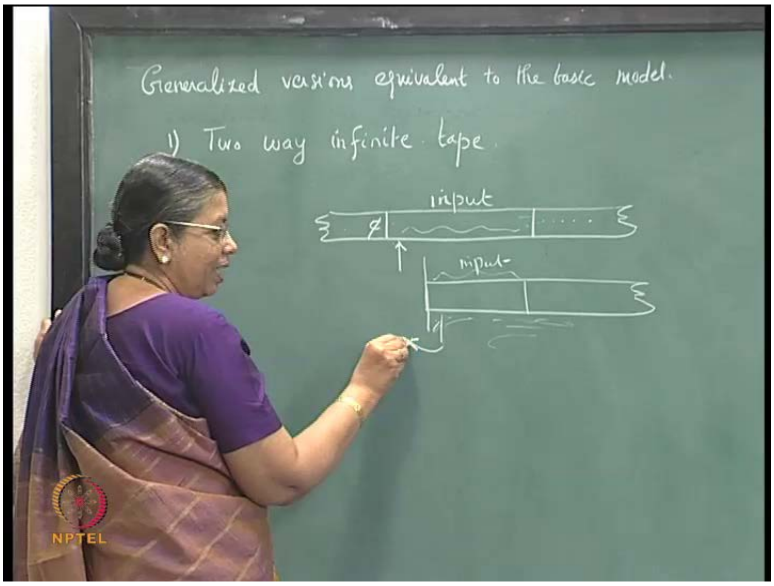
Theory of Computation
Prof. Kamala Krithivasan
Department of Computer Science and Engineering
Indian Institute Of Technology, Madras

Lecture No. # 29
Generalized Versions of Turing Machines

Today we shall considersome generalized versions of the turing machine which are equivalent to the basic model. Actually, when we were considering as ainput and output devicewe had a two way infinite tape.When we looked at it as a accepting device it was one way infinite left hand was fixed or we justified in doing that.Actually it does not matter whether you consider it as two way infinite or one way infinite you can do the same thing.

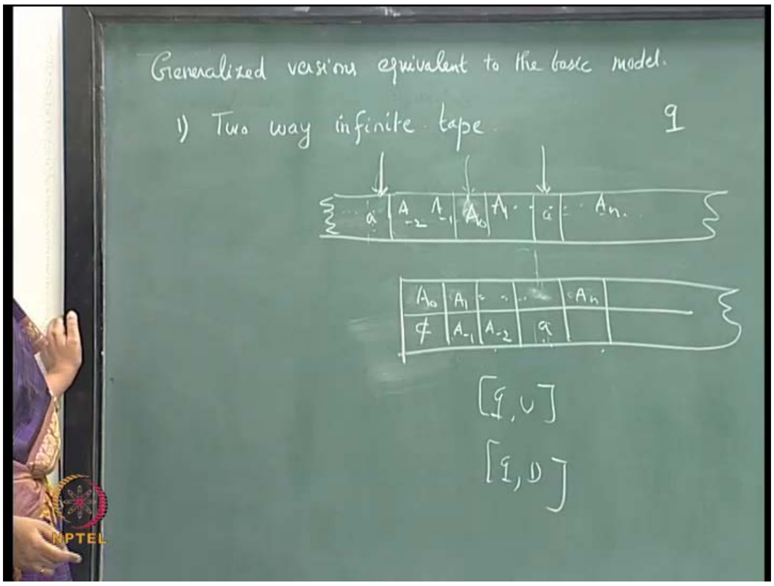
(Refer Slide Time: 00:46)



So, the first thing isyou make the tape two way infinitefor acceptance,two way infinite tapeis equivalent to one wayinfinite tape.How do we prove that?You have atwo way infinite tapeand the input is given here,when you look it as acceptance you start from hereafter making some movesit goes to the final state and accepts.We can move makeuse of some blank cells in this side; we can make use of some blank cells in this side also.Butif you look at it as a one way infinite tapethe input isgiven hereyou will start from hereand you can make some moves and then accept it.If itthe turing machine ever tries to moveof the left end then it will halt, it will reject the input you should not move of the left end of the tape, when you look at it as a one way infinite.

Now you can very easily simulate a one way infinite tape with the two way infinite tape,isnot it.Only thing is when the input is given you make a marker here,on this celland so, then you just simulate it as you behave here you can just simulate it.If at all it tries to move of the left end it willgo to this marker cell then you have to reject that is all.To simulate a one way tape by an two way tape is very easy.

(Refer Slide Time: 03:00)



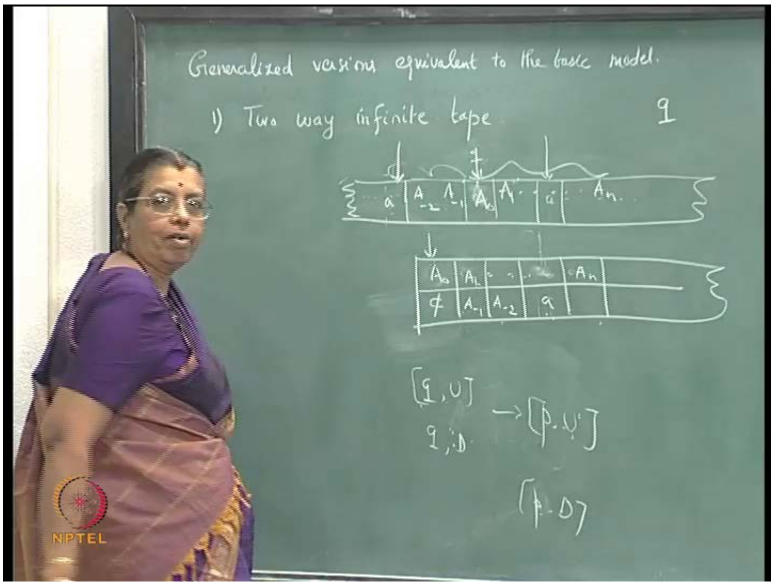
Now how do you simulate a two way tape with the one way tape?Now if you have a two way tape we can make use of some cells here alsofor that you take a tapewhich has got two tracks.Consider a one way infinite tape which has got two tracksif in the two way tape at sometime after some moves you started at this position say A naught some symbols are there A1, A2A n.You might have use some symbols A minus 1,A minus 2, etc. Here you start initial position was this then you have made use of some cell cells here input was here then you could make use of some blank cells here,you have made use of some blank cells to the left also.

Now in that case how do you simulateit with the one way infinite tape you start with thisA naught, A1whatever was the input suppose the input was saya naught. I will writea naught,a 1a n.The input will be herelike this, the lower track will be just blank at this stage then after making some moves see when you want tofirst move say A naught first here originally it was replaced by A naughtand you move right.This willbe replaced by A naught and you move right, but first what you do is you whenever you make the first move print a symbol some marker on the lower track at the first cell.Then after some time this is the content of the tape say after sometimethese symbolsA naught,A1A n will be there on the first trackA minus 1, A minus 2 these symbols will be there on the lower track in the reverse order A1 this way it will be like this.

So, at some timeyou are reading a symbol hereyou will be reading that symbol here, but you will be reading from the upper track only.So, whether you are going to read from the upper track or the lower track that information you have to keep in the state.In the original machine if q is the state in this machine you will takeq, U as the state or q,D as the state.Upper track q,Ddown or something like that or you can say q,L lower track.So, the state will keep the information we have earlier seen that you can make use of the state to store some information, here you are making use of that to store the information.Whether you are going to bother about the symbol in the upper track or in the lower track.

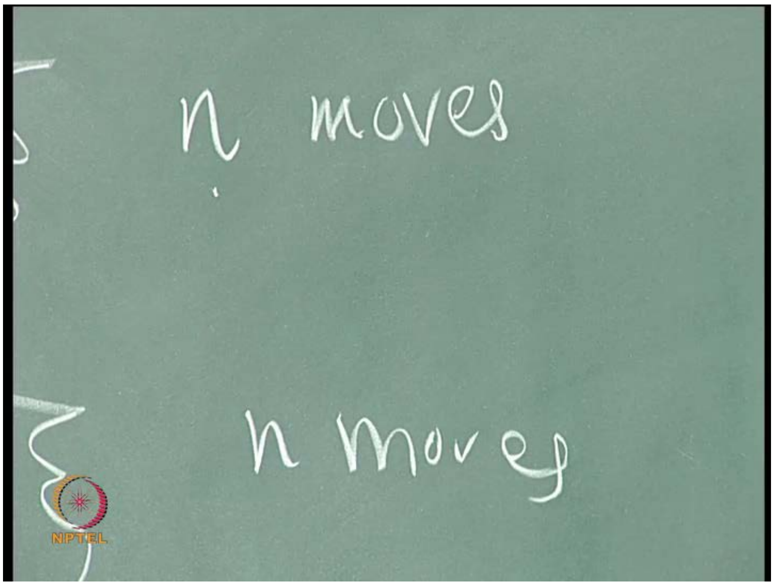
So, in this case supposeyou are going to read a symbol a that is herethen after reading that you move right means here also you will move right.If you move left means here also you will moveleft,on the other hand if you are reading some symbol heresay some athen after making the move if you move left here you have to move right,if you move right here you have to move left here.So, if you are reading the symbolthis symbolthe tape head here will bepointing to this.

