Approximation Algorithm

Prof. Palash Dey

Department of Computer Science and Engineering

Indian Institute of Technology, Kharagpur

Week – 10

Lecture 46

Lecture 46 : Primal-dual Algorithm for Minimum Weighted Feedback Vertex Set Contd.

So, in the last class we have seen how using cycles with small number of vertices of degree at least 3 could be useful for designing good or small factor approximation algorithm for minimum weight feedback vertex set problem. So, let us finish that analysis in this lecture. So, minimum weight feedback vertex set problem. So, let us write the pseudo code of our modified attempt for primal modified primal dual algorithm for this problem. So, as before we are starting with the dual feasible solution of y is 0 and primal infeasible solution to be empty set.

So, we first before starting we remove all degree 1 vertices. So, let me write here while there exists a cycle in G first I do the clean up to ensure that in the input graph also there is no degree 1 vertex. So, repeatedly remove all degree 1 vertices from G. Then I know that in the resulting graph there exists a cycle with at most twice ceiling of log n many high degree vertices.

So, find cycle C with at most $2\log n$ vertices of degree 3 or more and then for this cycle do the standard primal dual method that means, you increase the variable y_C as long as some dual constraint involving y_C becomes tight. y_C until there is a vertex $l \in C$ such that summation $C' \in \notin$ is the set of cycles in G such that $l \in C'$ $y_{C'} = w_l$ ok. Then you put that vertex l in your solution $S = S \cup \{l\}$ and remove l from G ok. And in the next iteration in the beginning itself we will repeatedly remove all degree 1 vertices from G. So, this is the pseudocode of the algorithm and then return S.

So, this is the our modified algorithm. So, we claim that this is a $4\lceil \log n \rceil$ factor approximation algorithm. Our algorithm achieves an approximation ratio of at least $4\lceil \log n \rceil$. So, whenever we do not write base it should be assumed that the base of logarithm is 2 proof. So, following the standard primal dual analysis we have $l = \sum_{i \in S} w_i$.

Now, we pick and vertex only if it is tight the corresponding dual constraint is tight. So, this is equal to $\sum_{i\in S}\sum_{C\in \mathcal{C}:i\in C} y_C$. Again we have double sum we should exchange the double sum and try to gain insight cycle $\sum_{C\in \mathcal{C}} |S\cap C| y_C$. Now, because of the lemma because cycle C whenever we pick a cycle C it contains at most $2\lceil \log n\rceil$ many high degree vertices and because each path can contain at most 1 solution in S the number of vertices picked in S in C can be at most $4\lceil \log n\rceil$. So, since C contains at most twice log n vertices of degree at least 3 and thus at most $4\lceil \log n\rceil$ paths maximal paths of vertices. of degree 2 and since every path of vertices of degree 2 can contain at most 1 vertex. in the solution C intersection S can be at most $4\lceil \log n\rceil$ these are for vertices of degree at least 3 plus $4\lceil \log n\rceil$ this many paths of degree 2 we have. So, this is equal to $4\lceil \log n\rceil$.

So, what we have now then? So, let us continue the analysis from here, ALG = $\sum_{C\in \mathcal{C}} |S\cap C| y_C$. And, $|S\cap C|$ is at most $4\lceil \log n\rceil$. So, this is less than equal to $4\lceil \log n\rceil \sum_{C\in \mathcal{C}} y_C$. Now, y_C is a dual feasible solution throughout the algorithm we maintain dual feasibility.

So, this is by weak duality this is less than equal to LP-opt of the primal. So, this is $\log n$ times LP-opt and LP-opt is less than equal to opt because LP linear program is the relaxation of the ILP formulation of the problem which is an exact formulation. So, this is less than equal to $4\lceil \log n\rceil$ times opt. So, what is the main take away message is the main take away message is that for set cover it was immaterial to choose or to decide which dual variable to increase, but for this minimum weight feedback vertex set problem it is important that we pick dual variable wisely to increase. in the primal dual based f factor algorithm.

for minimum weighted set cover problem, it did not matter. which dual variable we choose to increase. in minimum weighted feedback vertex set problem. it is important to choose a dual variable carefully. to increase ok.

And this is often the case in many other problems it is important that which dual variable you pick to choose to which dual variable you pick to increase and that seriously affects the performance or the approximation factor of our algorithm. So, the next natural question is what can we or can we have a better approximation guarantee. So, can we have an algorithm for minimum feedback minimum weighted feedback vertex set problem with better than $4\lceil \log n\rceil$ approximation factor. So, it turns out that that the integrality gap of this LP relaxation is $\Omega(\log n)$. So, this $4\lceil \log n\rceil$ approximation ratio may be improved up to constant factors.

Hence, using this LP relaxation to lower bound opt, one can have an algorithm with approximation guarantee at most or at best $O(\log n)$, but see this does not refute the possibility of an approximation algorithm with approximation factor better than $O(\log n)$ and indeed there exist better than $O(\log n)$ factor approximation algorithm. Note that this does not refute the possibility of a or smaller than $O(\log n)$ factor. approximation algorithm for minimum weighted feedback vertex set problem. Indeed there exist a two factor approximation algorithm which is based on primal dual method, but uses different linear programming relaxation. indeed there exists a primal dual two factor approximation algorithm using another LP relaxation called configurations configuration LP ok.

So, this is what we have for minimum weight feedback vertex set problem this primal dual method. Another remark is that you note that in the primal dual method we do not need to solve any the linear program. This is a very important point in primal dual method we do not need to solve the primal or dual LPs. We use LP to design and analyze our algorithm.

ok. This sort of algorithms are sometimes called combinatorial algorithm. This type of algorithms which do not solve which does not solve any LP are called combinatorial algorithms. Although there are fast LP solvers this type of combinatorial algorithms often run faster on real world instances. Although there are fast LP solvers, a combinatorial algorithm which uses LP only to design and analysis runs faster than LP based algorithms for example, deterministic rounding or randomized rounding for real world practical instances. Hence this combinatorial algorithms are more desirable than this their LP based counterparts.

This makes combinatorial algorithms more desirable than their LP based counterparts. So, in the next lecture we will see another primal dual algorithm for generalized Steiner tree problem and there we will see another new idea that it is sometimes useful to simultaneously increase multiple dual variables to get a good factor approximation algorithm ok. So, let us stop here. Thank you.