Lecture   03   :   Uncertainty   in   Financial   Markets   :   Idea   of   Hedging

Welcome to the third lecture of artificial intelligence for economics. In this segment that is in the next couple of lectures we will deal with uncertainty in financial markets or modeling uncertainty. Now in artificial intelligence a key idea is to train an agent to behave optimally in environment where the outcomes are uncertain. And in economics, uncertainty is an integral part. So in the next couple of lectures, we'll deal with uncertainty in financial markets, especially the idea of hedging and risk. Great.

So let's get started. Let's start with a very simple example of cricket betting. The T20 World Cup is coming up. So all of you must be excited about cricket.

So let's consider a following scenario. Let's say India versus Australia cricket match is happening. And there are two counters. You have entered a room and there are two counters. In counter one, Virat bets on India and offers the following bet.

25 is to 1. What does this mean? What does this mean? It simply means that if I bet rupee 1 with Virat and Virat is betting on India, so it means that Virat is hoping that India is going to win. So if I bet rupee 1 with Virat and Australia wins, so I am betting rupee 1 with Virat that Australia will win and Australia indeed wins, then Virat will pay me 25 rupees. and if India wins, I will pay Virat one rupee. So this is the bet or contract we can get   into.

That's what he's offering. In the other counter there is Steve. Steve on the other hand bets on Australia. So Steve bets that Australia is going to win and he offers a bet of sixes to five. So what does that mean? It means that if I bet rupee one with Steve, that India will win and India indeed wins then Steve will pay me 6 by 5 rupees and if Australia wins I will pay Steve 1 rupee okay so this is the scenario I have 100 rupees in my wallet and I have entered the room so what should I do should I bet with Virat Should I bet against Virat or should I bet against Steve? Which counter should I go to and place my bet? Let's see.

So if I bet all my money on Virat, against Virat, that is I bet all my money on Australia winning, then what is my payoff? If Australia indeed wins, then Virat will pay me 2500 rupees. But if India wins, then I'll have to pay Virat 100 rupees. So my payoff is minus 100 if India wins. On the other hand, if I bet all my money on India, that is I bet against Steve. Then what's going to be my payoff? If India wins, that is if Steve loses the bet, then Steve will pay me 6/5 into 100.

Remember, he was paying 6/5 rupees for every rupee bet. So if 100 rupees of bet is placed on the table, he's going to pay me 6/5 into 100, that's 120. But if Australia wins, then I will have to pay Steve the money which I was betting, which is 100 rupees. great so these are my two options I can either go to Steve or I can go to Virat but there is an inherent limitation with both these options what is that these are win all lose all options in the worst possible outcome I will lose all my money right now that's a little harsh I don't want that I don't want to lose all my money that's too much of a risk I don't want that can I mitigate my risk can I eliminate this risk that's the next that's the question I would want to address that can I eliminate this risk that in the worst possible case I will lose all my money let's see if I can do that let's consider this Let's say I bet an X amount of money against Virat and $100 - X$ amount of money against Steve. Okay? Great.

Now let's see what's gonna happen. Now I've bet X rupees against Virat, right? So, if Australia wins, when I'm betting against Virat, what am I saying? I'm saying that if Australia wins you will pay me 25x right so I put X on the table and Australia wins Virat pays me 25x so I get back 26x and against minus of course 100 which I have put on the table right because I'm losing the bet against Steve So if Australia wins, I'm winning the bet against Virat only and I'm losing the bet against Steve. So what is my net payoff? My net payoff is 26X which I have with me minus the 100 rupees which I put on both the tables put together. Similarly, if India wins, what will be my payoff? If India wins, I lose the bet against Virat, but I win the bet against Steve. But how much will Steve pay me for this bet? So Steve is going to pay me.

Steve will pay me this much. And this was the original amount which I put on the table. So I will get all this for myself. And 100 was the money I bet. okay so this is my payoff so this is my payoff net payoff if India wins this is my net payoff if Australia wins okay great now is it possible that I will make positive profits no matter who wins is that a possibility let's see When is my profit if India wins? This is my profit if India wins, 26x minus 100.

When is this positive? When x is greater than 3.85, this one. When India wins, what is my payoff? This is my, the first is my payoff when Australia wins. When India wins, this is my payoff. Now when will this be positive? This is always positive if x is less than 54.

