Performance Evaluation of Computer Systems

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Lecture No. # 20
Slotted Aloha Markov Model

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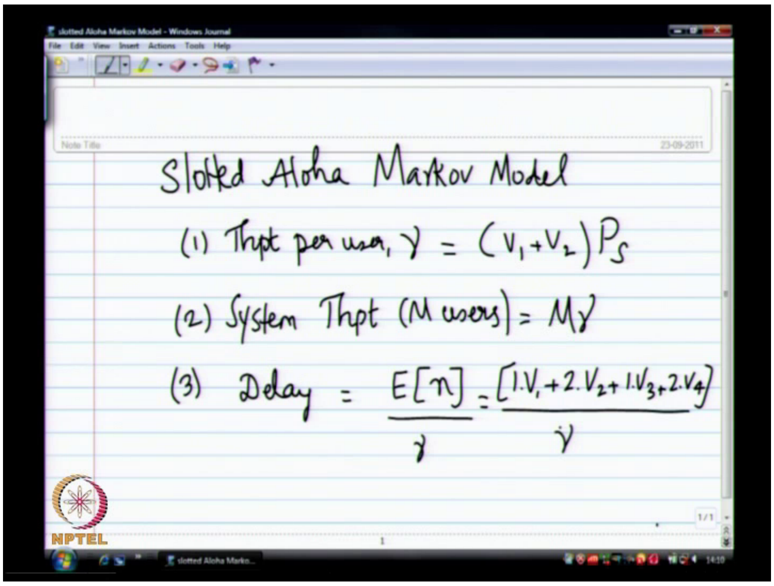

So, you have developing the model for that right. So, there are several minor variations you can make to that model to make it more accurate, and usually that is the trade off right. How accurate you want? If it gives you, reasonably good result with base model, and that is fine. Sometime some time there have inaccuracies also, so be careful with that. So, the model you remember from last time; so I want redraw that. We said we look at the two parameters right, so throughput per user we called as is gamma. So, throughput of the user is, so it is when the user is in state either 1 or 2.

So, that is when you are transmitting, and the probability of success is just P s. That is your user through put, this is my guess right. You should go back, then plug this numbers, get the values then go back to your simulation in to forpart 3, and see whether it actually makes sense right; 3 user, 10 user look at whether the numbers are actually matching. So, then the system throughput with M users is simply M into right, and the delay we can calculate by simply computing E of n, that is the expecting number of packets in the system right. If the if you are in state 0, then there is no packet state 1 or 3, there are there is one packet 2 and 4, there are two packets and so on.

So, now you know the steady state probability, you can simply compute that and there for will be simply right. So, this is no; lambda will be effect to throughput of the system, so mu is what? Looking at, mu is the effective arrival rate to the system; because mu is the effective throughput.We could check it out; because lambda we can have very large lambda. So, what the system is seeing in terms of departing from the system is the effective arrival rate for the system. But that is that is I have did it, I may be wrong will it go back and check whether lambda is actually the value. My paper I said I am mu and or gamma and right.

So, state 0 there are no packets. So, state 1 there is one packet.So, 1 into V 1; state 2 there are 2 packets and so on, this is the way I have number that. So, state two is transmit with 2 packets right. Let my sheets there. So, match which is and then V 3 plus 2 into V 4. So, that gives you one approximate analysis for aloha. They might ask, what is the point of this? Aloha, I can implement in two pages with any simulator. So, why I go through all of this? So, that is when we used to remember our first class;firstset of classes that is for cross validation.How do you know your simulation is correct?

Youhave some other model to validate it. So, this is what cross validation comes.How do you know model is correct? Will you get simulation results? Finally, at some point the two reasonably converge you know that we have seen. So, that is the aloha model and infact, this model actually developed for a system with multiple channels. Not just one channel, there aremultiple channels that will be sharedamong users andwe had this fixed receiver model and so on, that paper actually look at which are now put on the website is come in topaper.

The model will model as the same; there only thing is, the P s calculationswill change, because of multiple channels for example. There are several variances, this is the w d n local area network with multiple channels; that variance will look at fixing the user transceiver channel. User will receive one of its channels. So, if I want to startto the some user, I have to send on the users this home channels so called. Therefore, the load will be evenly distributed among all the siegein ideally, you should be if there are M users, then I have there are c channels.

M is larger than c.Say Mby c user will be sharing a given channel. So, the load per channel is M by c, yeah for unbalanced traffic, imbalanced traffic or whatever yeah then the channel whether lot of film. But we cannot do much with single transceiver. I have only one receiver at a node.It is like a base station.The base station can have a multiple transceiver if it wants to. But our single transceiver model, it is not suitable. You will have collision on the channel, yeah, V 1 will be divided by the groupingwhichone V 1 plus V 2.

