Discrete Mathematical Structures
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Lecture -13
Relations

Today we shall consider about relation. What is a relation? In ordinary English and in ordinary life you talk about relationships. See you have a collection of boxes and you say that one box is heavier than the other. That is the relationship among the boxes. And among human beings you say that one man is more intelligent than the other man. That is another relationship between men, the human beings. But you may also have relationship between different sets. x lives in the city y.

You can have something like: this x lives in city y or x was born in city y in the year z. So this brings out the relationship between different sets. x is a human being and y is a city and z is a year. So you can talk about relationship between different sets. And in the ordinary sense you talk about the relationship between human beings like x is the father of y x is the uncle of y. So all those are relations and in the common sense of the word and they also represent the technical sensor relationship between the same sets of human beings.

Now, why is it necessary to study about this? You know that in data base management system you can retrieve a record or you can find out something about the record and so on by searching a data base management system. And in relational there are several database management systems and in relational data base management system the whole thing is based on the concept of a relation. Everything is arranged in the form of a table where each table represents n-ary relations. May be I will mention about this a little bit later after we learn the definitions.

So it is very essential to learn the concept of relations. Also in several other places you talk about binary relations that are an ordered pair and in that order pair it will have certain specific properties like reflexive symmetric and the condition that relation should satisfy some of those properties is essential in many of the fields especially in computer science. And we will learn about the applications where it is used as and when we come across the definition.

Now let us start with the definition of an ordered n-tuple and what is an n-ary relation and so on.

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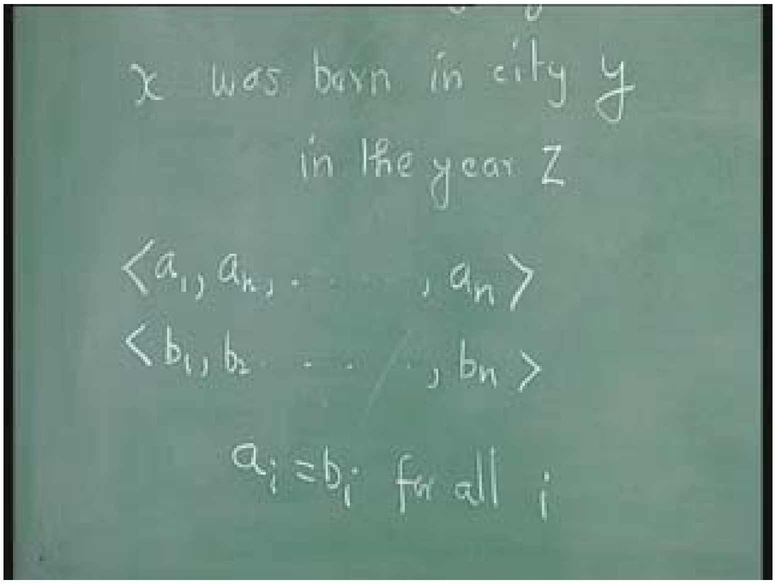

For n greater than 0, an ordered n-tuple or simply n-tuple with ith component a_i is a sequence of n objects denoted by a_1, a_2, a_3, a_n. Two ordered n tuples are equal if and only if their ith components are equal for all i. If n is equal to 2 the ordered n-tuple is called an ordered pair, if n is equal to 3 the ordered n-tuple is called an ordered triple. So an ordered n-tuple a_1, a_2, a_n is represented like this. And if you have two of them say b_1, b_2, b_n you can say that these two are equal if and only if a_i is equal to b_i for all i.

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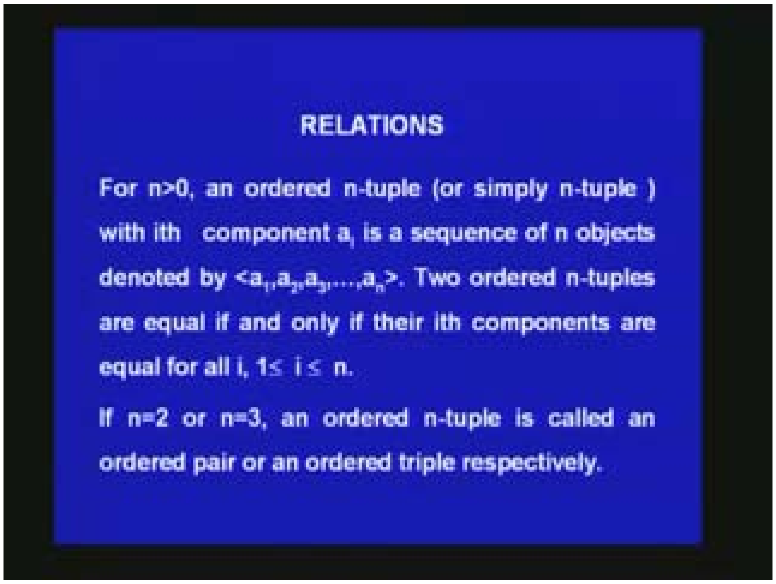

When you say ordered tuple the order is very important. For example consider the relation less and consider the ordered pair3, 7 this would mean 3 is less than 7, if we change the order and write it as 7 and 3 this would mean 7 is less than 3 and this is true while this is false. So the order is very important.

And similarly when you say x lives in city y or x was born in the city y in the year z you represent it as a tuple x, y, z that means x was born in the city y in the year z. x represents a human being y represents a city and z represents a year. Now, if you interchange the order y, x, z it does not make sense because it will mean some city was born in x that is a human being which does not make sense. So if you represented like this: x was born in city y in year z, you must write it as x, y, z only and if you write it as y, z it does not make sense. For example, Ram was born in city Madras or Chennai in 1972, this will be represented by the tuple Ram, Madras, 1972 and if you represent it like Madras, Ram it does not make sense, so the order is very important.

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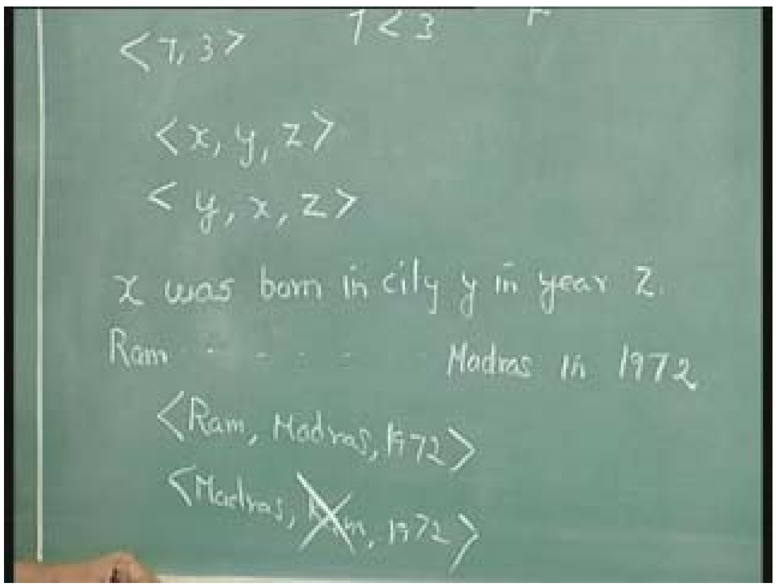

Especially in the case of relational data bases you represent them as tables and tuples like this. You have employee number, name, salary, department etc. So this will represent a tuple and you may have other components also and because of this you are able to define normal forms etc on relational data bases.

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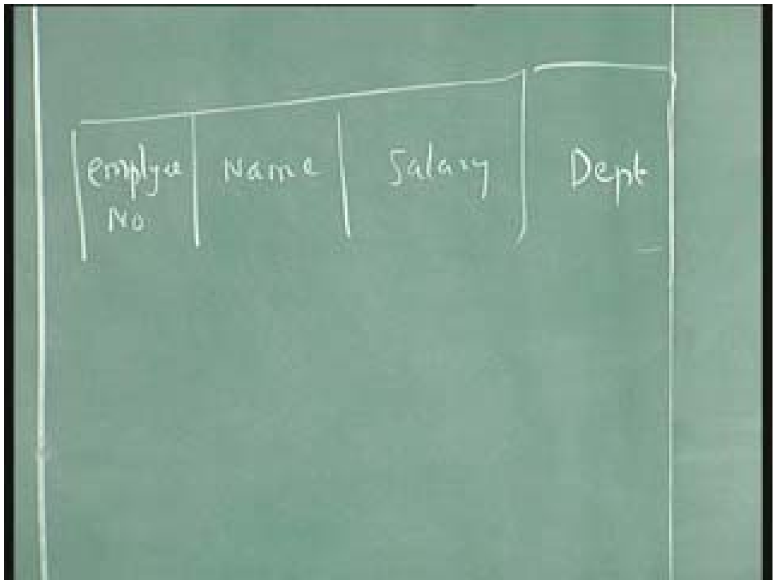

Now before going further into the ordered tuple let us consider what is meant by Cartesian product. Let A_1, A_2, A_3, A_n be an indexed collection of sets with indices from 1 to n where n is greater than 0. The Cartesian product or cross product of the sets A_1 through A_n is denoted by A_1 cross A_2 cross A_3 etc An or X_i is equal to 1 to n A_i. It represents the set of ordered n-tuples. So if you represent the Cartesian product A_1, A_2, A_n or you represent it as X_i is equal to 1 to n A_i. In the set of this x was born in city y in the year z you can represent it as A cross B cross C. A will represent set of human beings, B will represent cities and C represents years. This is known as Cartesian product.

