Computer Graphics

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Lecture - 24

Solid Modelling

Welcome to the lectures on computer graphics. We have discussed a few methods of representing solid objects or concepts of solid modelling in the last class. The two main concepts which we discussed in the last class were the method of regularized Boolean set expressions. We have seen the three different operators of Boolean set expressions that is union, difference and intersections. And we also had seen why we need a regularization operation to remove hanging surfaces, lines and points which are not part of the solid object. So you can use regularized Boolean set expressions to create complex object structures from simple primitives. The four simple examples of primitives could be a sphere, cone, a cylinder and a cube. So you can use these to create other complex structures with the help of regularized Boolean set expressions.

We had also seen in the last class the concept of the sweep representation where we took a two dimensional structure and swept it either along an axis of rotation or provided a translation in 3D and the volume which this two dimensional area sweeps due to rotation or translation or could be a combination of two transformations like translation and scale or even it could be shear then we land up with representations of what we call as wire frame diagram of the solid object with the help of a sweep were the bounding surface of the solid is now represented by a set of polygons or even it could be triangles or rectangles or quadrilaterals and the coordinates of the vertices are used to represent the surface of the solid with the help of polygons.

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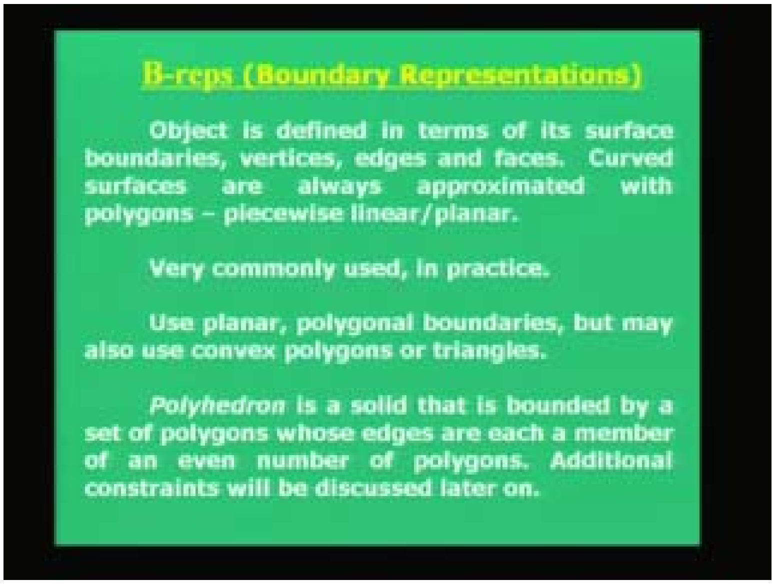

Towards the end of the last class I just introduced the concept of boundary representations which is similar to the concept of wire frame diagrams to represent the surface of the solids. So we continue from that point today where we discussed and have a look at the slide which we discussed about last class the boundary representations or B-reps as it is called where an object is defined in terms of its surface boundaries, its vertices, edges and faces.

Curved surfaces are always approximated in this case with the help of polygons were these polygons provide piecewise linear or planar patches approximating the curved surfaces or edges on the boundary of the solid object. It is very commonly used in practice the boundary representation of a solid structure and there are various instruments based on light intensities which use imaging technology to obtain the boundary representation of an arbitrary structure.

Let us say you are interested to represent a solid structure of a very arbitrary complex structure which could be the statue let us say or a three dimensional model of a phase or the entire scholar. So in these cases using imaging technologies either using medical scanned images or using laser sensors it is often possible to retrieve the boundary information or the bounding surface binding the solid. It is possible to extract that information and represent in the form of boundaries or using wire frame diagrams because for most complex structures you cannot visualize a sweep representation to represent that object because the structure may not be having very regular geometrical surfaces on it or you cannot model by simple primitive instances using the simple primitives of sphere, cone and all that. There could be fine undulations and corrugations on the surface so you need imaging technologies to obtain the boundary points on the boundary of the surface and then represent in a computer system using the boundary representations.

So coming back to the slide we use planar, polygonal boundaries but may also use convex, polygons or triangles to represent the surface on the solid. We also introduced the term called the polyhedron. A polygon in 2D you can visualize an extension in 3D is called as a polyhedron which is a solid that is bounded by a set of polygons whose edges are each a member of an even number of polygons. That is the one which we must keep in mind that which a solid which is bounded by a set of polygons and the edges of the polygons are each member of an even number of polygons. That means if we consider any edge it must be sheared by more than one polygon and it should be an even number. Additional constraints will be discussed later on as we go on.

Simple polyhedron we can also visualize that it can be always be deformed into a sphere using scaling or shearing. It is always possible to transform or deform a polyhedron into a sphere and vice versa and a polyhedron has no hole.

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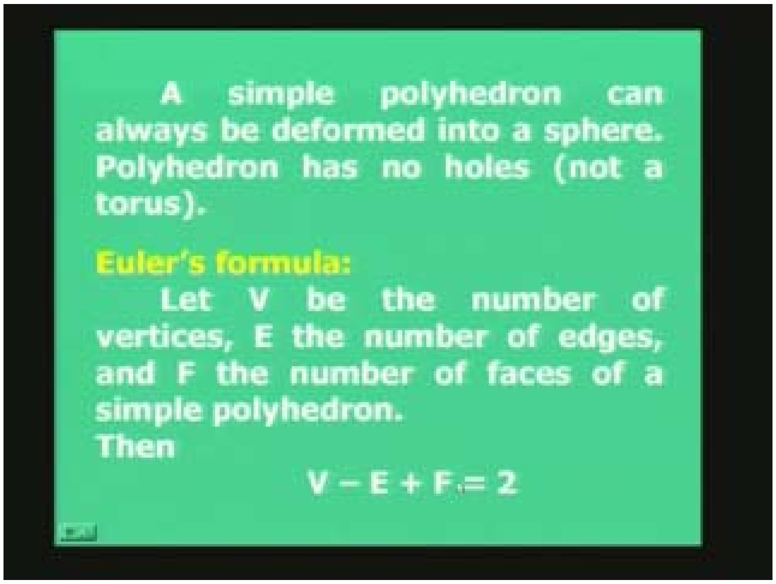

A typical example I gave of a torus in the last class where if you take a circular patch and sweep it along a ring and then the volume which is stretched by that sweeping circle about a line and of course you have to rotate that particular two dimensional structure as well. So it is a combination of translation rotation, if you provide along the circle of a track then the volume which you sweep you can visualize as torus but that is not an example of a polyhedron structure.

Typically a cube or a sphere can be visualized as examples of polyhedron structures. Now we introduce the concept of what is called as the Euler’s formula to verify whether a polyhedron represents a solid structure or not. Let us say as for example we have a typical solid object represented using a wire frame diagram or boundary representation and the total number of vertices which are used to represent that solid let it be V, E be the number of edges or lines of the polygons which are used to represent the surface and F being the number of faces of the polygons or the simple polyhedron.

