Artificial Intelligence for Economics

Prof. Palash Dey

Computer Science and Engineering

Indian Institute of Technology Kharagpur

Week – 07

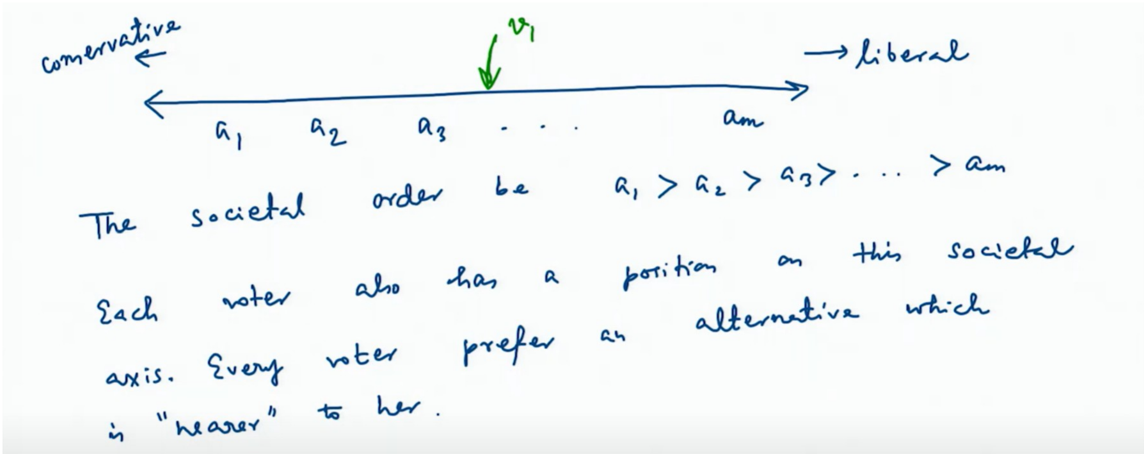
Lecture - 34

Lecture 34 : Single Peaked Domain and Median Voting

welcome. So, in the last lecture we have seen how in the last couple of lectures actually we have seen how the Gibbard-Satterwaite theorem can be bypassed using quasi linear environment and then single parameter domain and so on. But a problem with that approach is that we need a notion of money and that mechanism design is often called mechanism design with money. But, in many applications the use of monetary transaction for various reasons are not allowed. Examples of such applications are say matching or say voting. So, how can we tackle or bypass the impossibility implications of Gibbard-Satterwaite theorem using or without using money and that comes under the broad area called mechanism design without money.

So, in this lecture let us see a brief or high level overview of that of that area. So, the today's topic is mechanism design without money. So, for that let us see a concrete example of voting we have already seen an example application of matching or in particular stable matching and there we have seen that it is we can design stable matchings where you know there it does not have any blocking pair. So, in the context of voting let us see what we can get.

So, in particular today we will discuss area of voting theory which is called single picked preferences or single picked domain. if you recall the high level idea of quasi linear environment is to restrict the utility functions that the user can have. The same is going on here also in spirit that we restrict the set of preferences or rankings that the voters can have or players can have over the alternatives. So, we restrict the set of preferences or rankings that players also called voters in this context. can have over the alternatives .

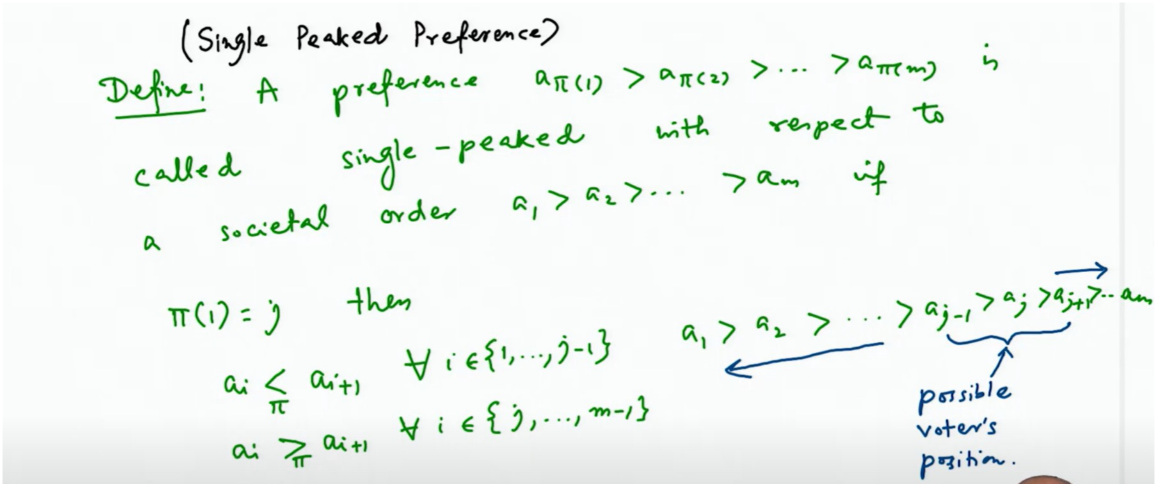


So, here we assume here in this single peak domain here in the single peaked domain, we assume that there is a societal axis on which the alternatives are ranked . So, this sort of assumptions make sense in many applications. For example, political elections candidates are often either conservative or liberal or somewhere in between. So, what we assume is that there is a societal axis. where the alternatives say a_1 a_2 are ordered we do not know their exact points up to precision, but we know their orders that a_1.

