Computer Graphics
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Lecture - 6
Transformations in 2-D

Welcome everybody. We continue the lectures recent Computer Graphics and today we start to discuss about transformations in two dimensions. Talking of geometric transformations or affine transformations the terms are used almost interchangeably. We are talking about transformations of points, lines and object shapes in a two dimensional screen. That is the topic of our lecture today, transformations in two dimensions or in 2-D. Two dimensional transformations and the use of linear algebra or matrix theory are almost very interlinked. In fact I should say for the first hour today we are going to have a little bit of mathematics in this course which is necessary to understand about two dimensional transformations.

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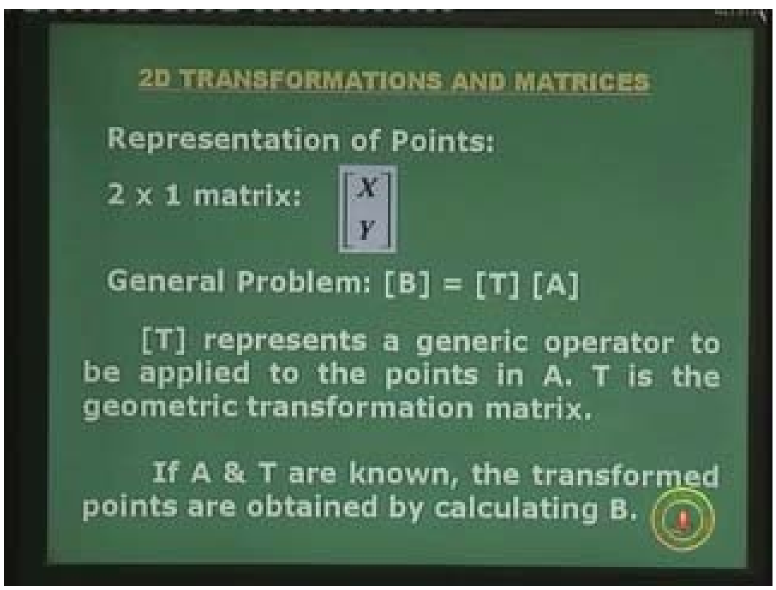

Matrix manipulations are mainly to do with multiplications and additions, a little bit of inverse transpose properties of matrices you must know I believe. And if we have not probably touched upon it in the resent past I will advise that you go back to your basic mathematical books or competitions in engineering let us say and look back into concepts of matrix and properties of determinants, matrix manipulations and simple things like that.

So we are talking of transformations. So if we look here we are talking about representation of a point in a 2 into 1 matrix that means it is a column vector with one column and two rows, the two corresponding elements in the column or just the x and y coordinates of the matrix. We assume in a graph paper, the origin in the left bottom there is a point x and y so in that also a matrix form is represented.

We are going to apply transformation on to this point, we are going to move it any where on the screen depending upon where you want, you could move a line, or a or a point, or a structure and we need to see how to apply these different types of transformations using basic matrix theory or using matrix manipulation techniques. The general method of implementing a transformation is to apply operator T which is again a matrix on the operator A. A is the coordinate of the points or point on which we are going to apply the transformation and T is the operator matrix and the multiplication of T with A gives you the position of the new point or the points with B. So that is the basic idea of transformations and it is basically matrix manipulation I was just talking about is what you have to know.

So the T represents a generic operator to be applied to the points in A, A could be a single point or more than one point we will see because we had seen in the concept of raster refresh graphics displays that typically when we have to draw objects including lines, polygons and areas typically all those are build up with points, it is a mixture of points and so when we transform the objects we have to transform all these points because the shaping function must be kept almost intact in general, of course there are certain cases where the shape information also may not be intact after the transformation takes place.

So we apply the T on point or points in A and T is called the geometric transformation matrix. It is also called the affine transformation matrix in general world it depends upon what functional form you are using and based on that you might get different types of transformations. So the right hand side of the equation B equals to A is known and if A and T are known the last line it says the transform points are obtained by calculating B and the points in B in which the transform points are obtained by a simple matrix multiplication of T with A.

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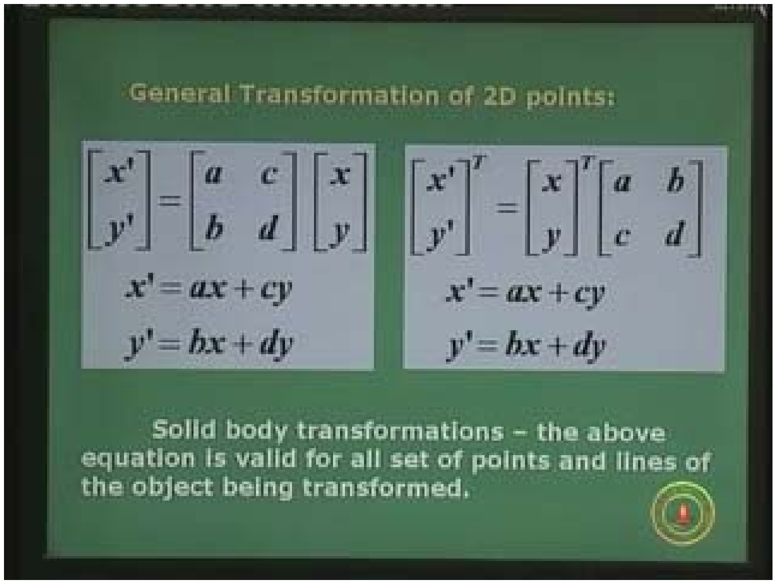

General transformation of 2D points: Well, before we move forward with the different types of transformations I would like to give you a little bit of the inside to this matrix multiplication. There are two ways you can multiply a matrix; either put the matrix B equals A T we can some times say when T A instead of A T. If you look into the general form of matrix multiplication here we are talking about xy the point coordinate in the graphic screen which is premultiplied, why premultiplied? Because this operator T has four parameters, the scalar four elements of the matrix T are a b c and d as given here.

And if we premultiply you get the new coordinates or transformed coordinates x prime and y prime. If you know the matrix multiplication which is obviously very simple, you can easily see that first you get two linear equations or scalar equations from the vector of matrix multiplication equations here where x prime the transformed coordinate, x coordinate of the transformed point is ax plus cy and the y prime is bx plus dy. So if you multiply you get this as you can see, the row and then the column multiplication the x prime is ax plus cy and if you take bottom row of T and the column of x that is we have y prime equals bx plus dy and that is typically what you see.

