Artificial Intelligence for Economics

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Week – 01

Lecture - 05

Lecture 05 : Uncertainty in Financial Markets : Idea of Hedging (Contd.)

Welcome to the fifth lecture of the course and we will continue with uncertainty in financial markets and the idea of hedging. In the previous lecture, we have looked at different kinds of interesting financial instruments. We looked at straddle, strangle, we looked at shorting with call. and we looked how we are hedging for risk, how we are mitigating risk. Towards the end of the last lecture we looked at straddle and strangle and they were essentially creating strategies where we bet on movement.

We were hoping that the price of the underlying asset will move significantly either to the right or to the left. right so that was straddle and strangle we can also design a portfolio where again I am not we are not sure in case of straddle and strangle of course in case of straddle we were not sure about the direction of the movement right now we can create another portfolio where we bet against movement Okay, and such a portfolio is called butterfly, a butterfly spread more precisely. So, let us see. So, we are going to talk about a long call butterfly spread.

So, what is a butterfly spread? What we are doing is the following. This is what we do. We buy a call at a really low strike price for the same underlying asset. We buy two calls with a slightly higher strike price. Sorry, sell two calls at a slightly higher strike price.

And finally, buy another call for an even higher strike price. All these call options have the same expiration date. They have the same strike price. Sorry, the strike prices are different, of course. And the strike prices are equidistant.

So let's see what I mean by this. Let's take an example. So let's consider any underlying asset, whichever you want to imagine. Let's say I have bought a call with a strike price \$55. Then I will sell two calls with a higher strike price of \$60.

And finally, I will buy another call with an even higher strike price, \$65. The strategy is based on the speculation or we are hoping for the fact that the price of the underlying asset will hover around the center strike price which is $60. It won't move much. So we are betting on no movement. Let's see.

Let's see why that's the case. Okay, before we move on, let's try to think what the payoffs are going to be. Let's say if S_T is less than 5, 55, then what will happen? Well, if the price of an asset is less than 55, the right to buy at 55, the right to buy at 60, the right to buy at 65, all these are useless rights. So all these call options are useless and they won't be exercised. So the payoff is 0.

it won't be used by anybody be it the holder be this so the payoff to everybody will be the seller will be 0 if S_T the price of the underlying asset at the expiration date lies between 55 and 60 then what will happen then what will happen well let's say if it let's say this 57 If the price of an asset lies between 55 and 60, the call option with strike price 60 and the call option with strike price 65, they won't be exercised. The only call option which will be exercised is this one, the one with strike price 55. and I have bought a call option with strike price 55. So, I have the right to buy the stock at 55 and sell it at S_T. So, what will be my profit? It will be $S_T - 55$.

So, my profit what if the price lies between 60 and 65 what's gonna happen then let's say it is 62 then well I can sell I have bought a call option with price with strike price 55 so I will definitely make a profit of $S_T - 55$ from that right so I will make a profit of 7 from this but then I have sold two call options which have a strike price of 60 so let's say I have sold it to you so you have the right to buy this asset at 60 rupees from me So I will have to buy it from the market at 62 and you will buy it from me at 60 rupees. You will exercise the right. So I will incur a loss of 2 for each of these call options with strike price 60. So what will be my loss? 4 rupees. So what's the payoff then? It's $S_T - 55$ plus or maybe it is actually minus.

So, you have S_T, so that is 60 or $S_T - 60$, this is what we are left with and this is multiplied by 2. So what's the payoff then? What's the payoff? The payoff is simply, so this is $120 - 55$, that turns out to how much? So that's $120 - 55 - S_T$. So it just goes down like this. What if the strike price is above 65? Then what will happen? when then something interesting happens if S_T is above 65 then something interesting happens then what's going on then let's say the 70 so I have sold you two call options you have the right to buy two units of that asset from me at 60 rupees okay but it is being traded in the market at 70 So will I buy it from the market and sell it to you? Probably no. So what I'm gonna do is the following.

I will simply use two of my call options which I have bought, these two. and I will buy two of these assets, two units of these assets. How much will they cost me? 55+65, which is 120. And then I will sell both of those units to you for 60 each, which is again 120. So my payoff is gonna be zero.

