Theory of Computation

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Lecture No. # 36

NP-Complete Problems Cook's Theorem

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We were considering what is a p?What is NP?NP is the class of languages accepted by non deterministicTuring machine in polynomial time.P is the class of languagesaccepted by deterministic Turing machine in polynomial time is NP is equal to P is still an open problem.Just recall the connection between problems and languages for example, does M accept w is the problemand the corresponding languageis denoted by strings having the formM w is the corresponding language.So, every problem we can have a language representation.So,is CFG ambiguousgiven a, CFG ambiguousthe language will be encoding of G which is encodingof G which are ambiguous.A language corresponding to that will be L which will acceptedall grammars which are ambiguousG is ambiguous.So, you can talk about problems usually we use the symbol pi and the corresponding languages L.

So you talk about pi in NP,pi in P or L in NP, L in P both are equivalent.Now, when you look at a language representation for a problem certain things you have to consider.For example, if you have a Boolean expression is the Boolean expression satisfiable.

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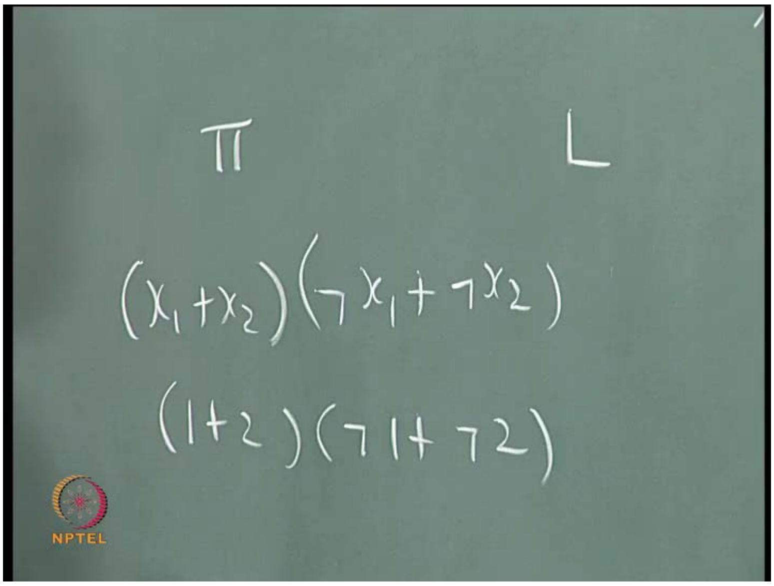

Then for example, x 1 plus x 2notx 1 plus notx 2 or something like that this is a Boolean expression.Now, how do you represent this is a string you can represent it as a string but, you must x 1 x 2 instead of writing x 1 x 2, you can use numbers for the variablesso for example, there are two variables.So,0 and 1 or you can use the binary representation or just decimal representation 1and 2you can use.So, this 1you can write as 1 plus 2not1 plus not2 something like thatyou can use each variable is represented as a decimal number if there are n variables up to n decimal numbers you will use.

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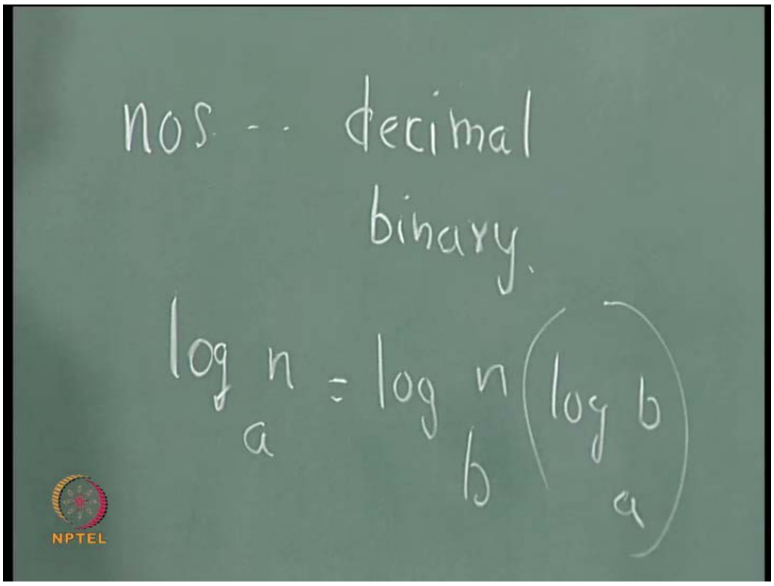

So, there are things which you have to becareful.Numbers you can represent as decimalorbinary does not matter or any other base,because from one base the length will be only increased by a constant.Logn to the base ais equal to logn to the base b,log b to the base a.

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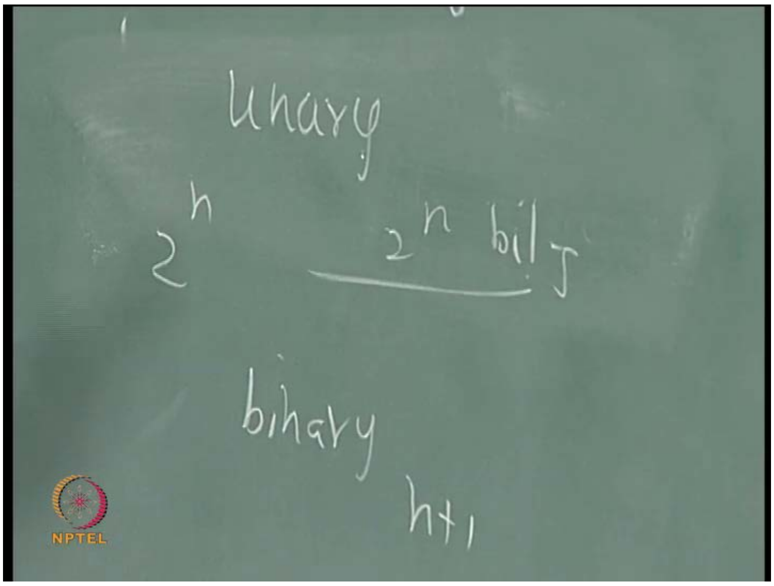

So this is a constant so, from one base to another the length will be affected only by a constant factor, but noise if you useunaryrepresentation2 power n will use 2power n bits.In binary this is in unary in binary it will usen plus 1drastically changes.

So you should not consider unary representation any other representation is,because you have to scan the whole input.If you are going torepresent it unary time will be exponential increase exponential.

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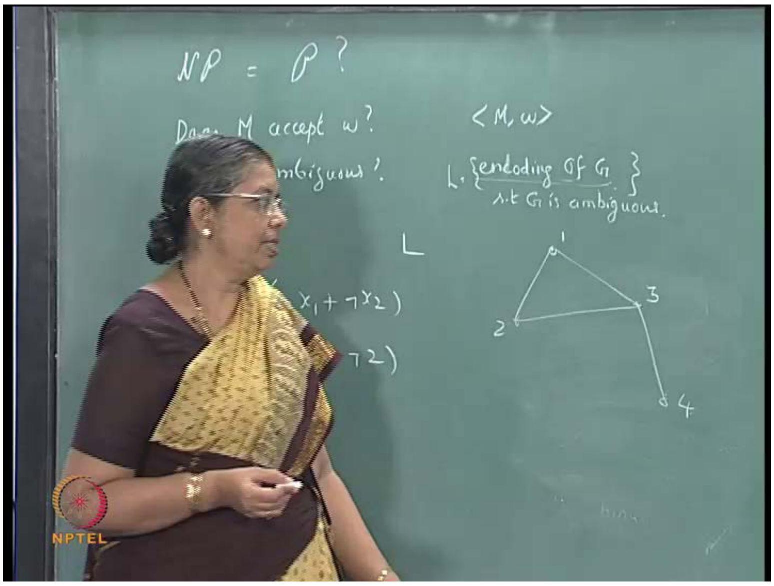

So, proper representation you have to choose.You can have graphsgiven a graph something like thiswith nodes 1, 2, 3, 4.The question is does it have a click of size three click is complete sub graph does this graph havea complete sub graph of size three.It has a triangle is there, but how do we represent this graph there are 4 nodes.

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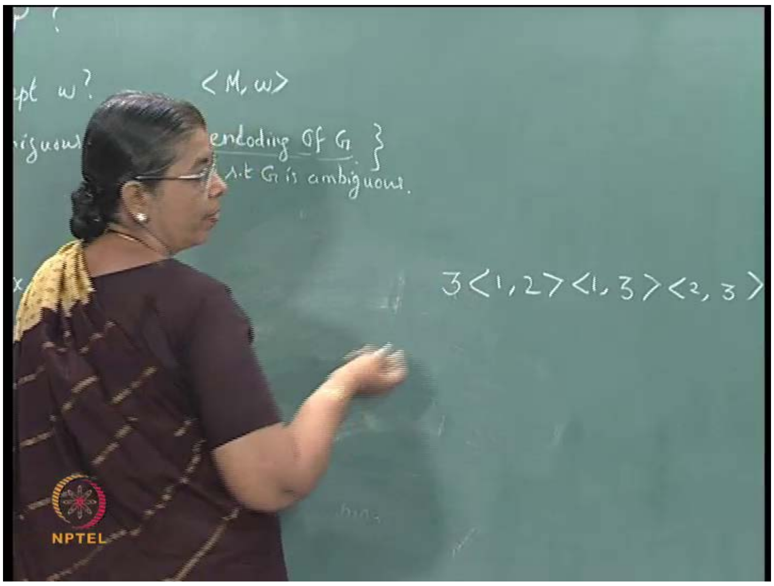

And some edges you choose some proper representation like1,2edges you represent 1,3,2,3,all edges starting from 1first and all edges starting from 2 then all edges starting from3,4 you can represent it as a string like this.

So, any problem you can represent in a proper manner as a language.Now, does it have aclick of size 3 that 3 you can put in front.So, any problemproper way you canconsider a language.So, you can talk about the problem beingin NP being a language being in NP without anydifficulty.