So, suppose this left side is conservative conservative right side is liberal And it is it is common knowledge that the candidate a_1 is most conservative among all the candidates followed by a_2 followed by a_3 and so on a_m. We assume that we have m candidates. So, and this order is sometimes called the societal order. the societal order be $a_1 > a_2 > a_3 > \ldots > a_m$ ok. Now, how about the voters? Voters also only vote as per this axis.

So, each voter also has a position on this axis. ok and voters preference drops as the candidates are further from the voters position. Every voter preferred a candidate or an alternative which is nearer to her. In particular suppose here is some voter v then I voter v_1 if it is if the voter is placed in between a_3 and a_4 then without even without knowing the exact position of a_1, a_2, a_3 and so on. we can say that a_2 is nearer to a_1.

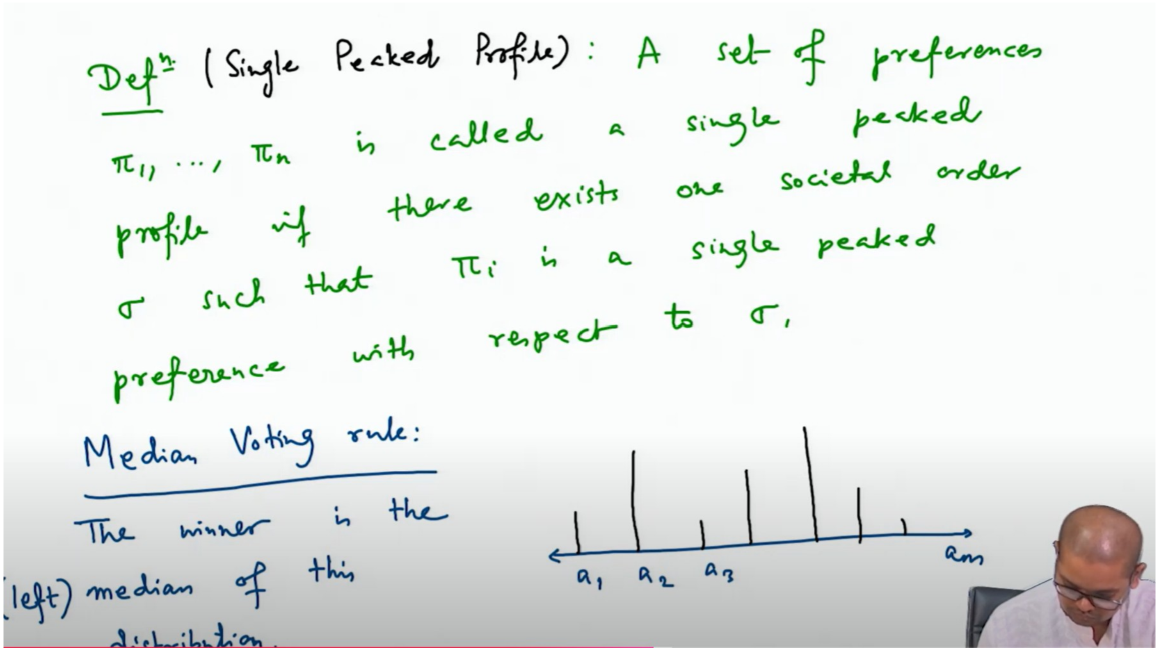
So, voter a_2 will prefer a_2 than a_1. So, if this is true for all voters with respect to this societal order, then that profile is called a single pick profile and the domain which domain of preferences which allow only single pick profiles are called single pick domain. So, let us formally define what is single pick domain. a



preference say $a_{\pi(1)}$ followed by $a_{\pi(2)},\ldots,a_{\pi(m)}$ is called single peaked with respect to a societal order. which by renaming we can assume without loss of generality $a_1 > a_2 > a_3 > \ldots > a_m$.

If you look at $a_{\pi(1)}$ if $\pi(1)=j$. that means, in this societal order $a_1 > a_2 > a_3 > \ldots a_{j-1} > a_j > a_{j+1} > \ldots > a_m$. We know that if the top preferred candidate or the most preferred candidate of the in this preference is a_j, then we know that the voter is positioned between a_{j-1} and a_{j+1}, this is the possible voters position. then the restriction is that here from a_j to left hand side a_{j-1} a_{j-2} and so on the more we go the candidates will be less preferred and same on the right hand side. If v_1 equal to j then a_i should be less preferred than a_{i+1} under π for all $i \in \{1,\ldots,j-1\}$ and a_i should be more preferred than a_{i+1} for all $i \in \{j,\ldots,m-1\}$.