Number of faces that means the number of polygons which are used could be F to define your simple polyhedron. So remember V for vertices, E for edges and F for faces. If these are the three quantities which we can compute for a simple polyhedron then the Euler's formula says that that V minus E plus F is equal to 2 that is a very simple relation we can keep in mind. The other way of visualizing this is V plus F minus E is equal to 2 or V plus F will be equal to 2 plus E and another way it does not matter, just remember this formula. Again I repeat V is number of vertices, E the number of edges and F the number of faces of the polygon polyhedron rather. Let us verify this Euler's formula with the help of these simple examples.

I have taken two examples separately. On the left hand side you have the cube or it could be a rectangular parallelepiped if the length, breadth and height of that particular structure are not equal to one another. Then of course it becomes a rectangular parallelepiped or a cube. We can also have pyramid on the right hand side. So let us count the number of vertices, edges and faces on the left hand side.

Can you tell me the number of vertices for this cube? We can easily count it, it is 4 plus 4 so number of vertices V is equal to 8 for the cube. Can you tell me how many edges we talked about this earlier when we counted the number of edges also for this particular structure of the cube? How many it should be 4, there are 4 on the front, 4 on the back side and 4 on the side so you have E is equal to 12. And how many number of faces? Well again 1 front, 1 on the back, 1 top and below 1, 1 on the left and 1 on the right so there are 6 faces. So if you look at this structure you have V is equal to 8, E is equal to 12, F is equal to 6. So V plus F that is 8 plus 6 is 14 minus 12 is equal to 2. So the Euler's formula holds good for this cube because if you put these values in the formula, again I repeat you have 14, 8 plus 6 is 14 minus 12 is equal to 2. So it holds good for a cube.
Let us look at this pyramidal structure with a square base.

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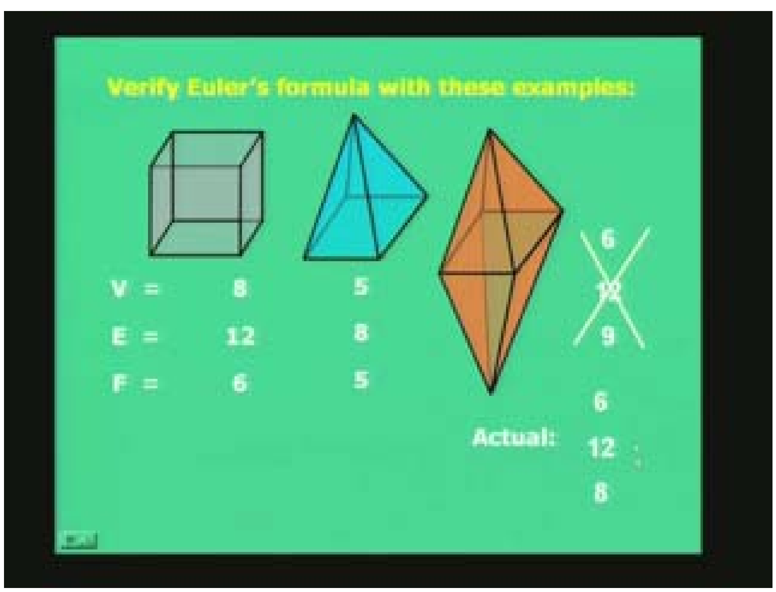

What is the number of vertices? V should be equal to 5, 4 at the bottom, 1 at the vertex at the top, so V is equal to 5 affects of the pyramid at the top. V is equal to 5 the number of edges E 4 plus 4 is 8 that is good 5 plus 5 and E is equal to 8, what about the number of faces 1, 2, 3, 4, 5 so 5. F and V are 5 each and E is 8 as we had just calculated. Again the Euler's formula holds good because we have 5 plus 5 is 10 minus 8 is 2. So again the Euler's formula holds good for the pyramid.

Again I repeat; V plus F is 10 minus E is 8 so you get 2 and that is the Euler's formula for you in the case of pyramid. In these two structures as you can see good examples to illustrate that the Euler's formula holds good. Let us look at the slightly more complicated structure which is the structure of the two pyramids joined together. We had seen pyramid on the top and middle and on the right hand side it is a diamond type of a structure with two pyramid that means we have 2 vertices one on the top and the bottom and so it is put together. If you count the number of vertices well we have 1, 2, 3, 4 plus 1 at the bottom 1 on the top so 6. V is equal to 6. If you count the number of edges 4 plus 4 plus 4 so you already had 8 and you have 4 more so you have 12. So you can take the pyramidal structure which is in blue color in left hand side add one more vertex so 5 plus 1 is 6.

You can look at the edges you already had 8 you add 4 more so 8 plus 4 is 12. What about the number of faces? We already had 5 and you are adding 4 more so it becomes 9. I hope this is very clear if we add in this blue pyramidal structure 5 vertices add 1 more at the bottom you get 6, 8 edges already in this structure add 4 more so you have 8 plus 4 equals 12 and the number of faces, you already had 5 in this case add 4 more at the bottom so you have 9. What do you see about the Euler's formula from these values, well 6 plus 9 is 15, 15 minus 12 is 3. Now what is wrong? The Euler's formula is not getting valid in this particular example because V plus F minus E should be 2 and we are getting 3. Please look at the diagram once more and check out if these values are wrong anywhere. I do not think so, 5 plus 1 is 6, 8 plus 4 is 12, 5 plus 4 is 9. But this formula does not hold good for the case of this diamond type of a structure.

Where did we get wrong? Can you pick up something where we went wrong? The number of vertices seems to be quite alright, the number of edges seems to be quite alright, can you pick up the hint? Something should be wrong somewhere. Well, I ask you to have a very close look at the structure and see the number of faces. Remember we are talking of a solid object with no hole. So if you look at the number of faces the one face which was at the bottom of this blue pyramid does not exist any more because it is a solid object so that vanishes. So you have basically 4 faces on the top and 4 at the bottom resulting into the actual value of V, E and F are 6, 12 and 8. That is were we went wrong that we cannot simply add 4 faces to the value 5 and got 9 and that is where we went wrong. So the actual number of faces is 8. So you should be very careful to count the number of vertices, edges and faces for a solid.

Try to visualize any structure as a polyhedron or a solid with no holes and try to compute the vertices edges and faces. So this was the test whether you are capable to visualize or your capacity to visualize it as correct or not. And if you are able to correctly visualize the number of vertices, edges and faces of this diamond structure without comparing it with the pyramid. So if you compare with the pyramid you could go wrong the actual value is 6, 12 and 8.