So, now aloha we have done. This is on user perspective and we have now let us, we have seen TDMA with that other, which is also yourlast tutorial let you saw.We can compute TDMA performance with that model. Now, I say I want to do TDMA with the markov model. So, let us straightly develop theTDMA with markov model andknowingly or unknowingly, I left my TDMA model at home. So, we will now have to develop that from scratch. So, how do you model? A TDMA behavior of a particular user.

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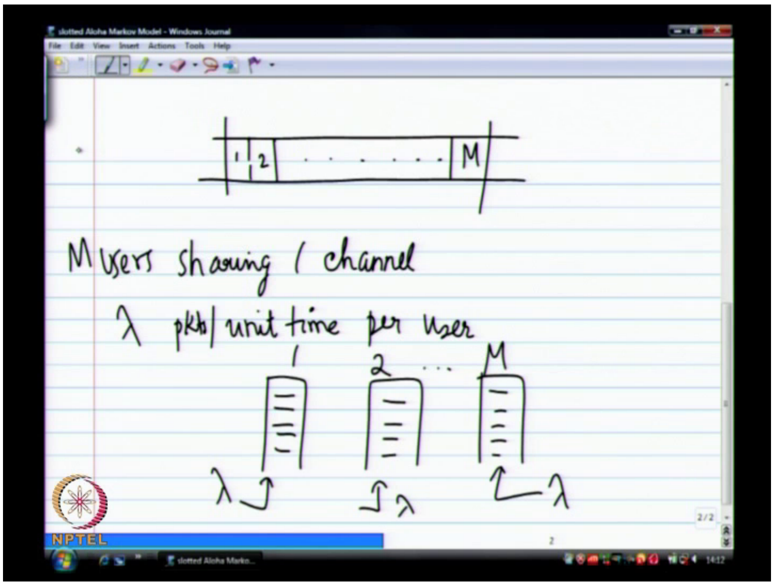

So, the systemdescription is as follows. I have M users sharing 1channel and each user is lambda packets per unit time per userandthis is broken down in to slots. So, there are M slots. Every user hashis or her own slot.We saw this couple of weeks ago. So, if you want to model this as markov; not exactly markov or semi markov. But let us try to model this and then we will see the definitions. So, let us try to figure out the states of this model.Again, this is looking at the each users queue.

So, I am looking at the user queue to which,brackets are coming at rate lambda and there are several such case. Some basically modeling the state of the system straight of each transmitter (No audio from 07:12 to 07:22) and we assumed that, this is the major assumption that, all the users are identical. Therefore, just looking at one users queue is representative of the system behavior. So, let us start with one state, state 0; what will be that idle state.

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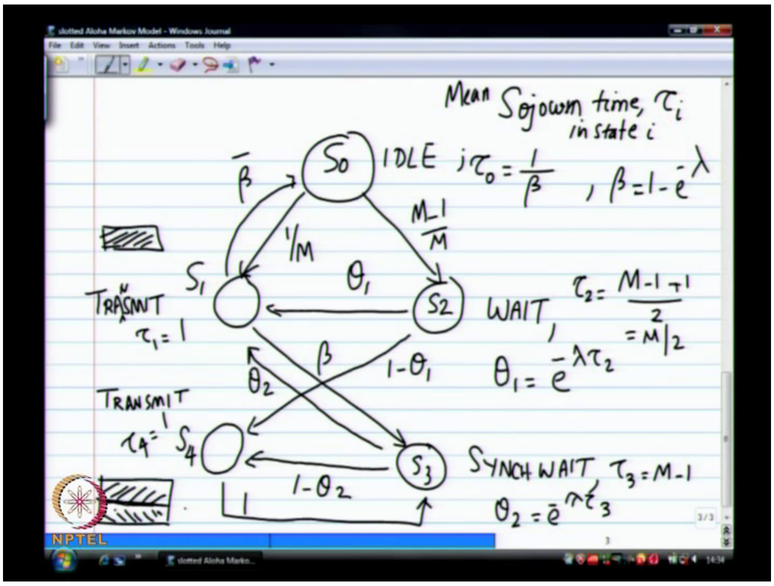

So, this is S 0, idle state. So, I will be in idle state for some time, for a while for a while. So, I can do the self loop like last time,where an idle state in every slot probability ofgenerating a packet is beta.Beta is 1 minus epower minus lambda. So, that same definition is holding. But now let us say that I do not want to right I do not want to do the self loop. I want to eliminate the self loop. So therefore, the time spent in this state S 0 is no longer unity; it is some number.It could be,it is a random number in this case it is ax geometric process. So, we know it is going to be 1 over beta is the time you spend in this particular slot.