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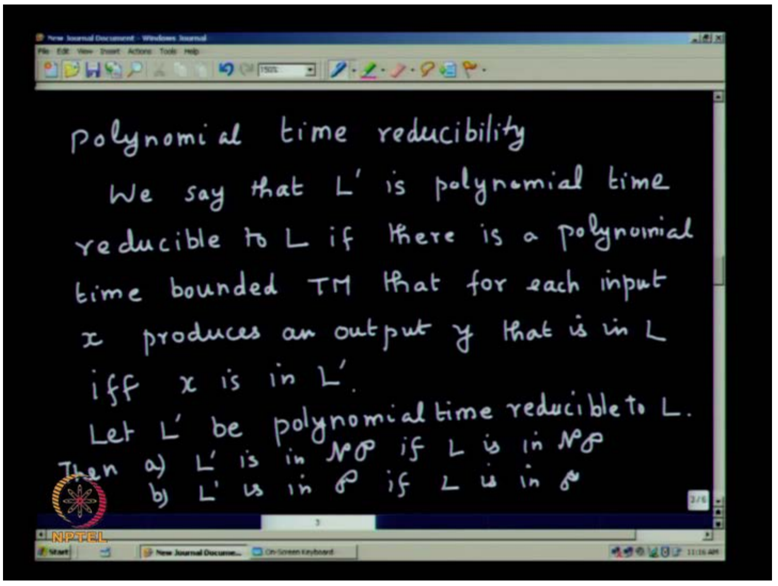

Now, let us see what is polynomial time reducibility?We say that L dash is polynomial time reducible to L.If there is a polynomial time bounded Turing machinethat for each input x produces an output y.That is in L,if and only if x is in L dash.So, from one language it is transforming into another languagethat is and the transformation is done by a deterministic Turing machine in polynomial time this is called polynomial time reducible.I willread again,we say that a language L is polynomial time reducible to L,Ldash is polynomial time reducible to L,if there is a polynomial time bounded Turing machinethat for each input x produces an output y, that is in if and only if x is in L dash.

If L dash is polynomial time reducible to L then, if L dash is in NP then,a L dash will be in NP if L is in NP and if L dash is in pL is in NP if one L dash is polynomial time reducible to L and if L is in NP L dash will be in NP if L is in p L dash will be in pa slightly.

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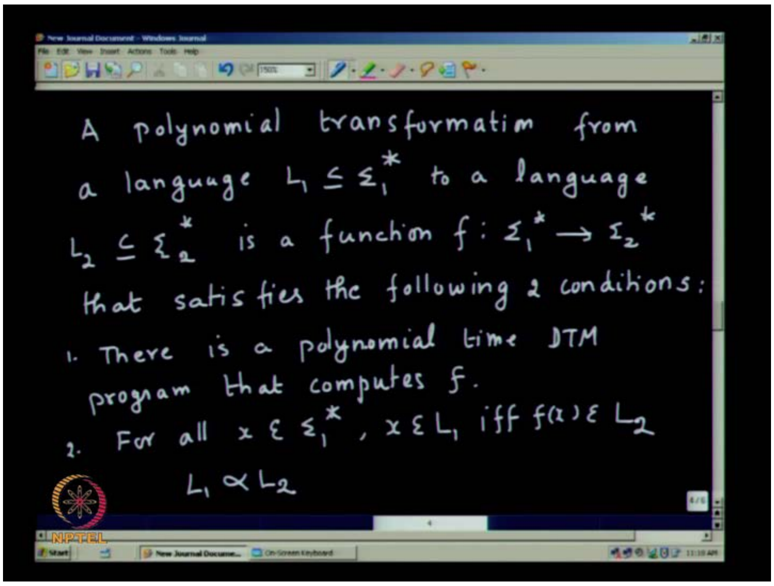

The same definition putting in a slightly different manner,same definition only is stated in a slightly different way that is all.A polynomial transformation from a language L 1 contained in sigma star to a language L 2 contained in sigma 2 star is a function f.Which transforms strings in sigma 1 star to strings in sigma 2star that satisfies the following two conditions,one there is a polynomial time deterministic Turing machine that computes the function f.That is it transforms L 1 into L 2for all x in sigma 1 star x belongs to L 1, if and only if f of x belongs to L 2then, you use the symbol L 1 is reducible to L 2, L 1 L 2this symbol is used.

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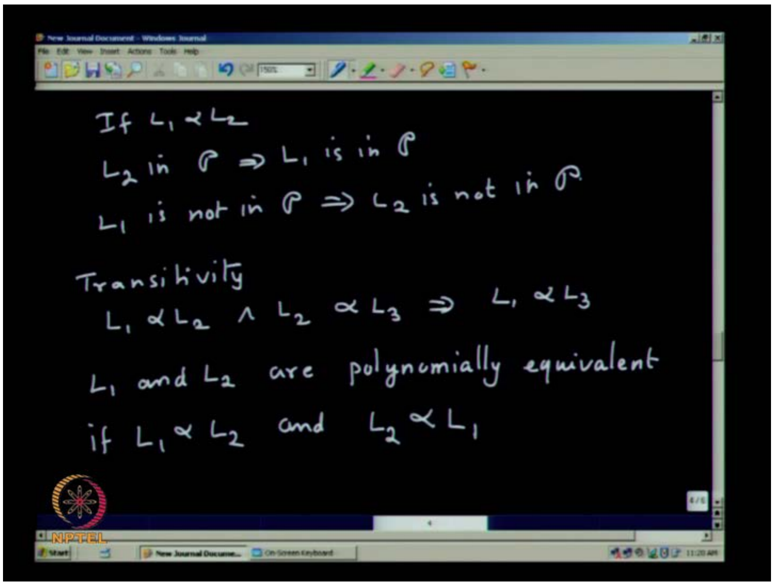

Now, havingdefined this let ussee some more things.If L 1 is polynomial time reducible to L 2then, if L 2 is in p then L 1 will be in p,because L 1polynomially you can transform to L 2 and you can accept L 2 in polynomial time.If L 2 is in p that means L 2 can accepted inpolynomial deterministic Turing machine can accept it in polynomial time. L 1 can be acceptedby a deterministic Turing machine in polynomial time, because from L 1 you can convert it to L 2 and then accept it L 2 polynomial time this is also a polynomial and acceptance of L 2 is also polynomial.So, L 1 can be accepted by adeterministic Turing machine in polynomial time.

Now, transitivity you can very easily see, if L 1 can be polynomially transformed to L 2and L 2 can be polynomially transformed to L 3 then, obviously combining the one Turing machine can transform L 1 to L 2.And that is a deterministic Turing machine working in polynomial timeand another Turing machine can transform L 2 to L 3.That is also done by a deterministic Turing machine in a polynomial time.So, combining the two you can have a Turing machine which transforms L 1 to L 3 in polynomial time.So, if L 1 is polynomial time reducible to L 2 and L 2 is polynomial time reducible to L 3then, L 1 is polynomial time reducible to L 3.When do you say L 1 and L 2 are polynomially equivalent, if you can reduce L 1 in polynomial time to L 2 andyou can also reduce L 2 in polynomial time to L 1.

There issomething else called log-space reducibility,I will not consider it now.So, this all this reductions which we are going to consider of polynomial time reductionso, with this we willproceed further.In a way what is acomplete problemyou have a class in that class you pick up some problems and show that they are as difficult as any other problem in that class those problem are called complete problems.

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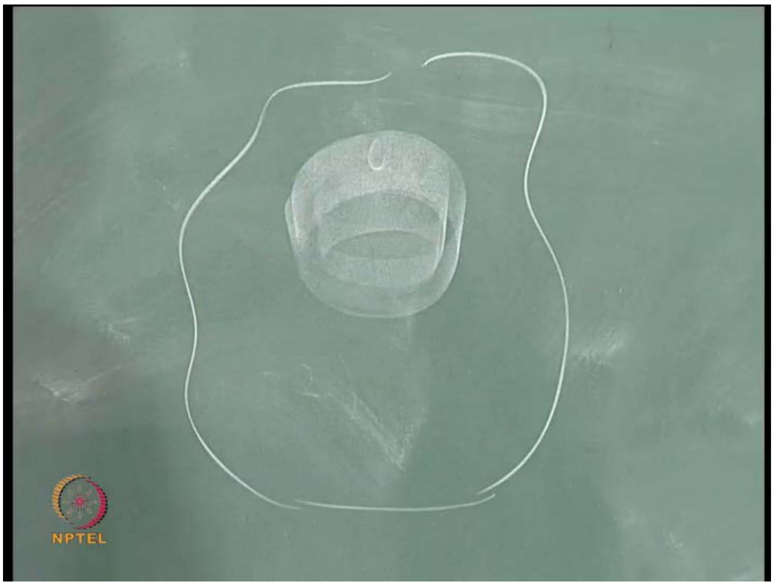

This need not be in NP we are going to study about NP by NP complete problems, but it can be other class also.Any class you take youmake youfind out some problems and these problems you show thatthey are as difficult as any other problems in that class.So, those problems are known as complete problems for that classyou have also have PSPACE complete problems and so on.

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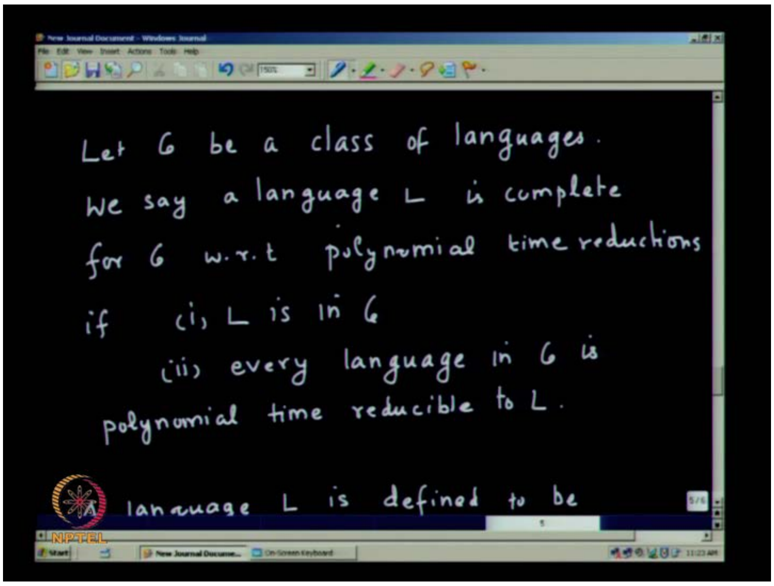

Let c be a class of languages it is a general definition then we will define for NP.What is an NP complete problem.