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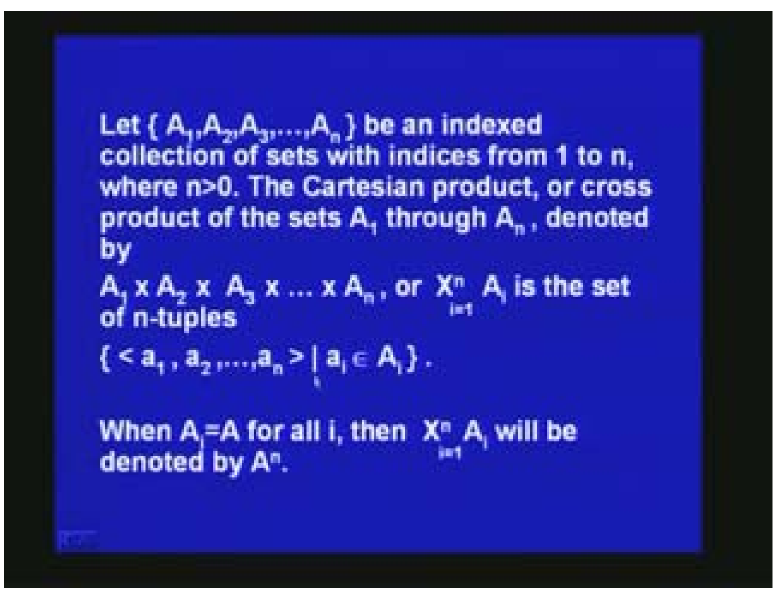

Again if all the As are the same for example ordered pair represent the relation A. Here in the ordered pair if you represent as x, y where x is an integer and y is also an integer. In that case A_1 is I, A_2 is also I the set of integers. So you represent it as A cross A or I cross I here. In general, if A_1, A_2, A_n are all the same. You represent it as I is equal to 1 to n A I is represented as A power n. Here it will be represented because the less than relationship is on the set of integers, here it will be represented by I square.

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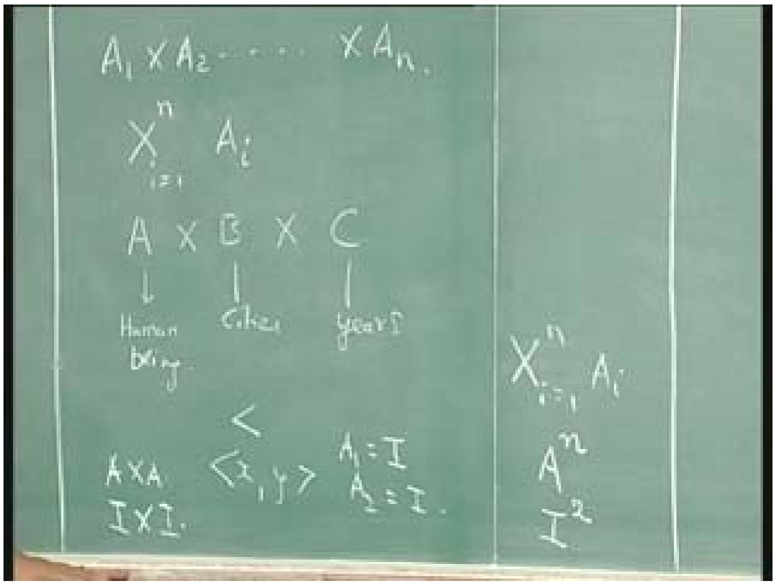

Now here again the order is very important. Let us take some example and consider what is meant by this. Suppose A is 1 and 2 and B is a and b, C is some alpha, D is phi, then what do you mean by A cross B? This represents ordered pairs where the first component will belong to A and the second component will belong to B. So it will represent 1, a, 1,b, 2,a and 2,b.

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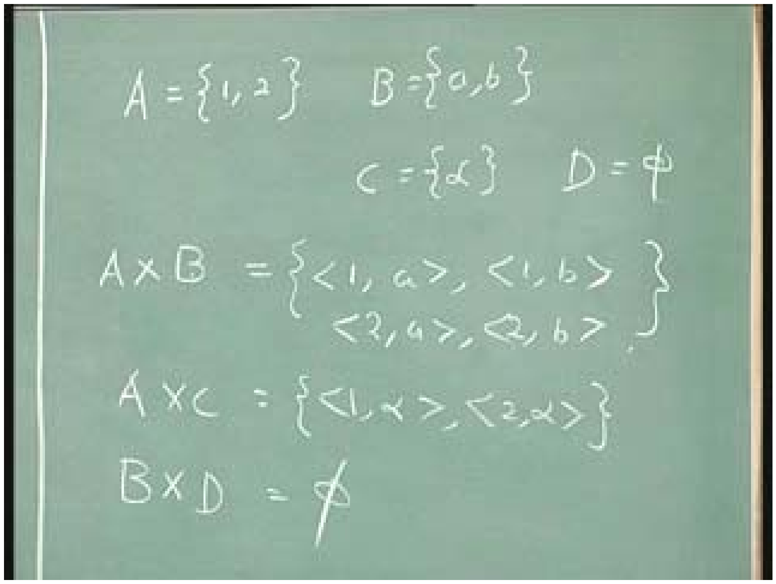

What do you mean by A cross C? A cross C will be 1, alpha 2, alpha it will represent this. What do you mean by B cross D where D is the empty set? In this case this will be empty. And here again the order is very important.

Suppose you have A cross B where A is the set of real numbers between 1 that is x belongs to A means it is a real number between 0 and 1 and y is a real number between 1 and 2 and y belongs to B, both are real numbers. Ao A represents the set of real numbers between 0 and 1 and B represents the set of real number between 1 and 2. What does A cross B represent? x varies from 0 to 1 and y varies from 1 to 2. So it will represent the set of points in this square.

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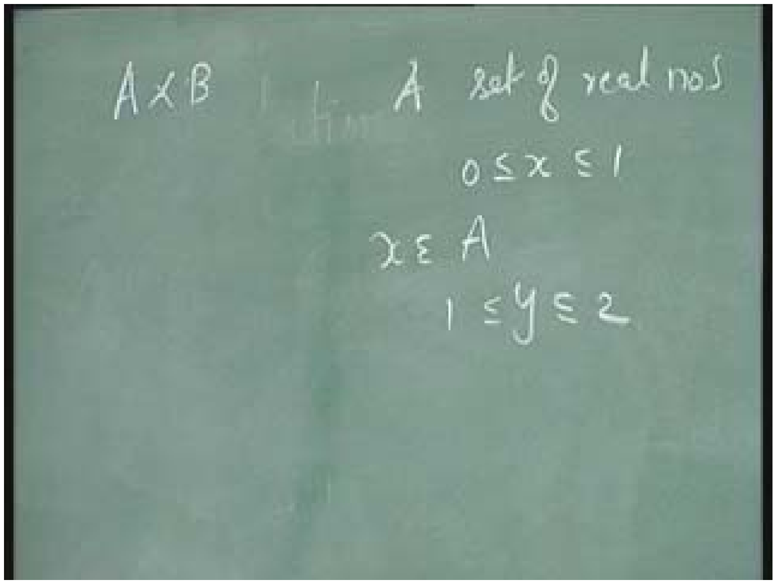

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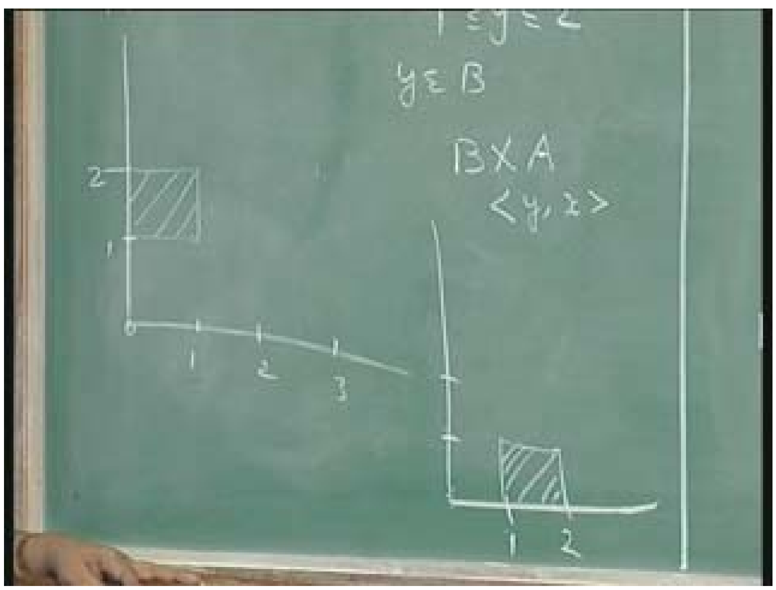

Now suppose I consider B cross A then it represents ordered pairs y, x. Now when interchanging the order the first component varies between 1 and 2 and the second component varies between 0 and 1. So it will represent the set of points in this square. So the set represented are different, they are not the same, so here the order is important A cross B is different from B cross A. So commutativity does not hold.

What about associativity?