So, from S naught, I will go to what state? I will go to 2 states S 1. So, what are the two possibilities thatI can have? It arrives exactly when the slotfeature. So, you will be able to send the base slot packets. In the previous slot, there are 2 slots; 0 and 1.Beginning of the slot 0, there was no packet in the queue; waited waited waited just sometime in between before slot one started, the packet is arrival.Therefore, it can be sent right away. So, there is no waiting there; that is one possibility. So, we will have one transmit.This is will actually will this is the transmit state.

So, this is S 1 transmit state with only one packet in the buffer only one packet. So, from S naught, I can go to S 1 or what is the other possibility? I will go to the wait state. So, there is the wait state here.This is waiting for a turn to arrive on the channel.So, now for each of these states, I will also list there. So, called sojourn time. (No audio from 09:53 to 10:06) So, basically the average time, this is actually mean sojourn time.This is themean sojourn time in each state. It is no longer unity. In aloha case, it was nice.But in this case,it isthe way that M model. So, what is the amount of time spent in this state?1 by beta, geometric process right.

The probability beta will leave that state. So, that is a probability of success;so, 1 overp. So, that is 1 by beta. Then what is probability of going to state S 1 from S naught? Yeah I am leaving, if there is only a packet arrival. So, that beta will not, it is it is either 1 is 1 into something what is the… I am leaving the state only, when there is a packet, yeah. So, from S naught to S 1, what is the probability of (( ))? 1 by n; 1 by n. So, there is M slot and exactly in the beginning of your allocated slot, your packet is arrived.There is the packet in the queuearrives with probability 1 by M. So, with probability 1 by M, I will go tothis. This is actually different from that model, you will see in paper.

Model actually, I donot do S naught to S 1. Isend everybody to the wait state. So, that is why, if you look at my graphs, there is a gap between the simulation and the analysis always will find a distinct gap. But we can be more accurate by looking at this special case. Ifthe packet is just arrived, that is the probabilitythere are M slots. So, packet can arrive known as the M slots.And since all slotsare equally probable, arriving exactly in that one slot that is allotted to 1 by M and this is M minus 1 by M, so this is my transmit. So, then it is my allotted slot. So, therefore, what is the... So, this is not TDMA.

Therefore, I will simply spend 1 unit of time and the packet is successful. Then after that what happens? So, once I send a packet in a slot, 2 possibilities. A new packet is arrived in that slot, whenever sending or there is no packet at all. So, if no packet has been generated, then where do I go to go back to idle state. So, that is this transition here, probability beta. So, S naughtto S 1, I slot I sent.In case a new packet has arrived, then I have to go to a state, another wait state. Will let us you want to do this wait state first, now there is there is another waitcalled synchronize wait or full weight.

How long will I wait in that state? Because my allotted slot is over, need to wait for? n minus 1 n minus 1 right. So therefore, this is going towill call this S 2, S 3. So, in synchronize wait;

packethas arrived, then that will be like that; that also does the sync wait. (( )) will come to that. So, now if you just look at this sequence, packet comes to my slot, I sent. Another packet comes, I am going to this full wait state, and the time for thisone is M minus 1. So, I have to wait M minus 1 slot for my slot to arrive; for my chance to arrive again; that is first part. This is the sequence so far.

Beta is probability of a one packet arriving inone slot. (No audio from 13:50 to 13:59) Because it is lambda packetsper unit time and slot is 1. So, this is 1 sequence of packet.(No audio from 14:06 to 14:20) So, now S 2 what is the time spent S 2?(( ))synchronize with mean that get us gone in 1 of this slot.Now,it will base for exactly M minus 1slot. (( )) wait that is not; it is a it is a probabilistic way. So, here now let us try to compute this. So, what is thissojourn time for this state? So, I am coming into wait state, because I missed my slot. So, now I could actually have S 1, S 2, S 3, S 4 up to S minus 1 you get a whole sequence of states.

By saying that, S 2 I wait two slots; S3 I wait 3 slots andsoon or S 1, I wait 0; S 2I have to wait for 1 slot. I could have one entire length of wait per state. Because it is only I can easily do that which I tend to…No, this is a uniform random variable. So, the time spent is either 1 slot or n minus 1 slot. So, the average time;so, now we areonly looking at the average time spent in this waiting state. So, what is the average waiting time? Either because I just missed my slot right, so what is the… I would have wait I would have wait for n minus 1 slot or I just came, there is only one slot left to go; I was in the previous slot before my allocation.

