Artificial Intelligence for Economics

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Lecture - 33

Lecture 33 : Sponsored Search Auction

Welcome. So, in the last class we have seen Myerson's lemma which characterizes the implementable allocation rules in dominant strategy in single parameter domain and we have seen that that set is the monotone allocation rules. So, today we will study an important application of Myerson's rule. That application is called sponsored search auction. So, what is the setting? In a typical search engines either in Amazon or Google or whatever there are we search for the keyword. So, here is the keyword keyword or phrase we give for search and then it returns the the best matches in this sequence and there are some ads are also shown.

So, and there are some ads ad slots there is a one ad maybe here is here is another ad maybe here is another ad. ad and so on. So, there are various ad slots is together with the search results we have also shown ads sponsored by various various companies or organizations. And typically this organizations are bid for keywords and the bids differ from differ from the slot ad slot.

There are various ad slots typically the each buyer will maybe value the highest the high the top most ad slot because human being will scroll from top to bottom and maybe from left to right. So, this ad slots have different prices and so on. So, how such a and such a thing is implemented and the answer is in online such places we run auction. So, what is the setting? So, let us first understand the setting and then we will apply buyer's sense lemma to get the payments and so on. So, sponsored search auction setting.

So, we assume that there are there are key slots to show sponsored links. ok on a search page. The bidders are advertisers. and that in the typical setting the advertiser have to pay only if the platform that the user clicks on their ad or their sponsored link. So, it is the job of the platform or the search engine designer to show the ads cleverly to the users that the user clicks and they generate revenue.

The bidders are the advertisers. who have a standing bid on the keywords keywords that was searched on that are searched. So, as we said that there are k slots and each slot if the slot is on that on the top of the page that is more likely that user clicks that slot than the slot at the bottom of the page. So, that we quantify as click through rates. So, we quantify the difference between between different slots using click through rates called CTRs in short ok.

click through rates and suppose those click through rates are $\alpha_1, \ldots, \alpha_k$ and we have sorted the slots as per the click through rates. So, we assume that $\alpha_1 \geq \alpha_2 \geq \ldots \geq \alpha_k$. And also for each ad there is a quality of the ad whether user clicks on the ad we assume that that depends on two parameters or on the first parameter is where the ad is shown in which slot and the second is the quality of the ad. So, each we quantify the quality of the sponsored link using a quality score β_i for every advertiser i β_i we assume it to be in between 0 and ok and we assume that these two things are independent.

We assume that β_i we assume that click through rate CTRs are independent of the quality scores. So, this implies that if an ad is shown if an add i is shown in slot j, then the probability that an user clicks on it is $\beta_i \times \alpha_j$ ok. So, this is the setting and again now the question is suppose there are k slots and multiple ads, then which ads we will choose? You see that it is not a win-loss scenario for each advertisers point of view and so, does not fit the basic framework of single parameter domain. But nevertheless as I said that is a simple framework the in its most general form you can think the single parameter domain as the domain where as the domain where the type of the player can be encoded by one real number which is which is satisfied in this case and hence this is also a single parameter domain. So, each player.