I can represent this transformation. As you see here by different order of multiplication or if you look into this on the right hand side I am saying that the coordinates of a point P given as x, y is now been post multiplied. The difference between post and premultiplied, the upright T is on the right hand side of b, in the previous case it was left hand side that means it was pre multiplied but in this case it is post multiplied but of course you must be careful here of two things that we are saying that the coordinates of a point P given by x and y or x prime or y prime is a row vector.

Did you see that transpose sign on top of x and y and x prime y prime on the equation given on the right hand side block and so we are talking of the row vector and to keep the consistency of the equation in terms of the interpretation of a b c d the four elements of the operator matrix T, what you see now that the position of the element especially the off diagonal elements b and c have been change in T.

You please notice that the equations in terms of x prime equals whatever y prime equals whatever ax plus cy and bx plus d y and that equation is kept unaltered and to keep that unaltered and to handle both forms of premultiplication and post multiplication of the point coordinate P with the operator T2 changes have to be made first of all the column vector of the point coordinates becomes a row vector. And the matrix T is basically a transpose of the previous matrix because the diagonal element are there where ever it is a and d but b c have interchanged positions. So basically it is a T transpose of the matrix. So you can use anyone of these representations given either on the left hand side or on the right hand side it is absolutely no problem. And I should mention here that some books follow the left hand side or the premultiplication notation and on the other hand some books follow the post multiplication.

But you should actually keep a watch on what form you are using and what you are comfortable with. You can use any one of them but never mix. Never mix in a equation when you are using it for implementation or driving anything analytically, please do not mix up the representations of pre and post multiplication. Interchangeably you can use any notation you like in terms of pre or post multiplication but be consistent. Be consistent in the sense that, if you start with premultiplication follow the premultiplication representation of matrix multiplication to implement geometric transformation throughout your derivations or program implementation. Whereas as if you are starting with the post multiplication, transpose and all that you better stick to that please do not keep swapping.

In the course I have probably tried to stick to it but once in a while I will probably be in the other example also to just give you an idea of what is the little difference. In terms of implementation and end result it is all same.

Why all same? Because you see the expressions here; x primes equals and y primes equals or whatever is given is the same. The corresponding coefficients of x and y after transformation is a x where a for a x and b and c for c and d for y respectively. That is kept unaltered because if you keep that unaltered the significance of the four elements a b c d of the operator T will remain the same. Their significance, their method of whatever they are going to transform is going to remain unaltered whereas in terms of matrix you can either use post multiplication or premultiplication which is absolutely no problem. But again I keep repeating this that you must follow it in a methodical manner.

The other point about general transformations of 2D points is the following that we are only going to discuss in this course and obviously in the lecture today and the remaining lecture on transformations whenever it comes. Of course after 2D we are going to 3D transformations but even in 2D as well as in 3D we are going to discuss solid body transformation. Thus you see in the slide, here we are talking about solid body transformations. That means the wave equation whichever you pick up, pre or post is valid for all set of points and lines of the object being transformed. This means when you transform an object from one position to another and give it any sort of transformations we say that the transformation equation is applicable not only for the entire object or the object by itself. But whatever you trace on the object for example; all points or lines it is also varied for all those points.

And solid body talks about that the body does not change its shape or size in general and its a solid body in the sense that the corresponding displacement or distance between any two points that is we draw a line on the solid object before transformation and then transform it. You start at one point and then transform it into the other point, the inter point displacement or the distance between any two points is going to be invariant with these transformation and that is the meaning of solid body transformation.

For any solid object we will follow these rules and norms. Of course what does it mean? What is the other part of the world? Yes we are talking of deformable object structures typically it could be a clock, it could be a paper, it could be a rubber sheet, any soft products such as a sponge etc are a few examples of deformable structures or nonsolid objects. We are not going to discuss about those because that is a separate area by itself. It is in fact the area of big research to represent those transformations of non-rigid structures represent transformations as well as deformations of those structures. But what we are going to discuss is only solid body.

You take any object it could be a dues ball, or a cricket ball being played on the field to the normal table, chair and whatever is in front of us, all solid objects undergo transformations and the entire object as well as all points, and all lines, curves which you draw on the object will basically follow the same pattern of transformations and the object will not typically change its structure except in one particular case. But in solid object typically we assume that the body does not change the structure although transformations in some cases may appear to change the structure.

We will look at some special cases of 2D transformations. The simplest one, you remember the transformation matrix T and the case when the diagonal elements, you remember the structure a b c d, it will come back again to that form. When the diagonal elements a and d is 1 and off diagonal elements or non-diagonal elements as it is called b and c are equal to 0 and we are basically getting an identity matrix. An identity matrix does not change the position of the objects. This new position x prime y prime of a point or all points of the object are going to be the same as the previous or old data point. The body does not change its position, identity matrix does not provide any transformations.

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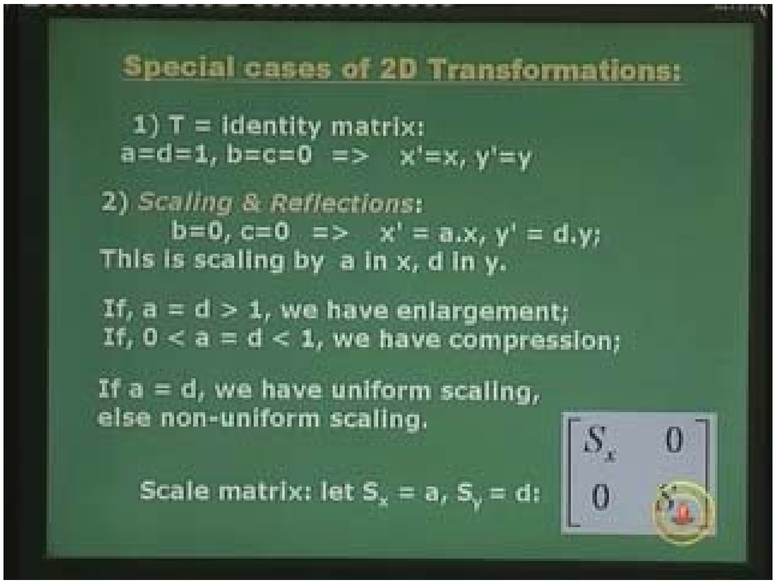

Virtual matrix is non-identity and it provides different types of transformations. We will start with the true transformation which is provided by similar type of structures or properties of the transformation matrix, one is scaling and the other is reflections.