(Refer Slide Time: 06:58)

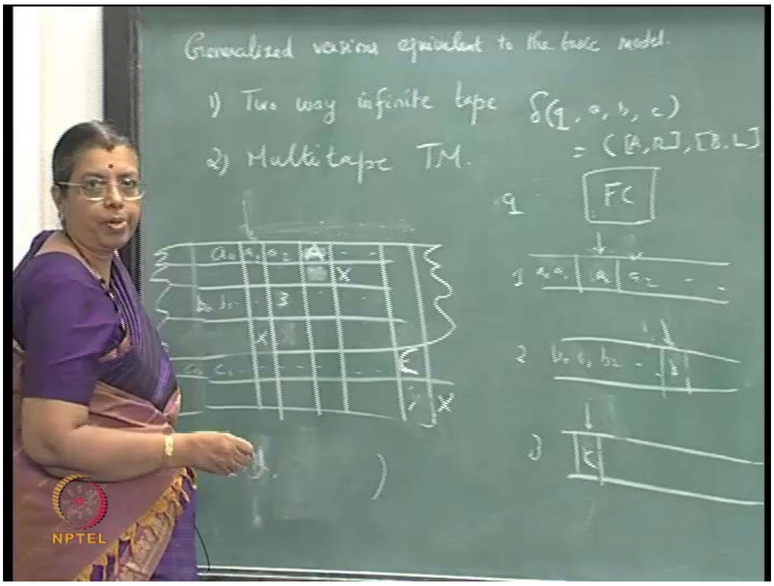


Now if the move is to the right in the state you will change it whateverthis when you are reading here if the state will be either q,U or q,Dwhatever it isthen if the move is to the right the state will change to somep,U.If the move is to the left then the state will change to move will be right whatever you read here the next move will be to the right, but in the originalone way tape I mean in the in one track if you move right then the state will change to p,U.If you move left the change willstate will change top,D.You are reading this in state qand you are going to change to state pand move right means here from q,U you will change the state to p,U.If you are going to move left means from this or this you will change the state to p,Dand move right.That way whatever happens in this portion you simulate in the upper track and whatever happens in this portion you simulate in the lower track.

(Refer Slide Time: 08:49)



So, two way infinite tape can be simulated by a one way infinite tape. So, it does not matter whether you take it as two way infinite or one way infinite both are equivalent the language accepting power is the same.Now if this two way infinite tapean input is accepted in n moves,how many moves will this make one way infinite tapefor one move?Here how many makes it moves just one moveonly one movehere is simulated by a move here it may be move left or moveslight change may be there, but one move here is simulated by one move here.So, if this makes n moves before acceptance how many moves it will make?Thisis also make n movesthe number of moves is the same.(Refer Slide Time: 10:20)



While printing this you are counting this printing the symbol that the first move itself it can print and move initialor separately you can use one move that in that case n plus 1, but in the first move itselfyou can print and move that does not matter. So, anyway it isn goes to n or n plus 1does not matter linear.So, number of moves is not very much affected by this simulation,the next iswe consider multi tape turing machinethis is different from multi track.We have already considered multitrackfor turing machine construction techniques,now we are going to consider a generalized version this turing machine has a finite control I willwrite here it has a finite controlandseveral tapessay for example, three tapes I can takeyou can have any finite number of tapes cannot be infinite it can have 1,2,3upto k tapes.

Now each tapeis provided with the tape head,each tape has a separate tape head multi track there will be only one tape and that tape is divided into several tracks and there will be only one tape head it will be reading a tuple that is the main ideathe tape alphabet is taken as a tuple that is the idea of multi track,multi tape turing machine you have several tapes finite number of tapes each tape will have a separate tape head.Suppose it is reading this aasymbol bhere a symbol as an example I have taken a three tapea, b, c then the move will be like this delta ofq, a,b, c will be some A R, B L,CR something like thatthe move will be something like this.

The state can change and that should also be taken care of that is instead of a it will print a Aand move rightinstead of b it will print aB and move left instead ofsmall c it will print a C andmove right .So, the move depends on the state and each one of the symbol read and the tape head can move independently one tape head can move left another one can move right and each one will print a another symbol on that.So, this is a way the moves are defined, now how is this equivalent to a single tape modeloriginally we had this.So, howcan you say that a multi tape turing machine is equivalent to a single tapeturing machineyou can simulate a multi tape turingof course, single tape turing machine is a particular case of a multi tapeturing machine that is one.

The other way round you can simulate a multi tape turing machine with the single tape, simulate a multi tape turing machine with the single tape,the single tapeit if you have k tapes here it will have two k tracks.So, here there are 6,3 tapesso,there will be six tracks.So, you can haveinfinite in both direction does not matter,ifso the ordertracks represent the contents of the tapes.So, here something like a naught a 1something is there that will be representedas a naught, a 1, a 2etc.At a particular instance the symbol a is being read and similarly, here if you have some b naught, b 1, b 2that will be there in the third trackb naught, b1etc.It will be leaving a symbol b at a particular instance.

This is the fourth trackthird track will contain thisb naught,b 1and etc,reading a, b and the fifth track will contain say the contents of the third tape that is some c naught, c 1etc, somethis reading c say at a particular instance.So, it ismultiple track like thisthe even number of tracksthey are all blanks except that one cell will contain a marker.Pointing the I mean it represent the tape head position for example, if a is being read there will be X here rest of this portion will be blank similarly, if b is read in the second tape at a particular instance there will be a marker here in the fourth track, but rest of them will be blank.

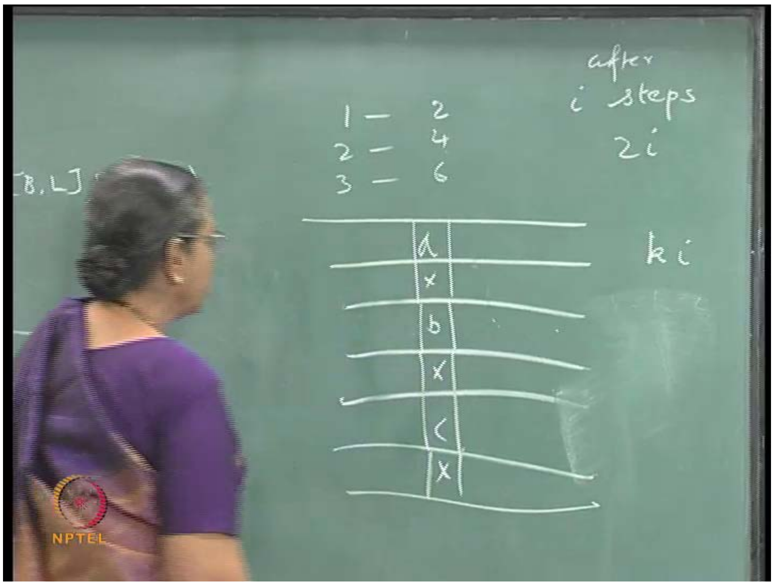
Similarly there will be a marker here so, how do you simulate one move of a turing machine, multi tape turing machine with a single tape turing machine you startmoving from left to right and then moving fromright to left you make one pass moving from left to right and make another pass moving from right to left.Start somewhereherethe state will have here the state is qsay the state will be q then there will be a counter here there will be a counterhow many tapes are there 3 tapes are here.So, there will be a counter which keeps the value 1,2,3 like thatup to k.It can have from 1 to k it can store a numberwhat is the move the move here is q, a, b, c this is AR,B L,CR.

So, it has toremember that it is reading aa from the first tape b from the second tape c from the third tape.So, it makes a pass there will be three positions in the state to keep track of that so, it makes a passand then when it comes here it sees that there is a X in the fourth track, that means the second symbol issecondtape it is reading thesymbol b.So, it will place that b here then move again now it sees another X here that means in the first tape it is reading the symbol a so, it will put awhenever it is filling this it is also incrementinginitially it was zero.

So, when it fills b it has seen one symbol so, it will increment to one then when it sees a it will this will increment to 2then it will move againand when it sees an X in the sixth track that means in the third tape it is reading a symbol c.So, that c will be put here and now the count reaches the maximum value 3that means it has read all the symbols.Once it knows that it has read all the symbols it starts movingthe move is determined now q a b c meansit has to go to a a should be replaced by A and move right b should be replaced by B and move left and so on.

So, what it does is it starts movinglike thisand this c will be replaced by Cmove is to the rightX will be placed there.Then it will continue moving leftand at this point again it is a move to the right this will be replaced by Aand this X will be placed here then it will move againnow this b will bereplaced by B andX will be moved to the left.Now while doing so, you can erase each symbol whenever it has been replaced we can erase that also and this three will be you will be decrementing that so, when you at this stage it will become after you do this it will become 2 after you do this it will become 1 and after you do this it will become 0when it becomes zero that means you have changed all the symbols and positioned the head in a proper way.