Associativity is A_1 cross A_2 cross A_3 is the same as A_1 cross A_2 A_3 where A_1 A_2 A_3 are sets. There is a slight problem here because usually if x belongs to A_1, y belongs to A_2 and z belongs to A_3 it will be represented as x, y, z and this will be represented as x, y, z.
There is a slight difference between these two but usually in many cases you will try to write this as A_1 cross A_2 cross A_3 without any problem. But you have to be a little bit careful what you mean is correct. If some thing is very particular and you want represent it only like this then you have to represent it like this only. But most of the cases will be without any problem and we will be able to write as A_1, A_2 A_3. So we do not talk about the closure and associativity. It is slightly tricky.

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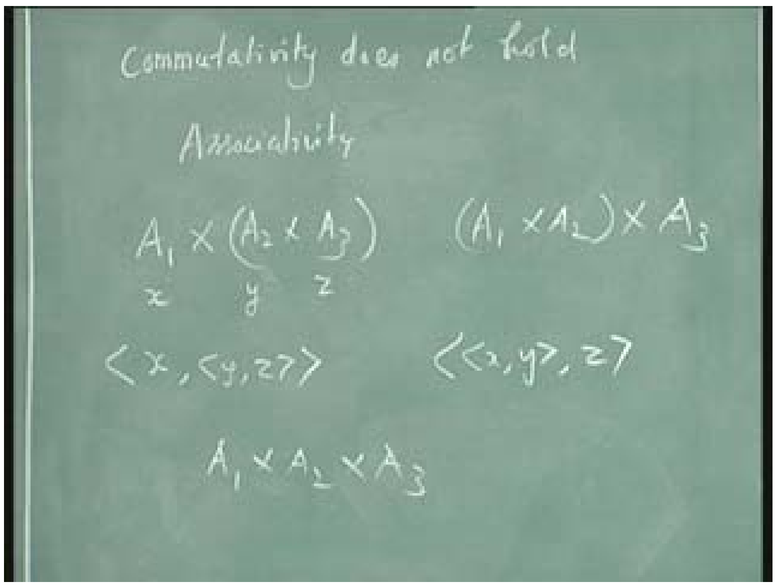

The next definition is what is a relation?
Before going to that let us see some relationship between the Cartesian product union and intersection. If I say this cross product the cross distributes over union and intersection. A cross B union C is equal to A cross B union A cross C. Similarly, A cross B intersection C is equal to A cross B intersection A cross C. The other way round A union B cross C will be equal to A cross C union B cross C. And A intersection B cross C will be equal to A cross C intersection B cross C.

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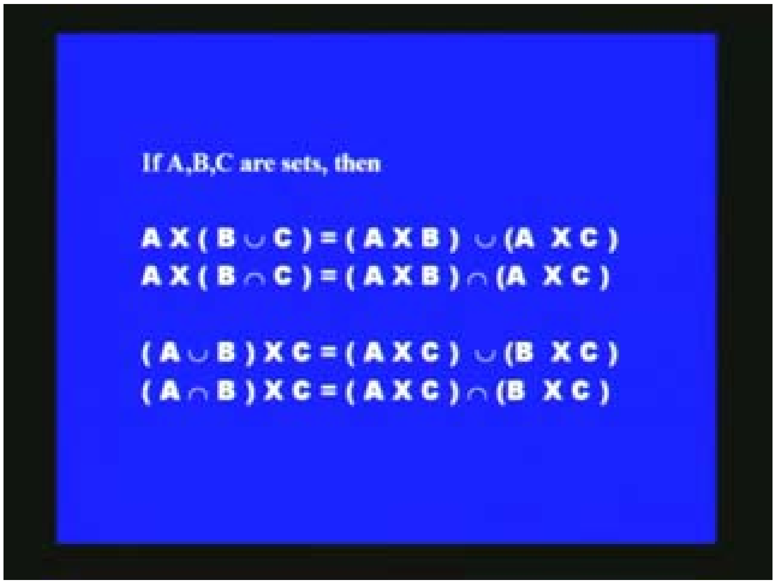

Let us prove this result. Let us take the first one and prove. The remaining can be proved in a similar manner. What is the first one? A cross B union C is equal to A cross B union A cross C. Let us see how we prove that?
Now when you say an ordered pair x, y belonging to A cross B union C this is equivalent to saying x belongs to A it represents ordered pairs x, y where x belongs to A and y belongs to B union C. That is equivalent to saying it represents a set of ordered pairs x, y where x belongs to A, y belongs to B OR y belongs to C.

Actually here it is x belongs to A AND y belongs to B union C and similarly x belongs to A AND y belongs to B OR y belongs to C. That is equivalent to saying x, y. For AND distributes over OR you can say as x belongs to A AND y belongs to B OR x belongs to A AND y belongs to C. And that is equivalent to saying, it represents the set of ordered pairs x belongs to A AND y belongs to B that means x belongs to A cross B AND x belongs to A AND y belongs to C means xy belongs to A cross C. So this one you can say as x, y where x belongs to A AND y belongs to B OR it represents the set of ordered pairs x, y where x belongs to A AND y belongs to C. And that is equivalent to saying x, y belongs to A cross B OR x, y belongs to A cross C. That is again equivalent to saying x, y belongs to A cross B union A cross C. So this proves the first part and second, third and the fourth part can be proved in a similar manner.

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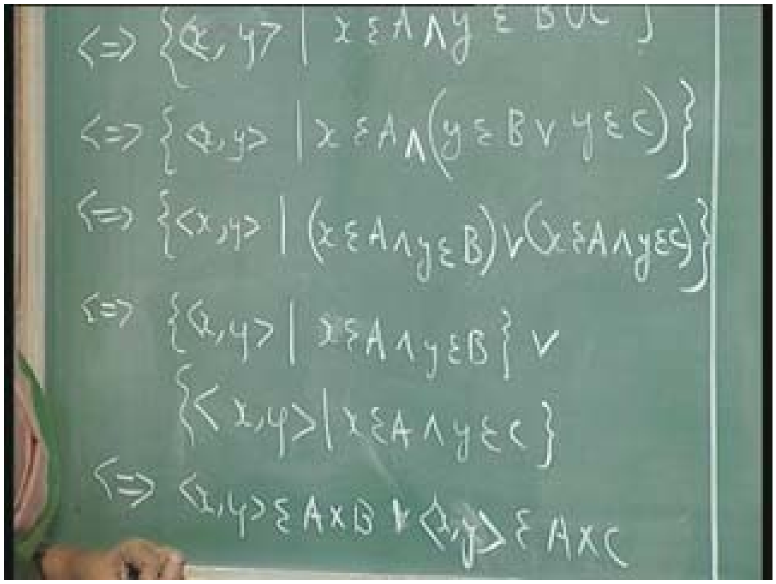

Now let us see what is a relation?
Now we have seen what a Cartesian product is. A Cartesian product is denoted by X is equal to I is equal to 1 to n A_i when you have n sets.

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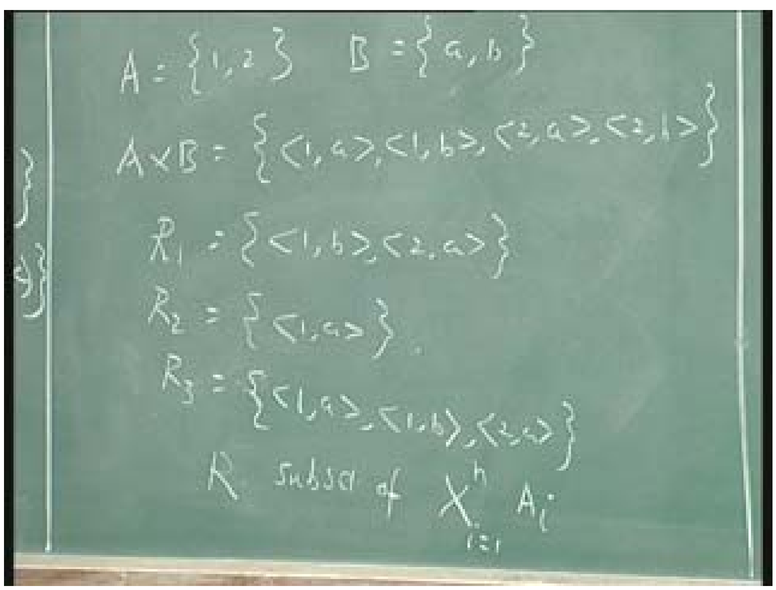

Now an n-ary relation is the subset of that. For example let us take A to B as in the earlier case it is 1, 2 and B to be say a, b. Then A cross B represents the elements 1, a, 1,b, 2,a, 2,b and n-ary relation is the subset of that. It need not contain all the elements it can just contain some elements.

For example, a relation R1 may be a subset of this; say 1, b, 2, a or another relation R_2 may just have 1,a alone. Another subset may have 1,a, 1,b and 2,a like that. So a relation is a subset of the Cartesian product. In this case we have taken ordered pairs.

In general, you can take n-tuples. A relation R will be a subset of the Cartesian product i is equal to 1 to n A_i. If it denotes that subset is the empty set, if R is equal to phi then it is called the empty relation or the void relation. The subset can be just the empty set also. And if the subset is the whole set, if R is equal to x, i is equal to 1 to n R, A_1 cross A_2 cross A_3 etc up to A_n. Then R is called the universal relation where all the elements are present. This itself represents a relation and that is the universal relation. If n is equal to 1, 2, 3 it is called a unary relation. If n is equal to 2 it is called a binary relation. If n is equal to 3 it is called a ternary relation. So for n is equal to 1, 2 or 3 R is called a unary, binary or ternary relation represent.