Let c be a class of languages we say a language L is complete for c with respect to polynomial time reductions.If L is in c and every language in c is polynomial time reducible to L.There are two conditions to be satisfiedgenerally, we take polynomial time reducible sometimes certain cases you have to take log space reductions and so on.So, that is for some other classhere generallywe are taking polynomial time reductions.So,I willrepeat this definition again, let c be a class of languages this is a class sayand a language L is said to be complete for that class,it is as difficult as any other problem in the class.

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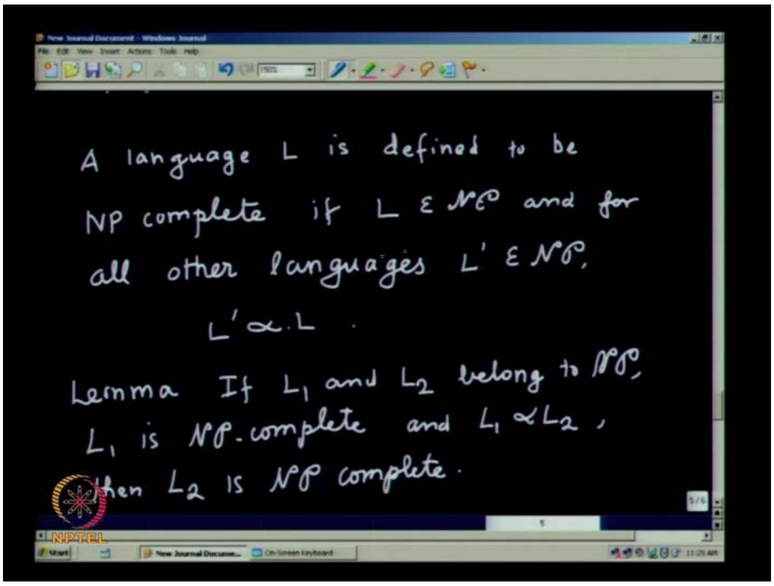

A language L is complete for c with respect to polynomial time reductions.If L is in c and every language is c is polynomial time reducible to L,two conditions have to be satisfied.

Now, let us go to the definitionof NP complete in a same thing, but we will specify NP specifically.A language L is defined to be NP complete if L belongs to NP and for all other languages L dash belonging to NP, L dash is polynomial time reducible to L.Now, we are therestricting our to one classthe class NP.A language I willrepeat again, because these definitions are important.A language L is defined to be NP complete,if L belongs to NP and for any other language L dash belongs to NP, L dash is polynomial time reducible to L.

Now, what is NP hard?There are two conditions,one is this, the second is this any other language L dash,Ldash is polynomial time reducible to L.There are two conditions, if the first condition is not there, but only the second condition is therethe language is said to be NP hard.The language means you can also talk for problems when, you talk about languages corresponding problems also you can talk.

Lemma if L 1 and L 2 belong to NP and L 1 is NP complete and L 1 can be polynomially reduced to L 2 then, L 2 is NP complete.

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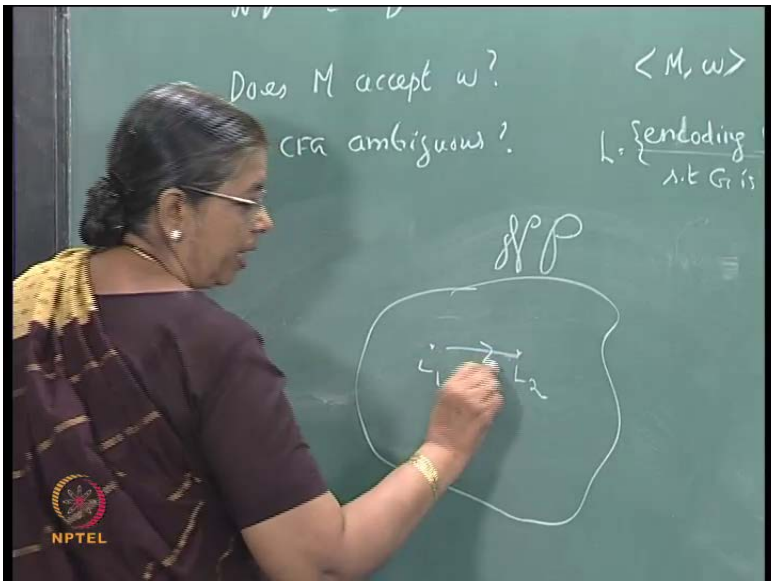

So, suppose this is the class,you are considering the class NP and this is the class L 1 and L 2 are in this L 1 is inL1 is NP complete and if you can reduce L 1 to L 2then L 2 is also NP complete.How do you prove this?Now,I know that L 1 is NP completeI want to prove L 2 is NP complete.

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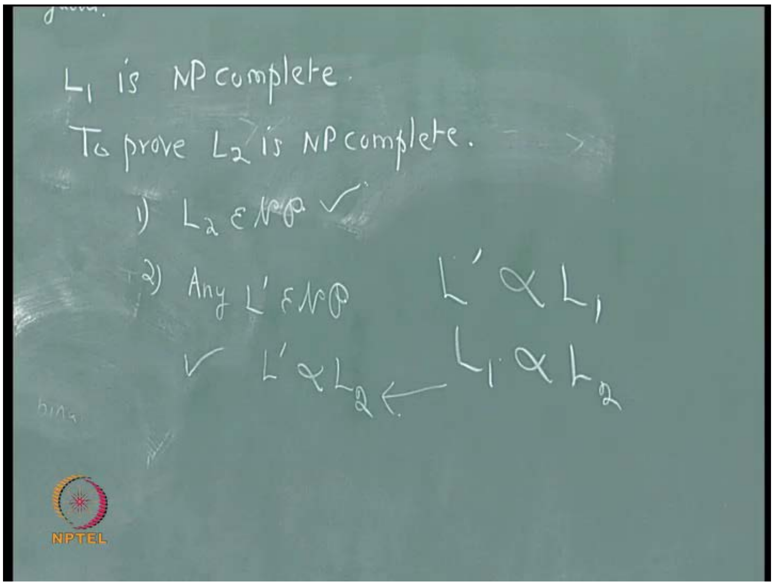

Now, there are two conditions which are to be satisfied.What are the two conditions?You have to show that L 2 belongs to NP first condition, but that is given L 1 and L 2is in NP.So, this is satisfied,the second condition any L dash belonging to NP for any L dash belonging to NP, L dash is polynomially transformable to L 2 this is what we have to prove, but L 1 is a NP complete, we already know L 1 is NP complete.So, L dash if you takeit is polynomially transformable to L 1and it is given that L 1 is polynomially transformable to L 2so, by transitivityyou get this.So, the second condition is alsosatisfied.

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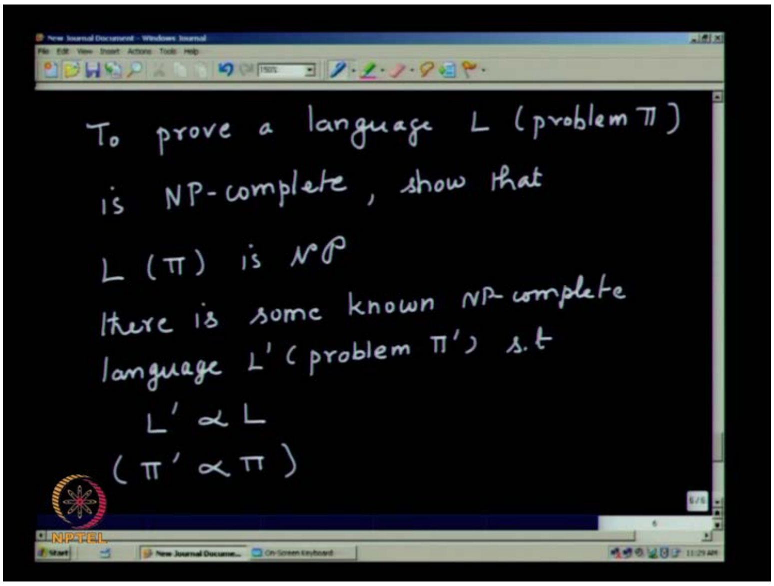

So, that thislemma now, what you have to doyou is given a language L or a problem pi a new problem pi then, you want to show it is NP complete.How do you show that first of all you show L is in NP or corresponding problem.We talk about pi is in NP there is some known NP complete language L dash such that L dash can be reduced to L in polynomial time.You can talk about in theas problema problem pi is said to be NP complete, if pi is in NP and there is some known NP complete problem pi dash just that pi dash is polynomial reducible to pi.

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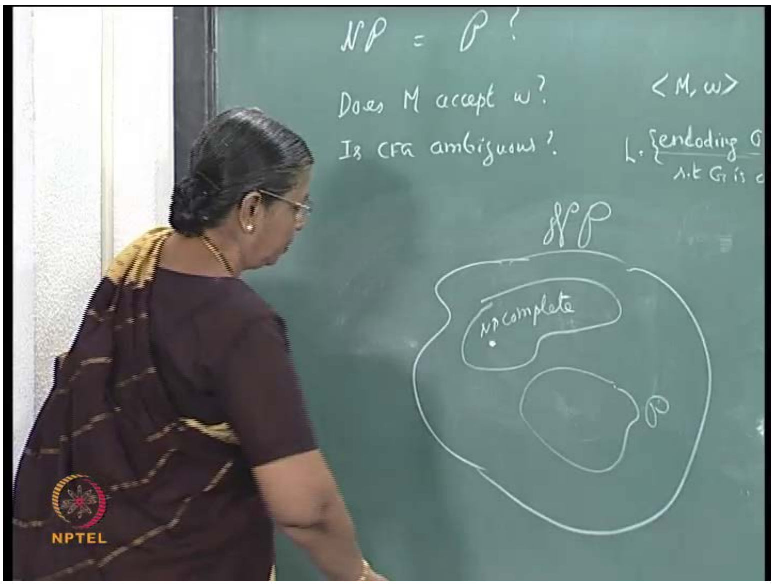

So, the class is like this NP is this classand p islike this NP complete problemscurrentthing looks like this.Whether this is proper inclusion or not we do not know it is still an open problem.