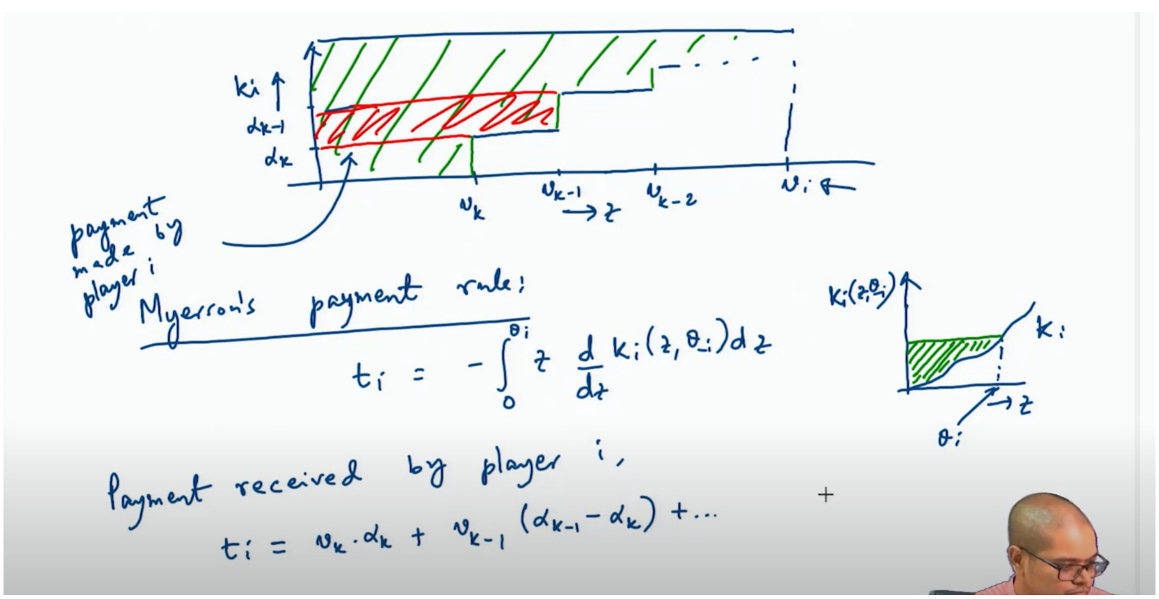
So, what is typeset? Each player has a valuation each player i has a valuation v_i if an user clicks on its sponsored link ok. So, which is a real number valuation v_i which belongs to R. we note that sponsored search auction is a single parameter domain. Now, again we use the any monotone allocation rules and we need to compute the payment. allocation rule ok.

So, for every allocation if I show the advert add of advertiser i to slot j then its valuation the valuation because of monotone allocation rule ok. So, the valuation the advertiser i if its ad is shown at slot j. is it is v_i which is its valuation if it is clicked, but the probability that it is clicked is $\beta_i \times \alpha_j$ ok good. Now, we need to compute the payments. So, suppose there are n advertisers with valuations $v_1, \ldots, v_n$ and there are k slots which is less than n because if $k = n$ it is best to show all the adds then there are k adds ok.

So, which k will be will be shown and so, we by renaming by renaming the advertisers we can assume without loss of generality that v_1 is the highest valuation $v_1 \geq v_2 \geq \ldots \geq v_n$ ok. Now, let us look at player i. Now, let us let us consider player i ok. So, player i what is its payment? So, the allocation function function of player i let it be $k_i(z, v_{-i})$ the valuation of other players remain same. These are the types the valuations are the types and again recall again observe that because it is DSIC because of Myerson's lemma this is this mechanism is dominant strategy incentive compatible.

We can assume again without loss of generality that every player bids his true valuations, reports his true valuations and hence the true valuation is known to the mechanism designer or auctioneer. So, what is $k_i(z)$? So, you see function $k_i(z)$. player i be $k_i(z)$ and it varies from as z varies it is a function of z it is think of the valuation of other players has remained same and as z varies k_i varies ok. So, let us plot k_i. So, here is here is z and here is k_i.

Now, if $k_i \leq v_k$ see and let us pick an allocatively efficient allocation rule any allocatively efficient allocation rule. and recall every allocatively efficient allocation rule is also monotone allocation rule, but it makes sense to maximize the sum of valuations of all the bidders instead of using any arbitrary monotone allocation rules. So, let us use any allocatively efficient allocation rule will allocate the k slots to the top k advertisers respectively. top k advertiser respectively. That means, whichever advertiser has the highest valuation they will get the first slot remember the slots are also have different click through rates are also different importance.