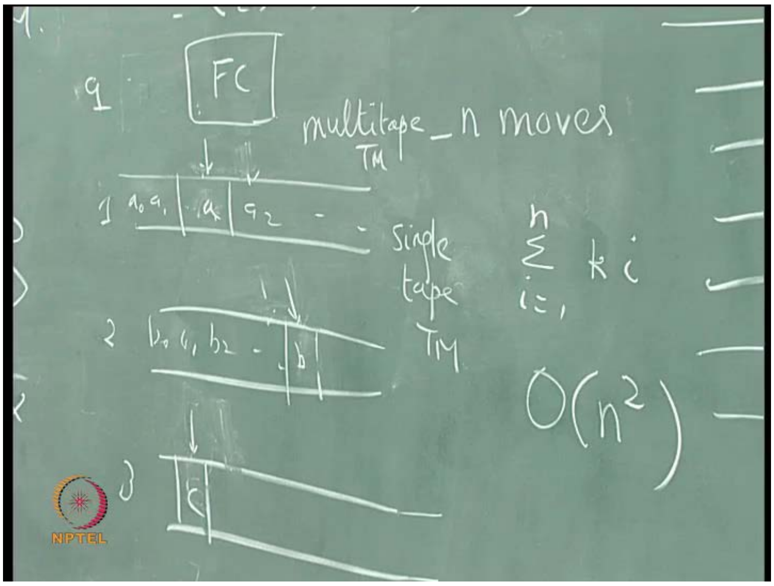
(Refer Slide Time: 21:02)



So, that means you have completed simulating one move of the multi tape turing machineso, one move of the multi tape turing machine you have simulated by making one path from left to right and again a pass from right to left.So, how much time it would have taken how many moves it would have taken to simulate one moveinitiallythe initial position you can keep like thisinitially we may start this after one move you may move right sometimes you may move left there can be two cells apart after second move there can be four cells apart after the ith move there can be i plus 2 cells apart or something like that is not it initially you start after one move they can be two cells apart the X can be maximum after one move you can move one cell to the right and one cell to the left.

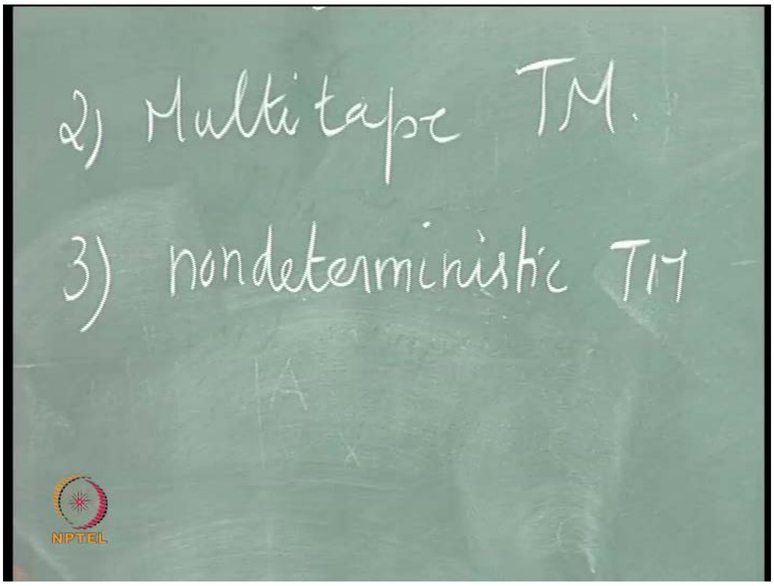
So, there X can be two cells apart after making the second move it can be four cells apart after making the third move it can be six cells apart and so on.So, after i stepsthe distance between two Xwill be maximum 2 ithere can be 2 i cells apart so, you have to make one move this way one move this way. Not only that some more moving right or left you have to make a 1 or 2 moves again extra.So, if you move 2 i cells to the right and then 2 i cells to the left you are making 4 i moves, but while writingthe X you have to shift while coming back if it is a move right you will come back here and then put it here and go back.

(Refer Slide Time: 23:21)

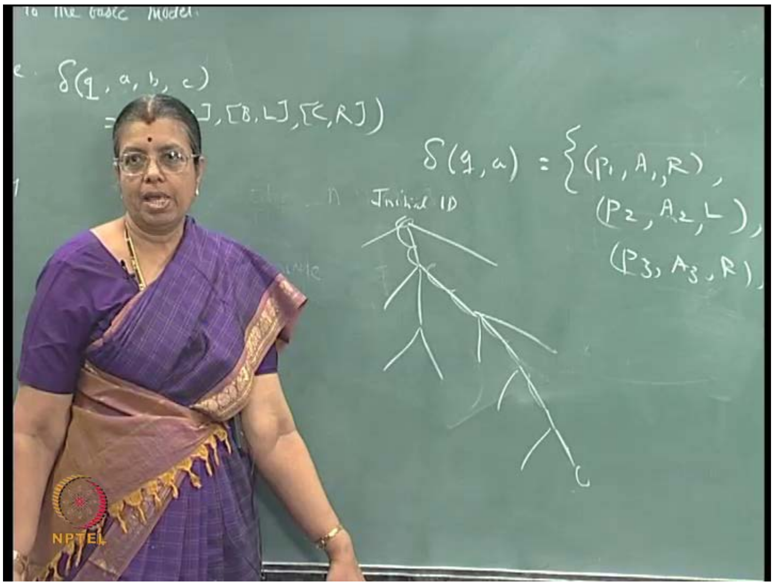


So, it will be something like know not exactly 4 i slightly it may be more than 4 i.So, maximumi will put is some k i,k will be k can bebetween 4 and 6for that matterany way it is finite.So, of if it makes n moveswith a multi tape turing machinen moves,single tape turing machine how many moves?itwill makeit will be sigmai is equal to 1to nsome k i.k is a constant so, it is ordern squaredterms.So, summing up will be i into i minus 1 by 2 so,k into i into i minus 1 by 2 i plus 1 by 2 i into i plus 1 by 2 so, it will be order n square constant does not matter.So, while simulating a multi tape turing machine the single tape turing machine makes quadratic number of movesso, this simulation does not affect the polynomial property if something is accepted by a multi tape turing machine in polynomial time it will also be accepted by a single tape turing machine in polynomial time.

(Refer Slide Time: 25:14)



(Refer Slide Time: 25:29)



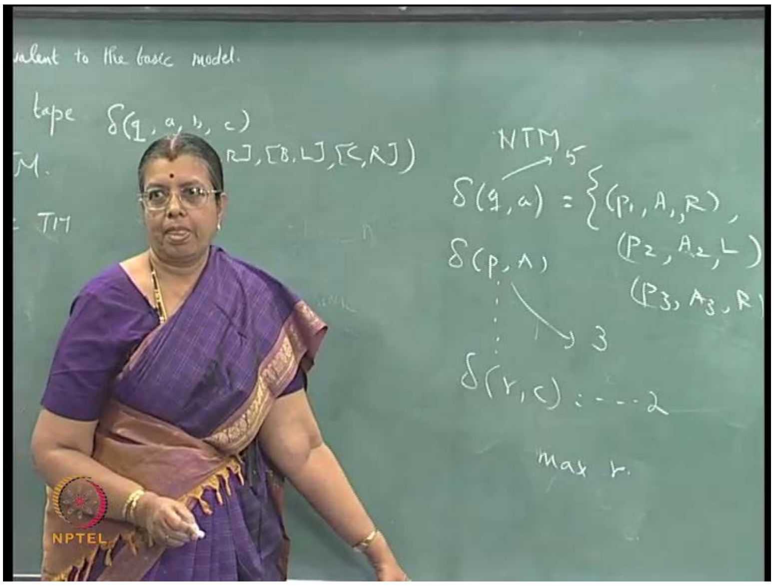
So, this is another generalized version that is a multi tapeturing machine.

(No audio from 24:53 to 25:12)

The third isnon deterministicturing machine,what is a non deterministicturing machine?So, instead of delta of q, ayou may have a finite choiceP1A1R,P2A2L,P3A3Rand so on.some finite number of choices will be possible for each move when you are in state q and reading a symbol athen either you can go to P1 print a A1 and move right or you can go toP2 print A2 move left or printgo to P3 print a A3 and move right it is like this.But please remember that it cannot go to P1 print aA2you cannot combine like this.The triples are specified it has to choose form one of the triples.

So,starting with the initial IDyou may have some choices from each one you may have choices from each you may have more choices and so on.So, when will the string be accepted if there is one sequencewhich leads to acceptance the string will be accepted.Other sequences may not lead to acceptance does not matter,if there is one sequencewhich leads to acceptance the string will be accepted just as we defined for a push down automata.So, how do you simulate a non deterministicturing machine with a deterministic turing machinehow can you simulate a non deterministicturing machine with a deterministic turing machine.Now the idea is like thisthere will bemappings for thenon deterministicturing machinewill be given like this delta of q, a delta ofp,Athere will be some moves defined for the non deterministicturing machine.