Scaling or reflections are caused by the diagonal elements of the matrix T. We are talking of the off-diagonal elements or non-diagonal elements b and c both equal to 0 and we have the diagonal elements a and d let us say they are non 0 0. Of course if all elements are 0 typically you are shifting the point to the origin and nothing happens basically. But in this case of scaling and reflection as you see the new coordinates x and y are given by the equation as given in the slide here.

Here x prime is a, multiplied by x, y prime is d multiply by y and you are talking of pre or post it does not matter, you get the same form and we are saying that we are talking of scaling in this case where the x coordinate is transformed by a parameter or the element a of the matrix T and the coordinates y of the point is scaled by the element d or the parameter d of the matrix, we can use the word element or parameter of the matrix because these four parameters a b c d are responsible or four elements of the matrix are responsible for any type of transformation you are going to implement.

Well, assuming b and c are both 0 as given here we are talking about scaling to start with, we will move to reflection and we know that the x coordinate are scaled by the parameter a and the y coordinates are scale by the parameter d. If both a and d are same and they are more than 1 that is they are greater than 1, we have enlargement that means the body zooms out and enlarges in size and scale, it scales up, it grows or expands or enlarges. That is the concept which we have when the diagonal elements a and d both are equal. If they are not equal we will come to that in a moment. But if they are equal then we have more than 1, we have enlargement and if both are equal but they are less than 1, not negative, you must very careful here, there is a fractional number a and d are both equal and a fractional number between 0 to 1 we have a compression or reduction in the size or the scale of the object that used to be compressed. This is what we have as scaling. Now when both a and d we have are same what we call as uniform scaling, uniform means the amount of scale, whatever happens either enlargement or compression, in both the x and y axis are same and that is why we say that the scaling is uniform.

You can have non-uniform scaling also, so that is what we are talking, if a is equal to d if the parameter diagonal elements a and d are same we have uniform scaling so we can have uniform enlargement and you can have uniform compression depending upon both being same and the value being more than 1 or less than 1 respectively, we have seen that in the equations.

And we can also have non-uniform scaling where a and d are not identical, they are not same. If a is not equal to d we have non-uniform scaling and if a is equal to d you have a uniform scale. So we talk of a scale factor usually when we do scaling and these parameters a and d can be replaced by the corresponding parameters Sx and Sy where Sx represents the scale factor S stands for scale along the x direction and Sy stands for the scale factor along the y direction. So we have scale factor along x which is the parameter a the scale factor along the y which is the parameter d and they are the corresponding diagonal elements. Here we are not concerned about the off-diagonal elements for the time being. Both are kept at 0 and so that is what the scaling will result due to the diagonal elements a and d. We will soon see an example of scaling.

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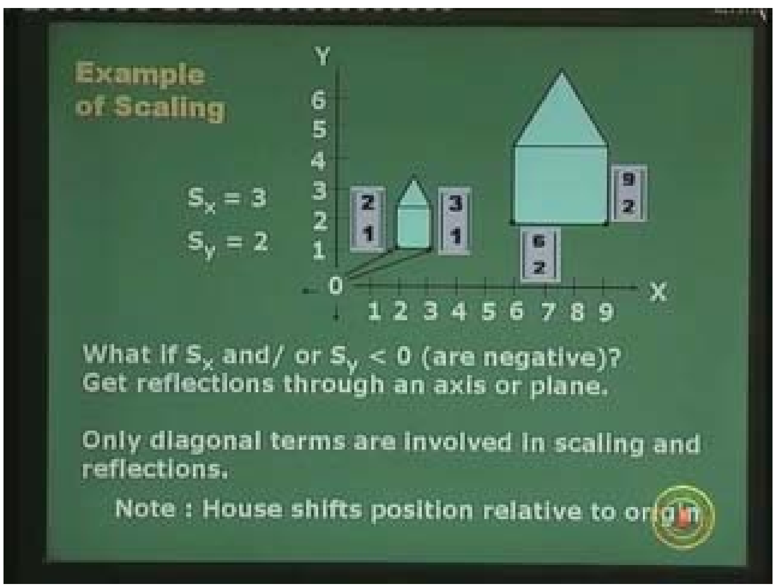

Example of scaling is a highest example I have taken. A simple 2D structure and the entire structure is scaled but what I have marked here is the base of this structure which looks like a church or a house whatever with a triangular roof on the top. I have marked these two points at the base of the structure here and the corresponding coordinates is also given which is 2, 1 that means x, y that means the first point is at 2 and 1 x coordinate is 2 and y coordinate is 1. The other point is x coordinate is 3 and y coordinate is 1. We can say the x and y axis is approximately aligned and is shown 3, 1 and 2,1 and what I do is this entire structure is scaled with a transformation matrix where this scale parameter along x direction is 3 and the scale parameter along y direction is 2. So basically in this we have a non-uniform scaling number 1 and since both these parameters are more than unity or more than 1 we have non-uniform expansion and non-uniform enlargement.

You have non-uniform because a is not equal to b or Sx is not equal to Sy and you have an enlargement or expansion because both the values are more than 1. If the both the values would have been less than 1 you could have uniform compression or non-uniform compression. So let us look at the example which shows an example of non-uniform expansion from the left to the right and this structure becomes something like this.

As you see here if you multiply this matrix elements 2, 1 and 3, 1 with this matrix with the corresponding diagonal elements 3 and 2 off diagonal elements 0 the 2, 1 will become 6, 2 and 3, 1 will become 9, 2 so that is the transformation. The values are shown for only two points but the same amount of scaling with respect to the govern equation of transformation will be applicable for all the other points and lines of the object.

Of course the lines are little bit tricky but I can assume that there are 5 vertices in this structure the bottom 2 are shown. The values are shown basically and then the top there are 3 again. So those 3 and the bottom 2, all this 5 are transformed in an identical fashion and the values for only two of these points are shown. So you can see, this is an example of a non-uniform enlargement or expansion. You can actually visualize the non-uniform compression also with this figure. How?