So these problems are such that even for one problem, if you give a polynomial time algorithm deterministic polynomial time algorithm everything collapses p becomes equal to NP,even if you are able to give a solutiondeterministic polynomial time algorithm for one of thisthen, NP becomes equal to p.

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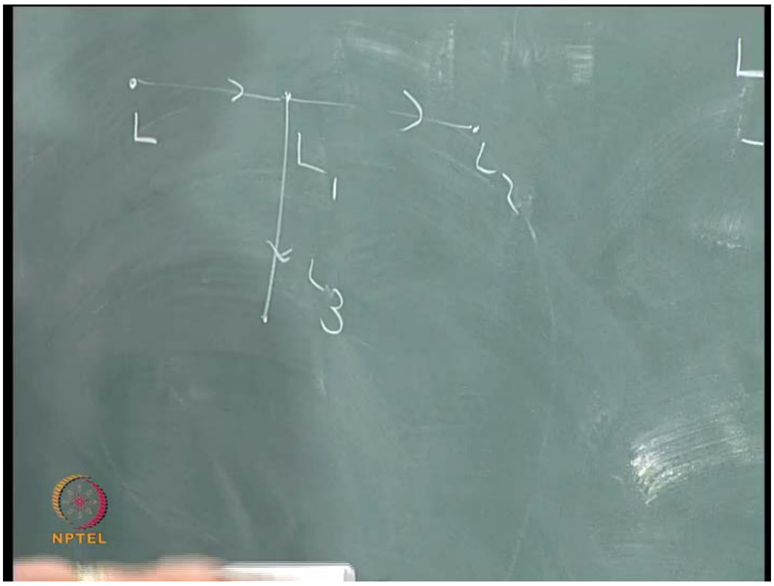

Now,if you know one problem to be NP completeother problems you can show to be NP complete bysome other problem by reducing this to this.If you know one problem to be NP complete you can reduce polynomial time reduced to another new problem and new problem is NP complete.Then once you know this you can reduce it to L 2you can reduce it to L 3and so on.

But first one problem you have to show up to be NP complete,that isthe Boolean satisfiability problem.

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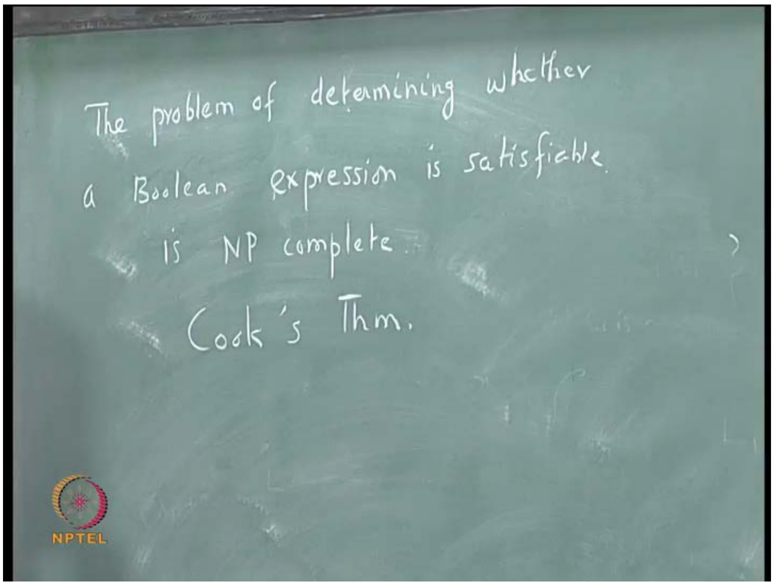

So, we will show that Booleansatisfiability is NP completeorthis what you want to show.The problemofdeterminingwhether,a Booleanexpressionis satisfiableis NP complete,this is known as Cook’s theorem.So, we will consider the proof for this now so,we will consider how to prove that.

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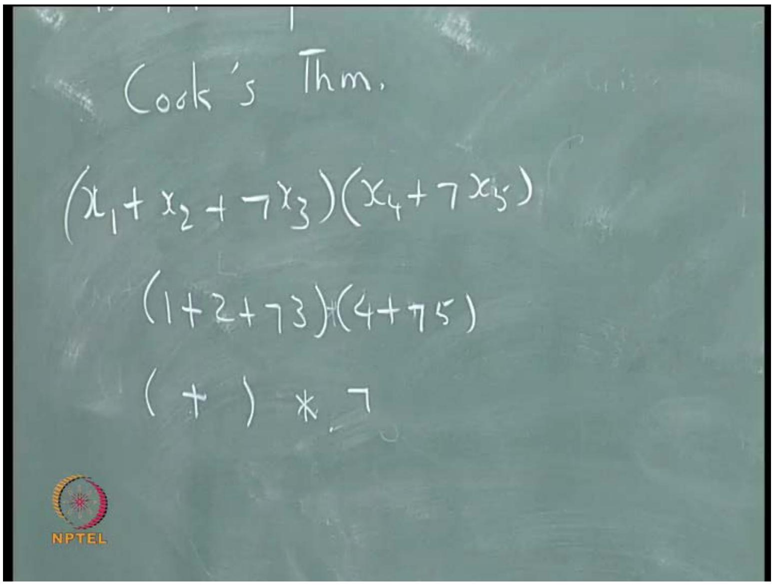

Now, in order to show that Boolean satisfiability is NP complete, first of all let us consider the representation of it.How do you represent the Boolean expression?As I told you can use something like say x 1 plus x 2plus notx 3into x 4plus notx 5something like thatas 1 plus2 plus not3,4 plusnot5.The symbols you use are left parentheses plus parentheses you may also use star here star is omittedfor and you use star for or you use plus and then the notsymboland the variables are represented by decimal numbers.

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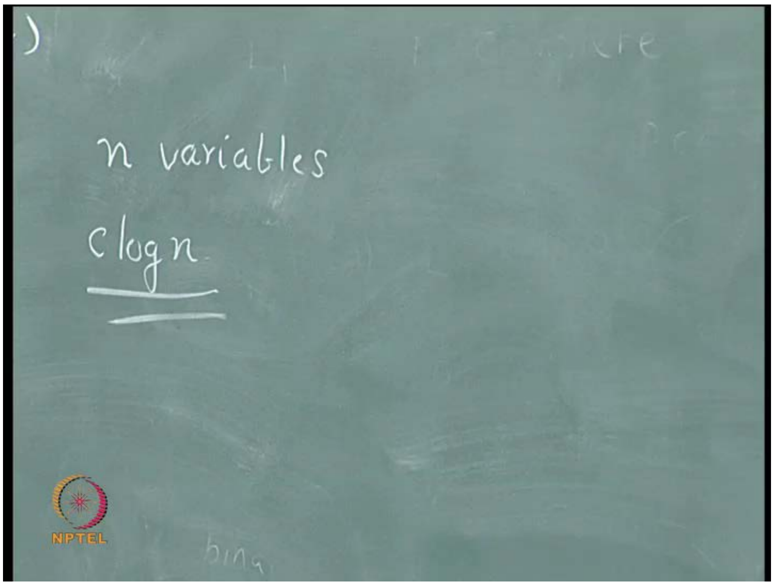

Now, if you haven variablesto represent each one of them in decimal or something like that.You require something like clog nbits,but let us not bother about this factor,because ultimately when you have a polynomial time multiplying by log nis notgoing to affect the polynomial or the non polynomial time of it.So, let us notworry about thatassume thateveryvariable is represented as ones symbol like this.

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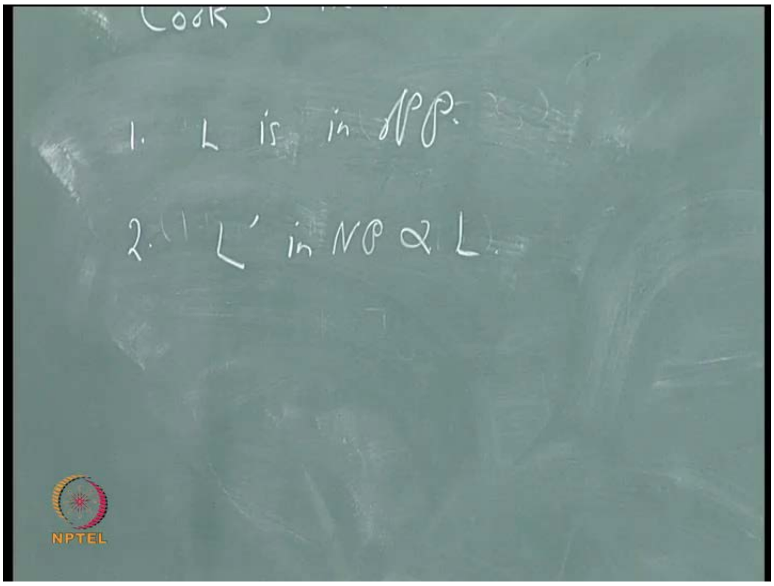

Now, there are two aspectsto proving something is NP complete you have to prove that L is in NPand twoany language L dash in NP is you can transform to L isn’t it.Now,how do you show that first this is in NP can you have a non-deterministic Turing machine which will take a Boolean expression as input and say whether it is satisfiable or not in polynomial time.You can have a non deterministicTuring machine,which will take as inputs.The given Boolean expression it willnon deterministically guess an assignment,if there are n variables there are 2power n possible assignmentsif you use a deterministic Turing machine one by one you have to try the 2power n assignments.