(Refer Slide Time: 27:50)

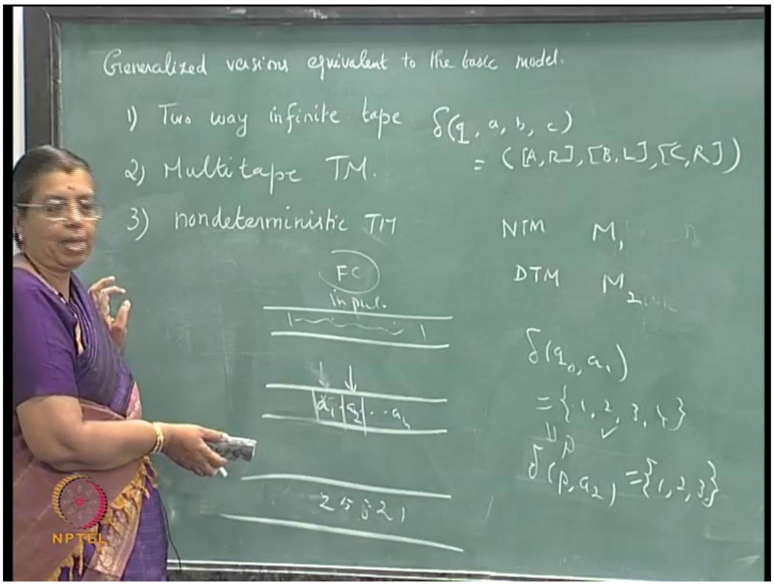


Now for this for example, there may be five possible choices for this there may be three choicesforsomething like that there may betwo choices only and so on. Take the maximum value maximum of these choices for one move there may be five for another move there may be three choices for another one there may be two choices and so on. So, take the maximum value maximum is r sayamong this.

(Refer Slide Time: 28:27)

Now from the letter from thisconsider sigma s is equal to1 to r and generate sequences over 1 to r in the standard ordering we have studied lexicographic ordering and standard ordering you remember.So, if we generatestrings in the standard orderingwhat will be the order1,2,3upto r,11,12,13up to 1r,21,22like that you can generate sequences over 1 to r in the standard ordering.

(Refer Slide Time: 29:29)



So, in order to simulate a non deterministicturing machine with the deterministic turing machine, the deterministic turing machine will have a finite controland it will havethree tapeswe are taking non deterministic turing machine to have one tape,you can extend the result when it has got multi tape also, in the non deterministicturing machinehas one tape whatever is there it is inputit is copied on herethe first tapethen what it does is it copies the inputon to the second tapeand 1 by 1it generates the sequence there.So,first it will be generate 1 then simulate the behaviour of the non deterministicturing machine if I call as M1the deterministic turing machine which is going to simulate I call it as M2,M2 has 3 tapes.

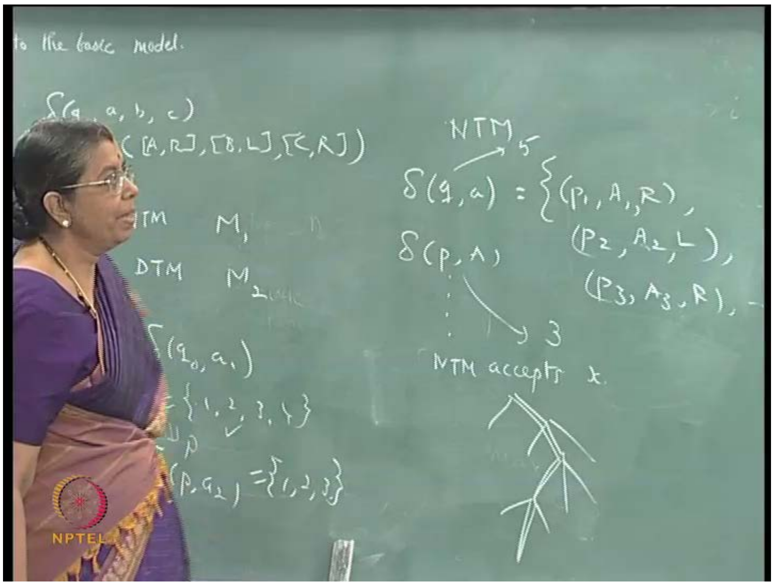
So, in one tape the input is there the second tape and the third tape are initially emptysecond tape and the third tape are initially emptyyou copy the input onto the second tapethen generate the sequence 1 by 1.First you generate one sequence and then simulate the behaviourof M1 by M2 you simulate M2simulates the behaviour of M1 on this input using this sequence.Suppose at particular time the sequenceis say 25321and the input is copied on herethe head will be pointing to that initially.So, in the initial state is q naught.So, delta ofq naught, a 1there will be1,2,3,4 choices sayfor thisthe number is 2 here.So, it will select the second choice and behave like that it will simulate the second choice on this so, suppose it is moving right printing something here and then the symbol read is 2state changes to Psay then the second move is delta ofP, a 2.

Now delta of P, a 2 this has gotsay 1,2,3,4,5 choicesthen this number is5.So, it will select the fifth choice and simulate similarly, for the next move it will selectfor that there will be some choices it will select the third.Now it may so, happen that for this there are only three choices possiblethen number 5 is appearing here that means it will halt I mean it will halt means it will discard that it cannot simulate the fifth choices there are only three choices for that, but the number 5 is here.

(Refer Slide Time: 33:34)

So, it cannotproceed further,now once if it accepts it accepts if it does not accept suppose after simulating the moves with this sequence 25321 it finds that it not accepting then what it will do is it will erase the n ththe content should have been changed by now.So, it will erase thiswrite the next numberin the next sequence in the lexicographicin the standard ordering that is 25322something like that.Then copy the input againheretries to simulate with this sequenceand so on.If the string is accepted by the non deterministicturing machinethenif asuppose some string is accepted by the non deterministicturing machine then there will besome sequencesome sequencewhich leads you to acceptanceand when you are creating the sequence 1 by 1 here at some time that sequence will appear here on thethird tape, then you copy the input on to the second tape and when you simulate it will be get accepted.

(Refer Slide Time: 33: 58)

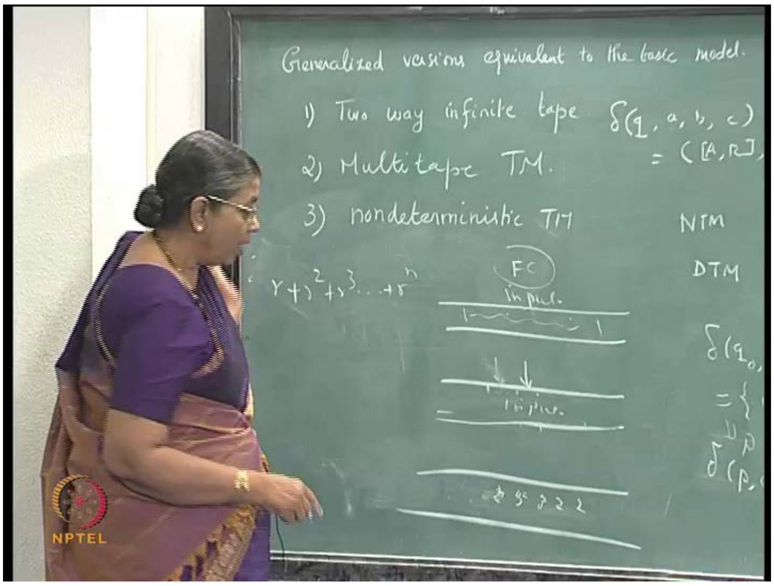


So, if a string is accepted it will be accepted through a sequence of moves and that sequence will be a sequence over the alphabet 1to rand sometime or the other that will appear here, because you are going to generate the sequence 1 by 1 herewhen that particular sequence appears hereyou copy the input on to the second tape and simulate the behaviour of M1 with this sequence then it will accept, but if a string is not accepted what will happen if a string is not accepted one string you will generate here it will not get accepted then you will try the next sequence it will not get accepted then you will try the next sequence it will not get accepted then you try it will not get accepted and so on.

So,you will go on and on it may not halt machine may not halt for strings which are not accepted.For strings which are accepted after sometime that particular sequence in which it accepts will appear on the third tape and when you are trying to simulate the behaviour of M1 with M2 using that sequence you will reach an accepting state so, the string will be accepted.So, in this way the non deterministicturing machine can be simulated by deterministic turing machine, but how many moves it will take too much isnot it because if it accepts in n move that sequence of length n has to appear here for that you have to write sequences of length 1, length 2, length 3, length 4, length 5 and so on.