Now let us look at the Euler's formula for the actual values of V, E and F. well it holds good because 8 plus 6 is 14 minus 12 is equal to 2. So the Euler's formula finally holds good for this structure as well. I would request you to take various other structures, let us say slightly more complex structures like even cylinders or even a very core sphere and try to represent it using a wire frame diagram boundary representation, count the number of vertices, take a coarse approximation of a cylinder or may be even a cone, you can take a cone with the bottom which is supposed to be a circle but you can approximate that circle with a polygon with higher number of vertices or faces.

Let us say a hexagon or octagon that is a polygon with 6 edges or 6 vertices is a hexagon, a polygon with 8 vertices or 8 faces in octagon. So consider the base of a cone, like for example in this case we have taken a pyramid with a square base that is a very coarse approximation or crude approximation of a circle. It is better to take more number of edges or vertices to approximate a circle. You can take 6 or 8 at the minimum and that is what you take for the cone and d join the vertex of the cone with each of these vertices and then count the number of vertices edges and faces.

I leave this as a problem for you to take it home as an exercise and to consider as a cone with a circular base and the circular base consisting of a hexagon or an octagon and then try to verify the Euler's formula for both the cases of cone. Of course you can do it also on a sphere it will be very nice. So whatever we did for the Euler's formula is also applicable for curved edges and non-planar faces. It basically means one of the structures or the number of edges becomes a curved line instead of a straight line and then the planar surface which sometimes approximates a curved surface but you can place any one of these planar surfaces with a curved structure for one of the surfaces or more and still Euler's formula holds good for curved edges and non-planar faces.

Let us look at this particular structure now. I have taken a simple example of a cube which you have taken in the previous example and I have extended one of the faces. I have just extended one of the faces. So, if you look at the vertices you have A B C D the original one and E F G H in front of you. A B C D is that plane which is on the back side and E F G H is on the front side. Front and back depends on the perspective nature we visualize that cube and what I have done is I have taken the plane consisting of CGFB and extended that and introduced another plane extension of that plane with two more vertices G J and I.

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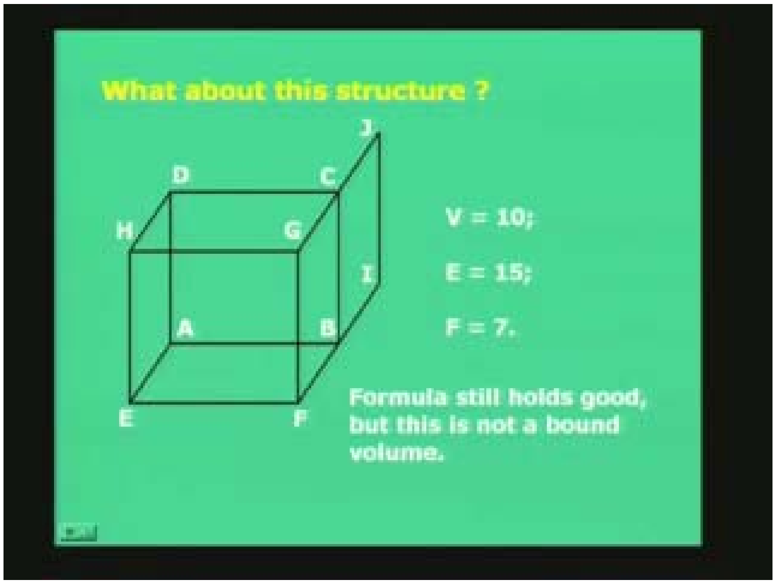

So let us count the number of vertices. Well, if you remember in the previous example for a cube we had V is equal to 8, 12 and 6. Just remember this. That for a cube we had 8 vertices, 12 edges and 6 faces, this keeps coming back. Cube is a very simple example to visualize and always remember 8 vertices, 12 edges and 6 faces. And now what has happened to this structure is we have added two more vertices we have added one more face and we have added three more edges. So the corresponding values now are V is equal to 10, E is equal to 15 and F is equal to 7. Why? We had 8 vertices when we add 2 so V equal to 8 plus 2 is 10. The numbers of edges were 12 plus 3 more so it becomes 15. The number of faces was of course 6 and we added just 1 more so it becomes 7. Interesting to know that the Euler's formula holds good also because we have V plus F as 17 minus 15 is equal to 2.

So if you verify this particular structure with this Euler's formula the formula still holds good but this is not a bound volume because we just have taken a cube and we extended one face of it it is not binding a solid. Remember, we are talking about Euler's formula being valid for a polygon or a solid which is bounded by faces, it should enclose that particular volume and that is not the case here.

Previous examples which we considered about for a cube, for a pyramid, or for the diamond shape structure, or cone, or cylinder whatever the case may be we were talking of a solid. So we have a set of polygons defining surfaces and the surfaces enclose a particular volume. And in this structure we have just seen, what I have basically done is extended just one face and a face like that cannot exist in reality and it is true that it is not binding any solid right now. So we need some additional constraint to get over this problem where the Euler's formula actually should not have been valid any more in this particular case because this polyhedron does not enclose a solid, the volume. It does not represent a solid structure bounded by certain surfaces and that should be the case for the Euler's formula.

So we need to put some other constraints where the formula gets valid for the solid structures which we have seen in the previous slide. And for the current slide whatever we had seen that is for the cube with one plane ejecting out on one side it is just the extension of one face, it does not bind the solid.

What we should have is that the Euler's formula should be corrected to ensure that this is not a valid structure. So what you do you? You need to have some modifications, you need additional constraints to guarantee that the formula is only valid for a solid object and is invalid for a representation of polygons or polyhedron which does not represent a solid object or enclose a particular volume. We put a first constraint saying that each edge must contain two vertices and that is true. Each edge runs from one vertex to another so eliminate hanging lines in this case. Second point each edge also must be shared by exactly two faces, this is an interesting phenomena. We say that the edge must be shared by even number of faces but we say that it must be shared exactly by two. So we do not allow even 4 forget 3 that is not an even number any more. So edge must contain two vertices and the edge must be shared exactly by two faces.

We will go back to the examples and see if this concept is valid, they are in fact. This is another interesting phenomena, at least three edges must meet to form a vertex. When you have a vertex you should have at least three edges. You should not have less than that you can have more than three. And faces must not interpenetrate, one face should not penetrate the other one. This is a typical case when we talk about clipping in 2D you must visualize clipping in 3D which we also visualize which you would need to do in the case of a canonical view volume. But we know, we do not do that in the case of a solid structure and we do not assume although in virtual reality a solid object can get inside the other one we actually do that.

Later on, in constructive solid geometry we will see how it is possible to visualize that one object is gone inside the other one. But typically we do not allow that for a polyhedron boundary representation for a particular solid where we cannot have a face which is interpenetrating into another one. We have a face and another face could just tangentially touch it but it should not lie in such a manner that those two planes intersect that should not be the case. So the faces must not interpenetrate.