But if you use a non deterministicTuring machine non deterministically you can guess avalue for all each of the variables that is you can non deterministically guess an assignment after guessing that assignment you have to evaluate that expression.How much time it will take to evaluate that expressioncan it be done in polynomial time?It can be done in polynomial time, because it depends on the passing algorithm you have for acontext free grammar can generate all Boolean expressions and how much time that parsing algorithm will have and so on.Or otherwise even look at it as a given string is of length n.

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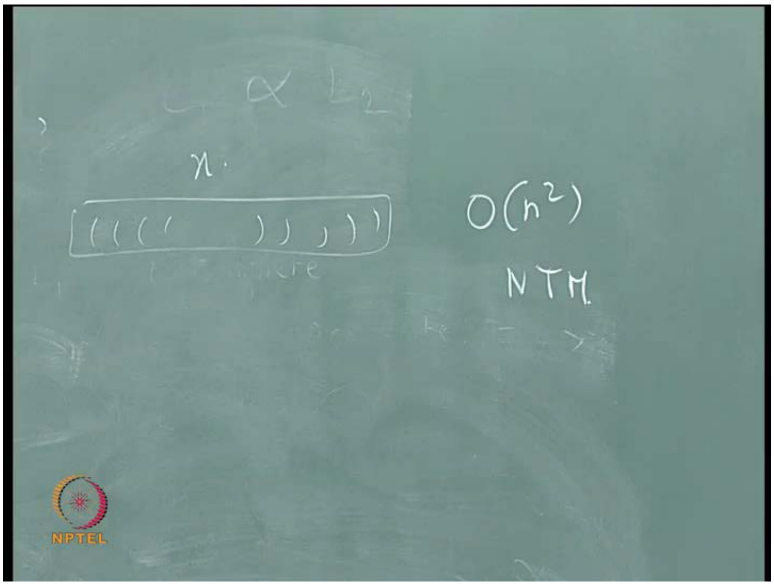

The Boolean expressionwe are considering in the terms of input isn't it.Input is of length nat the most you can havensort of a order n parentheses nested parentheses you can have.So, make one pass evaluate all the inner most things will be evaluated make another pass evaluate and so on.

In one pass will take order in timeat the most you have to maken passes.So, in order n squared time anyway you will be able to evaluate.Once your are able to guess an assignment evaluation of the expression will take order n squared timesome shifting this way that way all those things will be there that does not matterour aim is whether, it is a polynomial or not the constants do not matter so muchat this stage.

So, in order n squared time you can have anon deterministicTuring machinewhichwill evaluate the Boolean expression and say whether it is satisfiable if it reduces to 1it is satisfiable, if it does not reduced to if it reduces to 0.

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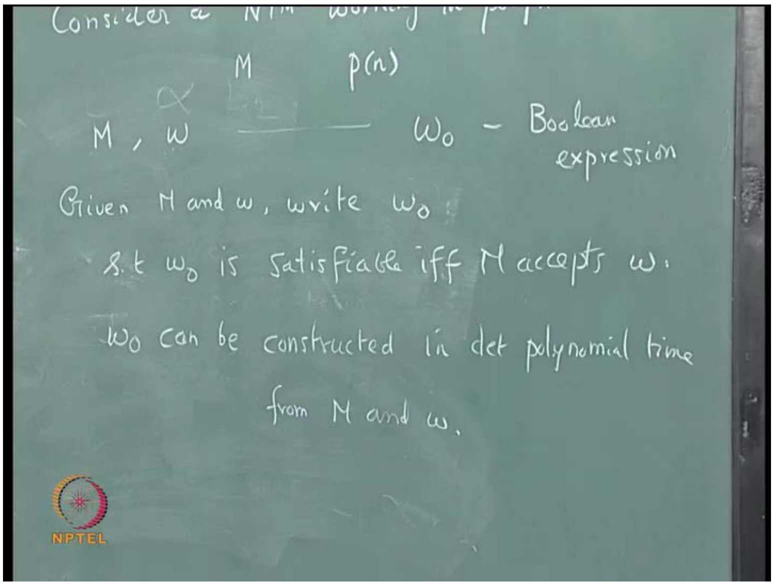

So, L is in NP this is very clear.The second is anyTuring machineany non deterministic any language L dash accepted by a non deterministicTuring machine is polynomially transformable to L this is the proof.What we do is consideringa non deterministicTuring machineworking in polynomial time.So, there is a Turing machine Mfor which the time complexity is a polynomial p n.

Now,given M andinput w,M is Turingnon deterministic Turing machineswhich will accept w in polynomial time accept or not.So, given M and wyou construct a Boolean expression w 0this is theBoolean expressionwhich will be satisfied,this w 0will be satisfied if and only if m accepts wgiven M and w you write an expression w 0a Boolean expression w 0such that w 0is satisfiable if and only if M accepts wgivenM and wwritew 0 form an expression w 0 such that,w 0issatisfiableif and only if Macceptsw.

But the point is given M and w to construct w 0it takes only polynomial amount of time there isa deterministic polynomial time algorithm which will construct w 0given M and w.So,there isw0 can bethis is the main point w 0can be constructedindeterministicpolynomialtimefromM andw.

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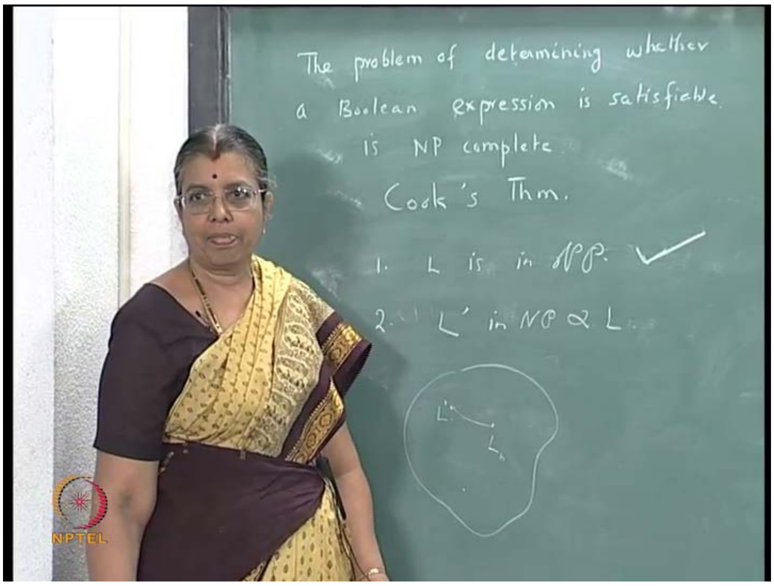

So, if you do that what you are showing isany L dash in NP this is some language a non deterministicTuring you can convert to this Booleansatisfiabity.If I put it as L 0in polynomial time.So, if you prove this then that means, we are showing that Booleansatisfiablity is NP complete.

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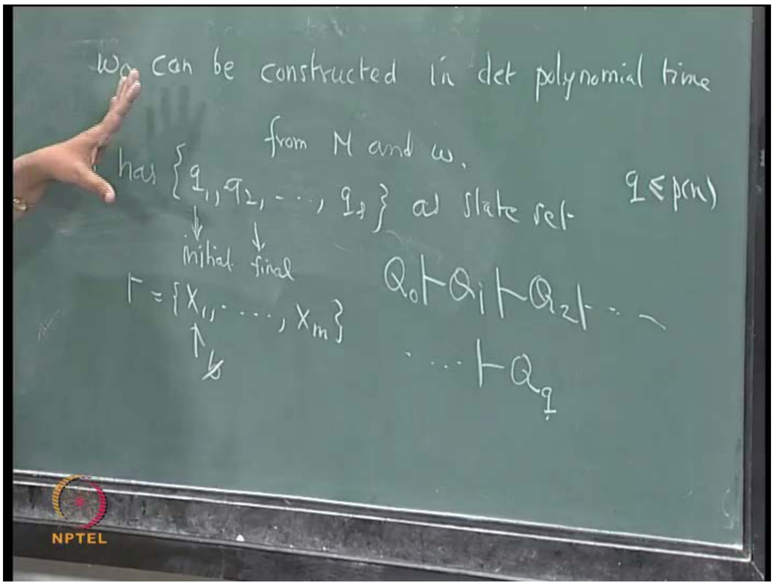

Now, we know how do we go about constructing w 0from M and wthere are some assumptions you have to make assumptions maybe I willwrite here itself M hasstates q 1 and q 2,q sthis is the stateset of states and this the initial stateand this is the final statewithout loss of generality you can assume thatthen the alphabet gammaisX 1,X 2,XM,M symbols are there in the tape alphabetand the first symbol you can take as the blank symbolone of them you have to take as blank symbol does not matter which one .

So, the initial ID of the Turing machine let us say some Q0 then the next IDwill beNext ID will be Q 2and so on.After someafter some stepsat Qq it will accept or reject.What is this Qq is less than or equal to p ofn maximums number of steps will be p of nwithin that step it willaccept.So, q is less than or equal to p of nthese are some assumptions we makeone more, two more things we have to take into account that I willmention.