So, this is M by 2 and this is no mean. So, it isamean time; mean waiting time. So, again a set of state, transition between the states only difference is now the time spent in each state is not exponential; not geometric; that is the only thing. (( )) S 1, S 0, S 2, why the end of page we are because I am circulate why beta as they appear in that because a make my I can make this self loop with 1 minus beta this beta by n beta. If I am saying that, the time spent in the state; average time spent in the state is simply 1 over beta. I amI ambasically struck in that state for should be return time that is the sojourn time.

So, rather I do per state, pertime slot transition, I simply do it (( )) Yeah, but this term this transition should also involved beta rate let because yourtau not equal to 1 by beta. So therefore, if you leaving that state, this means there is a packet. It is ageometric process, no packet no packet no packet finally, a packet has come. So, how long do you sent in the staying in the no packet side is 1 by beta. Say in our I am sorry it is not; it is not markov anymore; it is a semi markov model; let us if it is yeah. So, this will be asemi markov model.If I replace that self loop with this, sojourn time.

Now, in this case this is nice; this is geometric, which is memory less.But in this case, this is not tau 2 is not geometric. It is a uniform random variable; anywhere been 1 to M minus 1 plus 2 is possible. Therefore, the time spent is here and this is a fixed.This is deterministic random variable taken; we (( )) exactly M minus 1 slots. So, you haveall kinds of state. This is ofcourse1, which is also deterministic.But it is like our regular aloha model, where all slots are unit time. So far (( )) so now where do we go? So, now I have come here, I will spend my wait time. So, now I have to, I will go to transmit state.

So, it is my chance to send.But we will always we go to S 1 or should any direct another state. S 1,there is only one packet in the queue.I might it is possible that, while I was in S 2,one more packet might have arrived; because this is M minus 2 slot. So therefore, I need something else. So, what we have done is we have definedthing theta 1, theta 2. So, I will define theta 1 as,it is not consist with my beta definition.But unfortunately, my paperhas this irregularity there. So, this is e power minus lambda, which means no packets are arriving in in in tau 2; (( )) time of it. Therefore, with probability again I called theta 1.

I amcoming out of state 2and this is 1 minus theta 1. So, both are transmit statesand again numbering the paper might be different.I might have renumbered this 3 and 4 differently. So, we will just use this; they my paper does not have this 1 by M, which is slightly different. But you should be able to get the flow. So, from here to here, this is the probability theta 1, 1 minus theta. So, the sojourn time is again (No audio from 19:28 to 19:38) then from synchronize wait, I came there; because I startedoff with having only one packet; because I was in S 1 state.There is only one packet to begin with,in that is 1 period, 1 unit of time it another packet came.

But that packet is already finished transmission. So, I would have I am arriving here with only one packet in the queue.But during this duration it is possible that one more packet might have come. It is possible that, onemore packet might have come. So, I would then move to either here; I will again after define theta 2. (No audio from 20:11 to 20:27) So, this is 1 minus theta 2 and this is. So, I will synchronize wait state; no other packet came; I go back to transmission is only one packet. So, that is theta 2 probability; in S 3, I might have 1 packet or 2 packets. So, if I in the durational M minus 1 slots,I might haveone more packet, in which case I have to go to S 4, where there is one packet being transmitted; one packet waiting in the queue.

For the first packet is where I haveM minus 1slot. So, I am going to my my assigned slots. So, from 1, I go back to slot 1; I go to slot 1.The second packet has to do synchronize wait again.From transmit, I have to go back. So, that link we have not yet done.If I have 2 packets in the queue, first packet is being sent. There is no room for another, even thougha packet might come, it is going to get dropped; because of the fact that, I haveonly 2 buffers. So, I have finished my top packet, whatever packet comes is simply going to beta.At the end of the slot, what is my state? I have another packet to be sent with waiting period of M minus 1 slot.

I have to wait for the M minus 1 slot formy next chance.This packet waiting time is M minus 1;because it is exactly M minus 1.Yes. Now, when this packet is M minus 1 another packet is S 3. Yes, that is go to the S 3 to S 4 arrow is showing. See, S 3 to S 4 is probability 1 minus theta 2. If no packet has come, then there is onlyone;you are going to S 1. S 1 is simply transmissionwith one packet in queue. S 4 is transmission with 2 packets in the queue. So, from S 3, I go to either S 1 or to S 4.No no there is only one always only head of the queue is being sent.

Second is coming back see may S 4 to S 3, there is another loop, which is always 1; because I have to go and wait for the secondpacket.There is no probability because withthere isonly one way, I can go. I simply have to go back state S 3 and wait for M minus 1 slot for the next packet in the queue.Then, that packet during that period, if one more packet comes, then Icome to S 4or to S 3. So, best I used to take this model (( )) in to your first your program 1.Just observe the sequence of statesand see, how the system is moving;not steady state, actually use (( )) to see how it moves from state to state.