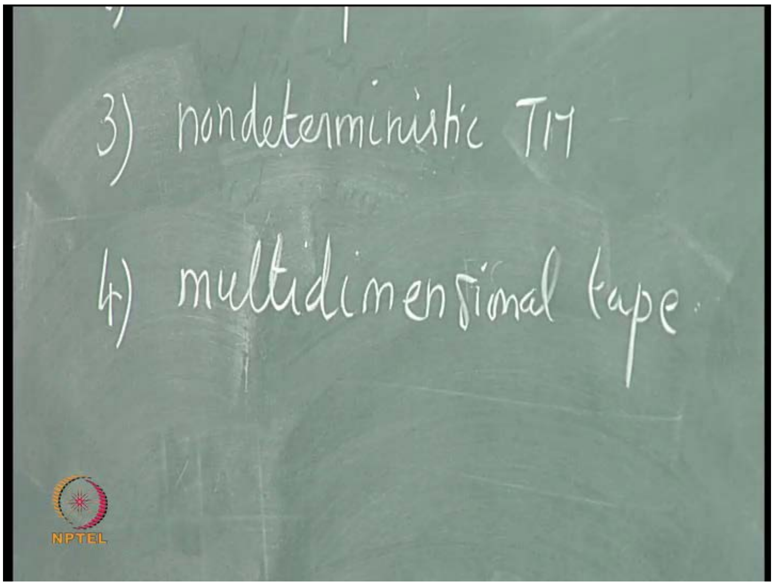
How many sequences of length 1 will be there it will be rlength two will be r squarethen r cube1 by 1r power n.One by one all these sequences have to be written here and each time you have to simulate the behaviour and so on.

(Refer Slide Time: 36:33)

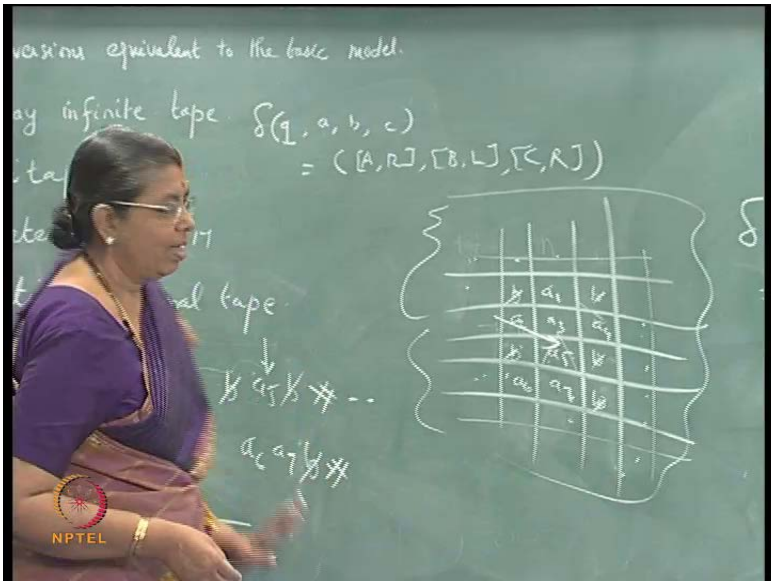


So, the number will be too much it will be the time will increase exponentiallyyou can simulate a non deterministicturing machine with a deterministic turing machine,but the time will increase exponentially, but the language acceptance power is not changedwhatever you can do with non deterministicturing machine you can do deterministicwith the deterministic turing machine. Whereas in the case of push down automata there are languages which cannot be accepted by deterministic turing machine, but here any type zero language can be accepted by a deterministic turing machine it can be accepted by a non deterministicturing machine.As far as language acceptance power is considered they are equivalent, but the difference comesin the number of moves.

(Refer Slide Time: 37:49)



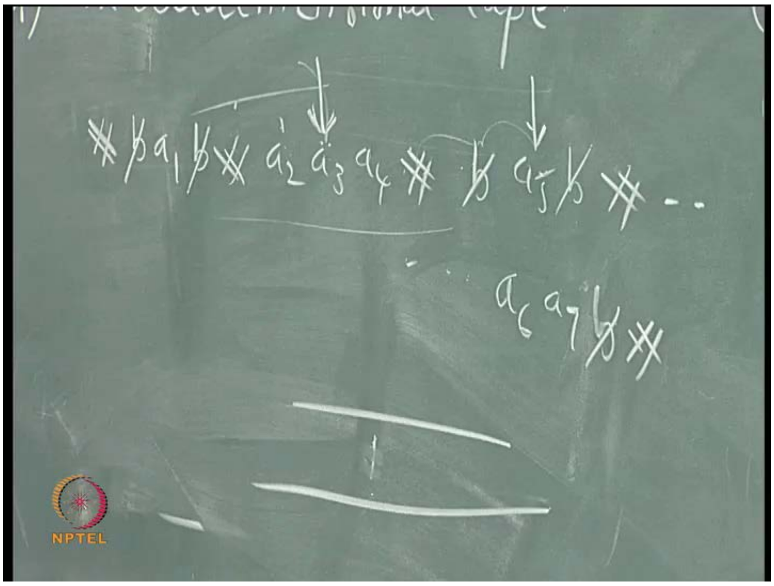
(Refer Slide Time: 38:11)



So, this is another generalizationanother ismultidimensional tapesI will consider two dimensional tape you can also have three dimensional tape in fact abstract way you can even think of k dimensional tape.So, when you have a two dimensional tapesuppose this is input a 1, a 2, a 3,a 4, a 5,a 6, a 7this is the input.Thetape is infinite in this direction, in this direction, in this direction, in this directionand the tape head will be reading one cell after reading that symbol it can change that symboland then move left right up or down.So, the mapping will be fromit will be of this formP,A,L, R, U, Dit can move left, right, up, down.

Thus it give a additional power will this give additional power, but will it will not give additional power,because this one suppose this is the portion non blank portion this is blank, this is blank,this is blank,this is blank,this is blank.So, this can be represented by say somehash blank,a 1 blank, hash,a 2, a 3, a 4,hash blank, a 5 blank,hash, a 6,a 7hash this is continuationsame tape.You can write it like this a 6, a 7 blankso, the whole thing you can write as strings within the hash mark each row you can write like this.

(Refer Slide Time: 39:38)



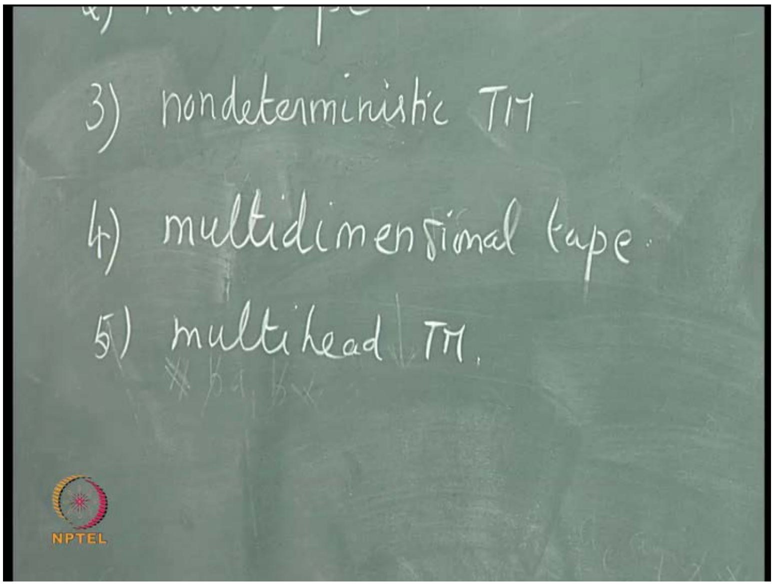
So, if you are reading a symbol a 5 the tape head will be here and after reading a 5 if you want to move left you can move left or move right hereno problem,if you want to move up or down how will you do? the tape suppose after reading a 5 you have to move up you have to go to a 3how will you do thatin the secondtape there will be this can be contained in one tape in the second tape when you move like this youhave a counter whichisincrements and say that you have movedtwo cellsthen when you go to the left one if it is move up means left block you go, then reach this hash symbol move two cells to the right that will show that you have to be in a three.

If you have to move from here to here when you move this head in this direction how many cells you have moved to reach this hash you keep a count in another tape and then move to the next hash symbol then by the same amount of same number you move the tape head to the right that means the tape head position is changed accordingly.Similarly, if you move down instead of moving to the left block you will move to the right block that is one thing then suppose you are here andyou move right that means you have to add a column one more column you may have to add one more column or one more row or up or one more row left.

If we are going to add one morerow up you have to create one new block on the left if you are going to create one more row down you have to create one more block to the right,if you are going to create one more column to the left each block you introduce one blank and use the technique of shifting over. You know how to shift the symbols to the left or to the right make use of that and create one blank symbol in everyblock to the left, if you are creating one column to the right shift again and then create one blank at the end of the row.

