Artificial Intelligence for Economics

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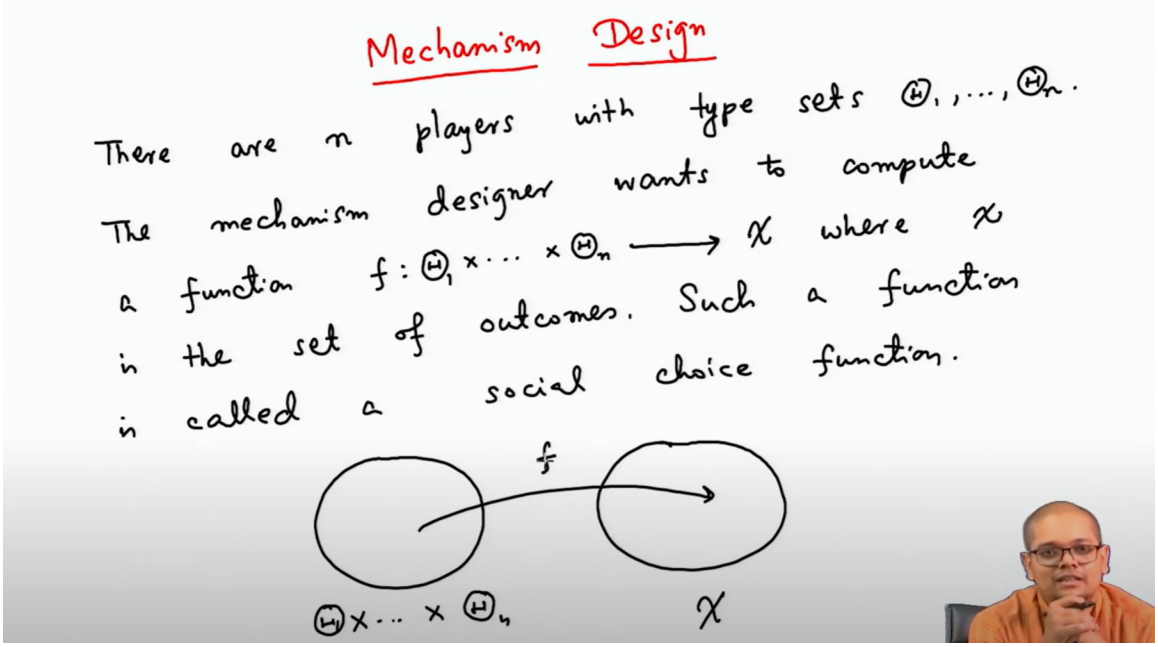
Lecture - 29

Lecture 29 : Overview of Mechanism Design

Welcome. So, in the last lecture we have studied the first price option and we have seen that if there are 2 buyers then bidding half their valuations is a Bayesian Nash equilibrium under some other realistic assumptions and then we have extended that result for n buyers and 1 seller. It is a nice exercise to do the analysis for the reverse auction which is there are n buyers and one sorry there are n sellers and one buyer then what will be the what will be the Bayesian Nash equilibrium ok. So, now, from today onwards we will see more general auctions and more general settings and those things are studied under what is called mechanism design. So, let me briefly introduce what is mechanism design. So, the setting is as follows that there are there are n players with typesets theta 1 to theta n ok.

And the mechanism designer who is like an organization who is who will define the rules of the game that is that guy is called mechanism designer. The mechanism designer wants to compute a function $f : \Theta_1 \times \ldots \times \Theta_n \rightarrow X$ which is the set of outcomes where X is the set of set of outcomes ok. Such a function is called social choice function. Now, for example, so this is the domain is $\Theta_1 \times \ldots \times \Theta_n$ and this is the range which is a set of outcomes these this function if. Now, what is the problem of mechanism designer? The problem of mechanism designer is that these inputs has n parts and each part is held by strategic players who may not reveal the true value of the i-th component of the function of the of the i-th component of the input if if he or he or she has incentive to misreport.