Print the state at every unit. We can do that in your simulate in(( )) isdoing that.What is the first state, second state andsoon? So, simply keepprinting the state as you are going along andyou will find that the system has to go through this particular model.That is one way; other is to implement this in the simulation and see how they simulated?What is the state change for a particular packet packet queue? What are the states it goes through? (No audio from 23:31 to 23:39) any other transition that I have missed?(( )) T 2 T 2 is an empty empty queue; packet is come and if I missed my turn M minus 1.

So, just missed myturn and wait for M minus 1or I just got the packets before justone slot before my expected slot. Then what I can do? I can do 1 over M hereandsend it to S 3 and then do M minus 2 by M.Then, this will become M minus 1by 2; M minus 2 plus 1. So, that is why, I said this S 2 can be blown up in to as many states as you want,one of which will be S 3. If you want to have full blown expansion, I can have S 1, S 2. I came in the first slot; I came in the second slot; third slot, 4th slot andso on; which is fine. Itis nothing wrong with that.Itis afinalgrain of states.

So, you will have more states in the system trying to capture every possible.There will be more accurate model. That will be your assignment3; because my class has this problem.There is big gap between whereas, this one. So, we are always going to (( )). This accuracy problem is, because this wait state S 2 is not an accurate representation. So, this is the kindof trade off, one has to do in a developing model. Sometimes if you want to describe a model and you know this case is a simple system.Sometime there will beso many such combinations.You cannot and you will go and explain acombinational explosion of trying to all possible combination is the state; it becomesaproblem.

So, you try to condense some states in to single state, you do some averaging.Averaging will lead to errors. Then there will be approximations and that is something but if you just want the approximate trend, I will show you the kind of graph, before you look at that. So, now is now is that question clear? Why we have M minus M by 2? It is basically half the channel, half the (( )) that you have to wait.Idea of S 4, where you putT 4 equal to 1. I will show you.What is the S 4 mean? This is simply the transmission state. See here, there is only one packetin the queue.

S 4 is this scenario, where there arethere is one packet being sent; this packet is wait. There are two packets in the queue. Say this is simply the transmission time that is all; One slot in these 2 are the transmit states. In S 4, there is only one packet in queue sorry. In S 1, there is only one packet in queue. In S 4, there are 2 packets in queue.But you are only sending the top packet at the head of the queue, which is 11 your time tag. (N audio from 26:24 to 26:33) So, this is again this arestart doing a model and you will find that you some time which makes some you know random assumptions sometimes.

For example, thatwhen I was doing this, the t w this this way this this state is this state was also tough. This one was easy; this is simply M minus 1 by 2. This state, because I was doing this multiple channel thing was much more complicated, because when I am in a particular state, I have sent my packet.Then, depending on what the next packetshome channel is depending on the receiver of the next channel.I can either way 0slots or 1 slot or 2 slots and so on. So, and of calculating the expression thousand times summation over this, summation over that and it is every time,it is there is some intuition; but half of it is also guesswork.

And you simply say,will this work?It works and just stops at that point.Then,you say I think intuition is correct. So, that difficulties there; in this for a single channel case,it is very simple.It is always M minus 1. So, this is model will get refined;that is why, you need the some other validation technique. So, we have covered.Every state has the probability of 1 and if you look thatfollow on paper for this, whichwe figure that, you know what? I can make even B buffer not just 2 buffers. I can extend this whole thing in to up to B buffers and simply generate the sameset of equations, I can do.

Only thing is I need my instead of theta 1, I will have a family of thetas;1 for 1 packet in a slot; 2 packets in a slot; 3 packets in a slot depending on that queue length will also keep on increases.We will put that for the final. I will make you look at that and derive that for the final. So, that is the basic model. This is not powerful, this is yes, this is if there are situations, wheresowe have in that right.

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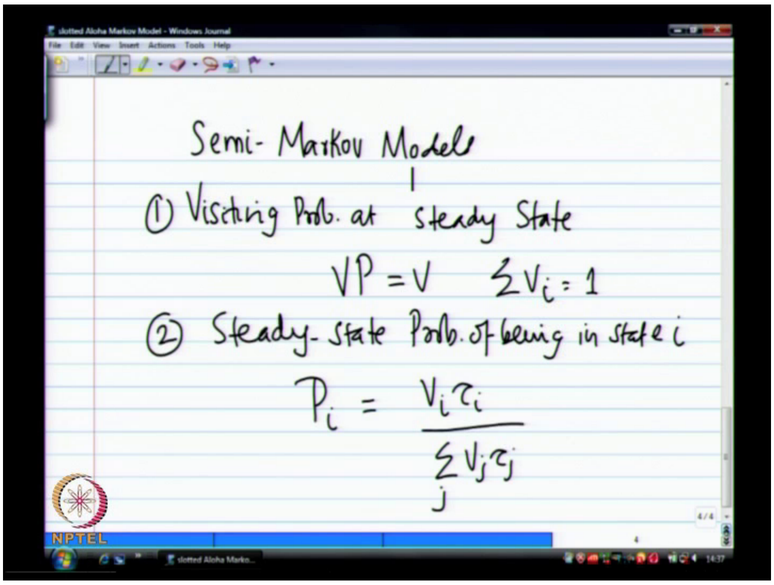