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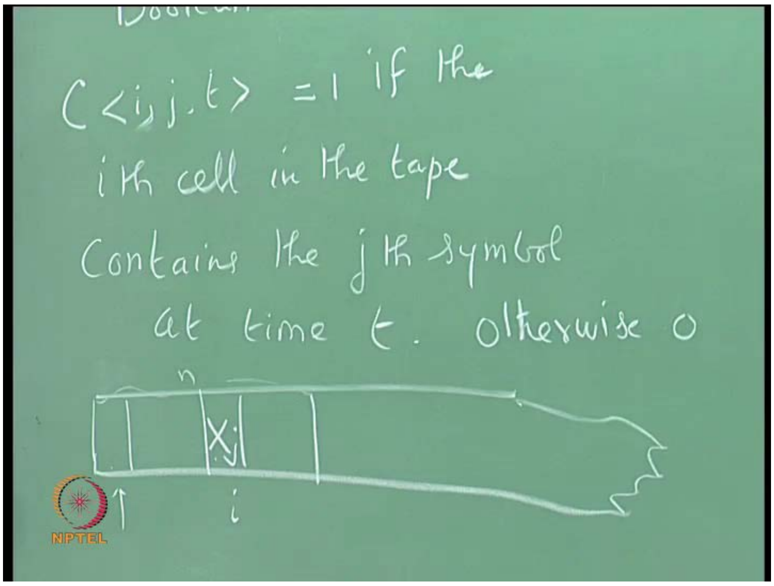

Now, in order tofind the expression w 0,we have to use some Boolean variables.What are those variables?You have a set of variables of the form CI ta collection of variables of the form CI t and what is the meaning of this variables isCI t is equal to 1if the ith cellin the tapecontainsthe jth symbolattime t.The tape will be like this,the input is of a length n initially it startyou will be starting at this point initially initial state will be q 1, if the ithcellcontains the jth symbol at time t then,CI j t will be equal to 1 otherwise 0.Now, what is the maximum number of cells you will requiresee machine starts here and it has to halt within p n steps.So, at the most it will need upto p n cells even, if it moves just forwardit will be going up to the p nth cell.

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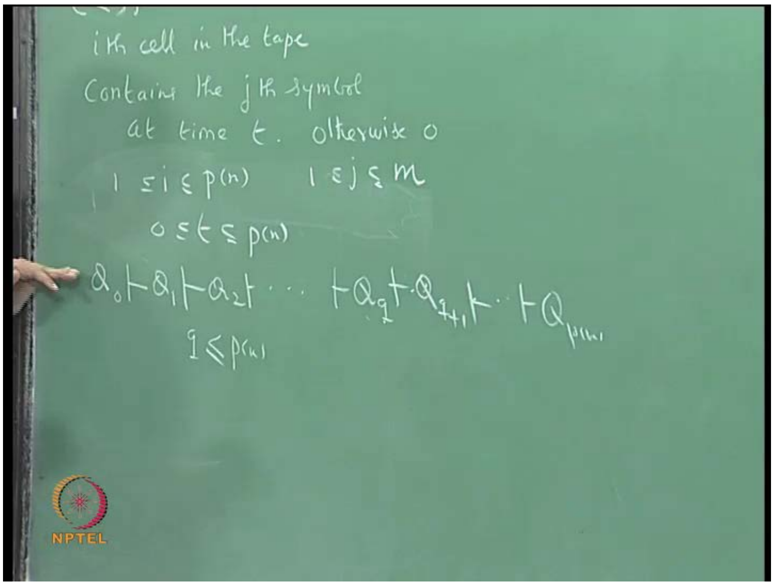

So,what is the range for I1top nfrom the first cell it can go upto the p nth cell it will not use more than that,because the time complexity is p of n.What is the range for j,j can be anyone of the M symbols x 1, x 2,x m,we have defined.So, it can be any one of theMsymbols.So, the range of j will be 1 to M. What is the range for t initial ID is time t is equal to 0 initially you start at time t is equal to 0 and you can go up totime p n there is a small thing you have to note herefrom ID q 0you go to q 1 you go to q 2 and after somestepit stops that q will be less than or equal to p n if it is less than p n we want all the ID’s to go up to p n steps,we do not want to distinguish between something which less than p n and equal to p n.So, the other ID’ssee hereall these are taken identical to Qqnext step it is also remaining the same no moveso that, we assume that everything takes p n steps.

So how many way such variables will be there I range is from 1to p n, t ranges from 0to p n, j ranges from 1 to M,M is a constant, s is a constant number of statesis q 1, q 2, q s, s is a constant.So, you have a orderp squared n variables of this form.

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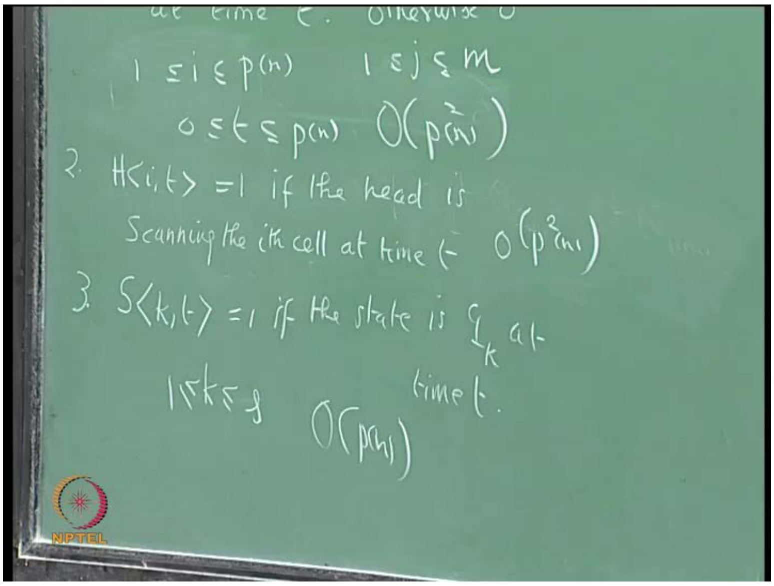

This is one setanother set of variablesHI t,HI t is equal to1 if thehead isscanning the ith cellattime t.You have variables these have meaning like this HI t is equal to 1 if the head is scanning the ith cell at timet.So, what is the range for I1to p n range for t 0to p n again order p squared nvariables will be there.

Third set of variablesSk t state S denoting the statethis is equal to 1,if the stateis q kattime t.So,of course, will vary from 1 to S range for kis1 to Sfor t, it is 0to p n.So, here we have orderp of n variablesnow, each of this variables we assume that when you write the expression it takes only oneunit or one symbol.It represent as one symbolactually since, there are order p squared n variables you have to represent them meanssome each one you have to represent byin a binary notation or something like that another log n factor will be there.So, we are not worried about that log n factor, because ultimately it will not affect the polynomial or the polynomialaspect of it.So, each variable is taken as one symbol for our calculation at present.Now,this expression W0,we have to write first given m and w you have torepresent w 0and w 0is satisfiable if and only if it represents a sequence of moves like you know.

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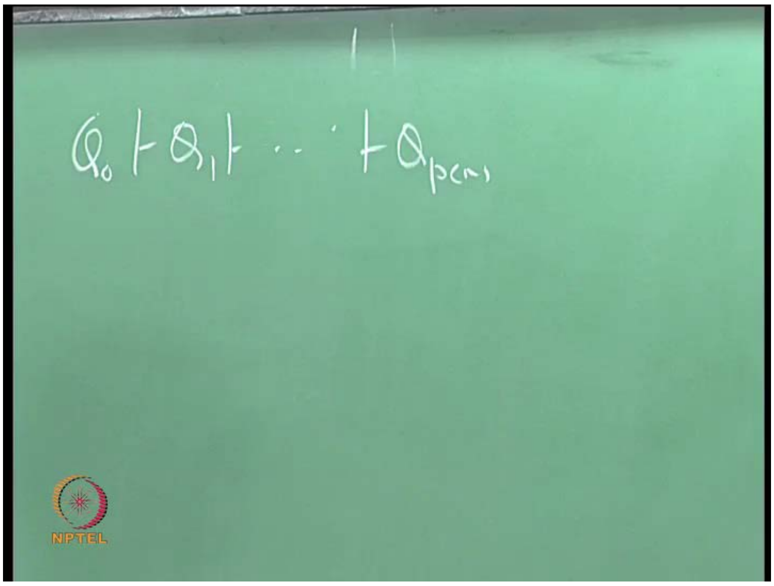

Which leads to acceptances q 0 Q 1 etcetera up to Q.W0is you have to write w 0in such a way that it will become satisfiable.If and only if there is a valid sequence of ID’sleading towards something.

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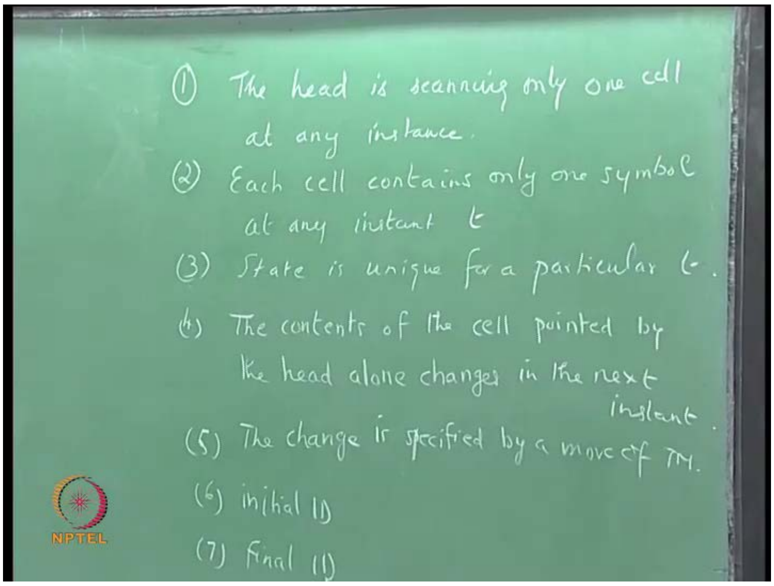

So,what are the conditions you have to look into that?These are the conditions you have to consider.The headisscanningonlyone cellat anyinstance,two eachcellcontainsonlyone symbolat anyinstant.Stateis uniquefor a particular tinstant t(( ))putfor a particular tthere can be only one state at any instant then machine can be only in one state.The containsof thecellpointedby theheadalonechangesin the nextinstant.The changeisspecifiedby a move ofthe Turing machine.6you have to take care of the initial IDand you have to take care of the final ID((no audio 44:19 to 45:00)).

