Artificial Intelligence for Economics

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Lecture - 31

Lecture 31 : VCG Mechanism

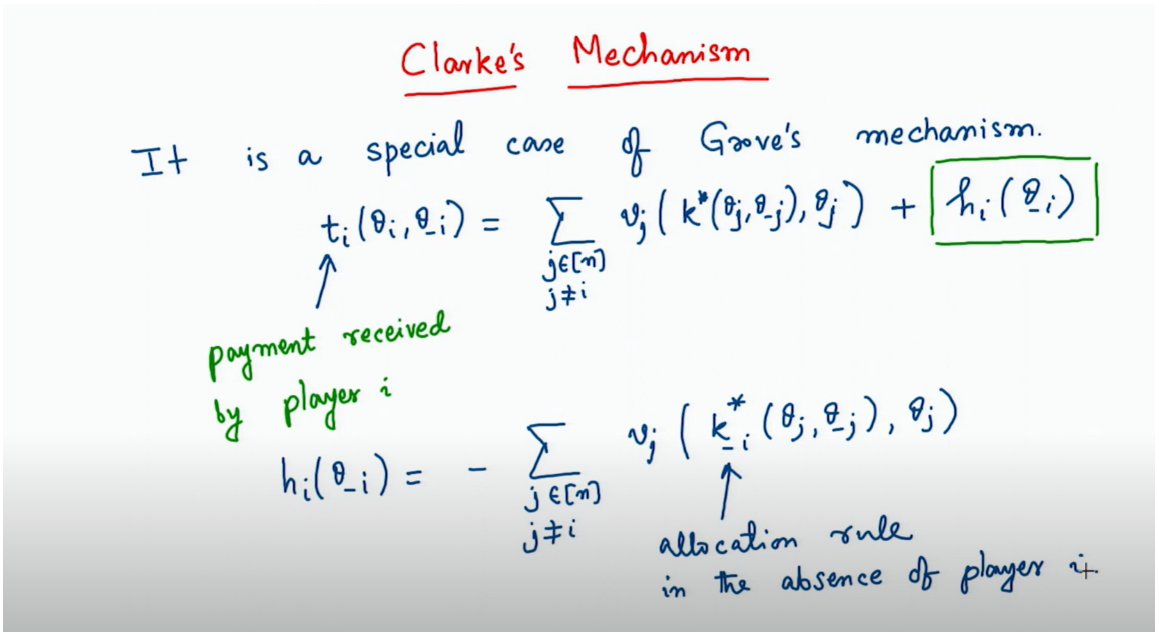
Welcome. So, in the last lecture we have seen groves mechanism and we have seen that every allocatively efficient rules allocation rules can be coupled with a payment scheme which is called groves payment scheme. So, that the resulting social choice function becomes dominant strategy incentive compatible. So, today we will study special class of Grove's mechanism which is called Clarke's mechanism and we will see. So, this is called Clarke's mechanism. So, Clarke's mechanism it is a special case of Grove's mechanism.

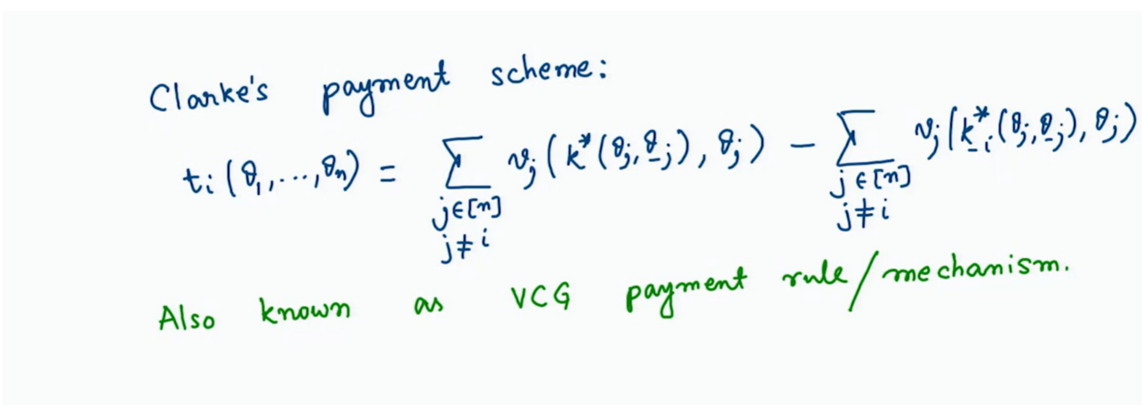
So, if you recall in Grove's mechanism the payment to player i payment made by payment made by player i which is $t_i(\theta_i, \theta_{-i})$ payment received by player i this is summation of valuations v_j of whatever the allocation rule you have chosen (θ_j, θ_{-j}). By the way (θ_i, θ_{-i}) is same as (θ_j, θ_{-j}) is all the tuple $(\theta_1, \ldots, \theta_n)$, we are just highlighting the type of player j. So, this is the sum of valuations plus any function on the type profile of other players $h_i(\theta_{-i})$.