So, this is an example ofsemi markov model.There are scenarios, where you try to model; where the time spent in the states is not memory less.This is memory less properties, not their question is can we use the sameproperty? So, to wait we would get the probabilities. So, we will first find out the visiting probabilities. (No audio from 28:43 to 28:56) So, theV that we calculated before in the case ofthat DTMC is also the steady state probabilities, so we solved this. So, with that same transition metrics, we can form the transition metrics.

We can compute the V,which is now the visiting probability.It is not the steady state probability of being in a state.To go the steady state probability of being in a state, the steady state probabilities of being in stateiis now P i. So, that is simply V i tau idivided by. So, this is the steady state probability of being in state. So, the the probability metric, transition metrics will only give you visiting probability ofeach state.But then you also have to look at the time spent in each state.Effectively to get your Pi,you have to do this. So, now youwill your get your P i in 2 steps.

And in this case, there is no iterationunlike the previous case, where P of s dependent upon sigma.In this case, there is no such iteration. It is asimple set of equations.Because there is no dependence on in none of these, but any of these variables dependent upon actually with; no this is fixed; this fixed; no. So, there isno real dependency. So, there is no need for iterative process on this, but you can verify that later. This particular system, the time is spent is not; is not markov. So, but you can still use the same approach to get to the P i' s, steady state probabilities we can get.

Then you have more complicated formula for finding out the tau 3, which should be an approximation and that iswilllead to errors in your final results.It will not accurately model the system.In that particular. So, you try to aggregate states, then you try to aggregate the waiting times or the sojourn times and that leads to. So, this is the first step. So, I have computed the steady state probability. Now, let us try to compute the throughput of the system. What is throughput of the system?

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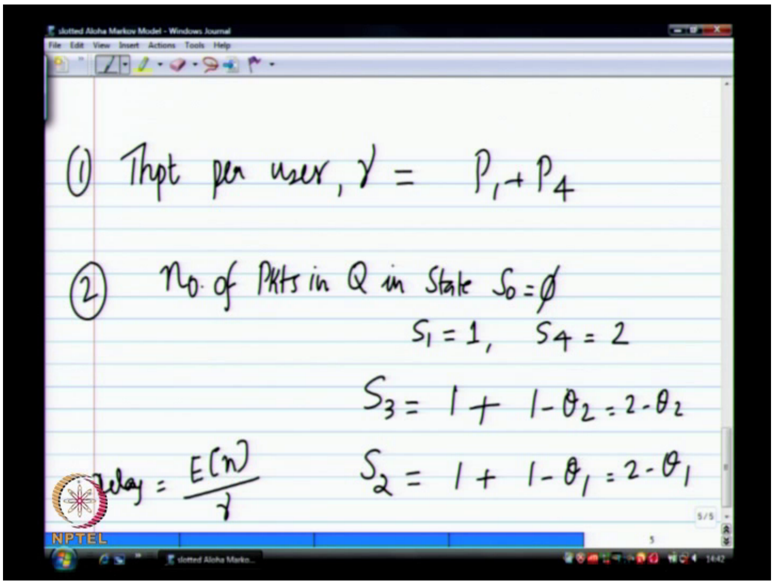

Throughput per user: So, what is the throughput? It is simply the fraction of stepbetweenThe 2 transmitting states, which was P 1 or P 4.My paper willsay P 2 I think, but in you know how to (( )). So, P 1 or P 4 and there is no question ofsuccess of every packet is successful as far as the MAC protocol is concerned.Errors might happen because of bitfailure, bit errors and so on. But if you send, this is your allocated slot; so P 1 plus P 4. So, that was easy enough. Now, second 1what is the number of packets in each state? So, the number of packets in the queue in state S.Some case is easy; some case isnot easy; S 1 is 1.