If I say that my Sx is 1 by 3 and Sy is 1 by 2 that means I am trying to compress this enlarged figure back to the original figure. That means when expansion I am moving from left to the right that means the small figure is enlarged to the right hand side figure. But I can also ask you to visualize the reverse process or the inverse process as it is called the matrix inversion will slowly come as we go along because every process is almost a reversible process here, so you should be able to start with the largest structure and give it a compression or a reduction and scale it down to the smaller structure.

So in this case you will need a non-uniform reduction or compression and the corresponding parameters Sx and Sy I leave it as an exercise for you and you can try it out, it will be nothing but just the reciprocal. You still have the off-diagonal elements to be 0 and the diagonal elements to be non 0 and those where as just 1 by 3 or 1 by 2 respectively. That means just 1 by Sx and 1 by Sy respectively. That is the inverse of the matrix with the diagonal elements.

You should be able to check it out and apply the same on the point coordinates to see that from 6, 2 now you get 2, 1 and from 9, 2 you get 3, 1. You can find out the other coordinates also. Here also some of them could be in a floating point or integer it does not matter but you should able to get back the corresponding coordinates. So we have been talking about scaling and the values of Sx and Sy which are the diagonal terms which we have talked about and that if it is equal or unequal or whether they give non-uniform or uniform scaling and so on. And if it is more than 1 you have an expansion, if you have less than 1 but greater than 0 that means fractional number gives ration or a compression.

What happens if I go to the other part of the scale that means if I make these diagonal elements negative? That is what is given in the slide, what if Sx and or Sy are negative or less than 0 that means either one of these or both. We will see examples when one of them is only 0 and the case when both are 0. We will check both of these Sx and Sy either or both are 0. You do not get a scaling but we will see with an example now that you get reflections through an axis or plain. What is the reflection? You typically have a reflection on a mirror when you stand in front of a mirror you see yourself.

Basically you see a virtual image, you do not see a real image. We will talk about the difference between virtual and real image, not now but later on because there are various concepts to these in terms of a virtual and real image which you see, in fact the image you are seeing now is basically a virtual image although it is being recorded live and real but when you are seeing it using a monitor or a devise or a TV you are basically watching a virtual image. But anyway, you can use a mirror to provide a reflection and that reflection image which you see with the help of a mirror either of yourself or for any other object is the classic and the best example of reflection or the mirror image. You do have mirror reflect effects when you are moving in a desert as well which is called a mirage. But in this case you can take a mirror and position it next to an object and you will see the reflection.

I am talking of those sort of reflections, but in this case since we are working in the two dimensional environment there is no question of a mirror standing and anywhere within the picture itself we have to visualize that the mirror is a plane which is sitting on your x, y plane because we are having a two dimensional plane, y is the vertical coordinate, x is the horizontal coordinate so it is the two dimensional screen or two dimensional plane and where is this mirror which was giving out reflection?

Well it is a plane which is perpendicular to the x, y plane. It stands or sits on top of this x, y plane, if x, y plane is the base then put a graph paper on the table and put a mirror just perpendicularly on top of that and you start noticing the graph paper or put an object on the paper and observe it through the mirror, you observe two things: First of all you observe the object itself and then you observe the mirror reflection which is in the opposite side of the mirror. You see that the object appears once again, put a mirror in front of the object and you see the object and the mirror difference.

So basically you always talk of a reflection about a plane or about an axis. What is this plane? Of course the mirror is the plane and what is its axis? The intersection of the x, y plane and the mirror plane, two planes will intersect and always intersect in a line or in this case it is planer surfaces or intersect on a line if it is a curve surfaces, you might get a curve or a conic section, we talk about that later on but two planer surfaces always intersect to form a line and so we are talking about that line. So we are talking about reflection through an axis or across a plane not even through a plane.

You can visualize that plane to be the mirror and the intersection of the plane and x, y plane and 2D in this case; the 2D plane which was intersected gives you the axis. So it is about at axis which you have the reflection and you can obtain that by geometrical transformation matrix when you put the two diagonal elements to be negative, or it is enough when one or both are negative.

Come back to the slide, the last point typically says that only diagonal term we must be careful here that we are starting to get into the matrix elements and see how they operate and we are talking about diagonal elements which are involved in scaling and reflection. We have not seen how the off diagonal elements work and we will work on that very soon. But remember now for the time been that we are into scaling and reflection and we already know that the diagonal terms are responsible for scaling and reflection. And when it is scaling? When is reflection, it is very easy now.

Bipolor logic, if it is all positive it is doing a scaling which could be uniform, non-uniform, it could expand, compress what ever it is and as soon as any or both of the terms become negative we have a reflection.

The last point with respect to the scaling or even reflection is that the house also shifts its own position relative to its origin. If you want to scale or compress and keep the object at almost the same or exactly at the same position you have to do something else. In this case if a simple scaling of an object such as expansion, compression or whatever it is it actually changes the relative position of the object or the structure with respect to the origin. Except if the object is around the origin then of course there is no problem but if it is anywhere else like in the house or the structure of the 2D object it shifts its position relative to the origin.

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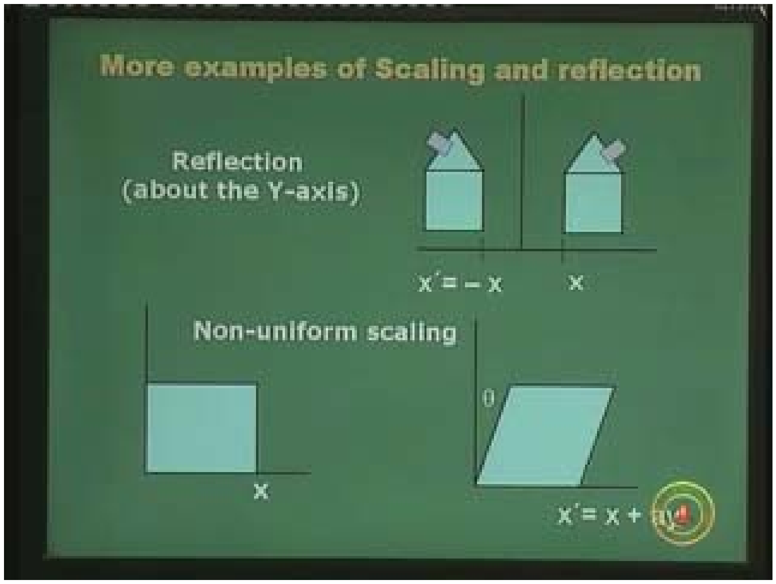

We will go through a couple of simple examples of scaling and reflection and of course I should also encourage you to keep trying these various things in a simple graph paper or write a small program to implement these transformations. Of course you need to use a small graphic standard or a package to implement this. But you can use a simulation tool and you have to taste this out and what actually it happens without worrying about standard like OPENGL figures and implementing c plus plus.