So, this is the payment for player i. So, this is this is payment received by player i. Now, you see there are lot of option in choosing this function $h_i(\theta_{-i})$, it could be an arbitrary function. Clarke's payment mechanism gives a special function $h_i(\theta_{-i})$. So, in in Clarke's payment rule $h_i(\theta_{-i})$ is a particular function which is called Clarke's payment function.

This is $-\sum_{j\in[n], j\neq i} v_j(k^*_{-i}(\theta_j, \theta_{-j}), \theta_j)$ ok. What is k^*_{-i}? This is the allocation rule in the absence of player i. So, it may not always make sense to have an allocation rule in the absence of player i, but if it makes sense for example, in auction scenario it makes sense such an allocation rule, then $h_i(\theta_{-i})$ is defined like this and this is the Clarke's mechanism. So, Clarke's mechanism is applicable or can be used only if the payment mechanism there are payment sorry there are allocation rules you know which makes sense in the absence of every player then only Clarke's mechanism makes sense.

So, what is Clarke's payment scheme then? is $t_i(\theta)$ payment received by player i $(\theta_1,\ldots,\theta_n)$. This is $\sum_{j\in[n],j\neq i} v_j(k^*(\theta_j,\theta_{-j}),\theta_j)$. This is the first part of Grove's payment scheme and then plus $h_i(\theta_{-i})$ which is $-\sum_{j\in[n],j\neq i} v_j(k^*_{-i}(\theta_j,\theta_{-j}),\theta_j)$ ok. So, this is called the Clarke's payment school payment scheme these also known as VCG payment scheme VCG payment scheme, rule or simply VCG rule or VCG mechanism ok.





So, let us see some examples of VCG mechanisms, go to next page with examples things will be clear. VCG auction is also called VCG auction ok good. So, till now the concrete auctions that we have studied are first price auction and second price auction and there there is only one single indivisible good that is being that is being traded. So, here now we can this mechanisms this groves mechanism or VCG mechanism we can apply to much more general kind of auctions.

So, our first example. So, suppose we are selling multiple identical objects. So, suppose a seller has 3 identical items. ok and say let us 3 identical items there are 5 buyers, there are 5 buyers each of whom wants to buy one item only. The valuations which are also called types the valuations of the item to the buyers are suppose these are the valuations 20, 15, 12, ok and let K star be any allocatively efficient allocation rule ok and VCG mechanism is used VCG ok.

So, we are using VCG mechanism, then who will get the items and so on and how much they will pay this is what we need to decide. So, first observe that this K star is a allocatively efficient allocation rule any its goal is to maximize the sum of valuations. The that will be achieved by allocating one object to first buyer, one object to second buyer and one object to third buyer because each buyer wants only one item. And so, and there are three items. So, the sum of valuations will be maximized by allocating the objects to the players to the buyers who values it highest.

So, and the magic is because VCG mechanism is dominant strategy incentive compatible, we can assume that players reveal their true type. since VCG mechanism is DSIC. why VCG mechanism is DSIC because VCG mechanism is a special case of groves mechanism which we have already shown to be DSIC. So, VCG mechanism is dominant strategy incentive compatible that means, for every player here the players are buyers for every buyer it is in the best of their interest to reveal their true type. So, since VCG mechanism is DSIC revealing true type is best for every player is use a grim theoretic term is a very weakly dominant strategy.

So, we can assume without loss of generality that every buyer reports their true type to the mechanism designer. So, now, because mechanism designer knows their true types they can allocate the they can pick allocation rule which maximizes the sum of valuations. So, and that is what any allocatively efficient allocation rule will do. So, any allocatively efficient allocation rule gives 1 item each to the first 3 buyers. So, this is the allocation.

Now, we need to compute the payments. So, payment received by buyer 1, let us call it t_1 . This is sum of valuations of the allocation. What is the allocation? Allocation is giving 1 item each to first 3 buyers. How much sum of the valuations of the allocation to all players except player 1? Second player is getting the item.

So, how much the second player values this allocation is same as his or her valuation 15. The third player values that allocation to 12 same as his or her valuation. Fourth player is not getting the item. So, his valuation of that allocation is 0. and the fifth player is also not getting the item.