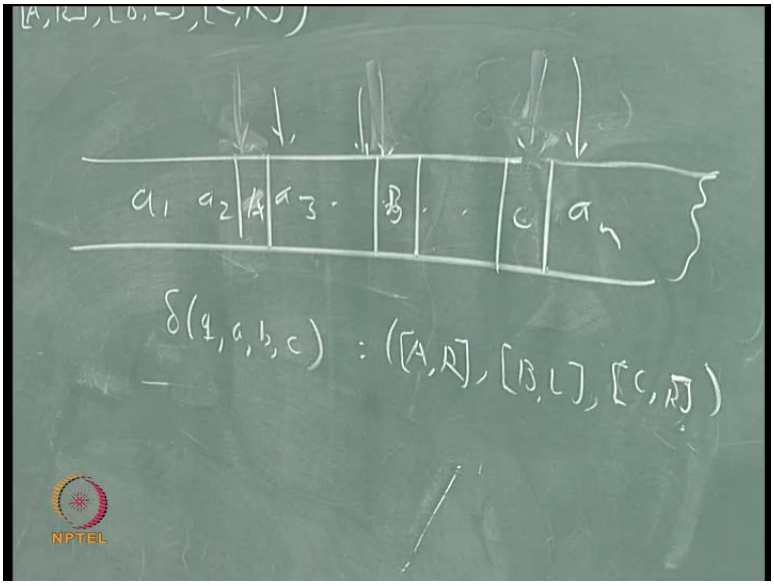
So,whatever you can do with the two dimensional tape we can do with one dimensional tape, but the number of moves will be more this number that will not be just, but it will not be exponentially increased.There will be more moves if you want to specially if you want to create a new column you have to do a lot of things, but it will not be as bad as simulating a non deterministicturing machine with thedeterministic turing machine.So, this is two dimensional tape we can have a three dimensional tape where you can move left, right, up, down or this way or this way.Six possibilities will be therein an abstract manner you can eventhink ofk dimensional tapes,this is another variation.

(Refer Slide Time: 43:49)



Then multi headturing machines

(Refer Slide Time: 43:58)



If you have only one tape, but there may be many headsthree heads reading the symbols a, b and c. Now depending on the symbol read state again the move will be like this delta of q,a, b, cis some Amove right,Bmove left and so onC move right.So, depending upon these symbols it will print aA and moverightprint asymbol and move leftprint a symbol andmove right and so on.

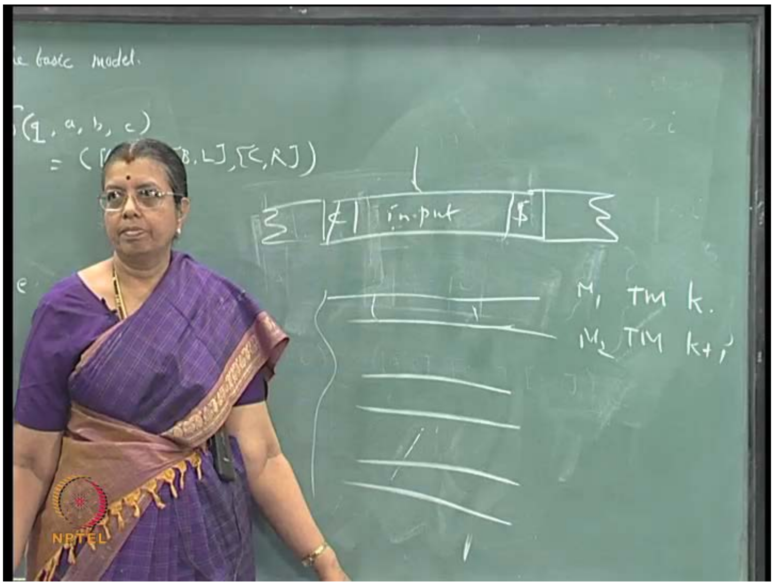
(Refer Slide Time: 45:14)

Now how can this be simulatedmulti track again the same idea of multi track if you are having k heads you will have k plus 1track.Suppose I have three heads input will be therein the first trackand the tape head position will be marked first tape head position will be marked in the second track, second head portion will be marked on theand so on.Again in a similar making a leftright move right to left move we can simulate the behaviour.There is one small point you have to note here what that is?

Conflict.

If there is a conflict.Supposetape head two and three read the same symbol then which one will you follow? move up the second head or third head you have to rewrite and move righthead can move left or right does not matter, but a symbol rewriting how will you do that you must specify some priority in that case, second head will have higher priority than the third one and so on. Some priority has to be specifiedso, you can simulate a multi head turing machine with the singlehead turingmachine.

(Refer Slide Time: 46:47)



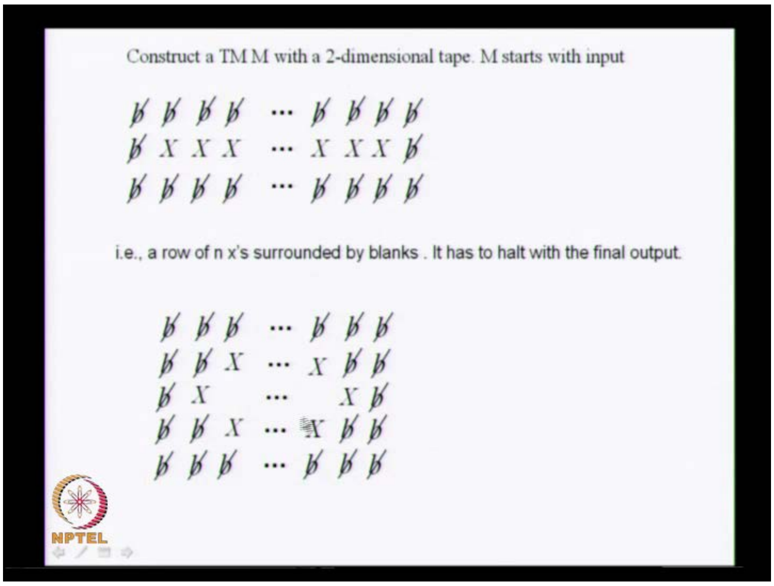
Then offline turing machines,what is an offline turing machine?offline turing machine is a turing machine wherethe input is given in one tapeand it is read onlythere is one tapewhich keeps the input withn markersand there will be other it is a multi tapeturing machinethere will be other tapes, but the input is read only you can only this on this tape the head only moves this way it cannot move beyond this it cannot move beyond that other tapes you can do for computation.So, this is a particular case of multi tape turing machine.

So, obviously it can be simulated by a single tape turing machine another thing iswhatever you can do with a single tape turing machine you can do with an offline turing machine any turing machine is you have the inputand thenone more tape suppose there is a multi tapeturing machinewith k tapes to make it offline have a turing machine with k plus 1 tapes,copy this on to this input then afterwards you do not bother while copying only use this copy this onto this and in this k tapes you be simulate the behaviour of the first machine.

So, you have a turing machine with k tapes M1you want to make it offline M2 will have one more tape which contains the input first stage is to copy this input on to this then just behave like M1,if it accepts it will accept if it does not accept it will does not accept.So, this way you can simulateasimulator ofany multi tape turing machine by an offline turing machine.Now these are some of the generalized versionssee you are making the machine more general and then you are saying that it is still is equivalent to the basic model.Now you can also have restricted version some restricted versions of turing machines which have the same power as the original turing machines.We shall consider them next and also you can look at the turing machine as a generating device.

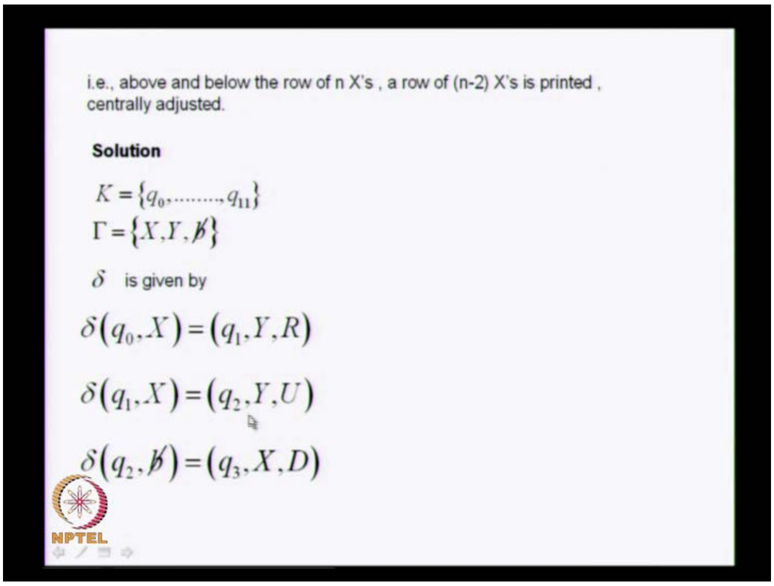
So, farwe have considered the turing machine as an accepting device or a computing device.We can also look at the turing machine as a generating device you have a tape output tape in which 1 by 1 the words will be printed between some marker symbols and because you are able to do that that is why it is called a recursively innumerable set the language accepted is usually called a recursively innumerable, because you can innumerate the strings using an output tape.

