Artificial Intelligence for Economics

Prof. Palash Dey

Computer Science and Engineering

Indian Institute of Technology Kharagpur

Week – 06

Lecture - 28

Lecture 28 : First Price Auction

Welcome. So, in the last couple of lectures we have been studying auctions and we have seen that the in the first price auction it is it may not be in the best interest of the players to bid their true valuations of the object, but in the second price auction we have seen we have proved in the last lecture that it is in the best interest of every player to bid their true valuation irrespective of what other players are bidding. So, this is a very strong notion of equilibrium and and this makes that second price auction very much applicable in use, applicable in practice both from theory and practice. But you know often we still use fast price auction, fast price auction is still very common. So, we may need to face a situation where we need to bid for the fast price auction. So, it is a interesting question that as a bidder how much you should bid in a fast price auction.

So, today we will see we will study fast price auction in today's lecture. So, recall the setting we have n buyers or sellers. So, again for concreteness let us fix something let us say buyers. who wants to buy an object from one seller who has one object who has one copy of that object.

So, typical example is say government is auctioning public resources like say mines or spectrum and so on and there are various buyers who are bidding for that object. And what is fast price auction? In the fast price auction. the player, the buyer, a buyer who bids the maximum wins the auction that is obtains the object from the seller. by paying his or her bid. So, if they are there are more than one buyers who bids the maximum value then any buyer can be chosen we can use some predetermined tie breaking rule among the buyers for example, lexicographic or maybe some we can randomly pick a maximum buyer all are fast price auction.

And the important thing is seller buyer pays his or her bid this is in contrast to the second price auction where the buyer pays the second highest bid ok. As we have seen that second price auction or every auction induces a game on the players and it turns out that for the first price auction there is no Nash equilibrium if we just tried it in a strategic form game complete information strategic form game. So, we need to widen our scope and what we do we study. the Bayesian game induced by the first price auction . So, what is the main theorem? Theorem in the Bayesian game.

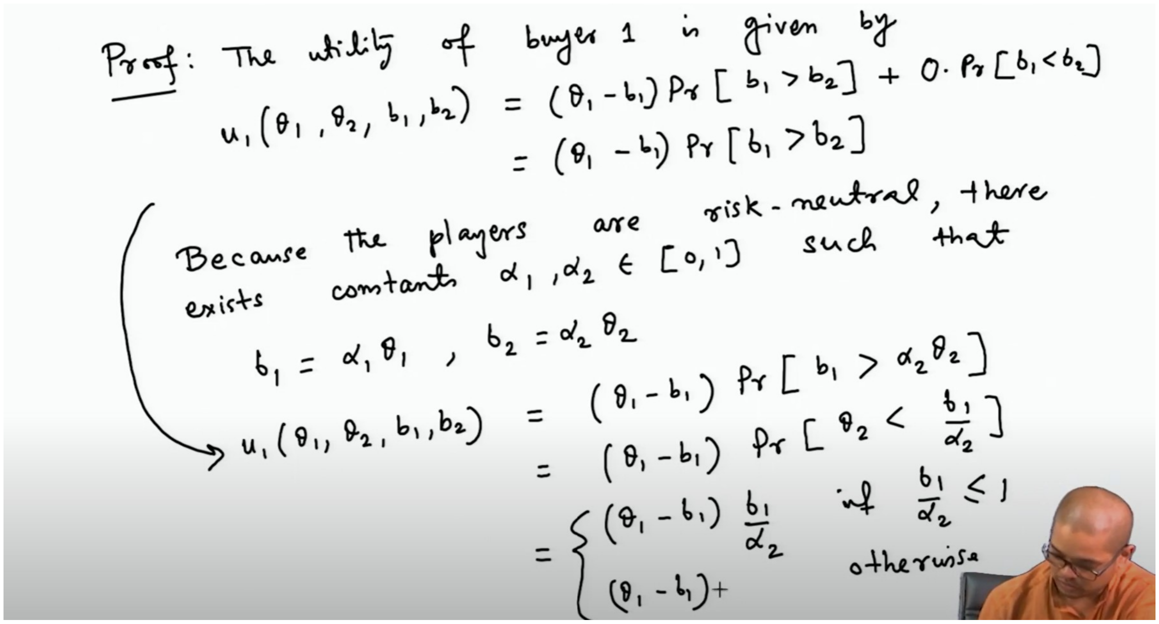
induced by the fast price auction, each buyer bidding half their valuations forms a Bayesian Nash equilibrium, Bayesian Nash equilibrium. You can take it as a easy exercise to show that fast price auction does not have a pure strategy Nash equilibrium or it does not have a weakly dominant strategy equilibrium or strongly dominant strategy equilibrium or very weakly dominant strategy equilibrium. So, this is the best known for fast price auction ok. is under assumptions forms of Asian Nash equilibrium very important assumption. There are some very important assumptions assuming assuming we have only two buyers each buyers valuation theta i is distributed uniformly randomly in the interval 0 to 1 and 3 each buyer is risk neutral.

So, these are the three assumptions that we have only two buyers each buyers valuation theta i is distributed uniformly randomly in $[0,1]$ and each buyer is risk neutral. So, we will we will get rid of this first assumption after this theorem and that the analysis will be similar, but let us do let us first prove this theorem only for two players. So, first assumption is really minor it is just a technicality. But the second and third assumption is very important. It says that you know from buyer 1's point of view buyer 2's valuation for the object could be any number any real number between 0 and 1 with which is distributed uniformly randomly in the interval 0 and 1.