I will probably cover OPENGL at a little point of time but before covering I thought I should cover a few basic things about computer graphics implementation and algorithms and then we will move to discuss about using one such example of a standard of OPENGL but you can use a simulation tool kit typically any simulation tool kits will have small manipulation examples in 2D and 3D. This is an example of a reflection about a y axis as you can see that this can be implemented, think about the transformation matrix T, what you have to do? b and c the off-diagonal elements should be 0 or non-diagonal elements should be 0 and you have to play around the diagonal elements that is a and d. And in this case since you are only reflecting around the y axis and the x coordinates only change y or not, you typically have to play around with the value of a.

In fact the values are very easy to guess, the transformation matrix T having four parameters at this which is a is equal to minus 1, b is equal to 0, c is equal to 0 and d is equal to 1. I immediately guessed it and you should also be able to guess it. So a is minus 1, d is 1, the rest of the off-diagonal elements are 0 sort of a thing. In fact for you I should say that whatever I am seeing is reverse, this is off diagonal elements for you and this is probably the diagonal elements for you. But what I mean is a and d are the minus 1 and 1 respectively b and c are 0.

Would you like to see an example for non-uniform scaling? You saw reflection, you probably saw scaling as well and this is an example of what non-uniform scaling can do.

Now this is a very simple example, but what it is doing is it is trying to violate the basic philosophy or principle of rigid body assumption or rigid body structure or rigid body transformation. The body does not change its shape and size what we talked about in general and that is number 1, all the points are transformed in a similar manner and that is also being valid here. But what happens is, it is only in a few specific examples of transformation that the body may tend to appear to change its shape or structure.

The previous examples of scaling, we will see other examples of rotation, translation, we have already seen reflection, we have already seen scaling and the structure of that house was intact in some sense, the structure in terms of appearance. The shape might change, size might change rather. In this case of non-uniform scaling which you see is done by putting a non 0 value of a, b and c is 0, d is equal to 1 and you have x prime changing linearly with respect to y. So, as y increases x changes and in fact y is unchanged, the typical value of y is unchanged. This is an example of non-uniform scaling. Typically the shape of the object appears to be a little bit different.

Special cases of reflections: As we see, I have put it in a table; we have seen when the diagonal elements are negative and when the off-diagonal elements are 0, which are typically responsible for reflection and reflection about an axis or a plane. And if you see, there are four examples here, now you should start to get little bit worried that the scenario is little different, in two out of the four cases do not worry, I will explain what is happening, if not now definitely in next few slides it will be clear.

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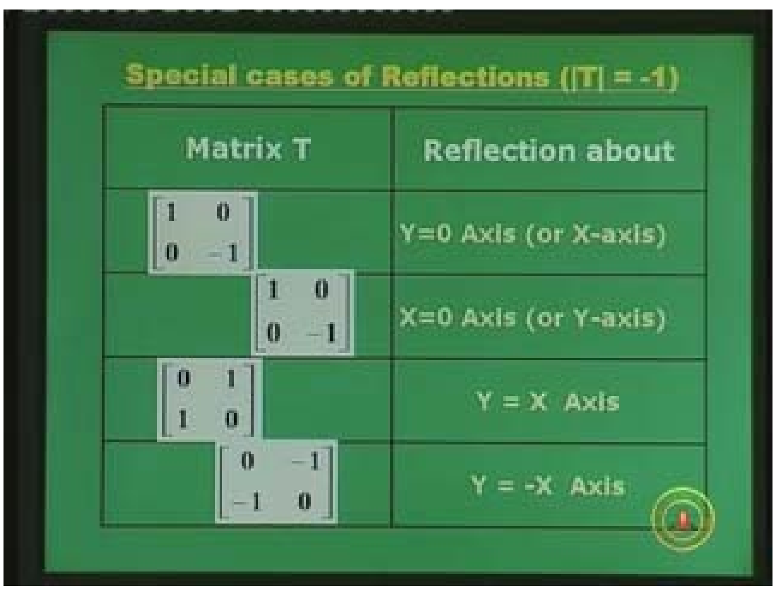

If you go back to the picture, the first row says that a is equal to 1 and d is equal to minus 1, you get reflection about the horizontal axis, y equal to 0 axis or x axis is what you get a reflection in a lot. And of course the previous example which you saw in the figure in the previous slide was a case with the second row where you are reflecting around the vertical axis or x equal to 0 axis or y axis. That is what you are reflecting it around and the both the elements a and d are equal to minus 1. That is what you have for reflection.

Now you would like to reflect it about any axis, reflect an object about any axis in a two dimensional space, there are two more special cases which were discussing now and the general case of transformation, either scaling or reflection about any axis will have to postpone it to later point of discussion may be if not in this lecture or in the next lecture definitely we will know that, but you look at two more special cases, very interestingly you see of the off-diagonal elements are now playing an important role in reflection.

Here we are trying to reflect the object about y equal to x axis, y equal to x axis is a diagonal line passing from the origin with the slope of 45 degree. Actually if you look into the screen and your left hand corner is your origin, the axis y equal to x axis will look like this, look like the line from your view point bottom left to your top right is what you will see. So, if you want to reflect about this y equal to x axis for you then the transformation matrix is as given here. So this is the y equal to x axis which you see here and the transformation matrix is as given by the off-diagonal elements. You can try this with the simple x, y and see what is all this.