So, if this condition if this conditions hold then we say that this preference is single picked with respect to this societal order. So, this is the definition of single picked So, what is single picked profile? Single picked profile. A set of preferences $\pi_1,\ldots,\pi_n$ is called a single peaked profile if there exists one societal order pi or sigma such that π_i is a single picked preference with respect to So, if there exist one societal order sigma with respect to which all the preferences are single picked, then that whole set is called single picked profile and the domain of preference profiles which allow only single picked preference. preferences with respect to some societal order sigma is called the single pick domain. Why do we care about it? First of all it is natural as motivated in political elections, not only that there exist a social choice function or voting rule in this context called median voting rule which is strategy proof and has many other desirable properties.



So, let us see median voting rule. So, in the median voting rule you first look at the societal order take the societal order again by renaming let us assume the societal order to be $a_1 > a_2 > a_3 > \ldots > a_m$. Now, you draw the histogram in the sense that for you ask how many voters or players have a_1 as their most preferred candidate. So, that bar you draw you ask how many voters or players has a_2 as most preferred this a_3 you have this bar chart like histogram and so on. And the median voting rule the winner is the median of this distribution is the median.

Now, if the number of voters n is even then there can could be two medians. So, for uniqueness let us take the left median So, if the number of voters is odd then the median is unique the winner is the median of this distribution proof or theorem. what is so good about this median voting rule? The median voting rule is dominant strategy dominant strategy incentive compatible. That means, irrespective of what other voters vote, it is the best interest of every voter to truthfully report their most preferred candidate. So, let us see the proof verbally and then you formalize it.

So, suppose a_i is the median. Suppose the current winner is a_i in a profile and there is so, this is the winner. What we will show is that irrespective of what is the top candidate of the voter, it is best to report that candidate. So, if so, pick any arbitrary profile and arbitrary voter and if it happens that the voters top candidate is a_i, then it makes it does not make any sense for the voter to vote for any other candidate than a_i because his top candidate his most preferred candidate is winning the election.

So, if so, let. a_i be the winner of any median winner that means, winner under median voting rule of any preference profile. So, if a_i and V any voter, now if V prefers a_i most then clearly voting for a_i is best for V, if V prefers a_j most for some j less than i. So, here is some a_j and suppose the voter prefers a_j the most this is less than i. then you see let us see what else the voter can vote. If the voter votes for some candidate on the left of a_j, then the winner does not change.

Then if the voter votes for any candidate in a_1 to a_j or a_1 to even a_{i-1}, then the winner remains the same. Hence, the voter does not been benefit by misreporting or by voting any candidate in a_1 to a_{i-1} instead of a_j. On the other hand, so if a_j is on the left of a_i, see voter does not benefit by misreporting any candidate or voting any candidate which is left of a_i, but he can still vote any candidate on the right of a_i, but will he benefit? If he votes for any candidate which is right of a_i, then the median of the new candidate election will be either a_i or any candidate to the right of a_i which is not better for a_i not better not better from the perspective of the voter. On the other hand, if the voter votes for any candidate in a_i to a_m then the new median winner is either a_i the median remains same or some candidate to the right of a_i, but the more right we go from a_i because the peak of the voter the voter prefers a_j which is on the left that new winner cannot be more preferred than a_i right of here which cannot be more preferred than the current winner which is a_i. See, here you see we have not assumed anything whether other voters are misreporting or not.

So, what this proof is if it does not matter what other voters are voting, if the voter which top candidate is to the left of the current winner, then it is best for the voter to make to correctly report its type its vote or top candidate. By similar argument we can show that if the if a_j the most preferred candidate of the voter happens to lie on the right of a_i, then also it is best for the voter V to vote for V vote for a_j. Similarly or similar arguments show that if a_j belongs to a_{i+1} to a_m, then also it is in the best interest of voter V to vote for V ok. So, this concludes the proof this shows that this median rule is dominant strategy incentive compatible. It also has another property here is another theorem at least 50 percent of the voters preferred the median winner than any other candidate.

So, if the winner is a_i and you pick any a_j at least 50 percent of the voters prefer a_i over a_jj which is easy to prove I leave it as a homework. Such a candidate is called a Condorcet candidate or Condorcet winner. Such a candidate is called a Condorcet winner. So, what we have shown here is that in a single picked domain in a or in a single picked preference profile they are always exist a Condorcet winner and the median voting rule always picks a Condorcet winner or we call it weekly Condorcet. why weekly? For normal Condorcet winner without weekly adjective the candidate needs to get or there should be more than 50 percent voters who prefer that candidate over anybody else.

If that happens then such a candidate is called a Condorcet winner otherwise it is called a weekly Condorcet winner if it is at least 50 percent instead of more than 50 percent. So, let us stop here. Thank you.