S 1and S 4is easy, 1 and 2. S 3, I have to spend some time to figure out. What is the probability of having that packet would have arrived? So, S 3, S 4, S 3and S 2 requires some extra computation. Because we started off in S 2 with S 3 with one packet, sorry in S 2 with one packet; S 2 is may sync that wait state of length n by 2. So, what will that be? 1 is there; 1 plus probability of there being of at least one packet right. So, basically 1 plus 1 minus theta 1 in this case; other case 1 plus 1 minus theta will 2 minus 1 2 minus theta 2. What is the probability of being atleast there being atleast one packet generated in S 2, 1 minus theta 1.

So, now I can compute that and theta 1 is againfixed. There is no inter dependence on something else. So, this is the way that I can. So, this isI am going to ask you to verify this. This is one packet in the queue.Probability of yetanother packetcoming in the duration of that was (No audio from 34:07 to 34:23) So, then my usual delay formula will be E of n divided by. So, delay equals. (No audio from 34:29 to 34:42) So, now we have 3 models for both SA and TDMA or at least 2 we have seen and 1 simulation that we will see next; that are have next validation. In S 3 probability, I am already having one packet in the queue.

I will havetwopackets withprobability there is probabilitywith probability 1 minus theta 1 I will have.This is a expected number of… It is 1 minus theta 1 into 1 plus theta 2 into 0. Therefore, it is simply 1 plus 1 minus theta. Because it is a Poisson process, I am saying, what is the probability of being at least there being at least one packet, basically there should be more than one packet, because more than one packet also this system will have essentially will will go to value of 2 right. So, I am just using one single value instead of saying packet. This is a duration perPoissonprobability of trying to get one at least one packet and the duration is 1 minus theta; 1 minus no packet. So, that is our first set of introduction to all thesedifferent models.

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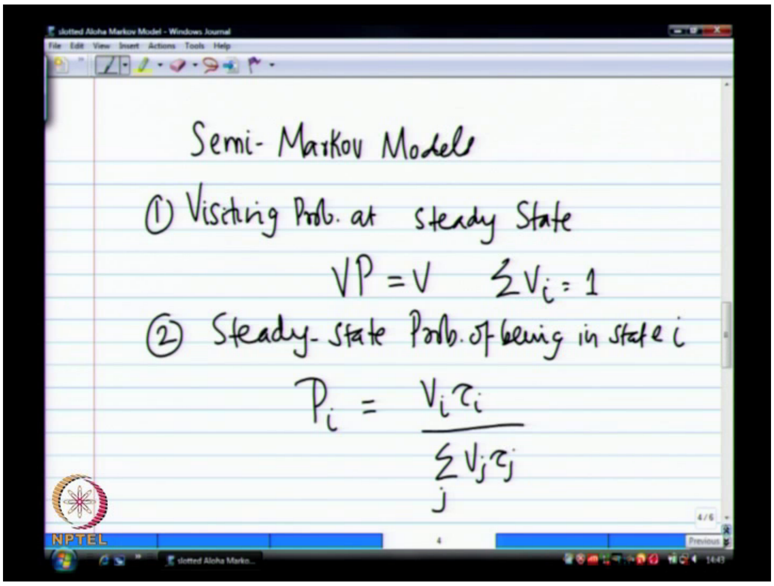

I nowreally looked into, I just took that as (( )), this case we look at like previous lecture be could be the previously multiplication by t of the probability set of probabilities in the next. This this p is the state that transition metrics right yeah. So therefore, we can do that. So, you are essentially getting the fraction of time that you are going to visit those states. But since the time spent in each of the states is going to be different, which is that moreor less like thereward function is what we are trying to do. So, tau iis reward function for being that and how does that equate to steady state probability?

I never really thought about giving explanation for that; I just took that as granted.(No audio from 36:51 to 37:01) So, now wewill slightly change gears to look at simulation because I introduced a bit ofdiscrete even simulation to you.tau 2 is the time spent in the given state.How long do I spent in each state? It is not there is the there is the average time spent in a given state.Yes, tau 2.Why do I go tau 2? Because I missed my slot; therefore, best case I my slot is the next coming slot. So, onlyone unit of time or I have M minus 1; because I just missed. I have doit for the entire duration of the frame, before I can get my term.

Waiting time is in the system itself. So, the the the delay that we have here is the. So, this will be minus 1 no sorry yeah, because the actual transmission time is only1.We did this before.We calculate E of w and then plus 1 is E of E of r. In this case, E of r is whatever you are getting minus 1 and will give you E of w.Waiting time is simply time until you get to the top of the queue.Once you go to the top of the queue; that means, that it is your turn. So therefore, it will be this 1. You simply add this, tau all terms. But we also need to know the probabilitythat we spent in the previous states.