So, the challenge or the goal the goal of the mechanism designer not goal let me write job. The job of the mechanism designer is to informally speaking is to design a mechanism which you can think as a game which incentivizes players to report their true valuations, true valuation also called types they are private information. For example, you can think of an auction scenario, so say selling one item to n potential buyers. Now, each buyer has a valuation for the item and suppose the job of the mechanism designer is to allocate the item to the buyer who values it most, who has the highest valuation for that object. Now, if the mechanism designer or if you organize a first price auction, then we have seen that bidding true valuations is not the best strategy.

So, it may be possible that and it has a Bayesian Nash equilibrium under some assumptions which is definitely stronger than the second price auction which does not need any assumption to have a weakly dominant strategy equilibrium. So, in the first place auction if the players are not does not meet all such assumptions every player does not meet all such assumption then it may be possible that the object is allocated to the player who does not value it highest. In that scenario we can say that mechanism designer has failed to implement the function that the social choice function. If the social choice function is is allocate the object to the player to a player who values it highest. On the other hand, if the mechanism designer implements second price auction, then then it does not need any assumption it just needs standard game theoretic assumptions to argue that it is in the best interest of the players of every player to report their crew valuations.

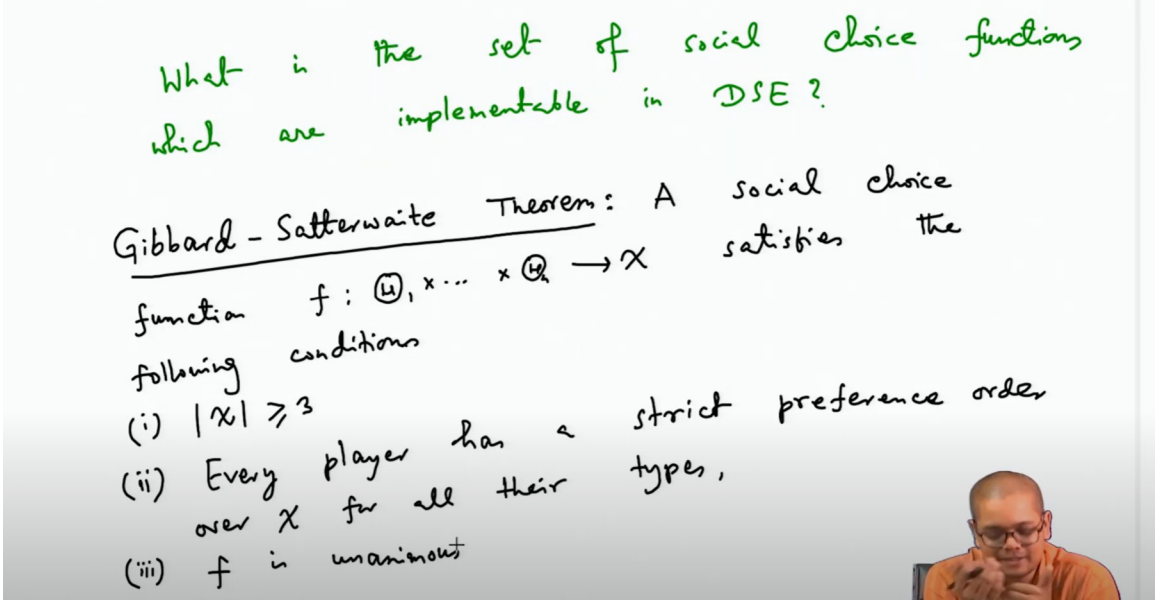
So, this is this is so, second price auction implements the max function what is in in mathematics what is this this function is called that you know allocating the object to the

highest highest to the player who has the highest valuation that is equivalent to a max function computing max of n numbers. The only challenge is these numbers are not known these numbers are held by strategic players and you need to design a game which is called mechanism to incentivize the players to report their these numbers to report these types. So, this is the this is the area of mechanism design ok. So, let me briefly sketch upon. a mechanism is called a direct mechanism there is something called direct mechanism.

A mechanism Again I am giving you a broad overview of mechanism design and then we will go to again auctions more general kind of auctions. A mechanism is called a direct mechanism if the mechanism designer simply reveals the social choice function $f:\Theta_1\times\ldots\times\Theta_n\rightarrow X$ simply reveals the social choice functions to all the players and ask them and asks them to report their type ok. Now there is something called an indirect mechanism where the mechanism designer tries to play smart and design some game which indirectly will help the mechanism designer to compute the social choice function f which he or she might want to implement . But there is a celebrated result in mechanism design which is called revolution principle which says that this is the revolution principle which says that a social choice function $f:\Theta_1\times\ldots\times\Theta_n\rightarrow X$ is implementable implementable by a direct mechanism if and only if it is implementable by some indirect mechanism. So, this is a very powerful principle and it is a theorem it can be proof, proof is easy.