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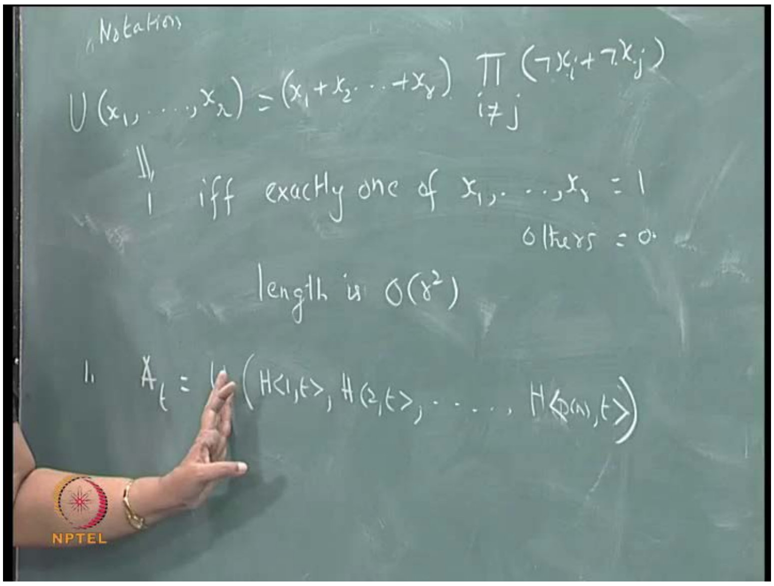

Now, we are assuming thateachvariableisrepresentedby a symbol.This is assumption we are making otherwise you have to multiply by a log n factor that is what I said.Now, let us consider for simplification we will use one more notation,an expressionusingrBoolean variables U,x 1, x 2,x r, this is defined like this it is x 1 plus x 2plus x randproduct of factors ofthe form x I plus 0x jI is 0equal to j.This is u, x 1, x 2,x n, is a Boolean expression.Which is of the form x 1, x 2,x r,into product of factors of the form 0x I plus 0 x jwhere I is not equal to jyou have all of them.So,for example, U,x 1, x 2,x 3,will be x 1 plus, x 2plus, x 30,x 1 plus,0 x 2,0 x 1 plus,0 x 3,0 x 2plus,0 x 3.It will be like that for r you can write like that.

Now, when we will this take the value one istrue.When exactly one of them is one and all the others are 0.When exactly one of them is one it will take the value of oneif none of them are one it will take the value 0,if two of them are one.So, it will take the value 0.Why if none of them is one all are 0 means this factor will be 0.So, the total expression will be 0if two of them are ones then there will be one factor if I j are one there will be one factor 0x i plus 0x j that will take the value 0.So, the expression will be 0.So, this is equal to 1if and only ifexactlyone of x 1, x 2,x r, is equal to 1othersequal to 0.Now, what can you say about the length of this expression there are r variables what can you say about the length of this expressionthis will be r.How many factors will be there.Nminusfor one If you take x 1 there will bea factor with x 2,x 3,x r,r minus 1 factorsthen, if you take x 2there will be factors with x 3,x 4, etcetera.So, that is r minus 2 and so on.Up to one,how many factorslike that it will be there1to r minus 1 that is r into r minus 1 by2 factors.How many symbols it will take 1, 2, 3, 4, 5, 6, 7, some constant number.So, it will be something like r into r minus 1 by 2into some constant k plus this one isr intor then some other constantl.So, anyway the length will be order r squarethe length islengthisorder r square this is important.So, we will make use of this notation for writing the expression of those conditions.So, there are sevenconditions which we have specifiedone by one we will write expression for them.

So, what is the first condition;The first condition is, the head is scanning only one cell at a time.So,A t is theexpressionat time t the head is scanning only one cell.So, this can be the condition, we can write like thisU ofH1 t,H2 t,hp n.What does that meanthe U is this expression. So, head will be scanning only one cell at a time.So, only one of the variables has to be one rest of them has to be 0 then only this will become one.

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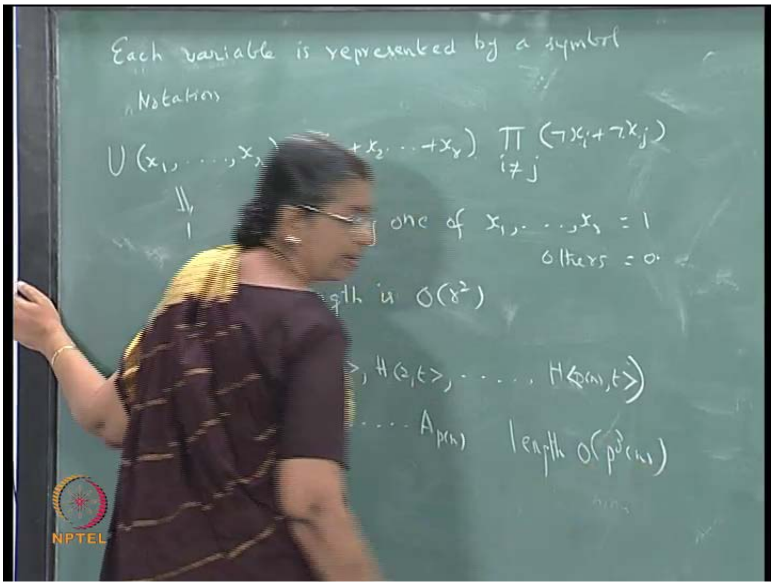

So,the whole expression is this should hold for instance t is equal to 0,1,2, up to p n.So, the expression A which satisfies the(( ))condition is A,0,A,1,A,2,up to ap n.Now, what can u say about the lengthof A,each A t is order p squared n it here, if it is r its r squaredhere it is p n.So, it is order p squared nand you have p n of them.So, the length will belength of such an expression will beorderp cubed.

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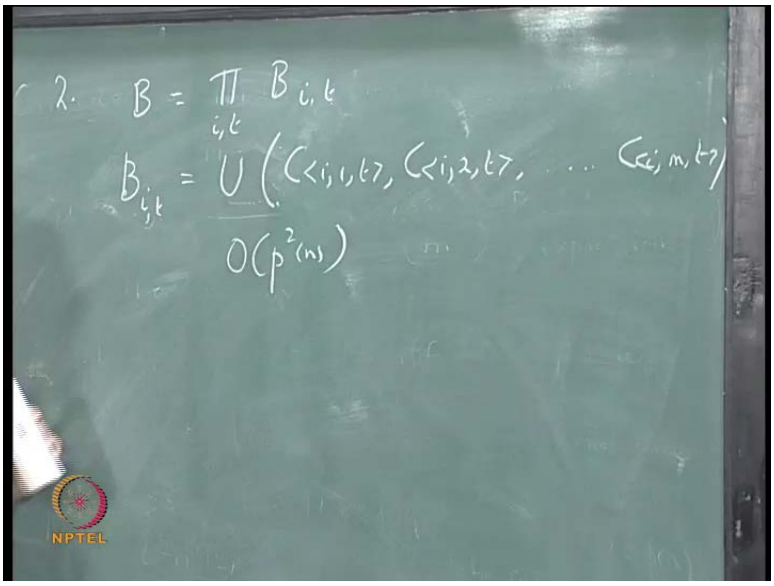

The second condition iseach cell contains only one symbol at a time.So, you write it as piBIthe expression for that will beBi t is at time t only one symbol will bethere that is this is U ofC i1tor Ci2t,Cimt.The ith cell can contain the first symbol or the second symbol or the nth symbol.Where the product is over i n t.Each cell can contain only one symbol at a time.So, at time instance tthe ith cell can contain only symbol one or symbol two or symbol m.So, this expression is equal to 1 only one of them is one and the othersare 0isn't it the way you have written U.So, this expression make surethat at any instance each cell contains only one symbol and what can you say about the length of B, B is the expressionthis is a product over I and t.

Where I will range fromone to p n, t will range from 0t to p n.The length of this will beorder m squared,because you have m variables here.So,thetotal this is a constant m square is a constantthis is constant, but thisI ranges from 1to p n, t ranges from0to p n.So, the whole length will be orderp squaredn.

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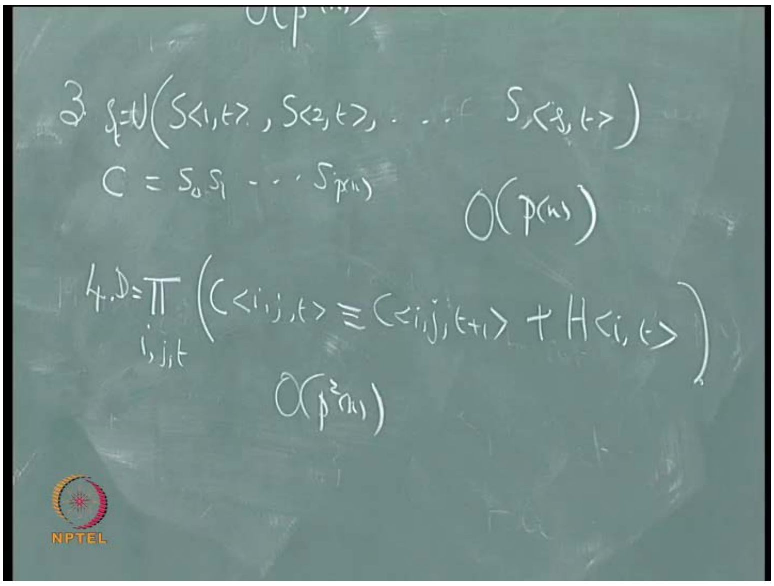

Then, the third condition.What is the third condition?The state is unique at any instance.So,S1 t,S2 tor S totally there are S states S tonly one of themI can put S t is equal to this U of this,U of this andexpression Cis S0,S1up to S p n that means only one of them will be 1 others will be 0 for a particular t and you have to consider t varying from 0to p n.The length of this is order S squared, but S is a constant rightthe length of this a constant but, this varies overfrom 0to p n.So, the total length of this expression will beorderp of n.

Thenfour fourth condition isonly the cells scanned by the headwill changethe contents of the cells, scanned by thehead will be changed the rest of them will remain as there isn't it.If you are if the head isscanning this one it will change this symbol, but rest of the portionit will be the same.So, that isCIj tis the same as CIj t plus oneor either the head is scanning the ith cellif head is not scanning the ith cell the contains of the cell I is the same at time t and the time t plus 1.This is the expression, but you have to take the product overIj tagain j will be 1 to m I will vary from 0 to 1to p n t from 0to p n this expression will also be oflengthp square n.