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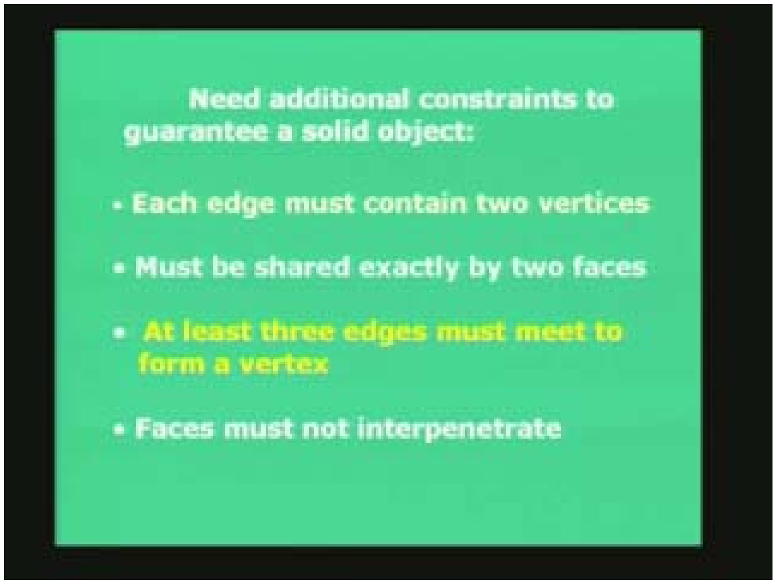

Before we look forward to understand this concept about conditions on solid object let us see the first three constraints on the previous solids. The previous examples of solids namely cube, pyramid, diamond structure and the other one which is the extension of the face from the solid, whether these conditions of edge containing two vertices, is very straightforward. We will see whether edge is shared exactly by 2 faces and at least 3 edges need to form a vertex. So let us verify this from the previous slide. Let us look back here.

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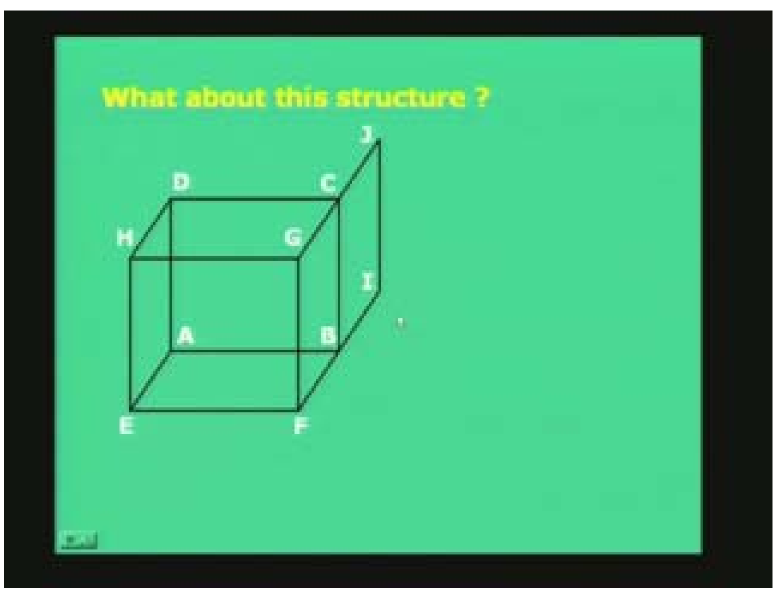

As you see if you look into the vertex of the cube there are 3 edges which meet to form a vertex at least 3 or more. And if you look at any edge there are at least 2 edge and definitely 2 edges exactly 2 edges in fact for any particular case. It is a same case for this pyramid where you have in fact 4 edges meeting to form a vertex on the top, 3 edges for any 1 of the 4 vertices at the bottom and when you take an edge it is shared by exactly 2 faces. So it holds good for the solid of the cube and a pyramid. I hope you can visualize and see that very easily.

If you look at the pyramid also the same thing holds good. Remember, the base of the pyramid of the diamond is solid, there is no plane in that so the top vertex and the bottom vertex has 4 edges which meet. In fact all the vertices in this case of a diamond structure has 4 edges or lines which meet to form a vertex. So the condition that 3 or more edges should meet to form a vertex holds good for the case of this diamond shape structure. If you look at the edge itself all the edges are shared by exactly 2 faces, so this is also true for this structure.

Now, let us look at this particular structure where we had a problem. We had a problem with this because the Euler's formula holds good. So, before verifying the Euler's formula we should now verify where the condition of the number of lines meeting to form a vertex or the edges being shared by two vertices holds good where you do not have to worry about vertices lines and edges which form the original cube.

Let us look at the extension point now. If you see this particular edge BC it is shared by 3 polygons or 3 squares or 3 rectangles whatever you may call it. What are they? If you look at that edge BC it is shared by the square or the polygon ABCD. It is also a part of GCBF and also between CBIJ. So that is one violation where this edge is shared not exactly by 2 faces but more than 2. If you look at the vertex J and I there are only 2 edges which meet to form vertex J and I. And the condition is 3 or more vertices should meet to form a vertex.

So if you look at the extension all of it goes wrong. The edges CJ, JI and BI are in fact now shared by only a part of 1 particular polygon face but not 2. So you see all constraints of the number of edges meeting to form a vertex and the edge or a line shared by exactly 2 polygons are all getting violated by this particular structure. I request you to draw it again yourself and verify that although I have used the screen to verify that where we had seen that edge belongs to 1 particular polygon or shared by 3 whereas the constraint says that it must be shared by exactly 2. That is number one. Then a vertex must have 3 or more edges which meet.

We have seen the example where the vertex has 2 edges which meets so that also is going wrong. So these cases of edges being shared are vertex form is all going wrong for the structure and hence there is no question of trying to use Euler's formula to verify this. In fact if you use Euler's formula you will go wrong we have seen that already so do not try to use Euler's formula because the basic constraint which we put before the Euler's formula of the number of polygons which are sharing an edge and exactly it should be 2 and the other constraint is three or more lines should meet together to form a vertex. Both of these constraints go wrong with this particular structure specifically with those vertices and edges which have come as an addition that has gone attached to the side of the cube. One particular surface is extra which is coming to all these edges and vertices violate those two conditions and henceforth this is not a valid structure and hence it is not bounding a particular solid structure.

So we throw out the structure and say that although the formula holds good this is not bound volume because the constraints which we talked about here that the edge must be shared exactly by 2 faces and at least 3 edges must meet to form a vertex hold does not hold good. And of course in this example faces must not interpenetrate also is another condition which is not shown in this example. There is another way by which we can verify whether edges are exactly shared by 2 particular faces only. This is a method which tries to illustrate the fact that whether edges are shared by exactly 2 or not. It is in fact is a con condition or boundary representation which talks about solids with boundaries with two manifolds.