Okay? Great. So this is how the payoff looks like. It goes up, and then it goes down, and after 65 at zero, after below 55 is 0, okay. Of course, this is without considering the prices of the options. If I consider the prices, it will just get scaled down or scaled up, whatever you call it.

Great. So, we are basically now hoping that the price will be around 60. Closer it is to 60, higher my profits. Okay, so we have seen straddle strangle, we have also seen a butterfly spread, and we see how the payoffs look like. Now, we'll try to look at any arbitrary portfolio which you might have, and we'll try to see how to write down the payoff of this portfolio. So let's say you have a portfolio of this kind, where you have sold two calls with strike price 50, you have bought two calls with strike price 70, you have bought again three calls with strike price 90, then you sell a call with strike price 110, and then you again sell two calls with strike price 120, so on and so forth.

ok so this is how a portfolio looks like what will be your payoff how will you plot your payoff as a function of the price of the underlying asset ok that's what we'll try to see in this so let's try to solve this so if let's say st the price of the underlying asset if st i am sorry if st is below 50 then what will happen remember what is a call a call option gives you a right to buy right so if st lies between 0 and 50 then That's the, every call option in this portfolio has a strike price above 50. So every call option involves a right to buy the underlying asset at a price above 50. But if the asset is trading at below 50, then all these call options are useless. So none of them will get exercised. So the payoff is gonna be zero.

What will happen if S_T lies between 50 and 70? Let's see. So why 70 suddenly? Because 70 is the next highest strike price which I observe in this portfolio. So if the price of the underlying asset at the expiration date lies between 50 and 70, it is 60, 65, whatever, which will be the only option which will be exercised? Well, it is of course this. Only this option will be exercised, right? Great. So what will be the payoff? I have sold, remember, I have sold two options with strike price 50.

So what will be the payoff? So I have sold it to you. You have the right to buy it from me at 50 rupees. But actually the price is above 50. So I will have a loss. So my payoff is going to be minus But then I have sold two call options, so it's going to be 2.

OK? Great. If it lies, now what is the next highest strike price? That's how we move on.

The next highest strike price is 70 in my portfolio. So what if S_T lies between 70 and 90. 90 is the next highest strike price. If it lies between 70 and 90, so it is still less than 90, then which of the options will be exercised? Well, it only will be the first and the second.

Only these two call options will be exercised. And what will be my payoff? Well, from the first option, my payoff will remain what it is. My first payoff will remain what it is, this. But, I also have this option coming into play now. I have bought, now I have two options in my pocket.

at strike price 70 so if the price of the underlying asset is above 70 then what will I do I have the right to buy at 70 and then sell it off at a higher price so I will make a profit now so what will be that profit so this is the payoff from the C50 the strike price with the call with strike price 50 now what is my payoff from the call with strike price 70 because I have bought them It is 2 into $S_T - 70$. Well, if you simplify these, this is simply 100 minus 2ST. This if you simplify, this simply becomes 40. Now, what is the next highest strike price in my portfolio? So, I am done with 70 and 90.

So, then is 110. So, if S_T lies between 90 and 110, then what is going to happen? Then what will be my payoff? If it lies between 90 and 110, what will happen? Well, all these three options will be Of course it's less than 110 and 120, so these options will still be defunct. They won't be exercised. So these three options, the first three options will be exercised. From the first category of options which I have sold, my payoff will remain the same from the first two categories. Now I have the third component coming in from this.

So how will my payoff look like? My payoff will look like this. I have bought another three more options at strike price 90, so this is what I have. And if we simplify this, this will simply become, I think by the way this is minus 40, if I am not wrong, this is minus 40, I am sorry. So this will become how much? This will, I think, become $3S_T - 310$.

If we simply simplify this. Okay. Now let's observe the next highest strike price in our portfolio. What is that? We are done with 110 it is 120. So, what if S_T lies between 110 to 120 that is the highest possible strike price which is there in my portfolio. So, what will be my payoff now? Well, the payoff from these calls will remain the same. So, I will have this 3ST minus 310 that is for sure.

And finally, I also had sold these 120 call options. Sorry, 110. This one is active now. This one is the additional one which becomes active.