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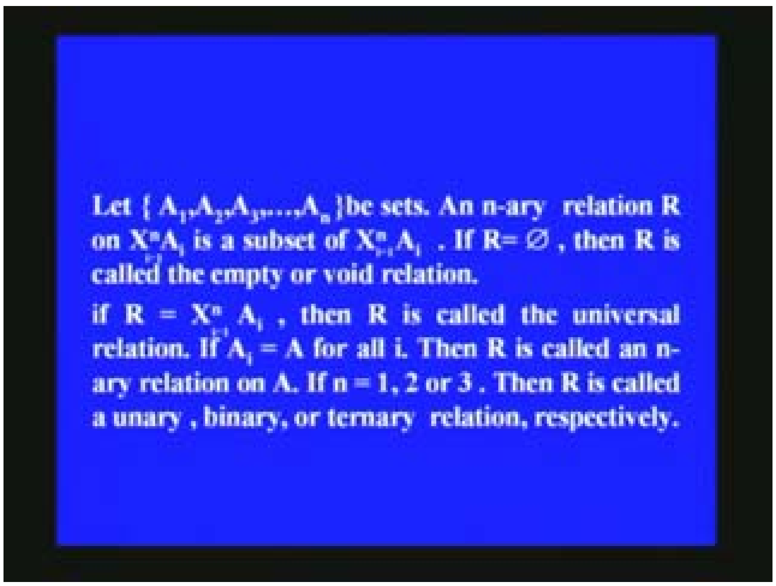

A relation can be represented by a predicate. For example, you take the set of non-negative integers N, then you can say that an ordered triple x, y, z belongs to the relation R if x plus y is equal to z. The sum of the first two non-negative integers is equal to z. So this is a relation on N cross N cross N where N is the set of non-negative integers. So you represent the relation like this: x, y, z and R consists of triples x, y, z such that x plus y is equal to z and this is actually a predicate P(x, y, z). You can represent by any predicate like that.

For example, x was born in city y in the year z is a predicate, so that represents a relation. So a relation may be represented by a predicate like this. Now in this case you can see that 1, 2, 3 belongs to R because 1 plus 2 is equal to 3, 2, 2, 5 it does not belong to R because 2 plus 2 is 4 and you are having 5 here. So this does not belong to R. So it is the subset of N cross N cross N.

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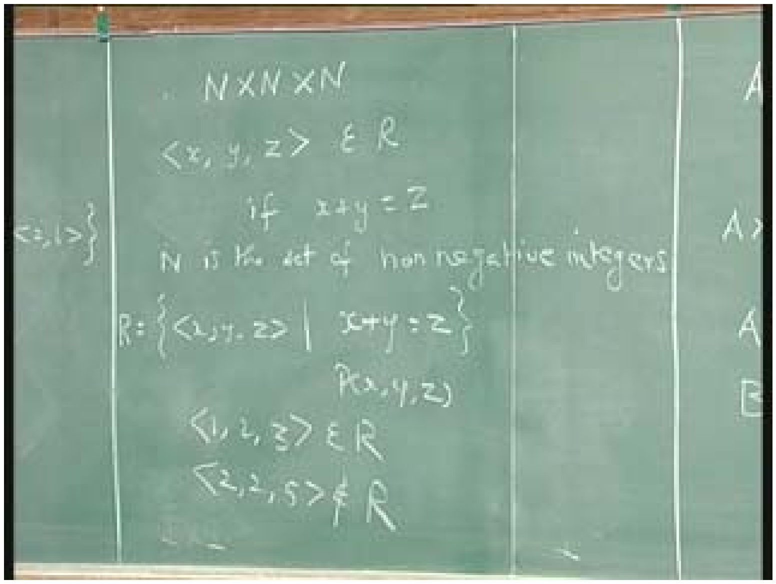

And when can you say that two relations are equal? You have one relation R_1 which is an n-ary relation on i is equal to 1 to n A_i. That is A_1 cross A_2 cross A_3 cross up to A_n and R_2 is an m-ary relation which is on A_1 cross A_2 cross A_3 and so on. We have taken it as Bi so it will be B_1 cross B_2 cross B_3 cross B_m. So you have two relations: one n-ary relation on A_1 cross A_2 this is R_1 A_n and R_2 is an m-ary relation which is on B_1 cross B_2 cross B_m. When do you say that these two are equal. When you say they are equal, when you say R_1 is equal to R_2 then you must have n is equal to m and A_i is equal to B_i for all i and they should represent the same subsets of the Cartesian product of i is equal to 1 to n A_i.

In other words, if m and n are different you cannot talk about the relations being equal and when they are the same they should represent equal subsets of the same set of Cartesian product. The underlying sets also have to be the same and only then you talk about the relations being equal.

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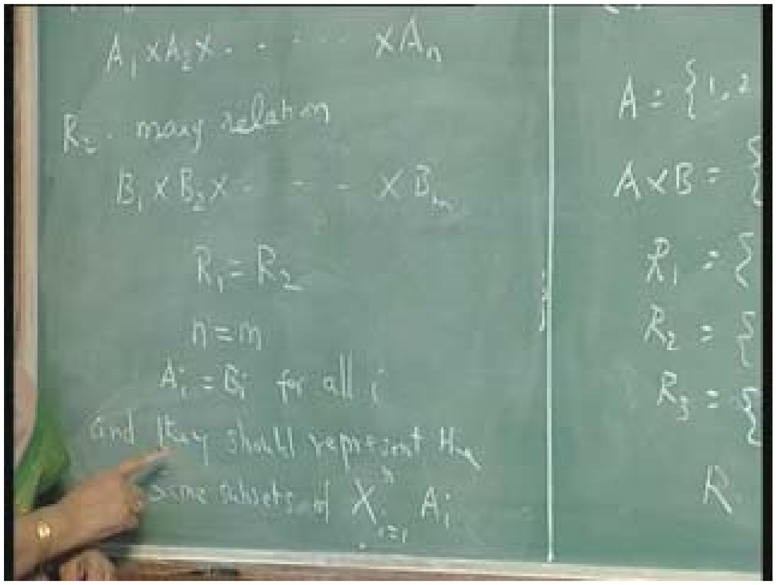

For example, here you have A B and say I have C is equal to a, b, c. Then suppose R_1 is on A cross B and that is 1,a, 2,b. R_2 is on A cross C and it represents the elements again the same 2,b. Even though they are same you cannot say they are equal because the underlying sets are different. So when you want two relations to be equal they have to be defined on the same sets A cross B and then they should also represent the same subsets. If R_2 is defined on A cross B then you can say that these two are equal. So, to talk about the equality of two relations R_1 is equal to R_2, if and only if n is equal to m and A_i is equal to B_i for all i.

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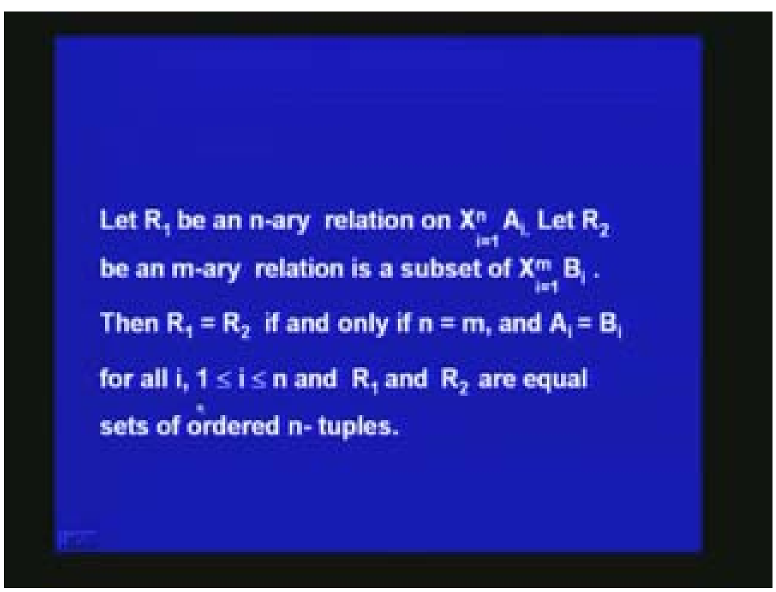

That is the underline sets also have to be equal and R_1 and R_2 are equal sets of order n-tuple. They should represent the same cross product of A_1 A_2 A_3 up to A_n.