What does this basically mean? If you look at these two particular structures and I have taken points on the polygonal surface. In the left hand side I have taken two points which are on the on each of the face, on two of the faces I have taken two points. And on the right hand side structure I have taken a point which is on an edge of a polygon. That polygonal edge is between shared by 2 polygon faces, that is fine. And now the colored, circular or elliptical area around that particular point shows that I am considering a neighborhood of pixels around that particular point. So I am talking about a set of points around a neighborhood of a point being considered and the left hand side figures I have a point which is on the center of a face and the right hand side I have the point on the edge.

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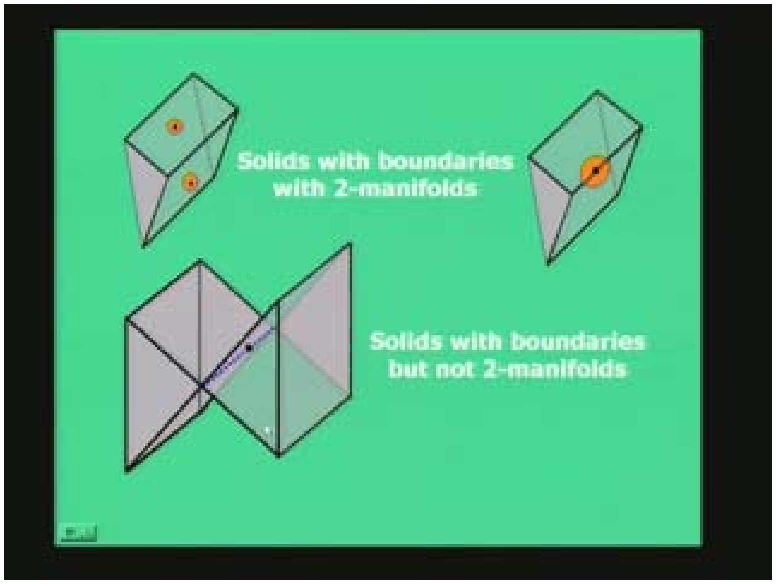

So what do I do with these neighboring points? Well you have to visualize this structure now in 3D and see that if I take the set of neighboring points around a particular point let it be either on the edge of a polygon or in the center of a face it does not matter. When I am considering a set of neighborhood points I can consider a neighborhood as a circular neighborhood or a square neighborhood that does not matter. As long as I take that and I try to map all these points on to any other plane, that particular plane itself or any transform plane, I will have a 1 is to 1 mapping of these points which are actually around the neighborhood of that point on 1 or 2 faces and the projection of those points on to any other plane.

I repeat, what you have to visualize is, I take a plane which is not one of this polygonal face let us say or it could be also and I project these points on to that plane, I will have exactly a 1 to 1 mapping between points on the planar face with the points which are on the neighborhood actually on the surface of the solid and that is true for these types of solids. If you look back that is what is going to happen. If I project these points around the neighborhood, around here to any other plane or even points here and projected to any other plane I will have a set of points where I have an exact 1 to 1 correspondence with the points projected and the points actually in the surface.

I hope you are able to visualize. This does not hold good for a structure like this. This is not a solid but it is something like as if two edges are joined at one of the polygonal faces. First of all if you see this is not the valid structure. Although the conditions of Euler's formula and all that, I leave it an exercise for you to verify the Euler's formula here. And if you see this particular edge which is the meeting point of these two edges at the center and I have considered a point in between but that edge is shared by even number of polygonal faces.

In fact it is 4 in number. Of course you can consider 1 here as well but it is 4, it is an even number and in some sense it is valid. But this is not solid why? If you take points around the neighborhood you can actually play two discs around this point, one on each of these faces. And if you project these points to any other plane you will not have 1 is to 1 correspondence because you may have overlapping points. Points adjacent to that point marked in the center of that line somewhere and if you take a neighborhood along two of the surfaces which are intersecting across that line to create two such surfaces on for the two edges intersect to form that line. So, if you have to consider neighborhood you have to consider two sets of neighborhoods.

When you consider two sets of neighborhoods along each of these planes and try to project them on to one particular plane. So, if you have neighborhoods here project on to the plane and if you have another set of neighborhood project them. You will not have 1 is to 1 correspondence of the projected points back to the points on the surface. So this concept of solids with boundaries with two manifolds does not hold good for this particular structure. This is another way to verify whether a surface is solid or not.

Look back here again, I ask you to visualize that you put this circular elliptical discs around that point and try to visualize that. I have two sets of neighborhoods now unlike one set here and one set there. But I have two sets and when I project them on to any other plane I will not have a 1 is to 1 mapping. So this is a solid with boundaries but not we will consider as two manifolds. So we looked at several constraints to verify solid structures based on boundary representations.

One is the Euler's formula, of course we look into more generalized Euler's formula right now but we had the Euler's formula, of course then we found that Euler's formula must be applied very carefully on solids. Then the second part of it is that the Euler's formula shows that it is valid for certain structures which are not a solid. That was the example when we took that cube and just put an additional plane on to that. We strangely found that the Euler's formula was valid. But it was actually the calculation was showing that it was valid for a structure which is not a solid.

So what we did, we put additional constraints based on the edges which must be shared by exactly 2 faces and 3 or more edges joining to form a vertex. Those constraints were not valid for that structure which we had seen. We apply that and then apply the Euler's formula and also apply the two manifolds concept to find out whether there is actually a solid which is binding or not. We should not have a line, we should not have hanging lines, points and surfaces on any solid because that does not bind or bound the solid volume and that is what we are looking at.

Try to visualize a sphere or a cone or a cylinder or what sort of structure, a solid box or whatever the case may be typically you have a surface around which bind the solid, we do not have a line or a point or a plane hanging out in space or in thin air as it is called and that does not bind the solid for us. These are the geometrical constraints, the Euler's formula, constraints on edges to be shared, constraints on vertex formed by the junction of 2 or 3 or more vertices and then this two manifold concepts which could all be used to verify whether the boundary representation which you have is valid or not. That means is it representing a valid structure, is it bounding a solid, is it binding a solid or is there a solid volume bound by these boundaries which is there in your representation.

We will extend this Euler's formula now and look at what we call as a generalized Euler's formula for object with two manifolds and faces which may have holes. We have seen the concept of object with two manifolds now and we will have to consider what we call as faces which may have holes. We look at this new formula where we already know this many of the terms or the variables V, E and F they are vertices, edges and faces. We have additionally the concept of H the variable H which is called the number of holes in the face number of holes in face is H.