So I've sold. So I'll have a negative payoff. What will be my negative payoff now? It will be $-S_T-110$. That's the loss island curve. If the price goes above 110, the person whom I have sold the call option with strike price 110 will exercise it and this will be my loss.

And this turns out to $2S_T-200$. Now finally, if the price of the underlying asset at the expiration date goes above the high strike price which is there in my portfolio, which is 120, then all the call options will be exercised in my portfolio. Even this will be exercised now. So what will be my payoff now? Well, all everything has been exercised. So it is 2ST minus 200 for sure from exercising the previous ones. And then you have from selling a call option at 120, this will be my loss.

But then I have sold two of them. So it's this. And if we simplify this, this simply becomes 40. Okay, so this is how the payoff looks like. So if we write it down a little more neatly, what do we have? Let me write it down a little more neatly here. So, or maybe I should write it down in another slide.

My profit is zero if S_T lies between, is less than 50. It is $100-2S_T$. if S_T lies between 50 and 70, when it is minus 40, if it lies between 70 and 90. this $3S_T-310$, this mean 90 and 110, this $2S_T-200$ if it lies between 110 and this becomes plus 40 if it's more than 120. So, if you just plot it in a diagram, how would it look like, if you just plot this.

So, this is the price of the underlying asset at the expiration date and this is your payoff. So, below 50 it is 0, then it goes down, so it is 0 here, then it goes down and this is minus 40, this is up till 70, between 70 and 90 it remains at - 40. between 90 and 110 it goes up and this slope is $3S_T$ between 110 and 120 again it goes up but the slope is 2 so it goes up at a lower rate like this so here the slope is 2 up till 120 and after 120 at 120 it reaches a profit of 40 and after 120 it just remains there. So, this is how the payoff function looks like, the profit function looks like. Of course, in this analysis we have ignored the price of the options, right.

So, we have sold some options, we have bought some options, we have got some net proceeds, it could be positive or negative, I do not know. So the entire payoff will get scaled up or down based on that initial net proceed which involves the price of the options. Great. So we have looked at how we can price or sorry how we can find out a payoff of any given portfolio. okay as a function of the price of the underlying asset as the price of the underlying asset varies we can we can plot the payoff of any portfolio so we have I have given you that tool fine now we have come to the last segment of talking about uncertainty in financial markets and in this segment we'll talk about we'll spend the last 10 to 15 minutes talking about option pricing now we don't have time to talk about option pricing in details so we'll just touch upon some basic ideas first idea the present value of money Okay, this is exactly the continuous version of compound interest which we have learnt in high school.

So what am I doing here? Let's say I have an amount of money empty in my wallet at time t, the interest rate is r and let's say the money which, the increase in money is in a period Δt is ok then what is ΔM_t which it is the interest earned let us say so that is M_t into r into delta t. So, as delta t becomes really small so we are having we are not having annual interest if we are having interest at every infinitesimally or a return at infinitesimally small time then $\frac{dM}{dt}$ is this and M_t is if you integrate both sides you are simply left with this ok. which means that if you have an amount M_0 now, if you have an amount M_0 at the beginning after time t you will necessarily have this much money. So, present value basically tells you that having a money, having amount M_0 now is equivalent to having $M_0 e^{rt}$ after time t or having an amount M_t at time t is the same as having an amount $M_t e^{-rt}$ at the beginning t periods before, okay. Having an amount M_t at any point t time t is the same as having $M_t e^{-rt}$ t periods before, okay, great.

Keeping this in mind we will talk about this extremely critical equality called the put-call parity equation. Okay, now what is this? This tells you, this basically connects the price of a call and a put option, given that they have the same strike price and expiration date. so at any point of time what is the price of a put option or a call option they are connected by this equation called the put call parity equation which basically tells you that if you manage to know the price of the put option you will automatically know the price of the call option at any point of time but why should that happen why should that happen let's understand that why should this equation hold true. We are going to prove this and we will have an no arbitrage argument and that is where we will end the lecture.

So, let us move on. So, why should this equality hold true? Let us see. Let us say it does not hold true. Let's say the left-hand side is greater than the right-hand side, let's say. Now what's the problem here? If it is so, then I will buy this particular, I will find this portfolio, I will do this, I will execute this portfolio, see what I am doing here.