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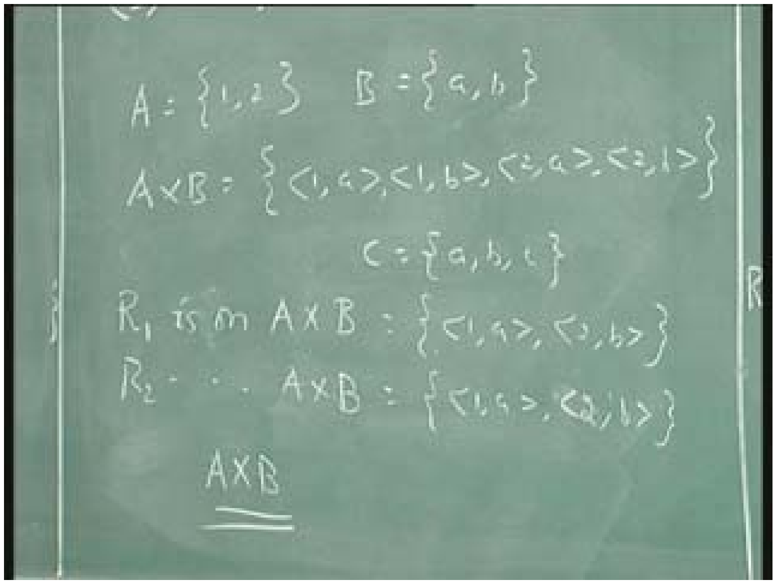

Now we are mainly interested in binary relations. Even though from the data base management point of view n-ary relations are more important. But for specific computer science fields you talk about binary relations and binary relations satisfying some properties and so on.

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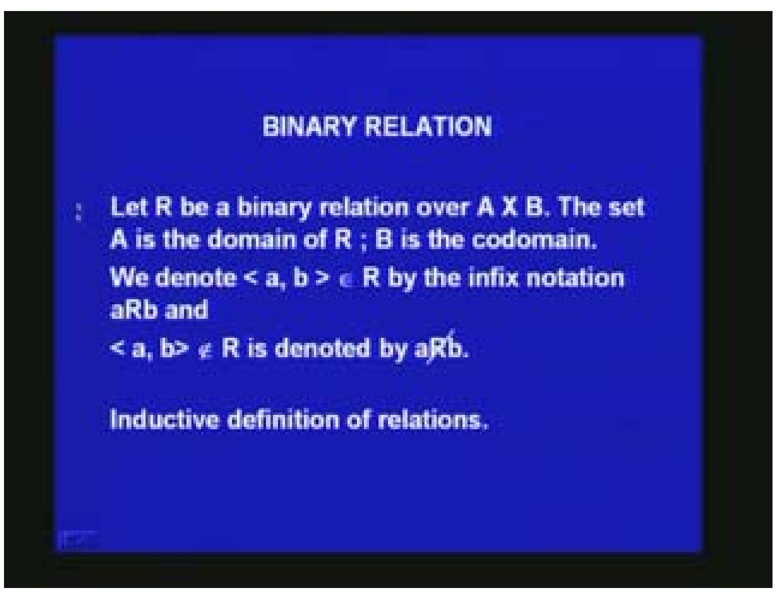

So let us talk about binary relations. Binary relations will be on sets A cross B. Let R be a binary relation over A cross B. The set A is called the domain of R and B is called the co domain. We denote a, b belongs to R by the infix notation aRb and a, b does not belong to R by aR with a stroke b. So, if R is the relation defined on A cross B then if you have an ordered pair a, b belonging to R you represent it as a related to b.

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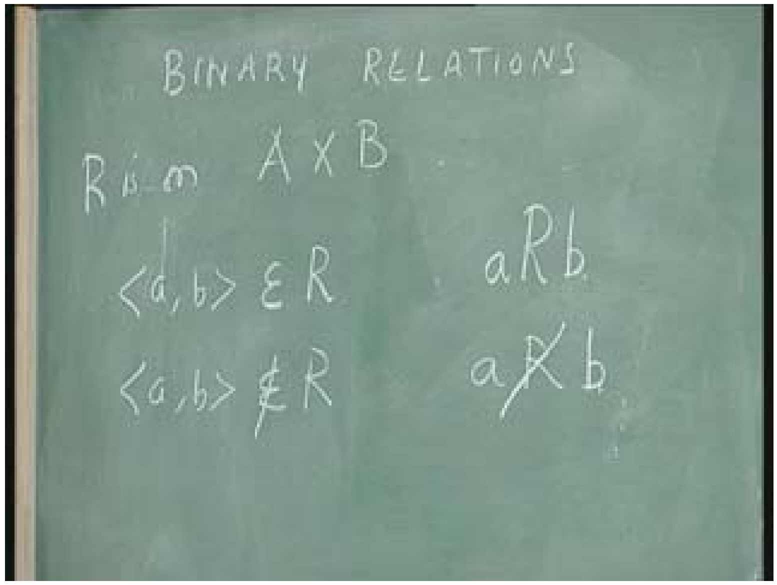

If this ordered pair does not belong to R, you write it as a does not belong to R. For binary relation this is the convention because you cannot extend this for ternary or other relations and only for binary you can use the infix notation. This is called infix notation. For example, take the less than binary relation then you say 4, 8 belongs to less than that is 4 is less than 8 usually you write it like this or 7, 2 does not belong to less than relationship or 7 is not less than 2. You can write like this.

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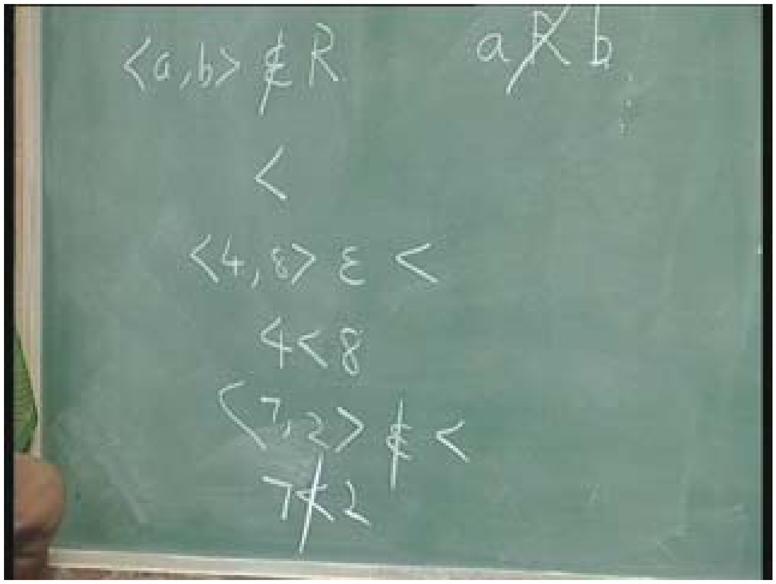

We have earlier seen that you can define a set inductively. What are the main points there? You define the basis clause which denotes the basic building blocks of the sets. Then you define the inductive clause which tells you how do build more and more elements from the already built elements of the set. And then you have an extremal clause. If you look at the relations they are also sets. Relations are also generally sets. They are the subsets of the Cartesian product A_1 cross A_2 cross An n-ary relations. So, as a set you can define them inductively that is when you look at the relations as subsets of the Cartesian product you can try to define them inductively.

For example, let us see how the less than relationship can be defined inductively, inductive definition of the less than relation. Now this is defined on the set of non-negative integers into non-negative integers. We are not taking negative integers but just the set of natural numbers or non-negative integers 0, 1, 2, etc. How can you try to define this? The basis clause is this 0 is less than 1. You can also represent it as 0, 1 belongs to the less than relationship. Then how do you build the more and more elements of this relation.

How do you define the induction clause?
It says that, if x is less than y then x is less than y plus 1 obviously. Also, x plus 1 will be less than y plus 1. If you write in this notation it will be if x, y belongs to less than, x, y plus 1 belongs to less than AND x plus 1 here also this is AND x plus 1, y plus 1. Let us see how we get some of the relations. 0 less than one so you will get 0 less than 2, then from this you will get 0 less than 3 and so on because making use of x less than y gives you x less than y plus 1. And when you consider this and use this, you get 1 less than 2 then 1 less than 3, 1 less than 4 and so on, make use of it 2 less than 3, 2 less than 4 and so on. So you will get all the less than relationship.

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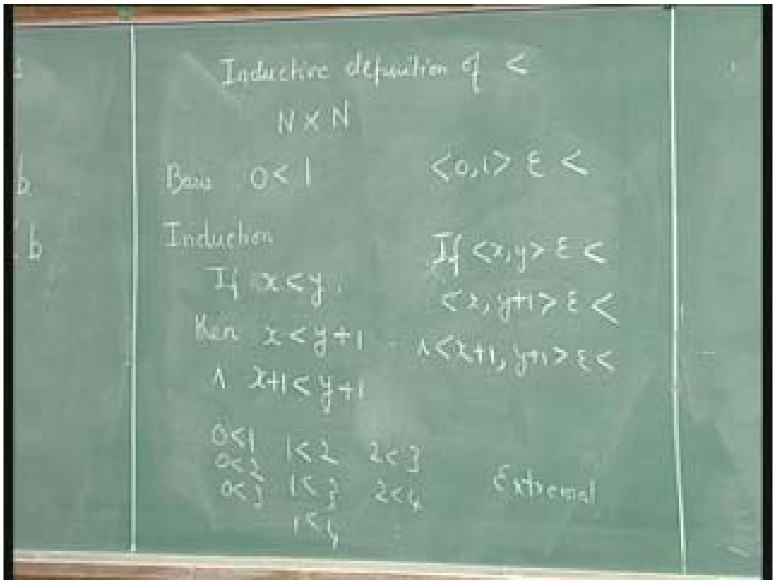

So this is the basic building block. And from this you can make use of the induction clause which tells you how more and more elements of the set can be built or more and more elements of the relation considered as a set can be built. You get all the elements of the relation and of course you have to mention the extremal clause also.