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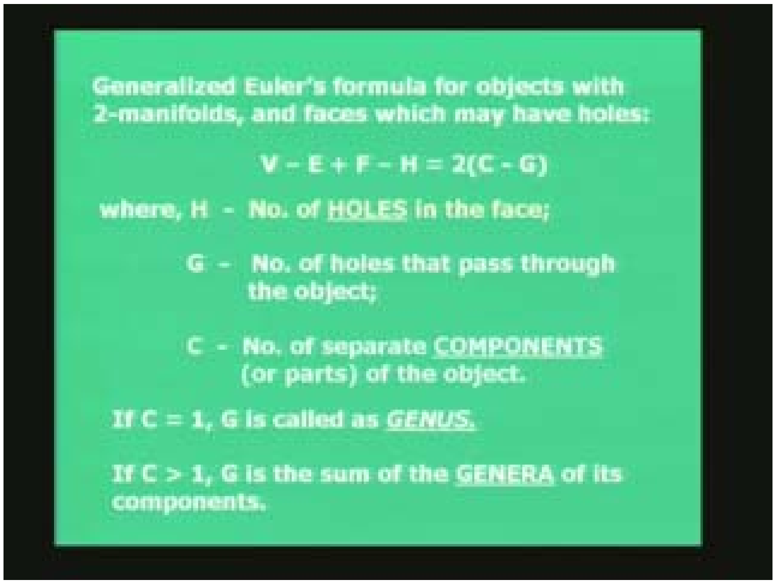

And we will also see the variable C and G which are may not be used in general for very simple structure where G is the number of holes that pass through the object, this is interesting. If H we have the holes in a face and G is the number of holes that pass through the object, C we talk of the number of separate components or parts of the object so that is the definition of C.

So let us look at the particular example but before doing that we typically consider that C is equal to 1 in general and in that case the G is called the genus and if C is more than 1 then G is the sum of the genera of its components. So just remember these words typically we will consider C is equal to 1. Let us look at the structure and verify the values which are given on the left hand side. Let us look at these values, you can see the structure, it is a rectangular parallelepiped.

You can visualize this to be a cone as well but a rectangular parallelepiped is probably a better visualization of this. And there is one hole which is again a sort of a rectangular parallelepiped structure type but it is a hole which passes through the object. So this is a hole, it is visible if it passes there at the bottom and it will get out of its or from top to bottom or from bottom to top that is a hole on the right hand side. Left one hole is not a hole, it is a hole but it does not pass through the object so it is a part of this box which is empty. So part of this box has a cavity here and that is the hole and that is another way of visualizing that it is a cavity. But it is a hole which does not pass through technically and this is a hole which passes through the object. So let us count the different values of the variables which we have here.

How many vertices do we have? Well, you remember that a cube has 8 vertices. That is the original cube which had 8 vertices. And now we can visualize that virtually as if this diagram has about exactly 3 cubes.

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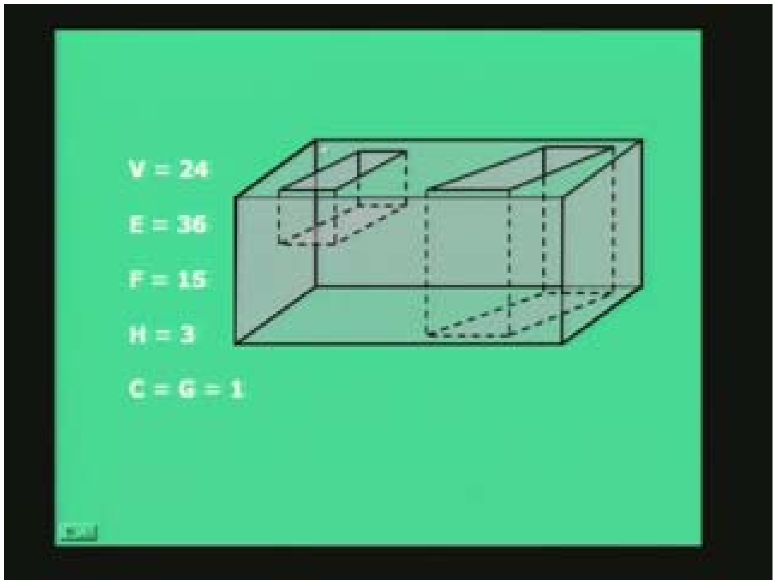

So if there are 3 cubes 1on the outside virtually, I repeat again 1 on the outside, 2 on the inside, this is the cube inside another hole also, if you can visualize that it is a cube and the outside cube. If we have 3 cubes each cube having 8 vertices so the total number of vertices which you have is equal to 24 so that is the total number vertices which is 24. What about the number of edges of a cube? A cube has 12 edges. If you remember correctly those were the values of vertices, edges and face which will always keep coming in our literature 8, 12 and how many faces 6 remember these values 12, 8 and 6. If there are 12 edges of a cube and again that there are actually 3 faces it is 12 into 3 is 36 because there are 3 cubes, 12 edges each 12 into 3 is 36. As we had 8 into 3 is 24, 12 into 3 gives you 36. So that is very simple to visualize.

What about the number of faces? Well we have 6 and that is true, we cannot simply multiply 6 and multiply by 3 and get 18. Why? It is because some of the faces do not exist as for as example, the original outside cubical structure the rectangular parallelepiped may have 6. But this cavity which is a hole which does not pass through it will have 6 minus 1 as 5 edges. So we have 6 in total plus 5 here and 4 there, this hole which passes through will actually have 4 edges inside they are all vertical. The top face and bottom face is missing so we have 6 plus 5 plus 4.

I repeat again, 6 faces for the outside which is as usual, this cavity which is here will give you 6 minus 1 which is 5, this hole which is passing through will give us 6 minus 2 which is 4. So you have 6 plus 5 plus 4 it is exactly 15, 11 plus 4 is 15. So I repeat again that you have 15 faces for the structure, 12 vertices, 36 edges and 15 faces.

Now, if you just apply the Euler's formula here without considering the holes we have 24 plus 15 which is 39 minus 36 which gives you 3. So the Euler's formula, the non-general one, the simple one is not valid. So we need to consider a hole that is why we consider holes here. So what is the number of holes; why is the number of holes equal to 3? What is H by definition? The H is the number of holes in the face. So how many holes in this? There are 2 holes on the top face and there is 1 in the bottom face, there are no holes elsewhere. Out of 6 faces of the outside cube the top face has 2 holes and the bottom face has 1 hole because that hole passes through the structure. So, you have to look at each face and find out how many holes exist for each face. Well the other 4 faces do not have holes. The top face has 2, the bottom face has 1 so there are 3 holes and that is why H is equal to 3.

What about C and G? Well number of separate components and parts I did inform you that C has to be considered as 1 and G is also considered as 1 in this case because that is the number of holes that pass through the object. There is only 1 hole which pass through the object and hence G is equal to 1 and C is equal to 1. So now we have the values 24, 36, 15, 3 and C and G are both 1. So let us look back into the formula. Let us compute the left hand side V minus E plus F minus H.0. So let us compute V minus E is minus 12 plus F is 15. So that is 3 minus H is also 3. So 3 - 3 is 0.