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The fifth condition isthe next move is specified this is the expressionD.The expression E 5thsatisfies the 5th condition.What is the 5th condition?The next move is specified by themoves of the Turing machine.So,either the head is not scanning the ith cell at time tor the state is notkat time t orthe ith cell is not containing the jth symbol at time t,I want to specify the move forthe case when the state is k and the ith cell head is scanning the ith cell and the contains is x j.So, ifone of this is violated, but if the head is scanning the ith cell the state is k and the ith cell contains j symbolwhat will be the next situation?Next situation will bestate will be someklt plus 1 state will change to some other state that is represented by kland head isC ith cell will contain some other symbol at time t plus 1and the head will go to I minus 1 or I plus 1isn’t.

So,HIlt plus 1this will be I minus 1, if the move is leftit is I plus, if the move is rightactually it is a nondeterministicTuring machine.What we are considering is a nondeterministicTuring.So, next move is not uniquethere will be some possible choice one choice is kljlIlthere may be another choices.So, consider all possible choices write them down.So,that is why we are using sigma here not pi sigma this or this or this one of the moves it will select.So, if the head is scanning the ith cell and ith cell contains the symbol j and the state is kthe next move will be given by one of them that will belpossible choice each one you writeotherwise, if it is not followed means head is not scanning the ith cell or the state is not k or the ith cell does not contain the jth symbol.

So, this expression tells you about the next move of thefrom one ID.How do you go to the next ID that is specified by this expressionthat is given byactually this is EI j k t is equal to this and you take the product e isI, j,again j will vary from 1to m k will vary from 1 to SI ranges from 1to p n t from 0to p n.So, thelength of the expression will be order psquare n.

The sixth condition is initial ID.What is a initial ID? H10 time 0 the first symbol will be (())state is 1 S10and first you C10, c20,cn0, this willget the input is A1,A2 depending upon thatyou will give the value here.Whichever symbol is there in the input you givethen, what can you say about cn plus 10c n plus,20 up to cp n0initially, you havethe input in the first n cellsand the rest cells are blankand blank is x 1 we have taken so, this will be 1 as a notation.

We have taken the blank to bex 1 then the final ID is just the state becomestwo at timeq 1is taken as initial state q 2is taken as the final state.Seventh condition is given by this expression these are all fixed lengththis is of length n this is only 1Iam sorry this is of lengthP N.

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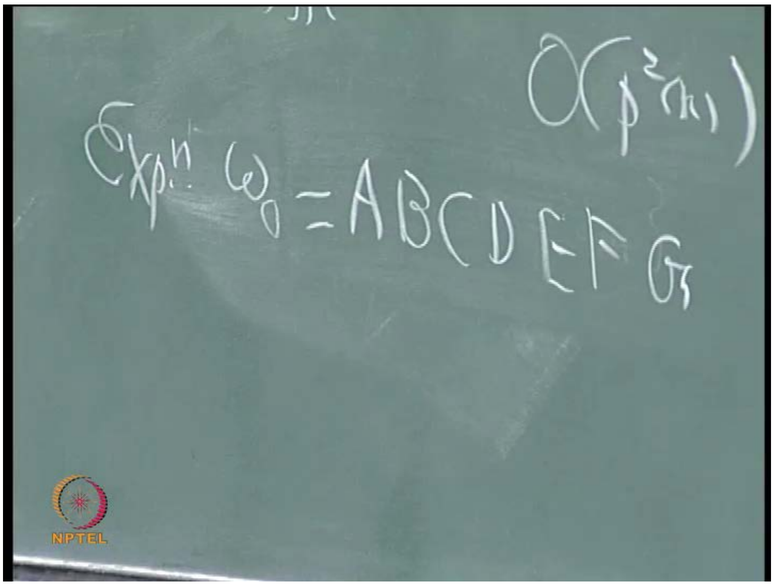

So, totally you have an expression Boolean expressionw notwhich is equal to A, B,C, D,E,if I denote this sixth condition by Fand the 7thcondition by Gthe expression is A, B, C, D, E, Fand G.

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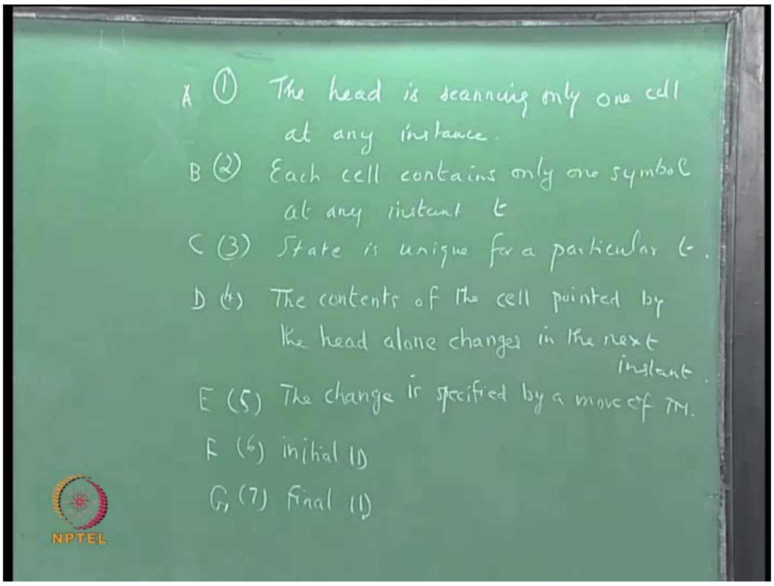

So, what we have done is, we havewritten expressions A, B,C, D,E, F,G,for satisfying the seven conditions.

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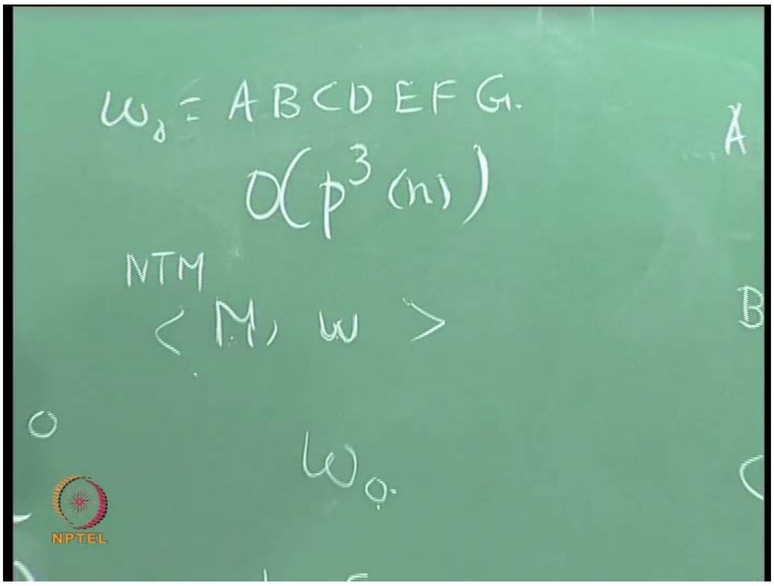

And we consider w 0to be equal to A, B, C, D,E, F,G.What can you say about the length of a it is Aorder p cubed n,B, C, D, E,they are p squared n Iam sorryC is p n B is p squared n D is p squared n E is squared n,F is p n G 1.

So, the total length of this isorderat the most its order p cubed n that is a polynomial.Here we are assuming that one variable is represented as one symbolyou may have to use some representation for represent variable in which case there will be one more logfactor.Suppose, you are having n variables you require log n bits to represent them.So,here the number of variables is order p square n log to that will be requiredthat does not matter any one more polynomial factor will be one more expression will be there.So, given M and wyou can writew 0 and the length of w 0is polynomial in n,nis the length of w and since, the length of w 0is like that and there is you know, how to write this w 0see there is a systematic way of writing it.

S, w 0can be written down from M and w in polynomial time its length is polynomialand it can also be written downin polynomial time.

So, any nondeterministic Turing machine M if you takeany nondeterministic Turing machineMyou take and the input is w from this, you can write w 0in polynomial amount of in polynomial time.

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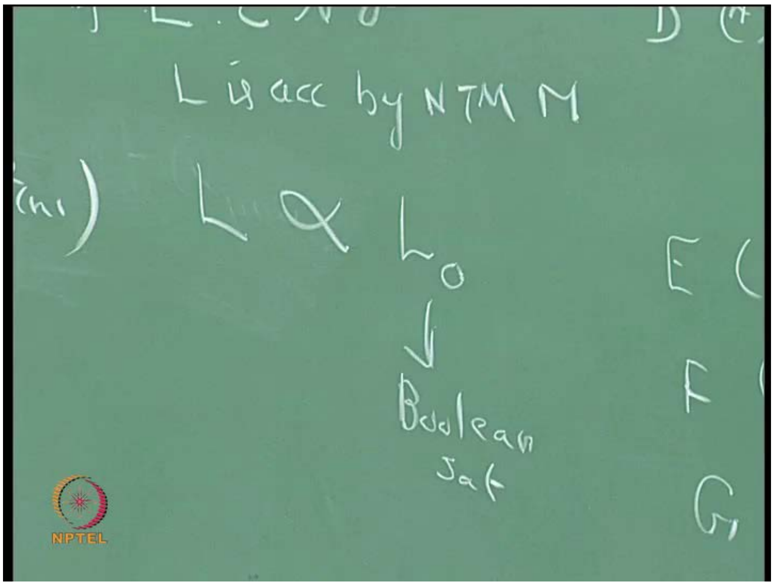

Deterministic polynomial time that is if L belongs to NP then, it is accepted by L isaccepted bynondeterministic Turing machine Mand you know that from this.We can write down w 0 such that w is satisfiable if and only if M (( ))accept wthat is L is polynomialtime reducible to L not L not is the Booleansatisfiability problem this is Booleansatisfiabilityany language L in NP can be transformed in polynomial time to L 0 so, this shows that Booleansatisfiability is NP-complete.So, we will continue in the next class lets tomorrow11 o'clock.