So the last example, special case of reflection, if you go back to the slide here the last row talks about a special case of reflection about y equal to minus x axis. That is if this is equal to y equal to x just rotate this axis by 90 degree you will get the other axis which is y equal to minus x are inclined at minus 45 degree angle. Now you can see that both the off-diagonal elements have reversed their sign, both are minus 1, the diagonal elements are basically 0 and the non 0 elements and the off-diagonal. Now, the last two rows are typical example which is violating our previous condition that the reflection is typically governed by the diagonal elements and the off-diagonal elements do not have any role to play.

In the last two rows, last two examples when we talk of y equal to x axis y equal to minus x just the reverse happens. It is interesting to note that the diagonal elements are made 0 and the off-diagonal elements are made 1 or minus 1. In fact we will see later on that this two special case of reflection about the y equal to x axis that is the third case and the fourth case of y equal to minus x axis. These two examples are equivalent to what we will see as the next transformation equivalent to rotation by a large degrees or large amount. So these can be visualized or conceived to be rotation about a point, so that is why the matrix nature is of that. So we will come back to that.

Before going to rotation we will talk about another typical off-diagonal element which are b and c and they are involved in shear. Shear is a case which is not basically similar to an example, typically you can shear an object which is soft. You can take a typical example of a rubber sheet or a cloth or a paper and you should be able to provide a shear.

Take a very large book dictionary which is fairly large in height let us say or a book, a large volume of a book and then what you do is put it on the table and give it a force, drag force on the horizontal direction on the top of the book, what you will see is that the book which is probably sitting upright will typically tend to bend towards the direction of the force and that is the good example of a shear, you can do that. You can do that with the help of a book, a dictionary for that matter and the book which was straight upright sitting will typically bend and shear itself along the direction of the force. So that can be given with the help of this equation as you see here.

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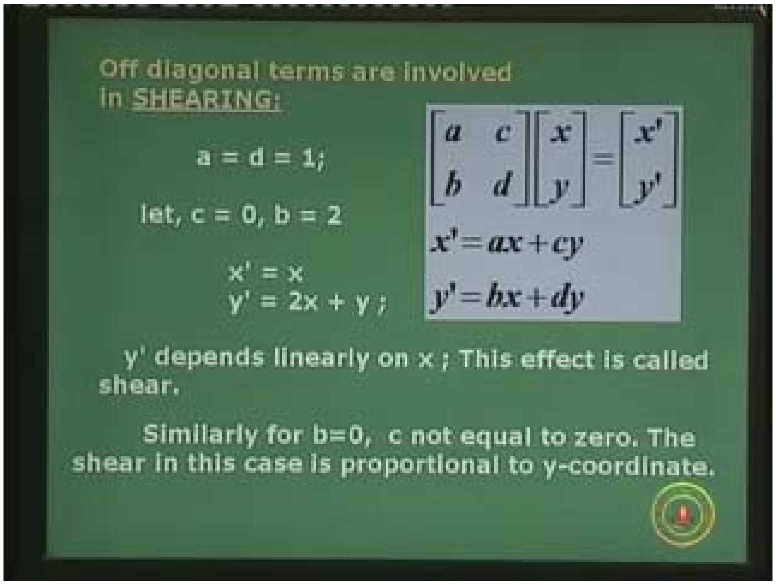

I have just reversed the rotation of the equation which does not matter. We are basically still talking about x prime equals ax plus cy. We are talking about post multiplication or rather premultiplication of the coordinates by the transformation matrix T, x prime is still ax plus cy, y prime is bx plus dy and we have done in the following that we have put the diagonal elements equal to unity and the off-diagonal elements are having a role to play. In this case one of the element is again 0 and b is equal to 2, we have x prime equal to x. You can easily solve this out and find out this equation y prime equals 2x plus y.

You see that this y prime which is the new coordinate in y, it linearly depends on x. This effect is called the shear. If y depends on x and x depends on y it gives an effect of a shear in some sense and you should be able to try that that means more you move towards along y prime, the more the x also is changing. Similarly, you can have an effect of d by putting the value of b equal to 0, we are talking about off-diagonal elements b equal to 0, c could be meant 2 or any value which is not equal to 0, then the shear in this case is proportional to the y coordinate. So we have an x prime which will change proportional to the y coordinates, so that is the example of shear.

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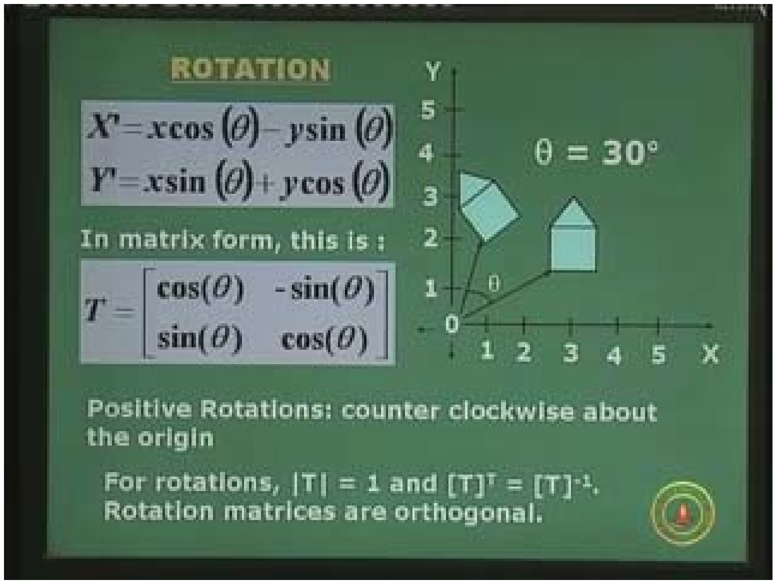

We have probably come back to the last topics of discussion for today's part of the lecture on computer graphics and in 2D transformation. We look at rotation, now this is probably the most commonly used and very interesting transformation which you must look at. And if you look at the equation you can actually derive this is. I leave this as an exercise where you start with a point, any point you take of this house on the right hand side, one of the point is marked by an arrow but you can take any of the five points of the house and we are rotating this point about the origin and the rotation must be about a point, why? Because our earth moves around the sun and the moon revolves around the earth, we also spin around the earth's axis. So we are talking about across an axis or a point and the amount of rotation is equal to theta and this case the typical example of theta is approximately about 30 degree is what I have tried to show you in this figure.