(Refer Slide Time: 50:06)



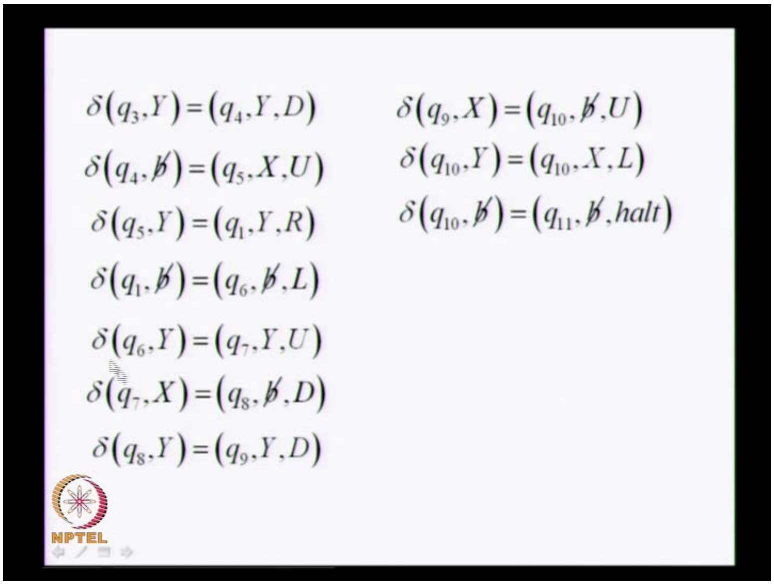
So, we shall study about this later in the next lectures.So, we have seen thatthere are several generalized versions of turing machine as an example we will consider a turing machine with a two dimensional tape the problem is like this.Construct a turing machine M with a two dimensional tape and it has the input like this or row of n X’s and finally, you want a pattern like thisthat is the row of n X’sis preceded by a row of n minus 2X’s and followed by a row of n minus 2X’s.So, you want to generate this pattern,from this pattern this is our idea and you want to construct a turing machine for thisyou know that a two dimensional tape turing machine hasgot four types of move depending upon the state and the symbol read it can move left, it can move right, it can move up or it can move down four types of moves are possible.

(Refer Slide Time: 51:18)

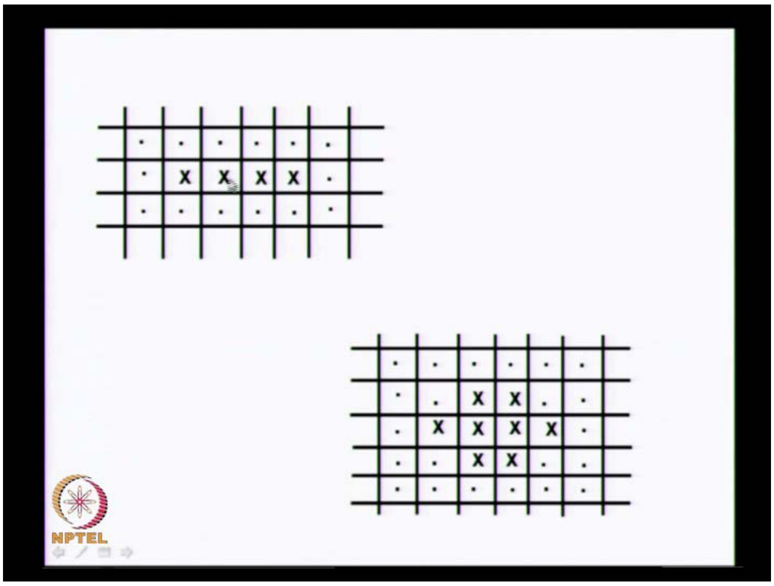


So, with this idea let us see how we can obtain this pattern from this pattern.So, the corresponding turing machine has 12 statesq naught to q 11, q naught is the initial state we use three symbols X Y and blank X is occurring in the pattern the blanks are denoted by dot in the figurein the two dimensional tape we denote by a dot the symbol blank.The mappings are given by this we will see how we are using the mappings.

(Refer Slide Time: 51:51)

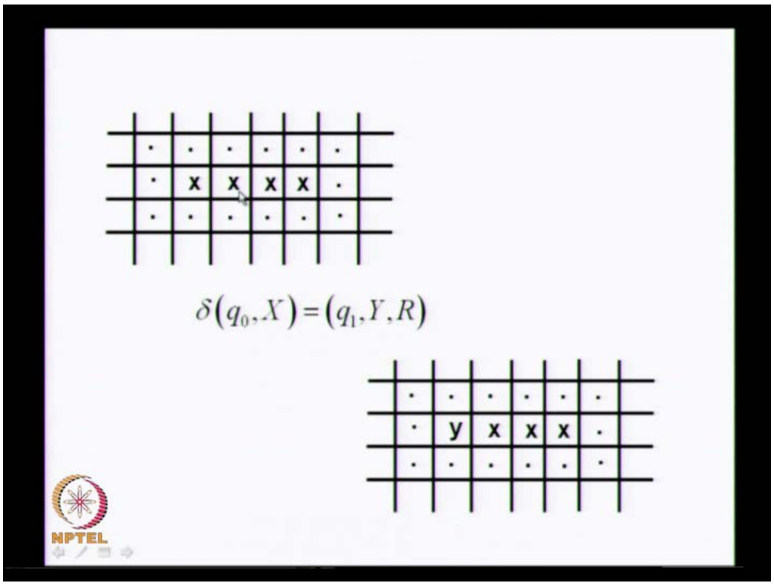


(Refer Slide Time: 51:57)

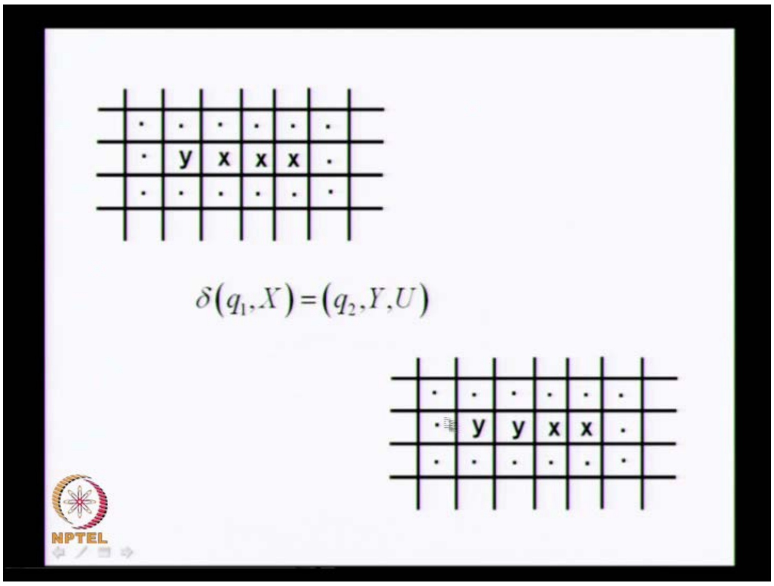


So, some more mappings these are the mappings which define the moves of the turing machine.So, let us consider apattern where you have 4X's initially the machine begins in this position in the state q naught and it ends here after printing 2X's above and 2X's below.So, it ends up with this pattern and a halting position is this it starts with the initial state in this position.Let us see how we get this pattern from this pattern.

(Refer Slide Time: 52:36)

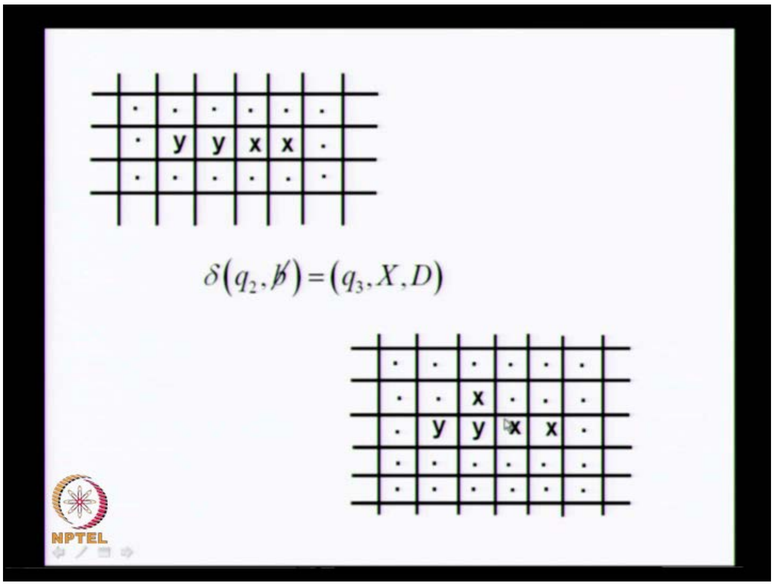


(Refer Slide Time: 52:59)