What does the extremal clause say? It just says all the elements can be got from the basis and the induction clause and no other element will be important. So you can also define relations inductively. Let us consider one or two more examples to see how to define a relation inductively. Consider this problem: For each of the following give an inductive definition for the relation R on N. In each case, use your definition to show that the particular ordered pair X belongs to R. So in the first example the relation is defined as this a,b where a and b are non-negative integers and a is equal to 2b so you can see that the pair 6, 3 belongs to the relation.

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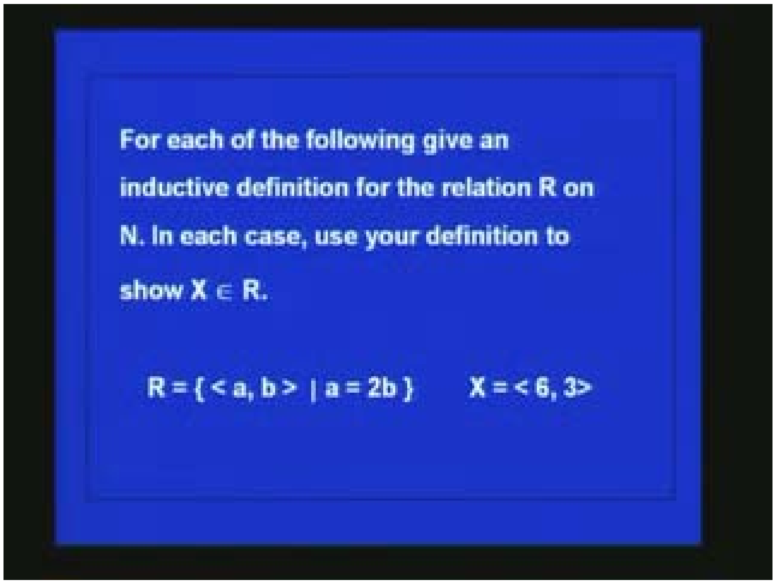

Let us see how to define this relation inductively. So the first basis clause will be 0, 0 belongs to R. You can see that two times 0 is 0. Then how to define inductively? The induction clause will be like this. If x, y belongs to R then x plus 2, y plus 1 belongs to R. So if x, y belongs to R then x is twice y and if you add 2 to x and 1 to y then again x plus 2 will be two times y plus 1. So this is the way the induction clause is defined. So extremal clause is the same for everything. Now let us see how we can derive the ordered pair 6, 3 from this. First from the basis you have 0, 0 then add to the first component and add 1 to the second component where you will get 2, 1. Then do the same again add 2 to the first component and 1 to the second component, you will get 4, 2 then you will get 6, 3. So like this you can get the pair from the basis clause using induction and this is the way the relation R is defined inductively.

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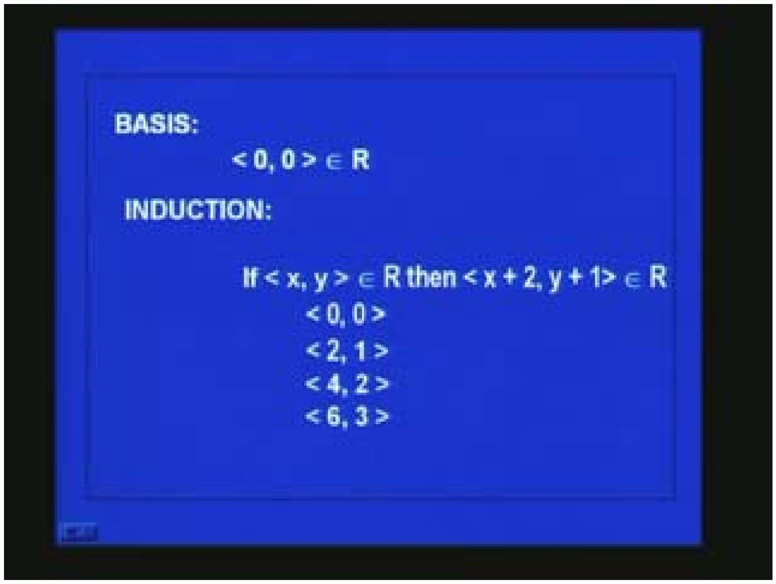

Let us consider one more example. The relation R is defined like this: R is equal to a, b, c where a plus b is equal to c where c is the sum of a and b. And you have to define this inductively and see that this particular triple 1, 1, 2 can be derived from the basis and the induction clauses. So let us see how we can define this relation R using basis and induction clauses. Basis clause will be 0, 0, 0 belongs to R, the sum of 0 and 0 is 0. So this is the basis clause. And how do we define the induction clause? If x, y, z belongs to R then x plus 1, y, z is equal to 1 belongs to R. That means if x, y, z belongs to R means, z is the sum of x and y that is x plus y is equal to z.

Now, if you add 1 to x then the sum will be increased by 1 that is what is given by this. If x, y, z belongs to R then z is equal to x plus y and when you add 1 to the first component the sum should be increased by 1, that is the third component should be increased by 1. Also, keeping the first component the same if you add 1 to the second component then also the sum will be increased by 1 that is the third component has to be increased by 1. So if x, y, z belongs to R, then x plus 1, y, z plus 1 belongs to R and x, y plus 1 z plus 1 also belongs to R. This is the induction clause. Of course, for all examples the extremal clause is the same. It is the smallest set built from the basis using the induction clauses.

Now let us see how this particular triple 1, 1, 2 can be derived from the basis and the induction clauses, 0, 0, 0 belongs to R and this is the basis clause. Then, add 1 to the first component and 1 to the third component. That is if x, y, z belong to R add 1 and 1 to the first and the third component, then you get this. Then by the second method you add 1 to the second component and the third component. So 1, 1, 2 belongs to R. So 2 is the sum of 1 and 1, that is 1 plus 1 is equal to 2 which belongs to R. So you are able to derive this triple starting from the basis using the induction hypothesis. This is how relation can be defined inductively.

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Now a binary relation can be represented as a graph, a directed graph. What is a graph? A graph has a set of nodes what are called as nodes or vertices. Here a b c d e f g are all nodes. Then, you have an arc between nodes, you have a directed edge they are called arc or edges between certain nodes like this. So a graph consists of a set of nodes and a set of arcs.

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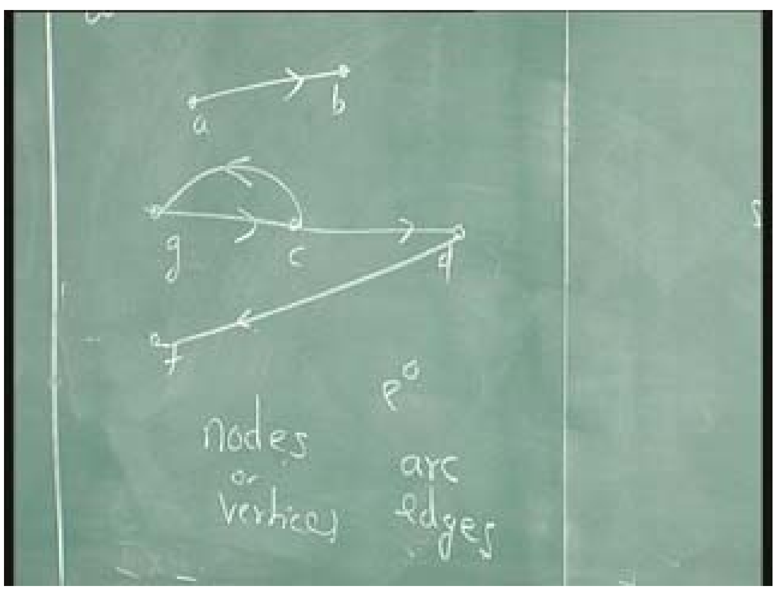

A binary relation can be represented by means of a graph. That is what we have to consider next. A direct graph or digraph is an ordered pair D is equal to A, R where A is a set and R is a binary relation on A. The set A is the set of nodes as points and vertices of

D and the elements of R are the arcs as edges and lines of D. The relation R is called the incidence relation of D.

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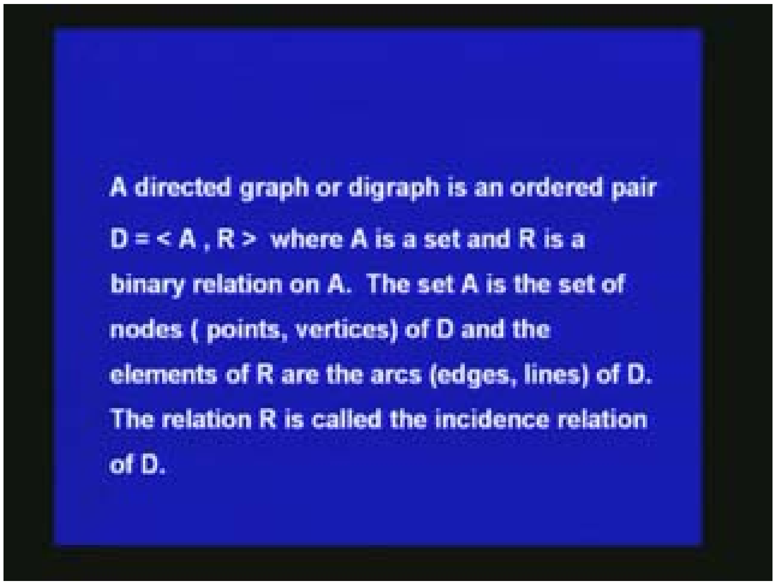

Now here, there is one point we have to note that we are taking a binary relation R on A cross A. We can also take A cross B. I will just come to that point in a moment. Now A cross A represents a set of elements. For example: take the elements 1, 2, 3, 4, 5 and the less than relationship. You have already considered the less than so let me again take less than relationship. So 1, 2, 3, 4, 5 will be represented as nodes and 1 is less than 2 so you have a directed edge between 1 and 2, 1 is less than 3 so you have a directed edge between 1 and 3, 1 is less than 4, 1 is less than 5, 2 is less than 3, 2 is less than 4, 2 is less than 5, 3 is less than 4, 3 is less than 5 and 4 is less than 5. So this is a graph which represents the binary relations R among the set of integers 1, 2, 3, 4, 5.