Therefore, when you compute the left hand side with the values for this particular structure I again repeat it will be 24 minus 36 plus 15 minus 3 we put this value in this left hand side expression left hand side is equal to 0. What about the right hand side? Well since C and G are both 1 the right hand side is also 0. So the generalized Euler's formula holds good for this particular structure.

I hope you have understood how to apply the generalized Euler's formula for the case with an object which has holes. If you understand that is sufficient. Do not worry about the concept of G where we consider objects with more than one components. Right now let us consider only objects with single components hence C is equal to 1. So we will talk of a genus particular case. So that is how you apply the generalized Euler's formula. I again repeat that you should take different types of holes for structures say we can take a structure of a cylinder or a cone and visualize a hole in between or a hole which passes through an object.

Or even you can take this cube and consider different type of holes take these two and add one more which passes from the side either which goes through the object or enters a part inside that object and what will typically happen is you should compute this generalized Euler's formula and see if this structure is valid or not. So now we leave the Euler's formula and go back into the boundary representations and we look into as to how do you store information about the boundary representation because boundaries are in the form of polygons or polyhedrons typically a triangles or quadrilaterals.

Again a quadrilateral can be decomposed in two triangles. We can do triagularization on quadrilateral faces and represent all the polygons with the help of triangles to bind a particular surface. Let us take a simple example of not a complete solid structure because if we take a solid structure it will have typically, if we take a cube you know that the number of faces is going to be 6 so there will be 6 faces and lots of vertices and edges so the table will look very long. I have just taken a simple structure let us say do not worry for the time being whether it is a solid or not.

Assuming that you have verified the generalized Euler's formula and ensured that the solid structure is bounded by a polyhedron or a set of polygons and it is a solid, all our constraints are satisfied, we are only taking a part of that representation of the solid and taking two faces or two polygonal faces, that is what we look here.

Let us visualize that as if we have two surfaces or two polygons S_1 and S_2, S_1 is this triangle which is this grey brownish in color and the blue quadrilateral surface S_2 and you can verify the Euler's constraints here at least where the edge E_3 which is common to surface S_1 and S_2 is exactly shared by two polygons.

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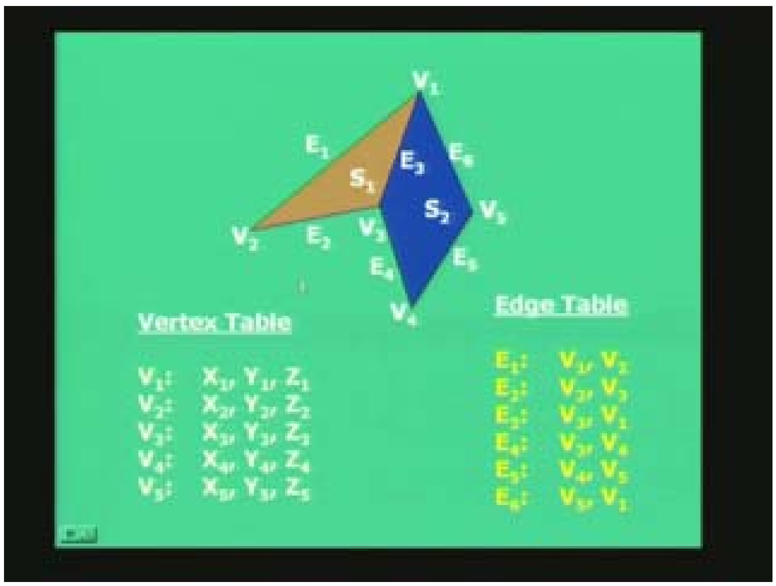

If you look at the vertex V_1 at least and vertex V_3 there are three edges which meet to form this vertex V_1 and V_3. And of course if you look at other vertices V_2, V_4 and V_5 do not worry since you only see two edges which meet to form these vertices. But there are other faces adjacent to E_6, E_5, E_4, E_2 and E_1 edges which will form additional faces, vertices, lines and hold this constraint will be satisfied. So do not worry about those constraints right now, just assume how do we represent this using a suitable data structure, this particular part of the solid with two faces only.

So how many vertices we have? There are V_1, V_2, V_3, V_4 and V_5 so there are 5 vertices and we use what is called as the vertex table. We use the vertex table with 5 vertices in that table and so it is basically a list type of data structure which we can use and there are 5 vertices so each node will have information about the X, Y and Z coordinates of those vertices. Go back to the vertex table and have a look.

As you can see here the each entry of that list will contain the corresponding label of V_1 of that node let us say and the corresponding coordinates X_1, Y_1, Z_1. That means for the vertex V_I I running from 1 to 5 in this case we have for each vertex V_I the corresponding

X_I Y_I and Z_I the X Y Z coordinates of the vertex. This is the case with only two faces for a solid.

If you have a set of n faces which bind a particular solid or a polyhedron representation with n number of faces and m number of vertices then the vertex table will actually have m vertex entries in that list. And each vertex entry will only have the X, Y, Z coordinates for the time being. That is how we construct the vertex table. So construct the vertex table and put all the vertices which you need to represent all the polygonal faces. After you put the vertex table you typically construct what is called the edge table.

Since we have taken a simple structure and let us count the number of edges. Well, we have E_1, E_2, $E_{3,}$ E_4, E_5 and E_6 there are six edges which are used to form these two different polygonal surfaces S_1 and S_2 and the edge table is again a list where it has a label for the corresponding edge E_1 and E_2 and so on for each edge E_I you have which are the pair of vertices which are joined to form.

Let us take a simple example of edge E_1 where it runs from vertex V_1 to V_2. Let us take edge table entry E_2 that runs from V_2 to V_3 transform V_2 to V_3. So this goes on till the last edge E_6 which runs from vertex V_1 to V_5. So that is how you do. So for each edge table entry E_I you have a vertex VK to VL which shows the pair of vertices which are connected to form the edge. This is how you form the vertex table entry and edge table entries.

Just two lists, the vertex table list and edge table list will contain the label of the vertex and the edge and information about in the case of vertex table you have the X, Y, Z coordinates for each table entry for a vertex table and for a edge table you have the labels of the vertices. The pair of vertices which are joined to form that corresponding edge. That is the structure for the vertex table list or a edge table list. Well, we have the polygon surface table now which will contain similarly the list of polygonal surfaces.