What happens is all these points get rotated by the same transformation matrix, T which is dictated by theta, the angle of rotation. Theta is the angle of rotation and we are talking of possible positive rotations and also negative rotations. When we have positive rotation the house moves from the original position on the right to the left and if you have a negative rotation then the house will come back from the left position to the right position. And what is this positive negative concept which you must keep in mind or the method by which you find out what is positive and negative.

When you place a watch on the paper or the screen on over which you are implying the transformations and observe the direction of the movement of the second hand or minute hand. The direction which, if it is clockwise, the counter clockwise or the reverse direction is basically conceived to be or taken to be positive rotations. So the direction of movement or rotation of the hands of the clock, typically the second hand minute or even the hour hand is the negative rotation. So the counter clockwise rotations about the origin are conceived to be or considered to be or taken to be positive rotations and you can easily derive, I leave this as an exercise for you, where given a point coordinate xy, after you give it a rotation by an amount theta you get the new coordinates x prime y prime which you can obtain by premultiplying or post multiplying the coordinate x, y by the transformation matrix T which is given in this slide here.

The matrix form is as given here, cos theta minus sin theta, sin theta and cos theta terms and the expressions of x prime y prime are given at the top for you. x prime equals x cos theta minus y sin theta, you should be very careful of one negative sign which appears here and y prime is x sin theta plus y cos theta.

Now I must tell you that these transformation matrix in the case of rotation has certain and very interesting properties which other transformation matrices may not have. First of all the determinant is equal to 1 for rotation. If you look at this transformation matrix the determinant is equal to 1.

What is the determinant in this case? If it is 2 into 2 matrix it is easy to visualize the determinant of this matrix. It is cos square theta minus, minus cancels out plus sin square theta, so you all know that the addition of cos square theta and sin square theta is 1. The other important property of the rotation matrix is that the transpose of the matrix is equal to its inverse or the inverse of the transformation matrix in this case rotation is equal to its transpose.

Now, based on the properties of matrices we know that these are called the special types of matrices which are called the orthogonal matrices which will satisfy this constraint that the inverse is equal to its transpose. And that is a very important property which is used in many concepts of mathematics and engineering disciplines where if you know that the matrix is orthogonal, you can easily compute its inverse by taking the transpose.

Transpose as you all know is making the rows to columns and columns to rows. And any orthogonal matrix, if you know that the matrix is orthogonal you can always compute the inverse by using this concept. The compressional complex in the time is saved enormously using this because typically matrix inverse computations are very costly operations. Those with computer science background will know that the computational complexity costs of the order of about ordering queue for that matter and for orthogonal matrices the complexity can go down to almost the order end by taking the transpose to be the inverse. That is the fundamental property of orthogonal matrices. If you look into the slide, the orthogonal matrices also satisfy one more property. Just coming out of graphics I will give you little mathematical tools with which you can play around.

If you take the sum of the squares of individual rows or columns, it is a simple two by two but you can experiment this with any n into n 3, 4 or higher values of n. Take an orthogonal matrix only and which the transpose equal to the inverse, you take the sum of the squares of the individual rows or columns, what is the value you are getting?

You take the left column, it is cos square theta plus sin square theta, the sum of the squares. You take the bottom row or top row also you will get cos square theta plus sin square theta so that means sum of the squares of the individual elements of a column or a row is equal to 1. That is one property of orthogonal matrix. The other is, if you multiply, take any two rows or any two columns or rows, you take any two rows, do not mix up, any two rows or any two columns of the matrix multiply the respective elements, it is called the dot product or inner product in the definitions of the mathematics. Take the inner product that means multiply the respective elements and then add up you will get the resultant sum to be equal to all are 0 and this is also interesting.

You look at the matrix again T equals so you multiply cos theta multiply by sin theta and take the next two elements so you get a minus sin theta cos theta and the terms cancel out. You take the two rows or two columns you get the same result. That is a very very important property of orthogonal matrices. In fact, that property helps you to build up an inverse which is equal to the transpose of the matrix.

How that helps us in computer graphics?
It is very easy because if you have a transformation matrix which is the case of a rotation and if you have given it a positive rotation in this figure where the upright house on the right hand side of that plot or that figure has inclined and moved, inclining left is the positive rotation. If you want create an inverse rotation you can start from the inclined house, give it a negative rotation, just the complementary or inverse of the rotation which we have used so far and we will get back to the upright house. So you can make a positive rotation or a negative rotation depending upon the counter clockwise convention which we just talked about when you looking into the graph paper counter clockwise is positive and clockwise is going to be negative and when given a transformation you can always consider it inverse in this case by taking the transpose of this matrix.

Transpose of this matrix T, it is easy to guess. What will happen? The negative sign on top right element will come to the lower left element. So minus sin theta will become plus sin theta and plus sin theta on the left bottom element will become minus sin theta.

You can also visualize this by a transpose. You will get the same result if you replace theta with minus theta because the angle has become negative. So plus theta has become minus theta so cos does not change its sign, the sine does change its sign and hence the minus sin theta on top right will become plus sin theta. That is how it is. It is a very good example of how the inverse equal to transpose or transpose equal to inverse for orthogonal matrix which is the case for a rotation.

Special cases of rotation: These are only special examples and I think in a few slides back somewhere in the middle of my lecture today I had cited special cases of reflection and couple of them was actually rotation of a certain amount. But if you look into these, these are only rotation matrices where theta equals on the left hand side 90 degree, 80 degree, 270 degree or minus 90 because you can move positive wise or negative wise, it does not matter or you can have it to be 360 degree or 0 degree. Or you can talk in radians; pi by 2, pi 3, pi by 2 and 2 pi radians respectively. I can take these values of theta, plug into the radiation of the matrix which we have just seen.