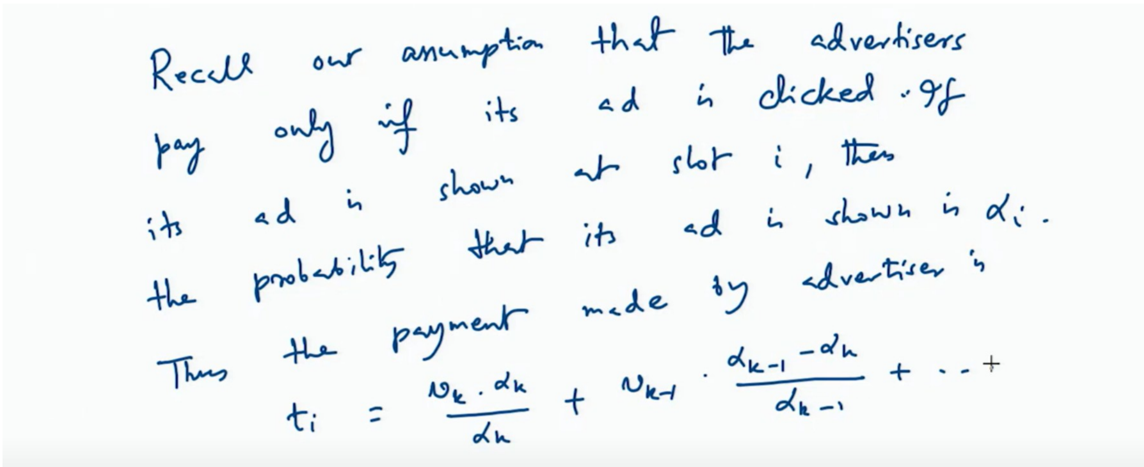
So, now, you plot. So, when z crosses v_k till v_k here is v_k it does not get any slot. So, that valuation is 0. So, v_k. So, we will extend v_k if it crosses v_k and till v_{k-1} its valuation it gets the last slot which it values at say α_{k-1}.

k. So, this is α_k ok. Once it crosses α_{k-1} it and between α_{k-1} and v_{k-2} it gets to the second last slot which is which is α_{k-1} and so on. Now if you recall the Myerson's payment formula Myerson's payment formula that $t_i = -\int z \frac{dk_i(z, \theta_i)}{dz}$. Now, what is this? This is nothing, but if you plot k_i. this is your k_i here is z here is in this direction you have $k_i(z, \theta_{-i})$ then and here is somewhere is if this is theta i then if you draw this line this is this area the payment is nothing, but this area.

So, now, you come back here and you see what is this area this is this area and so on till here how much you go you go till till v_i wherever it is you have to compute this area this is the payment to player i payment made by player. So, this is the payment made by player i. Now, if you recall if you now apply this formula here in sponsored such auction then payment received by player i now apply Madison's lemma which is t_i this is what this is summation. So, you first write you see v_k. So, this is the you divide into this triangle this rectangular regions and then it is the first rectangle the bottom most rectangle is $v_k \times \alpha_k$. Next one is $(v_{k-1} - v_k) \times \alpha_{k-1}$ no I am doing wrong this is v_{k-1} this rectangle.

one side is v_{k-1} these times $\alpha_{k-1} - \alpha_k$ and so on ok. So, if you write it succinctly this is summation and it goes till v_i. So, I think this is better because it depends on v_i. Now you see that recall our assumption was that better recall our assumption that they pay only if its add is clicked ok. And if it is shown at slot i if its add is shown at slot i, then the probability that its add is shown is α_i right.

So, and advertiser will not pay anything if its add is not clicked. So, this is the payment if this is the payment sorry this is the payment if advertiser is willing to pay if its ad is shown. right if the if the advertiser is willing to pay if its ad is shown then this is the payment, but this is not the case. So, hence in this case thus the payment of advertiser payment payment made by advertiser is which is t_i is summation not summation with the same formula, but you have to normalize it. So, here in this region at k-th slot it is shown which is which is clicked with probability α_k.



So, this is you divide by α_k first term, the second term in this in this space it is shown with probability $k-1$ you divide by α_{k-1} so on. $t_i = \frac{v_k \alpha_k}{\alpha_k} + v_{k-1} \frac{\alpha_{k-1} - \alpha_k}{\alpha_{k-1}} + \ldots$ So, this is the final payment of VCG mechanism sorry using Myerson's lemma this is a Myerson's payment in sponsored search auction ok. So, let us stop here. Thank you.