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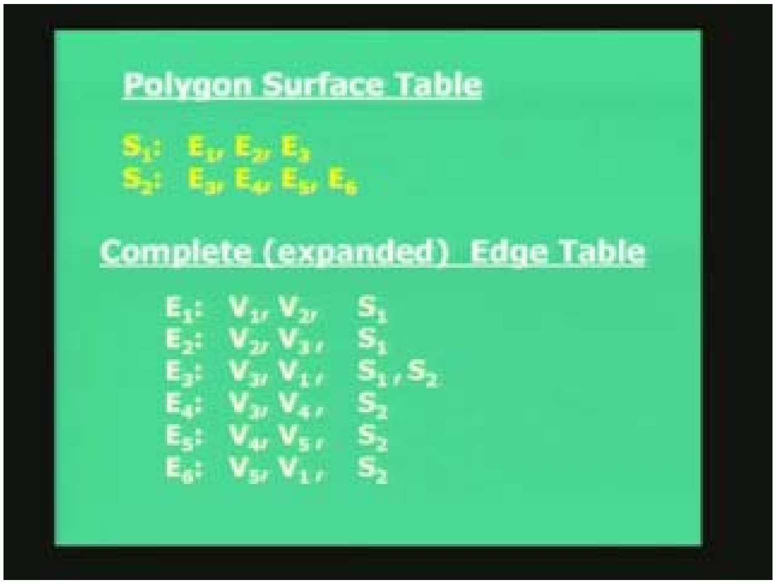

If you look back into this particular simple structure there are two particular polygons S_1 and S_2. There are two polygons S_1 and S_2 that is what the polygon surface table entry will contain. In this case there are two entries S_1 and S_2 and the entry of the surface label S_1 will have the list of edges which belong to that particular polygon. So if you go back the S_1 surface is formed by these three edges E_1, E_2 and E_3. Surface S_1 this brown color surface is found by edges E_1, E_2 and E_3 so that is the entry consisting for the surface S_1.

What about surface S_2? Well if you see surface S_2 we have edges E_3, E_4, E_5 and E_6. So the polygon surface table entry for surface S_2, label surface S_2 will have E_3, E_4, E_5 and E_6 as the case for the surface S. So you have already seen three lists; vertex table, edge table and polygon surface table. There are three lists and they contain information about vertices, edges, edges will in turn contains information about the pair of vertices which are joined to form the edges and the polygon surface table entries will have information about the surfaces which are used to form that solid. So there could be V number of vertices, E number of edges and F number of faces if that is so the vertex table entries will have V number of entries for the vertex table, E number of entries for the edge table and hence F number of entries for the polygon surface table. We always visualize a cube which has 8 vertices, 12 edges and 6 faces that is a very simple example for you to visualize a cube and also count in mind or almost remember it by heart 8 vertices, 12 edges and 6 faces.

Euler's formula simple case gets valid and those will be the number of entries in your vertex table. So, for a cube you will have 8 entries in a vertex table, 12 entries in your edge table and 6 entries in your polygon surface table. That is a simple case I chose. But there is one information which is probably missing in this particular polygon surface table entries and the edge table and vertex table. It does not tell you directly. Of course you can always compute as to how many polygons or faces share a particular edge. Of course in this particular simple structure which I have chosen to illustrate is a part of a complete object structure. I must again repeat it is not binding a full solid, it is a part of a full solid surface binding a particular solid object structure.

The left hand side only edge which we can view here is shared by two surfaces is the edge E_3 which is shared by two surfaces S_1 and S_2. Of course the other edges including E_1, E_2, E_4, E_5 and E_6 will also be shared by two surfaces S_1 S_2 which are not considered in the example but they will be there. We have taken a simple structure, Again, repeat to make the number of vertex table entries and edge table entries looks simple and straightforward. But at least we have on edge E_3 which is shared by the two surfaces S_1 and S_2. That information can be obtained of course by combining the information of edge table and polygon surface table. If you carefully see the common edge between these two entries of S_1 and S_2 is the edge E_3. So one can run through these list entries and come out with the common edge E_3 which is shared by this surface S_1 and S_2. But what is done in terms of data structure is you could talk of or build a complete expanded edge table. In the previous slide we had seen an edge table entries where the edge table entries will only contain entries of the pair of vertices which are use to form the edge.

But if you see this complete expanded edge table, have a look at it, you see I have added with respect to the previous edge table entries the vertex values V_1 V_2 and so on which is already there the entry for the surface. The entry for the surface is in no order. That means which particular polygon surface table is utilizing this edge or consisting of that edge. If you see in the previous example the edge E_1 is used by only surface S_1. E_2 is used by surface S_1 also.

Similarly, E_4, E_5, E_6 they are all shared by or used by the surface S_2. It is only the edge E_3 which is shared by the surface S_1 and S_2 and that is what appears in the complete expanded edge table. So you add the list of surfaces also along with the list of vertices for edge table entries which will show not only in your edge table entries where two vertices or pair of vertices are joined together to form the edge or the line but also will tell you the surfaces or the polygon faces which share that vertex.

Since we have put a constraint for the solid object that exactly two surfaces, even number the minimum even number to share a particular edge there is just one edge which will be shared by exactly two surfaces and if that is so, so in the complete expanded edge table entry each table entry will have a pair of vertices and you have exactly a pair of surfaces. All of them will have it for a complete structure.

In this particular example of course what we have chosen to simplify the example and because the entries of the vertex edge table and the complete expanded table will go very long. To give you visualization I have just taken a case of two polygonal surfaces sharing a particular edge. So, if you look back into the entries you will see the edge E_3 is having an entry of a pair of vertices and a pair of surfaces. Although all the other edge entries E_1, E_2 and E_4 to E_6 shows a pair of vertices in only one surfaces S1 that is not true because this is just shown for the example. It will have other surfaces which will also share these edges for a complete solid object representation using boundary representation.

So again complete expanded table is one way you can come with your own structure which is optimal to represent the boundary representation but a complete expanded edge table should not only have the pair of vertices which are joined to form the edge. But it should also have the pair of surfaces which are common and meets to form that edge. Two surfaces will intersect or meet to form an edge so that also the entry should be there in the complete expanded edge table. This is how the boundary representation data structure is used to represent a particular solid.

We have spent this class to talk of boundary representation because that is the most commonly used in practice and we have looked at lot of constraints. Based on this boundary representation data structure we will see how the surfaces and other properties are computed. That is number one which will be used for rendering and shading algorithms in later on lectures. And also we have a look into at least two or three other representations of solids. Namely as for example we have to look into Octree representations of data structures and if possible one or two other representations.

So we will wind up the lecture today saying that this is what we have learned today in terms of boundary representations. A polyhedron structure consisting of various polygons which bind a particular structure generalized Euler’s formula, data structure of vertex table, edge table, polygon table and a complete expanded edge table. And we also looked at constraints of edges and vertices shared between lines and polygon surface which we must verify to check whether you have a complete solid bound volume enclose by your representation using pairs.

We will stop here and continue in the next class about boundary representation, solids and other models for representing solids. Thank you very much.