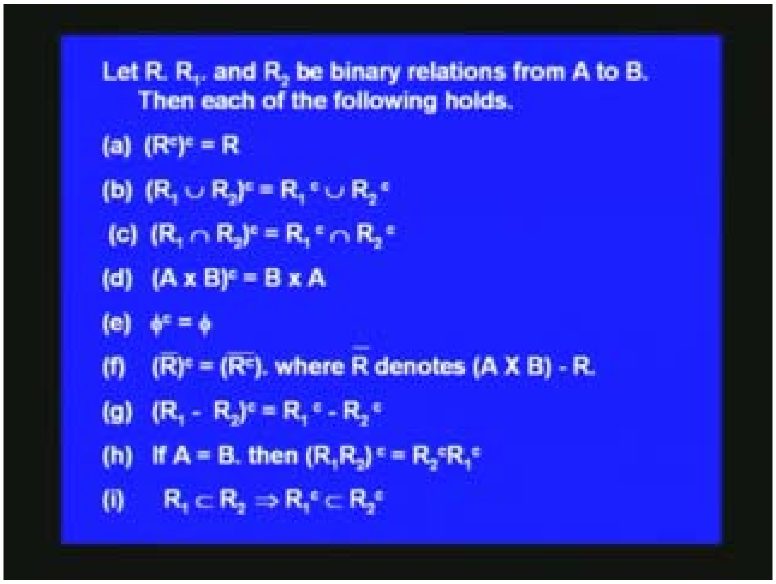

If D is the diagraph of the relation R the diagraph of R, c can be considered like this; it can be constructed from D by reversing the direction of all the arcs of D. So if you represent a binary relation here we have not even taken the same thing, a binary relation from A to B there are some elements here so the binary relation can be represented like this, the converse if R is this what can you say about R, c? The arc should be reversed the direction should be reversed so it will be on B cross A.

(Refer Slide Time 48:05)



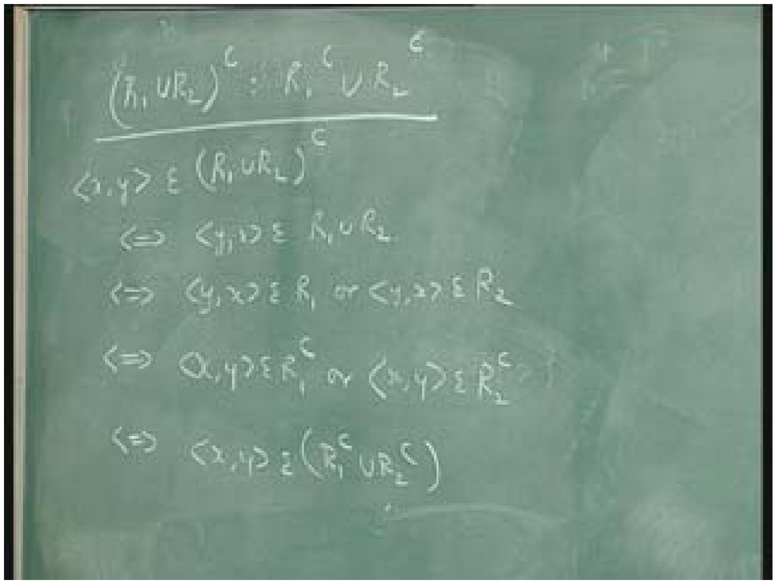
Let R be a binary relation so R, c is the set of ordered pairs y, x where x, y belongs to R. And if D is a diagraph the R, c can be constructed by D by reversing the direction of all the arcs of D.

(Refer Slide Time 48:29)




We have some results about converse, R $R_1$ $R_2$ are all binary relations from A to B then each of the following holds: R power c is equal to R that is if you take the converse you have to reverse the direction of the arcs. Then to take the converse again you have to again reverse the arcs so you get back the original relation. $R_1$ union $R_2$ power c the converse of $R_1$ union $R_2$ is the converse of $R_1$ union converse of $R_2$. And similarly these etc, we shall prove the B part it says $R_1$ union $R_2$ power c is equal to $R_1$ power c union $R_2$ power c what does that mean? if x, y belongs to $R_1$ union $R_2$ power c this is equivalent to saying y, x belongs to $R_1$ union $R_2$ and that is equivalent to saying y, x belongs to $R_1$ or y, x belongs to $R_2$ that is equivalent to saying x, y belongs to $R_1$ power c as y, x belongs to $R_1$ and x, y belongs to $R_1$ power c or here y, x belongs to $R_2$ means x, y will belong to $R_2$ power c and that is equivalent to saying x, y belongs to $R_1$ power c union $R_2$ power c. So we get $R_1$ union $R_2$ power c is equal to $R_1$ power c union $R_2$ power c the converse is denoted like this.