55. So we can conclude that if my x lies in this interval 3.85 to 54.55, my profit is positive no matter which team wins. So what am I doing here? I am basically betting on both sides. okay instead of betting on India against Steve or betting on Australia against Virat I'm actually betting on both and thereby I'm reducing my risk okay and X is the amount I'm betting with Virat so if that lies in this particular interval we see that I will make a positive profit no matter which team wins.

So now I have zero risk. In the previous slide we saw that we had huge risk. I ran the risk of losing all my money. But now I see that I will not lose even a single penny Okay, no matter which team wins. So I have completely eliminated my risk.

Very good. Let's move on. So if you just graphically plot it, you can see the yellow line is the payoff if Australia wins. It's a profit function as a function of x. The x which I choose. So remember what is x? x is the amount I'm betting against Virat.

100 minus x is the amount I'm betting against Steve. So the yellow line is the profit function as a function of x when Australia wins. The blue line is the profit as a function of x if India wins.

Okay. Fantastic. And we see that if x lies between 3.85 and 54.55, my profit, so let's say x is here, then my profit is positive even if Australia wins and even if India wins, it's positive. So it's positive in either case, no matter which team wins. So my next question would be, okay, I can choose an x between 3.

85 and 54.55, that will lead to no risk. Very good. But the next question which I would want to answer is, I will get a guaranteed profit if I choose an x in this interval, but what is the maximum guaranteed profit which I can have? Right? That's the next question which we would want to answer. We have eliminated our risk, great. Now we want to figure out, given that we have eliminated the risk, what is the maximum we can earn? Maximum risk-free gain which we can have.

That's our next objective, next natural objective. Okay, so let's see. By the way, what is the minimum guaranteed profit? If we go back to the previous slide, look at the diagram. So let's say x is here, let's say this is 4.5, okay? If Australia wins, what is my payoff? My payoff is this much.

And if India wins, what is my payoff? My payoff is this much. Right? So what is the minimum payoff which I'm going to get here? It is this, this payoff if Australia wins. On the other hand, if I'm here, let's say this is 49 or whatever. Now the payoff if India wins is this much. And if Australia wins, the payoff is this much.

Now what is the minimum guaranteed payoff? It's this much. The payoff if India wins. Here the minimum payoff is if Australia wins. Here the payoff is if India wins.

The minimum payoff. The minimum guaranteed payoff. And both are clearly positive. So the minimum guaranteed profit function is thus this. This is my payoff if Australia wins and this is my payoff if India wins. I hope you remember this from the previous slides.

This is my payoff if Australia wins, this is my net payoff if India wins, net profit. So this is if Australia wins, this is if India wins. and the minimum guaranteed payoff is thus minimum of these two because I don't know who is going to win great and this is the minimum guaranteed payoff which I want to maximize so I'm the gambler so what is my optimization problem the gamblers optimization problem well it is to maximize this minimum payoff in the risk-free interval which is X belonging to this interval right that's my optimization problem now correct great now if we solve this optimization problem we will see that this function this function which we have is maximized when the when the two arguments of the min are equal okay and it turns out that it's maximized when x star is 7.80 That is the choice of x which will lead to the minimum guaranteed profit.

Fine. By the way, if I choose x star equal to 7.8, what will be that risk-free profit? Well, it will simply be something like this. If we calculate, it will simply be 26 into 7.80 minus 100. and if we calculate it turns out to 102 okay and that's the minimum guaranteed profit which I can earn how much money did I have initially in my wallet 100 rupees and I see that if I enter the gambling room with 100 rupees and Virat and Steve are offering the bets which they are offering then I can end up with 102 rupees of guaranteed profit ok, which is more than 100 percent fantastic that is wonderful right and it is guaranteed ok.

So, that is what we see x star is 7.8 and that gives me a 102 that is my profit Now let's understand these terms. The idea which we saw here is the idea of mitigating or eliminating risk by betting on both sides which I spoke about a few minutes earlier. A strategy which generates guaranteed returns is called an arbitrage. Now, what is arbitrage? Arbitrage is basically, in a way, finding a loophole. By the way, can you tell me what's the loophole in the previous example which we just worked out? Why can't it exist in reality? Why can't two people offering such a bet in a gambling room, it can't exist?   Can   you   tell   me   why?   Try   to   guess.