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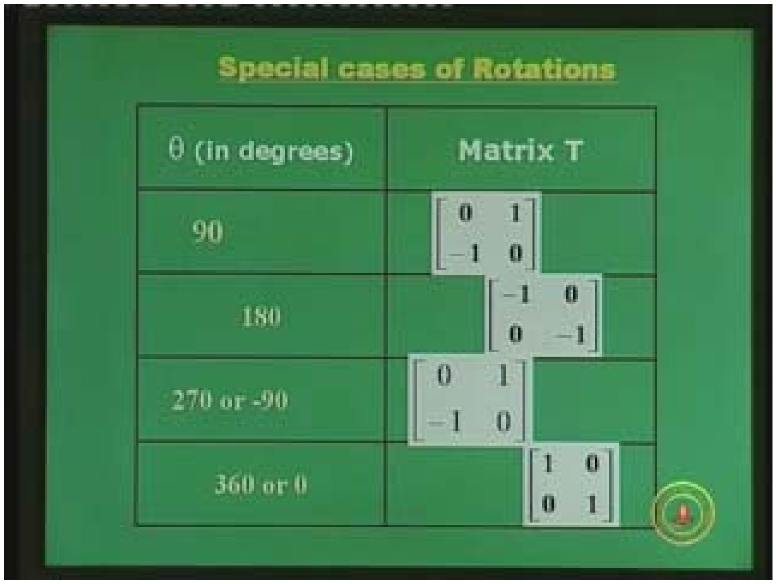

We have just seen in the previous slide and you are bound to get these values of the matrix T. These sort of rotations you can see almost that these are of course itself the last row 360 degree does not provide any rotation, it is an identity matrix which does not give you any rotation which is obvious because we are actually moving it by 0 degree but all others are going to typically give you a reflection in some sense with off-diagonal terms and diagonal terms playing an important role. So some amount of rotation by pi or pi by 2 or 3 pi by 2 also basically gives you a reflection or deflection can be conceived to be a rotation where all seems of some sense.

We will just take an example of transformation of a unit square. But before that I must admit here that throughout this lecture we have discussed four types of transformations. What are they? Scaling, reflections, shear and rotation, I repeat we started with scaling and reflection, then we move to shear, diagonal, off-diagonal terms is what we have seen and of course then we have seen shear and we had also seen some examples of one pair of a term is involved out of the four elements. And the rotation where first is an example of orthogonal matrix where we have seen some properties. Please go and read about orthogonal matrices.

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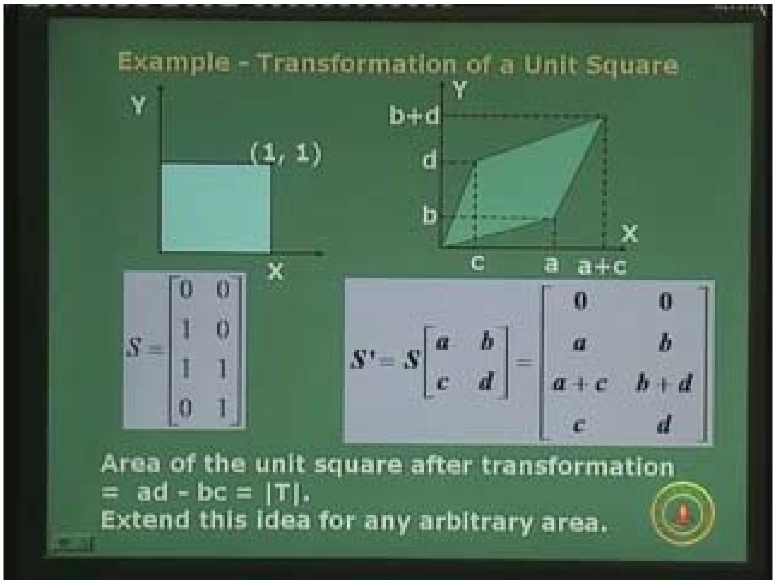

You should be surprised and should be wondering what is wrong that I have not discussed translation. The most simple example of transformation is just very simple, you move yourself, you know you are moving from Delhi to Chennai yourself or from Calcutta from Bombay, it is an example of a translation anywhere yourself or in the screen. It is of some reason that I want to postpone the discussion of translation for some reason. Let us concentrate on the operator T with four parameters and the four elements with this sort of transformations.

Let us take this example of the S, this figure shows a unit square on a two dimensional plane and the S is a vector of four rows talking about the four different vertices of the square. We have the 0 0 vertex at the origin then you have 1 0 which the point on the X axis, we have the third row which is the point here marked as 1 1 and we also have 0 1 which is the point on the Y axis.

So what I am going to do now is transform this S using the transformation matrix which could be any general transformation matrix a b c d. It could be for rotation, reflection, scale, shear or even translation and I have not discussed that. But we are post multiplying these arrays of points in this case by a b c d and the net result will be as given on the right hand side. The point at the origin stays where it is, the point at 1 0 shifts to a, b, you can multiply S with this T and check for yourself, 1 1 point goes to a plus c b plus d and the 0 1 goes to c and d respectively. This is the end result which you see that the origin is at where it is, the 1 0 has moved to a, b, the 1 1 point at the top right has moved to the point a plus c b plus d and 0 1 has moved to c, d respectively. This you can check it out and draw for yourself. But the interesting point is that the area of this unit square, I leave this as an exercise for you as a problem at the end of the lecture today.

You can find the area of this unit square which was one area which has been transformed to a rhombus type of a structure. Please check it out, you can find that the area of this transformation, the formula is given here as the determinant of the transformation matrix T as given here which is ad minus bc. But you can check the area by visualing this with the outward square which is b plus d multiplied by a plus c minus this triangle and you have to subtract these areas from the overall area and you get the structure of the transformed square or this rhombus and you should be able to get the same formula as given by ad minus bc.

You can extend this idea to an arbitrary case where an arbitrary shaped polygon can be conceived to be composed of unit areas and each of these unit areas if after transformation gets a new structure, a transform structure with an area given by ad minus bc I let this formula sum of the area of the total polygon can be conceived to be sum of the unit squares, the total area of an arbitrary structure also gets multiplied by the determinant of a transformation matrix ad minus bc.

So we stop with this point here in this class today saying that an arbitrary area when it gets transformed by a transformation matrix T given by a b c d elements, the overall area of the structure also gets multiplied by determinant of this matrix that is the old area, the existing area multiplied by the transformation matrix determinant ad minus bc will give the area of the transformed or the new structures. You can extrapolate these ideas very easily but overcome this unit structure as we have just seen in the previous slide. And once you are able to obtain this as ad minus bc extrapolation is a summation of unit squares that gives an arbitrary area and that is what you get.

We will stop here and come back in the next lecture when we start to discuss about translations. Thank you very much.