Please remember that these need not have straight length, you could also draw like this it does not matter it could also have a curved line. So this is called a directed graph because the edges are all directed. If you have nodes connected without the direction which is some thing like this then this is called an undirected graph. Later on when you study graphs we will talk about undirected graphs also. So a binary relation on A cross A can be represented as a graph like this.

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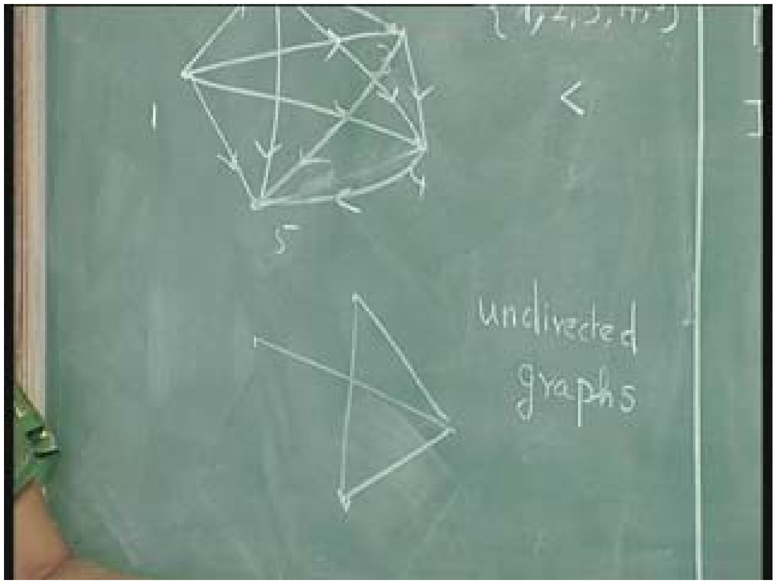

Let us see the definitions once again. A directed graph or diagraph is an ordered pair where A is the set and R is the binary relation on A. The set A is the set of nodes and the elements of R are the arcs or edges of D. The relation R is called the incidence relation of D. Now you may consider why we have taken A cross A, what happens if we consider A cross B? There is no problem, as in the ordered pair you can represent the elements of A and represent the elements of B like this and any ordered pair will be a directed edge from this side to this side which is like this. It can also be represented like this.

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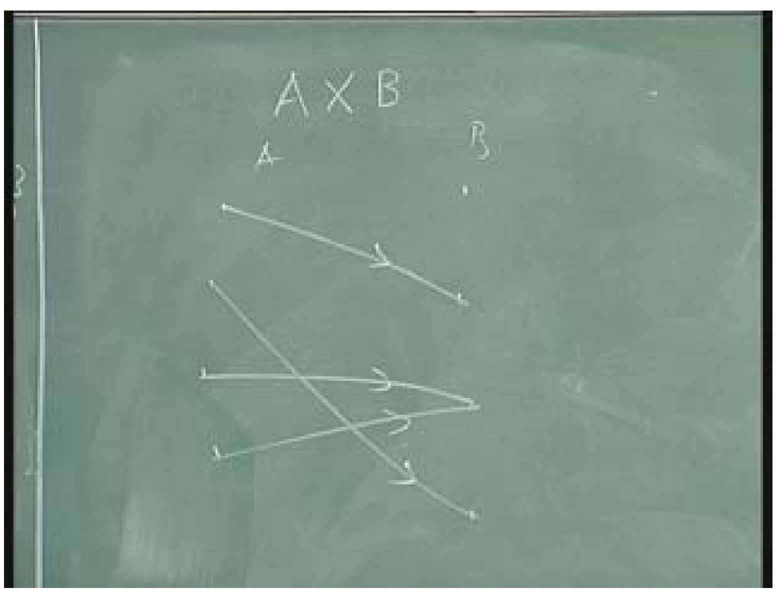

Now, when you have a directed edge from a node to another node for example: from node a to another node b which is a directed edge like this, then a is the source node and b is the destination node. In a graph, from node a, you may have some 3 or 4 nodes starting from that so from a you may have some arcs leaving the node a, that is called the outdegree of the node. And for node b you may have some arcs entering into it like this. The number of arcs entering in to that is called the indegree of the node. If you have a node and an arc like this, it is called a self loop.

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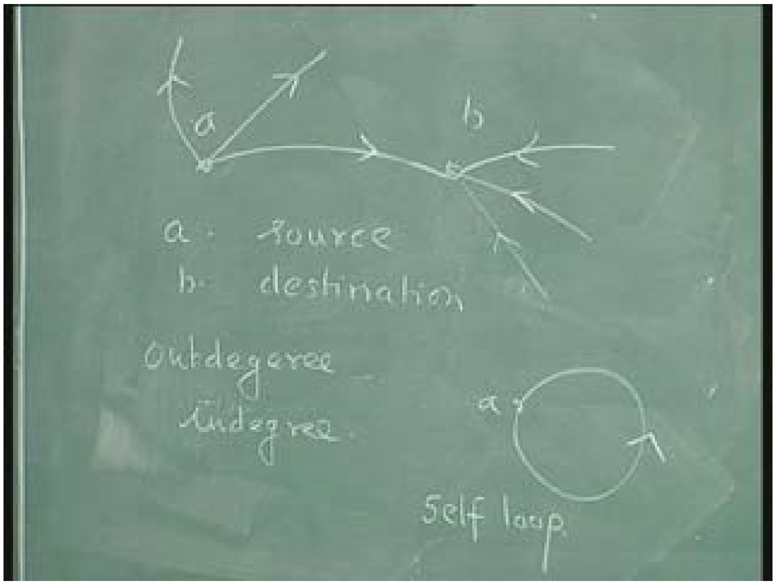

Let D is equal to A, R be a diagraph. If a is related to b then the arc a, b originates at a and terminates at b. in an arc of the form a, a is called a loop. The number of arcs which originate at node a is called the outdegree of the node a. That is the number of arcs leaving the node a is called the outdegree. The number of arcs which terminate at a or the number of arcs which enter into node a is called the indegree of node a.

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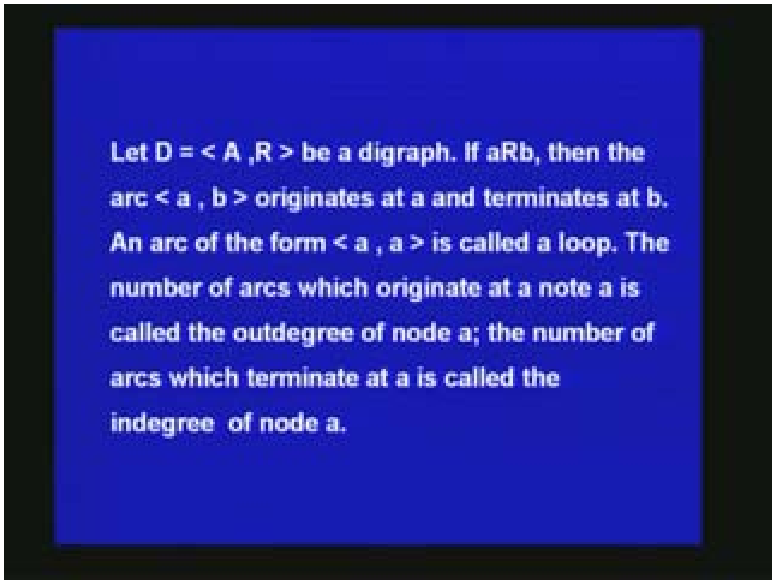

Let us take some more examples when we represent binary relation as a graph, one or two properties of graphs. Now look at this graph abcde. Then you say that abcd is a directed path. Let D is equal to A, R be a digraph with nodes a and b, undirected path p from a to b is a finite sequence of nodes c_0, c_1, c_2 such that c_0 is a and c_n is b and for all c_i such that i is equal to 0 to n either c_i is related to c_i plus 1 or c_i plus 1 is related to c.