The reason is, think about it. Let's say I have no money in my wallet. I just borrow 100 rupees from somebody, I go to the gambling room and I bet the way I did and I will make a profit of 102. So I will not only get my 100 rupees back, the principal, but I will also earn guaranteed 102 rupees and the 100 rupees which I borrowed I can give it back.

which means I can make 102 rupees out of nothing at any point of time if that's the case I will keep on making such 102 rupees every hour or every minute which will make me a trillionaire or a multi-trillionaire in five days but that can't happen that will that will bring the world crashing down okay so such a scenario is called an arbitrage Example of financial arbitrage which we see in the financial markets is BSE, NSE arbitrage. Often a particular stock trade at different values in the Bombay Stock Exchange and the National Stock Exchange.

Which should not happen. If it happens, what's the problem? The problem is simple. I will buy a unit from the Bombay Stock Exchange. Let's say at 10 rupees. And let's say if it is trading at 12 rupees in the National Stock Exchange, I will sell it in the National Stock Exchange. and I will make a profit of 2, I buy from BSE, sell in NSE or vice versa and make profits and I keep making profits till all the money in the world belongs to me, which is impossible.

So, we have talked about hedging and arbitrage. Now, a little extension of the example which we just talked about. We saw that if we enter the gambling room and Virat and Steve are offering the bets which they are offering, 25 is to 1 and 6 by 5 is to 1, I can make a guaranteed profit of 102. But then I am not happy with 102. I am a greedy man, I want more money and what if I decide to take a little bit of risk and earn more profit, can I do that? The answer is of course yes, if I bet all my money against Virat then I can make 2500 if Australia wins but I also have the risk of losing all my money.

okay so this is the risk and return which I am torn between which every investor in the financial market is torn between okay that here I can make a risk free return of 102 but on the other hand I can make 2500 which is a hell lot more but I am also susceptible to losing all my money okay great I don't want that that is why I wanted the risk-free return but now I say that okay fine risk-free 102 is very good but let's say I can't afford to lose all my money but I can take a risk of 10 rupees so I will simply take a little risk okay such that I will lose 10 rupees at max I will allow myself this small window of losing 10 rupees in the worst case. If that's the scenario, what's the maximum profit which I can earn? Given that my maximum loss is 10 rupees. Let's see. Let's try to solve this now.

If I choose X star at 7.8, my guaranteed profit was 102. Now let's say if I move a little bit to the right and I choose my X here, somewhere here maybe. Let's say this is my choice of X.

To the right of 54.55. So I am now leaving the safe bet zone. the risk free zone and I am choosing an X to the right of 54.55. Now what is the maximum profit I can earn? The maximum loss which I can have on the downside is if India loses then I will lose sorry if India wins I will lose 10 rupees minus 10 this is my maximum loss and if Australia wins I

will get 1436. So if I choose this X, which is out of the safe zone, this is my safe zone, remember?   3.

85254.55, this is my safe zone. Now if I go a little bit to the right of the safe zone, the risk-free zone, if I choose an X here, then the maximum loss which I can incur is 10, and the maximum profit which I can earn is 1436, okay? So huge profit and a little risk. Now whether I would do it or not depends on my psychological makeup. So it's a risk. If I choose to eliminate my risk completely, my profit is 102. If I choose to take a little bit of risk of 10 rupees, that's the max I can lose, then my profit shoots up to 1436.

So this is the risk return trade-off, the risk and profit trade-off which you see in financial markets and in life. Great. A little term which I want to introduce which you will encounter in all finance and in probability textbooks. It is fair bet and implied probability. Now what is a fair bet? A bet as you saw is a random variable or the winnings from a bet is   a   random   variable.

A bet or winnings from a bet is called fair if expectation of that random variable is 0. Now in our example, let us assume that Virat and Steve are both offering fair bets and they also think they are offering fair bets. Then, what is Virat's expected payoff? Well, Virat is also having some probability in mind which he thinks India will win. When he is betting, he must be having something in mind, some probability in mind. So, what is Virat's expected payoff? If India wins, he will get a payoff of plus 1.