The average packet might they have might in simply have come gone to S 1, S 2 gone back to 0.Therefore, waiting time is simply 1.So, you are saying that,use tau 2 as the as my reward function. I have tried that. They know that,they are trying to say that usesigma P 1 into again tau and again, P i tau iandsee that gives me the delay. I donotknow, that is going to workbut one can certainly try. We can try that, when we is to implementation that; very easy to get thisset of equations done; whether that is going to be correct. So, for a given packet I am spending time in this state with this probability.

But the thing is, it does not take into consideration the queuing aspects. See, this when we try to do this E of n E of r equals n by mu and so on. That I would not really fully worked out. Some of E of r equals n by mu, n plus 1 by mu works this similar thing you are asking for. I have to work it out. I am not sure, if I can whether that is going to be correct in all the cases. Can I agree for S naught tau naught equal to 1 by beta make sense that; you can use the markov model for it. But in the uniform distribution case for S 2, how can yousay correlated to another markov model; where the states generally time spent in a stateis like exponential in the markov model.

That is why, we can switch tosemi markov model, but here it is uniform. So, the semi markov model is defined for the case, where we donotworry about the distribution of the time spent;

only you worry about the mean sojourn time. That is that is that is what you doing. So, there is no necessarily core correlation necessary correlation to a exponential time. It is simply some random distribution for which only knows the mean time.

That is that is that is the and there is the say that, this works; that you can use the same approach to solve. It of the face of it may analyze go back and dig in to some of the old books talk about, why this is actually this property this probabilities are indeed correct? I will have to do that; I have not really. Intuitively it seems that we are expected memory less property for the other two. This case, we are taking any distribution. Therefore, where memory will be there in the system; that we can still expect this kind of results. But that is why; if you look at my graphs I am going to show you. I do not have the actual paper.

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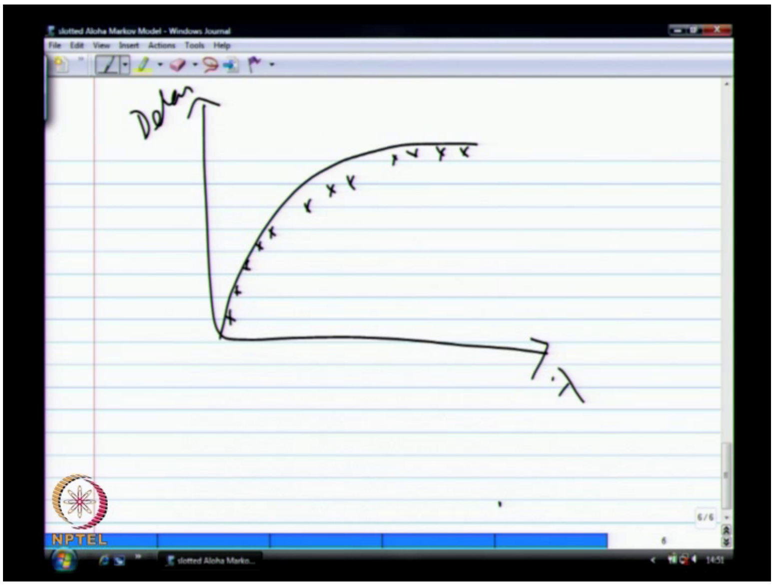

But when what we normally do is, I vary the load on the system. This is my lambda, and this is the delay and then my analytical model will give me some perfect system like this. Then when I try to actually compute with simulation, for low loads I will always find that, the values are reasonably close; for very high loads, I am able to accurately predict. But here you will find in the knee of the curve will always find gaps there; that is because of thissome of theseassumptions. So, if I reallywant to have more accurate one, then go for more states; multiple states and the waiting state.

Then,then you will have noissue at all.Then, there is no state which is got average waiting time. So, S 1 will be only 0 waiting time; S 2 will be just 1 slot; S 3 is 2 slot and so on and that is very easy to (( )). So, if I done that then I would in fact, in my paper the other thing I (( )) I did not even go from S naught to S 1, that paper will not have S 0, S 1 transition. It simply goes to S 2.We simply say I arrive, I have to wait (( )) M by 2; because if it is it will if you look at it will be M plus actually you should be M plus 1 by 2.How many times I figure it out? I would tried it out M plus 1 by 2 or M by 2finally, just M by 2.

But that half slot will cause errors in a final solution, because especially when the queues set - queue length increases that will contribute some more errors. So, whether this M because you may I just arrived. So therefore, my waiting time is 0 or I just missed; therefore, my waiting time is M. So, it is M plus 0 by 2 again the M by 2, whether there should be M plus 1 by 2. I have spent all download our trying to figure out, but if it is M by 2; finally, is what settled on, but yes this was my first attempted markov modeling whether knowing any basics of markov chain. I just sit here; give transition, give probability, the (( )) time here are numbers. So, we just look at on (( )) blind faith.