I will short a unit of the underlying asset. How much money will I get if I do that? This much. I have told the broker, hello, sell one unit of the underlying stock. He sold it, gave me the proceeds, which is S_T. That's the price of the stock at time, now, which is time T.

Okay, great. I will sell a put. If I sell a put, what is the price of the put? P_E. So, I get that money. How much is that money? This much. Very good. now I buy a call so what is my expenditure in buying a call this much and the call and the put of course have the same expiration date in strike price also I keep this much amount in the bank here okay I have done this by doing this I also have some amount of money left remember I earned this much money how by shorting a stock and selling a put this much money I have spent on the call and this much I have kept in the bank but the left hand side is greater than the right hand side so I have a guaranteed payoff already right I have got some money already great but then you will say that hang on hang on you have the port and the call so the payoffs might vary let's see what's going to happen something funny will happen now let's say the expiration date has come remember i have some money in my wallet already now the expiration date has come let's say the price of the asset at the expiration date is above the strike price e okay is above the strike price E.

Now if the price of the underlying asset at the expiration date is above E then which option will be exercised, the put or the call? Naturally it is the call, the call option if the price is above the strike price then I have a right to buy the asset at a lower strike price, so I will exercise the call. So, the call will be exercised, the put will not be exercised. So, the call is exercised, the put is not exercised, very good. Now, I have bought the call, right. So I will exercise the call and I will call the other person and say that, hey, you are obligated to sell me an asset at rupees E.

Or I have the right to buy one unit of the asset at rupees E from you. Okay? So he will say, yeah, yeah, yeah, please sell me. How will I get that money E? Well, I had kept this much money in the bank initially. so this was my M_0 so what will be my M_T it will be $M_0\ e^{r(T-t)}$ which is in this case E so the amount which I had kept in the bank has now become E so now I have E rupees with me I will take it out from the bank I will exercise the call option By using the call option, I have the right to buy a unit of the asset at rupees E. So I will place that E which I have withdrawn from the bank and I will get one unit of the asset.

Fine. Now I have one unit of the asset. Remember, I had also shorted one unit of the stock initially. That is the broker had sold one unit of the stock on my behalf. So I will have to repay him one unit of the stock. And I have just bought one unit of the stock exercising the call option.

And I will use that stock and I will pay it back to the broker. I will give that unit of the stock back to the broker. And my position with the broker has been squared off.

Great. Now I have no obligations in the market. I am clean. Fantastic. But what happened in between? I made this guaranteed profit. Okay. and I have no obligations now. So if the left hand side is greater than the right hand side, I will keep making such guaranteed profits forever. Like we saw in a previous lecture, I can make zero risk guaranteed profit by betting optimally with Virat and Steve.

Remember two lectures before, just like in that case here also I can keep making guaranteed profit till I suck up all the wealth in the world. So this is an impossibility. So this is called the no arbitrage, this is an impossibility because this leads to no arbitrage, this leads to an arbitrage condition. So this being greater than this leads to this problem. What about the other way around? Can that happen? Can the left-hand side be less than equal to the right-hand side? Let's see.

If that happens, then I'm going to implement the following portfolio. What will I do? I will buy one unit, sorry, I will sell a call and get this much money. I will borrow this amount from the bank. $e^{r(T-t)}$.

So how much cash do I have in hand right now? This much. This much. The right hand side. The right hand side amount of cash I have with me right now.

Okay. Now I buy a unit of the stock. So this is my expenditure. I also buy a put. That's my expenditure. So the amount I had in cash is more than my expenditure because the right-hand side is more than the left-hand side. So I have made some guaranteed payoff to start with.

Now let's see what's going to happen on the expiration date. By the way, I think I missed one little point here. In the previous case, we talked about if S_T is greater than E, right? I'm sorry about that, I should also mention what will happen if S_T is less than e. This case I have missed out, so let me cover. What will happen if S_T is less than e? Then which option will be exercised? The call option will not be exercised, the put option will be exercised.