If India loses, he will get a payoff of minus 25. That is the bet he is offering. So, minus 1 into probability of India winning according to Virat ± 25 into probability of Australia winning according to Virat. This PV end, this is the probability of India winning according to Virat, this is Virat's implied probability that India will win. If we solve this, we get that Virat's implied probability is 25/26 that India will win. So, assuming that Virat believes in offering a fair bet, then that is the implied probability of India winning according   to   Virat.

Similarly, we can compute the implied probabilities which of India winning and Australia winning according to Steve. Great. Now, we have got an idea of risk and profit. So, let us move on to financial markets. Let's move on to financial markets and see what instruments   we   have   got   in   order   to   mitigate   risk.

So first in today's lecture we'll talk about two key financial instruments, the call and the put and we'll talk about other kinds of instruments in the subsequent lectures. First, let's take an example. Let's start with an example. Consider this TCS stock price.

Let's say the TCS stock price today is 3416. Now, I want to buy a TCS stock right now,

but I have a feeling, I have a hunch that the market will crash and the TCS stock price will go down to 3316. Instead of 3416, it'll go down to 3316. That is by 100 rupees. okay so I'm tempted to wait and buy later by tomorrow on the 10th of September so it seems that waiting till 10th of September might be good but what if the crash doesn't happen what if the price goes up to 3500 then I will regret right then I will think that ah I wish I had bought it for 3416 right so I'm confused should I wait till 10th of September or I should buy it now now if there is a option which allows me to buy a TCS stock on 10th of September at today's price which is 3416 then my fear will be gone right what is my fear that the price will rise to 3500 but if somebody gives me the option gives me the assurance that don't worry you can buy the TCS stock at 3416 on 10th of September then I will be relieved of this pressure and I will take the risk of waiting if the price comes down to 3316 fantastic if it goes to 3500 even then it doesn't matter I have a insurance that I can buy it at 3416 okay this insurance or right is called a call option okay this right is called a call option where the expiration date is 10th of September and the strike price is 3416 Now why should anybody, let's say why should TCS give me this right for free or anybody should give me this right for free? No, they won't.

 Nobody would. So I'll have to buy this right. So a call option is a right to buy a particular asset at a particular price at some point in the future. Somebody who buys a call option, that is somebody who buys this right, is said to go long on a call option these are just terms used and somebody who sells this right to some other person so if I give you the right to buy TCS stock at 3416 from me on 10th of September then I am selling the call option then it is said that I go short on a call option okay great So more formally, this is what it is. A call option is a contract which gives the owner the right to buy an asset at an agreed upon price. It is denoted by CEST. E is the strike price, that is the price at which the call option allows you to buy the asset at the expiration date.

 The expiration date is T. that is after T months or T days and S is your underlying asset which is being bought or sold, in this case it was the TCS stock. Great, so if let's say if I go long on a call option that is if I buy a call option, what is my payoff? well let's say I buy a call option which allows me to buy one unit of an asset or one unit of a stock at E rupees at some point in the future at the expiration date now if the price becomes E plus 10 If the price at the expiration date, this is the stock at the expiration date, if it becomes E plus 10 then what will happen? Well, I have the right to buy the stock at what price? So I will buy the stock at price E and sell it in the market at rupees E plus 10. So what will be my payoff? 10 rupees. I will make this much of profit. So as the stock price at the expiration date goes up, as the price of the underlying asset at the expiration date goes up, my payoff will go up.

 Who am I? I'm the buyer or holder of the call option. ok what if the price of the asset or the stock price goes below e well then I will not exercise the call option at all because it

will be silly of me to do so ok so let's say if at the expiration date the price is e minus 10 that is the stock is available in the market that particular asset is available in the market at price e minus 10 Now the call option is giving me a right to buy that asset at E rupees. But why will I use that right? I can anyway buy it in the market at E minus 10 rupees at a cheaper price. So this right which I have by the virtue of owning this call option is useless.

So my payoff is 0. So it is 0 here. So this is how the payoff looks like. Of course here I am assuming that the call option is call option does not have any price. If we take the price into account then the entire payoff graph will shift down by an amount P which is this PCE that is the price of the call option. So I will stop here. We'll talk about put options in the next lecture and we'll look at different kinds of financial instruments which we can design with put and call.

But that will happen in the next lecture. So I'll see you in the next lecture. Thank you.