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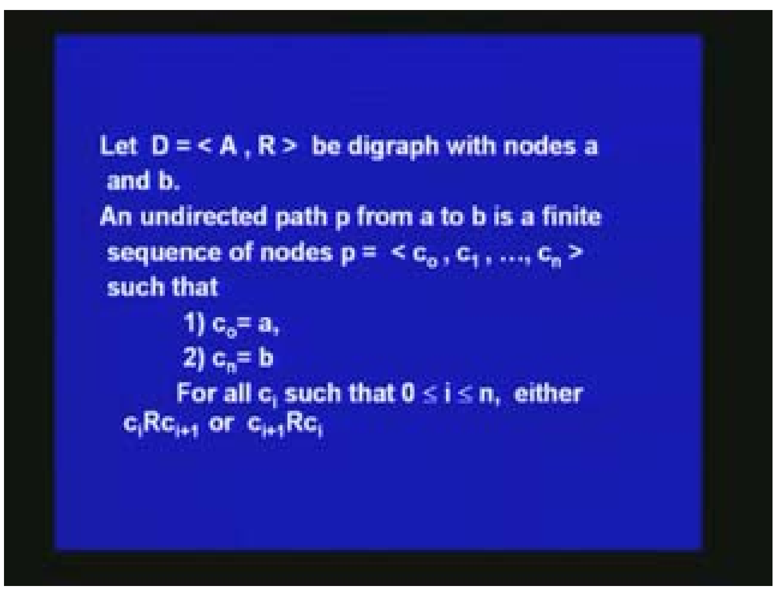

For example, here I may add one more R like this. So abcde is an undirected path because abcd is directed because a is related to b, b is related to c, c is related to d, so abcd is a directed path. But you have e related to d not d related to e if you consider this path the direction changes here so abcde is an undirected path but abcd is a directed path. It may include the loop also. So you have abcdd which is another path, abcdd where you can go through this loop several times then e this is undirected and abcdddb is directed and so on.

Now in that path c_0 to c_1, if no node is repeated it is called a simple path. Now if you look at this bcdb it starts and ends at b but no node is repeated here. This is called a cycle. And if no node is repeated here, the beginning node and the end node are the same. But if no node is repeated in between this is called a simple cycle.

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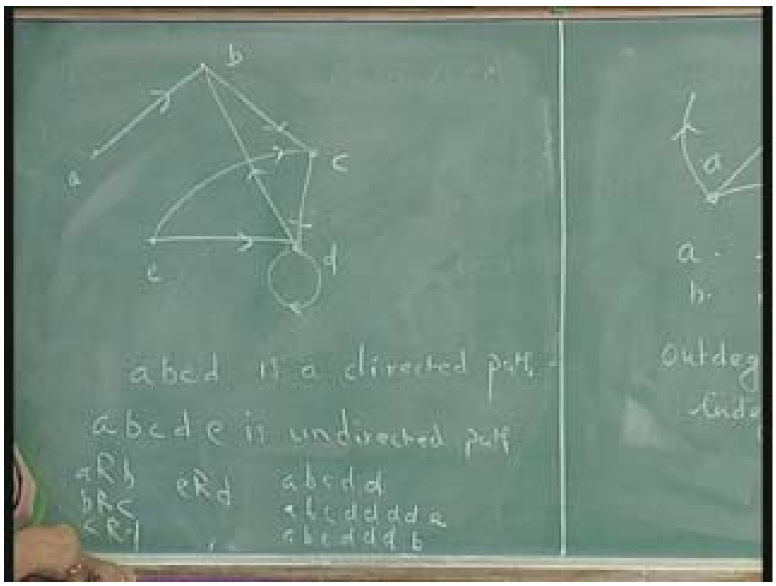

Let us consider this example. For each of the following sketch a diagraph of the given binary relation A. State whether the diagraph is disconnected connected strongly connected and state how many components. We shall see this portion after defining disconnected, connected and strongly connected components. But let us see how to draw the graph for the binary relation. Take this particular example. The set consist of 1, 2, 3, 4 and there are three ordered pairs 1, 2, 1, 3, 2, 4.

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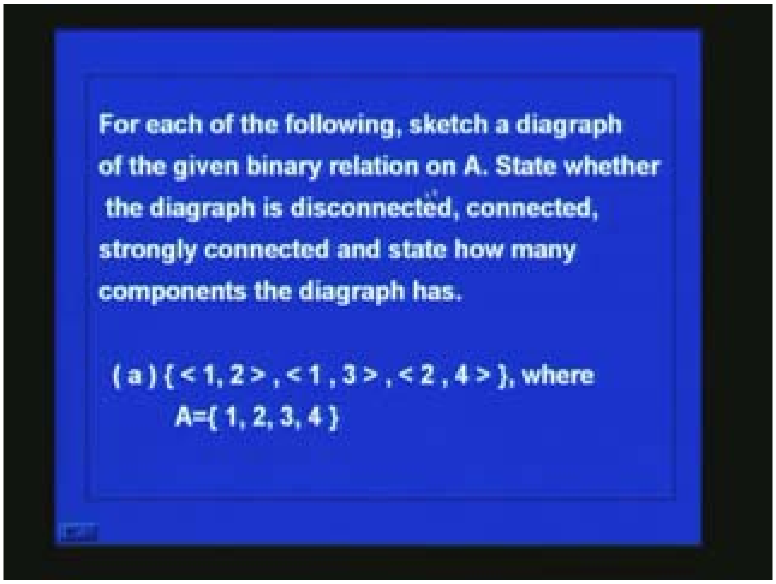

So how do you draw the graph?

It has got 1, 2 so there is a directed arc from 1 to 2, it has got the ordered pair 1, 3 so there is a directed arc from 1 to 3, it has got the ordered pair 2, 4 so there is a directed arc from 2 to 4.

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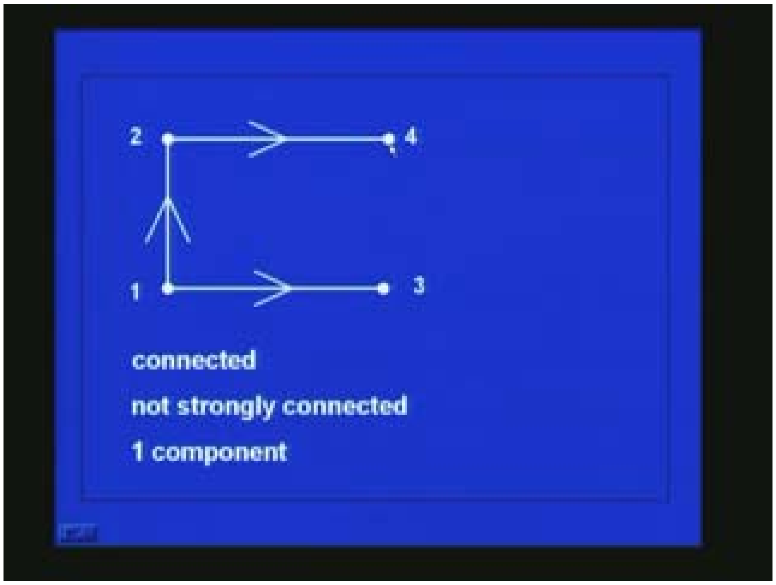

Let us consider the next example x, y where x, y can belong to 0, 1, 2, 3, 4 and the condition is that x should be between 0 and 3, y should be between 0 and 3 and x should be less than y. So how do you draw this? The underlying set consists of 0, 1, 2, 3, 4. So there is the node for each of the element of the set and the ordered pair (x, y) belongs to the relation if x is less than y and x, y are between 0 and 3. So there will be an arc from 0 to 1 because the ordered pair 0, 1 will be there, ordered pair 0, 2 will be there, the ordered pair 0, 3 will be there, the ordered pair 1, 2 will be there and 1, 3 will be there. This will satisfy the condition and also 2, 3 will be there.

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So the arcs can be drawn in this manner. There is an arc from 0 to 2, there is an arc from 0 to 3, there is an arc from 1 to 2, there is an from 1 to 3, and then there is an arc from 2 to 3 because the condition says that x and y should be between 0 and 3. If you have included four also there will be more arcs. But it is not included. So this is the diagram representing the relation defined by this.

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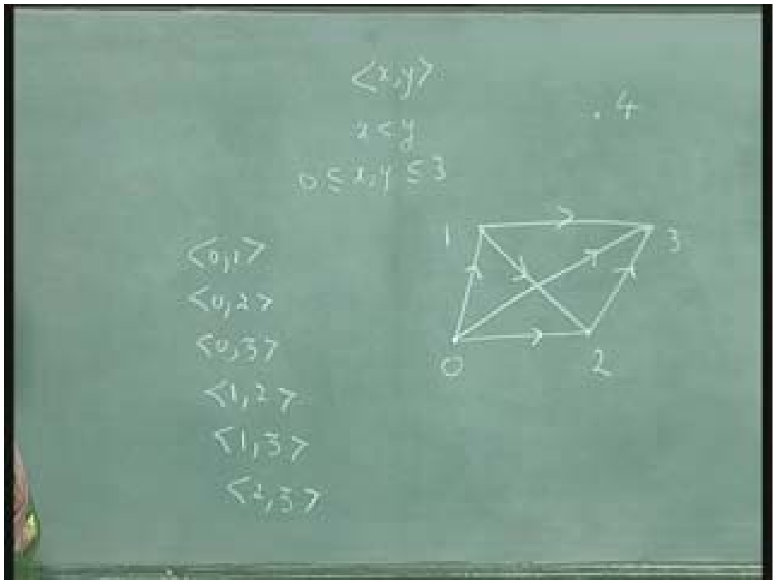

These are some of the basic terminologies about a graph. The main point here is that binary relations are represented as graphs. And we have seen a few terminologies about graphs. We shall consider more about binary relations and a little bit more about graphs in the next lecture.