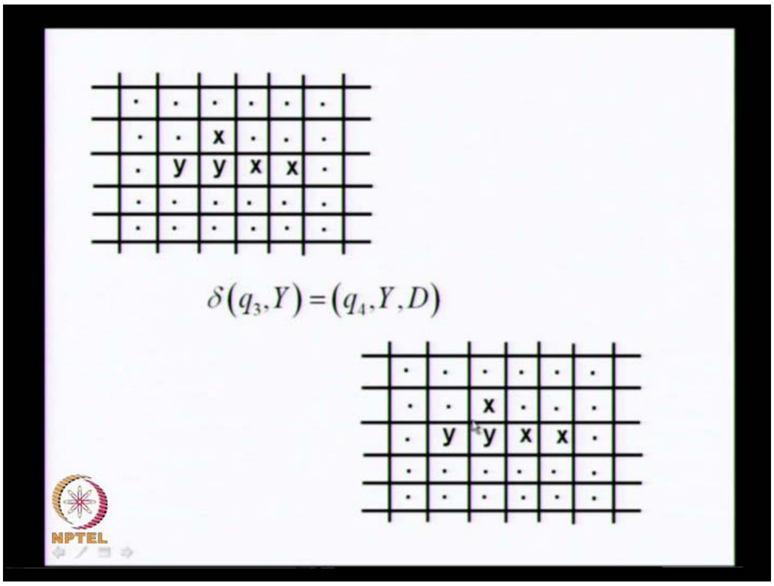


So, initially we have this positionthe tape head is here and the mapping used is delta of q naught, X's equal to q 1,Y,R.So, it changes this x into a y and moves right, now the tape head is in this positionthe second mapping is delta of q 1,X is equal to q 2,Y,U.The second x it changes this into ay and moves up so, the next situation is this x has been changed to y and the tape head is in this position.Now we want it to print a x here we also want to print a x here, let us see how this happens now the tape head is here and the state is q 2 the move is given by this mapping delta of q 2, blank is q 3,X,D.So, it prints a x over this blank and then moves down.

(Refer Slide Time: 53:33)

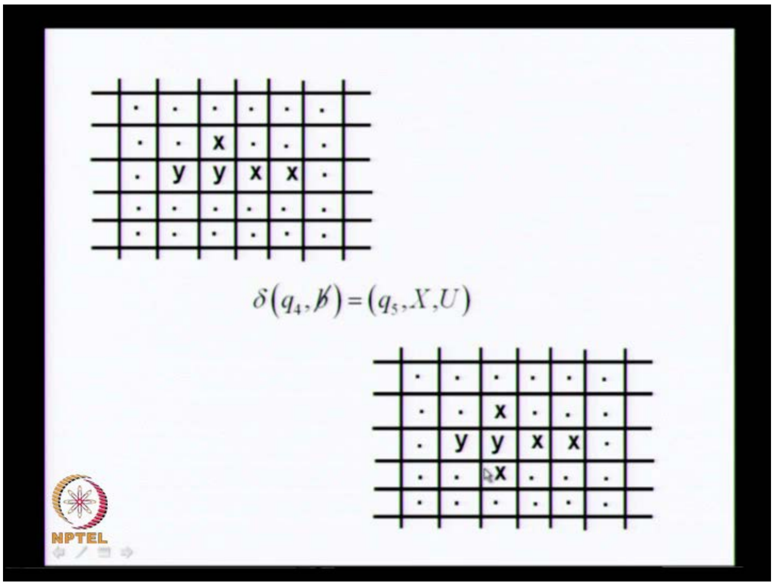


(Refer Slide Time: 54:00)



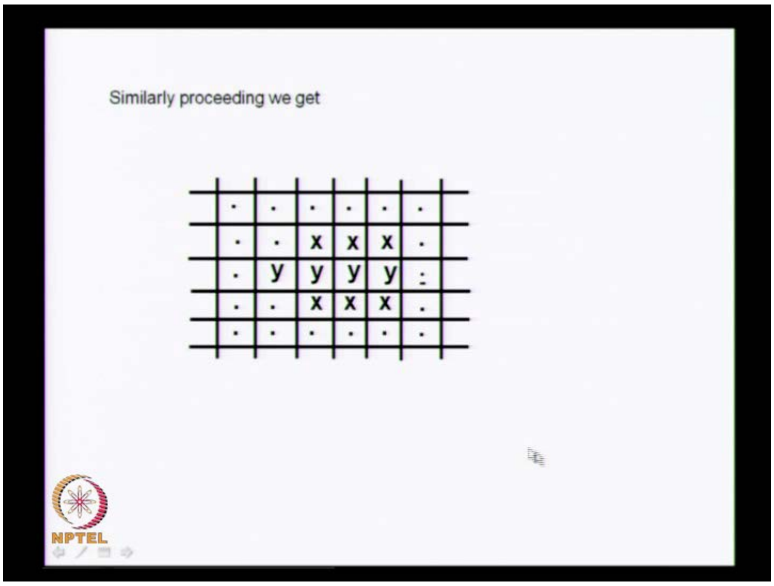
So, the next stage it prints a blank x here and then moves down.So, the tape head position is here nowso, the tape head positionis here and the map used is this delta of q 3, Y is equal to q 4,Y,D. It does not change this y, but then moves down in state q 4.So, it tape head position is here and in q 4 it reads a blank here and then goes to state q 5 changes the blank into X and moves up.

(Refer Slide Time: 54:19)



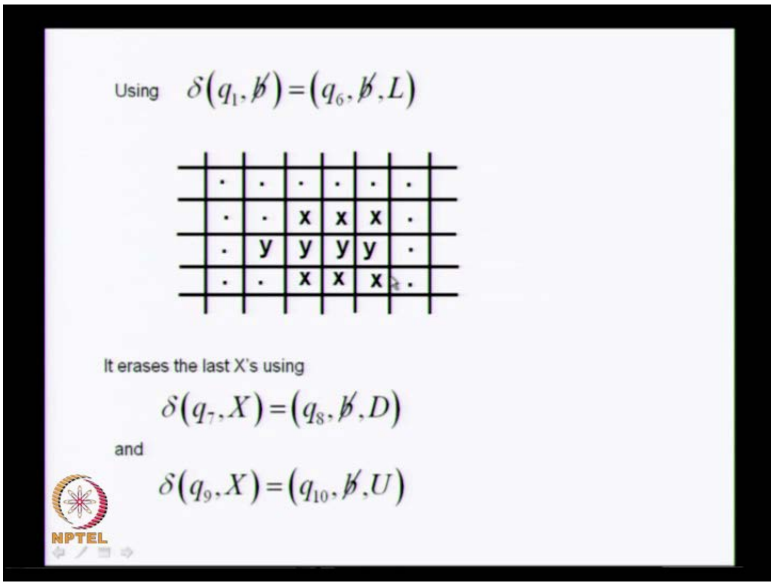
So, at this stage it prints a x and moves up hereso, in this column it initially change the x into a y and then moved up printed a x moved down again moved down printed a x andit came to this position.Now moving to the right it does the same thing in the next column and in the last column also though we do not want x initiallyit prints a x and then afterwards it starts it erases this x and this x.

(Refer Slide Time: 55:08)



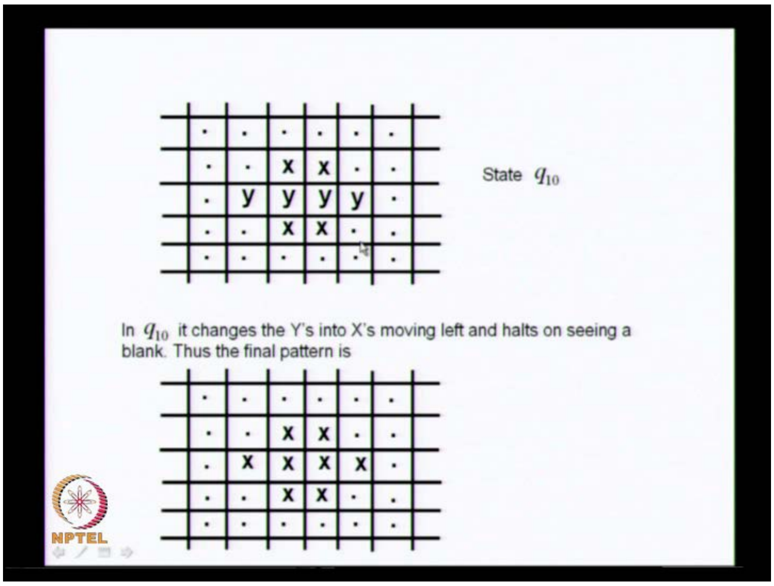
So,moving to the right and repeating the same thing it prints a x above this and below this changing the x into a y similarly, changing this x into a y it prints a x above and x below.Now when it moves right it does not see a x, but it sees a blanknow it knows that it has come to the end of the firstrow of x’s and so, it moves left and erases this x and erases this x.

(Refer Slide Time: 55:42)



So, that is done by this mapping when it sees a blank here it moves left and then it moves up and then it erases this x using this mapping coming down hereand then going down here it erases this x and goes up.Now these 2x's have been erased and it the tape head is here, but this whole of this is y and we want to change the y's into a x that is done by this mapping.

(Refer Slide Time: 56:33)



The last oneand so after changing the y's into a xwe get this pattern in q 10 it changes the Y's into X's moving leftit keeps on doing this and when it sees this blank it knows that theit has exhausted the rows with which it started position and halts on seeing a blank thus the final pattern is this way, we can get a patternfrom another pattern this is a very simple example just to illustrate how a two dimensional tape turing machine works.