Right? Okay. Now, I have sold a put option in the previous case I am talking in this situation I have sold the put option remember here so the put option will be exercised so the person I sold the put option to will come to me and say I will sell this unit of underlying asset to you at rupees E I will say yes please sell it to me I kept this much amount in the bank which has become E now. I will pay him E rupees, get that stock from him because he is going to sell it to me at E rupees. Well I got the stock, the call option is not exercised so that is not there. So I used this money and bought the stock at E rupees and I also had shorted the stock So I owe one unit of the stock to the broker so I will he will sell that unit of the stock to me the put guy I will take the stock and take it to the broker and square off my position. So again even if S_T is less than E even then I will have no obligation on the expiration date and I will keep my guaranteed profit in my wallet it will be untouched.

Okay. Okay. Anyway, so now let's move to the next situation where the left-hand side is less than the right-hand side. In this situation, I'm implementing this portfolio. I'm selling a call, getting this much money. I'm borrowing an amount this much from the bank and I'm keeping it in my wallet.

Then what am I doing? I'm buying a unit of the stock. This is my expenditure and I'm buying a put. and the cash which I had and my expenditure, the Δ of that is positive. So I have already made some guaranteed money which has been kept in my wallet safely. Now let's see what's gonna happen at the expiration date. If the price of the underlying asset at the expiration date is greater than E, which is the strike price, then which option will be exercised? remember whenever the price of the underlying asset is more than the strike price the call option is exercised right and the put option will not be exercised so the call option is exercised I had sold a call remember I had sold a call so the person I sold the call to will come to me and say that hey I want to buy a unit of the asset from you at rupees E.

At rupees E. I'll say okay. Now I have a unit of the asset. Remember I had bought one unit of the asset. That is with me. So I'm going to give it to the guy whom I had sold the call option. So once he exercises the call option, I will give this unit which I had bought to the guy.

He's happy. My position is squared off. The put option is not exercised at all. Now, so what did I get? I gave him the unit of the stock and he gave me E rupees. Fine, now I have E rupees. I had borrowed this much money from the bank this this so this much money if I borrowed from the bank how much will I need to pay the bank well I will need to pay if this is M_0 I will need to pay I will need to pay E amount to the bank but then the person I had sold the call to Paid me E rupees and took the stock from me.

So I have this E rupees with me. I will take this E rupees and pay the bank. So the bank is happy. The person I sold the call option to is happy. Everybody is happy. And the guaranteed payoff which I made, that remains untouched. Again, I made risk-free guaranteed profit if this inequality holds, if this strict inequality holds true.

Okay. Again this is, if S_T is greater than E, what if S_T is less than E? If S_T is less than E, we can similarly prove that I will make guaranteed profit. Let's see how. If S_T is less than E, then the call option will not be exercised, the put option will be exercised. So now what will happen? I have the stock with me which I had bought, this one. now I will exercise the put option and sell the stock at rupees E okay I have the right to sell the stock at rupees E by exercising the put option so I will sell the stock get rupees E now I have rupees E with me this rupees E I will pay the bank off because I had borrowed this much money from the bank so I owe the bank E rupees I have got E rupees from selling the stock by exercising the put option, that E rupees, I'm gonna pay the bank and square my position off.

The bank is happy, everybody is happy, I owe nothing to anybody. And my guaranteed payoff which I earned to start with is untouched. Again, so whether S_T is greater than E or lesser than E, I will make guaranteed profit. And making guaranteed, risk-free guaranteed profit leads to arbitrage which leads to complete market failure. So, both these inequalities cannot happen which leads us to the conclusion that this equality must hold true and this is a simple reflection of a no arbitrage condition. So this is the put call parity, which connects the price of a put option and a call option, given that their expiration date and strike price are exactly equal.

So if we manage to find the price of a put option at any instant of time, we automatically find the price of the call option at that point of time. Which call option? The one which has the same strike price and expiration date. But how do you find price of any put option? right for that we have Black Scholes Merton modeling for option pricing I mean there's a whole whole bunch of literature if you want to if you want to study this falls under the branch of mathematical finance if you are interested that's that's a very interesting area of study area of research anyway so this brings us to the end of the first segment The first five lectures where I have given you a glimpse of the overlaps between artificial intelligence and economics. I have tried to come up with examples from network data.

I tried to give you a glimpse of stable matching. I took a lecture on the stable matching algorithm and then a few lectures on uncertainty in financial markets. Great, I hope you're enjoying the course. See you in the next lecture. Thank you.