It says that if there is any indirect mechanism which allows the social choice function which allows the mechanism designer to implement the social choice function f, then the social choice function can just reveal the social choice then the mechanism designer can simply reveal the social choice function f. And, and ask the players to report their true type and players will players will report their true type and the and thus thus the mechanism designer will implement the social choice function. What does the word implement means? Mechanism implements a social choice function a mechanism implements a social choice function f, if reporting true types a mechanism implements a social choice function f, if reporting true types is best for every player ok. That is reporting true types forms an equilibrium in the underlying Bayesian game.

ok . So, if reporting true types form a very weakly dominant strategy equilibrium . If reporting true types form a very weakly dominant strategy equilibrium, then F is implementable in dominant if if then f is implementable in dominant strategy equilibrium it is called DSE and f is called dominant strategy incentive compatible. incentive compatible DSIC in short ok. On the other hand if reporting true valuations or true types. forms a Bayesian Nash equilibrium, then f is called Bayesian Nash incentive compatible.



BIC in short ok. So, the an important question is given or what is the set of social choice functions which are implementable in dominant strategy equilibrium in DSE. You know dominant strategy equilibrium is the is the most desired one because if the number of players is large, then and if reporting true types is a dominant strategy equilibrium, if it is a very weakly dominant strategy equilibrium, then for from every players perspective every player knows that reporting true type is best for him or her irrespective of what other players are doing. which is not the case for Bayesian Nash equilibrium. Reporting true type is a is the best option for the player if or only if or if the other players are all reporting their true types.

So, it is really interesting the important question is which social choice functions one can implement in dominant strategy equilibrium, but here the bad news is there is a celebrated result called Gibbard-Satterwaite theorem, which basically says that a social choice function $f:\Theta_1\times\ldots\times\Theta_n\rightarrow X$ satisfies Without going to technicality let me write there are 3 conditions let me write it first satisfies the following conditions. The first condition is the number of outcomes is at least 3 very easy to understand. Second is there is something called you know every player has a strict preference order over X every player has a strict preference order over X for all their types right. That means, if look at from player 1's perspective suppose player 1's type is true type is theta 1. for player 1 the among the outcomes of X, 2 outcome is same for player 1.

So, that is the thing and the third is that f is unanimous which says that if there is a particular outcome which is best for all the players, then the social choice function must pick that outcome. So, under these three assumptions say that then f is dominant strategy incentive compatible if and only if. if and only if, if is a dictatorship. Now, what is a

dictatorship? Which social choice functions are called dictatorships? A social choice function f is called a dictatorship if there exists a player d that player is called called a dictator there is a player d called dictator such that the social choice function if always outputs the outcome from X which d which the player d likes the most.

So, that means, it disregards the types of other players it just focuses on just considers the type of the dictator which is a player d and among and for that type there is one outcome in x which the dictator d likes the most and the social choice function f outputs that outcome and so, that there is dictator. So, obviously, dictatorship social choice functions are DSIC because very easy proof. So, for a dictator there is no incentive for lying because the social choice function is picking the outcome which is best for the dictator f or other players also there is no incentive for misreport their true type because the social choice function is simply discarding, disregarding their outcomes, their types, disregarding those players. So, social every dictatorship function is dominant strategy incentive compatible reporting their true types is a very weakly dominant strategy equilibrium. But, sadly these are the only dictatorship functions if my outcome function outcome set has at least 3 elements and the other 2 assumptions which are also very mild assumptions very very natural assumptions or expected from social choice function f.

So, under these assumptions you know it says that the dictatorships are the only social only social choice function which is dominant strategy incentive compatible. So, in the next class we will see how we get rid of this assumption by assuming extra structure which is called quasi linear setting and we get very useful and nice dominant strategy incentive compatible social choice functions ok. So, let us stop here today. Thank you.