(Refer Slide Time: 50:49)

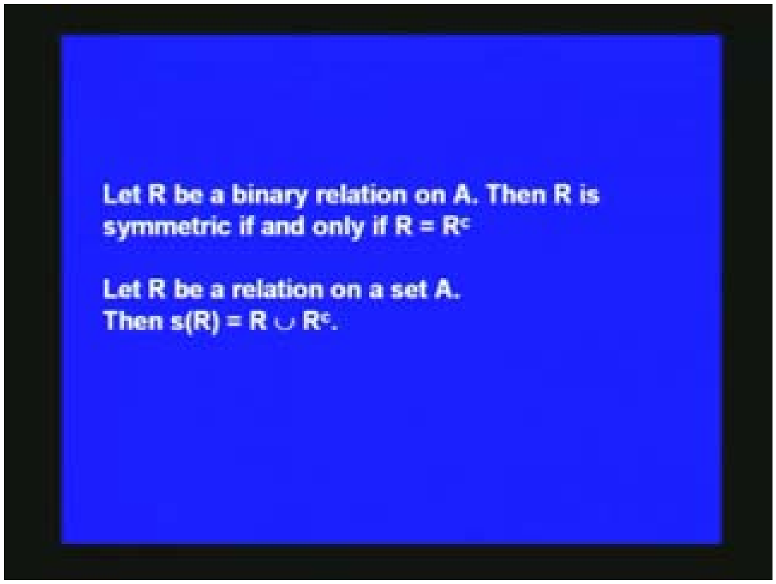



The other results can be proved in a similar manner. For intersection also you can prove in a similar manner. this A cross B denotes the whole Cartesian product from A to B taking the converse it will be the whole Cartesian product from B to A. And empty relation means there is no arc at all representing the graph that means you need not have to change the direction of the any arc so the converse of the empty relation is empty relation itself. And similarly, the other results follow R b, a R power c is equal to R power c bar and so on where R denotes the complement. And $R_1$ minus $R_2$ power c is $R_1$ power c minus $R_2$ power c.

If A is equal to B then $R_1$ $R_2$ power c is equal to $R_2$ power c $R_1$ power c. This you can see like this A is equal to B so $R_1$ $R_2$ are binary relations on A so $R_1$ $R_2$ is represented like this if there is an arc representing this is $R_1$ this is an arc from $R_2$ this is b to c; a, b belongs to $R_1$; b, c belongs to $R_2$. Then by our definition a, c will belong to $R_1$ $R_2$. Now b, a will belong to $R_1$ power c; c, a will belong to $R_2$ power c. If you reverse the directions c, b will belong to $R_2$ power c; b, a belong to $R_1$ power c.

Now what about an element belonging to $R_1$ $R_2$ power c? c, a belongs to $R_1$ $R_2$ power c and this you can get by combining c, b and b, a then c, b and b, a you can get. That is, if you combine c, b and b, a you get c, a and this belongs to $R_2$ power c and this belongs to $R_1$ power c. So you get that $R_1$ $R_2$ power c you can get by combining or by a applying the composition of operation on $R_2$c and $R_1$ power c that is if you have these two arcs $R_1$ $R_2$ will contain this arc and when you see the converse there will be an arc from here to here and that you can obtain like this that is $R_2$ first $R_2$ power c and then $R_1$ power c. The last one is if $R_1$ is contained in $R_2$ then $R_1$ power c is contained in $R_2$ power c.

(Refer Slide Time: 54:20)




It is also not very difficult to say that R is a binary relation on A then R is symmetric if and only if R is equal to R power c. So whenever you have the ordered pair A, B if you have also have the ordered paired B, A that is symmetric that means if it is symmetric means when you have the ordered pair A, B if it is symmetric if you have a, b you will also have b, a belonging to R. if a, b belongs to R b, a also belongs to R if it is a symmetric relation in that case the converse of R will have b, a it is also in R so R power c becomes is equal to R. It is not very difficult to see this result. If R is a binary relation then it is symmetric then if and only if R is equal to R power c.

Of course you can very easily see that if R is equal to R power c whenever it contains a, b it also contains b, a so it has to be symmetric. Then we have to also consider what a symmetric closure is and we find that if R is a relation on a set A then the symmetric closure is given by R union R power c. We have to look into the proof of this and we have to also consider what is meant by a transitive closure and some properties of the transitive closure. We shall consider this in the next lecture.