So, the valuation of this allocation to the fifth player is also 0. So, this is the sum of the valuations of all the players except player 1 of the current allocation. From this we have to subtract Now, remove player 1 from the system and again find out what is the what will be the allocation made by an allocatively efficient allocation rule. Now, if player 1 is not there then the top 3 valuations are 15, 12 and 10. So, the items will be given to second player, third player and fourth second player, third player and fourth player.

And so, their valuations will be the valuation of second player remains same 15, valuation of third player is 12 remains same. They are getting the object in both the allocations, but the fourth players valuation is now 10 because in this allocation the fourth player is getting the object. and fifth player is not getting the object. So, his valuation remains 0. So, this becomes minus 10.

So, so payment received by buyer 1 is minus 10. This minus sign indicates that buyer 1 makes the payment does not receive the payment which matches with our intuition because buyer 1 will make is getting the object. So, he pays for the object 10 is not his bid 15 or not second highest bid he pays 10 ok. So, similarly payment may received by buyer 2. t_2 the first term remains same sum of valuations of all players except player 2.

So, it will be then 20 plus 12 it does not remain same plus 0 plus 0 and in the absence of second player the allocation goes to one object to first player, one object to third player, one object to fourth player. is also minus 10. Similarly, you can compute similarly one can compute one can show that the third player also makes a payment of 10 t_3 is minus 10 t_4 is 0. 4th player does not make any payment and t_5 is also 0, 4th and 5th player does not make a payment ok. So, this is the VCG auction this one example.

Let us move on to second example which is very important called combinatorial auction combinatorial So, suppose there are multiple items being sold and players can bid their valuation for each bundle ok. So, suppose multiple non identical items are sold by a seller and there are multiple buyers and there are say 3 buyers. So, suppose the valuations of the buyers. So, suppose there are two items A, B. So, the possibilities are either he gets A or B or AB both ok.

Here you have buyer 1. So, suppose buyer 1 is not interested in only one item A. So, that means, buyer 1's valuation if he only gets A is 0, he is not interested in B also, but he is interested in the bundle and that bundle he values it at 12. buyer 2 suppose buyer 2 is only interested in item A. So, that he values at 5 he is not interested in item B and he is not interested and if he gets A B that also he values it 5 because he is interested in A only and buyer 3. but he is interested in say suppose B values it at 4 and if he gets the bundle A B he also gets B which he values it at 4.

Now, what is the efficient allocation rule will allocate? So, there are many ways to allocate and if you iterate over all such things you pick the allocation rule which maximizes the sum of valuations of the buyers. So, any allocatively efficient allocation rule will allocate the set the bundle A, B to buyer 1 ok. So, this is the allocation now what is the payment? Payment received by player 1. t_1 is the sum of valuations of the allocation to all the players except player 1, but in other players buyer 2 and 3 does not get anything they value the allocation at 0. So, it is 0 plus 0 minus now assume player 1 is buyer 1 is not there then the allocation the any allocatively efficient allocation rule will give item A to buyer 2 and item B to buyer 3.

So, that allocation buyer 1 will value it at 5 and buyer 2 will value it at 4. this is minus 9. So, again negative sign indicates that buyer 1 actually makes a payment. So, similarly you can show that T 2 is also 0 and T 3 is also 0 both are 0 ok. Now, our next example is strategic network formation, strategic network formation.

So, suppose there is a network, here is a node S, here is A, here is B, here is T. These are the links S to A, S to B, B to A, A to T. and b to t ok. And this is held by player 1, each edge is held by player 1 and if you need to buy that edge from that player and the valuation of player 1 is 10, player 1 values this edge s 2 a at 10. Similarly, suppose player 2 is holding the edge a to t the valuation is 7, player 3 holding the edge from s to b valuation is 5, player 4 holds b to t valuation is 10 and player 5 holds b to a valuation is 2.

and what do we want? We want to buy a S to T path. So, allocations what is the set of allocations? Set of allocations is so, let us call this edges by the index of the players. So, one possibility is you buy S to A and A to T. So, that way you buy the first edge and second edge and not others. This is one allocation or another possibility is S to B, B to T.

So, 0 0 1 1 0 or S 2 B 2 A 2 T 0. S 2 B 2 A 2 T. So, second player you have to buy from second player, from third player you have to buy and from fourth player you do not need to buy, fifth player you have to buy. So, these are the possible allocations. Again you find take it as a nice exercise what will an allocatively efficient allocation rule will pick and what will be the payment to each of the players. This is a very nice example of again a VCG auction.

So, let us stop here. So, in the next class we will again continue to a special class of auctions which is called single parameter domain that is a special domain of of auction theory. Thank you.