And each by each buyers buyer is risk neutral. Now, a buyer there are 3 kinds of attitude towards risk, one is risk neutral, another is risk averse, another is risk seeking. So, let me slightly discuss it. So, think of 2 options that getting say rupees 100, this is one option versus getting rupees um 200, where it INR 200 with probability 0.5 and nothing with probability 0.

5. Now you see that in the second option the expected amount of money that one will get is again 100, but he has a chance of getting 200 rupees which happens with 50 percent probability, but he can he may not get anything with 50 percent probability. So, if a player is given these two options which one a player will would choose. So, here comes three attitude one is risk seeking player prefers second option. That means, if a player is risk seeking he prefers the second option of getting rupees 200 with probability 50 percent and nothing with probability 50 percent. On the other hand if a player is risk averse risk averse player who wants to avoid risk the first option and the player is risk neutral risk neutral player risk neutral player.

is indifferent between these two options. So, this gives you a sense of the risk behaviour,



of buyer 1 is given by let us write u_1 it is depends on $\theta_1, \theta_2, b_1, b_2$ which is their bits.

Now, this is this is what this is player once bids b_1 and if he wins then he has to pay b_1 and he obtains a object which he or she values at theta 1. So, it is his gain is θ_1 - b_1 times probability that he wins that $b_1 > b_2$ right plus if he loses then that is 0 times probability if he loses he does not pay anything he does not gain anything this $b_1 < b_2$, but because this is 0 this term we can ignore this is θ_1 - b_1 times probability $b_1 > b_2$. Now, here comes the assumption use of the assumption of risk neutrality because the players are risk neutral. then because there exists constants α_1 and $\alpha_2 \in [0,1]$ such that their bids $b_1 = \alpha_1 \times \theta_1$ and $b_2 = \alpha_2 \times \theta_2$ ok. So, this is because the players are risk neutral.

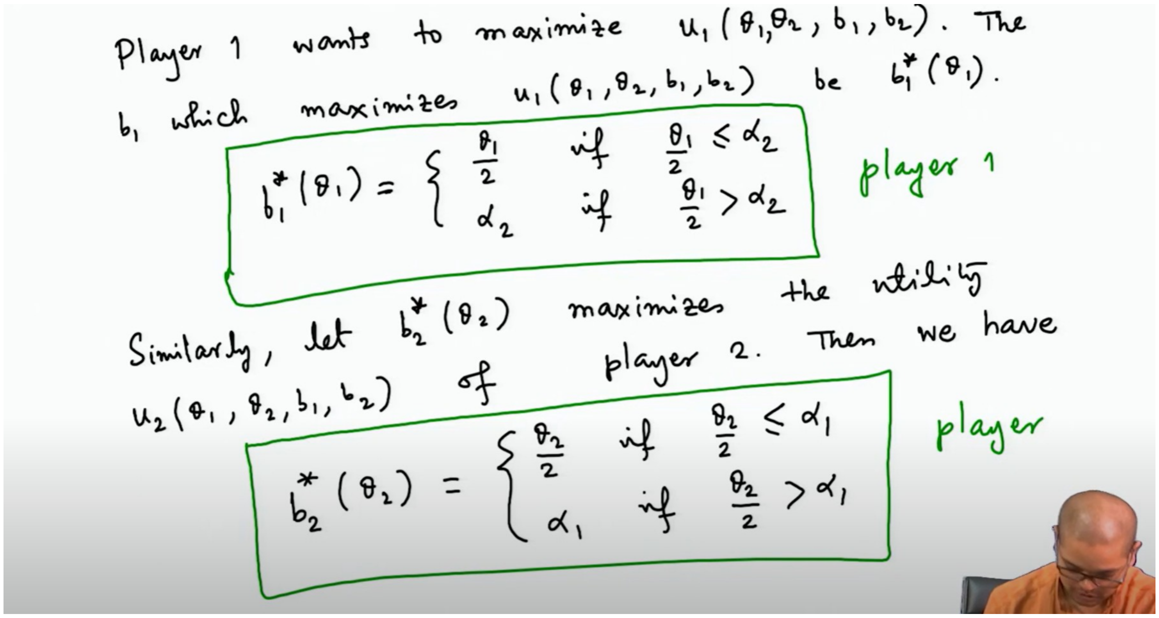
If the players were risk seeking then this function will not be linear function it will be a non-linear function for risk seeking and risk averse function risk averse players. Now, for one player if it is if he is risk averse then it will be either convex or concave or and the other case it will be the opposite. So, it is a nice exercise for you to think about. So, if the player is risk seeking then how does the bit function will look like will it be a concave function of θ_1 or it is it will be a will it be a convex function of θ_1 this you can think ok, but here it is risk neutral. So, this is where we are using risk neutrality.

So, now, we continue our computation from here then $u_1(\theta_1, \theta_2, b_1, b_2)$. This is $\theta_1 - b_1$ probability b_1 is greater than. Now, player look at from player 1's perspective because we are analyzing player 1, player 1 knows that there that player 2 is risk neutral and and hence player 1 knows that whatever be the be the value of θ_2 and α_2 it will be this alpha 2 his bid will be $\alpha_2 \times \theta_2$. this is from player 1's perspective.

Player 1 does not knowα_2 does not know θ_2, but player 1 knows that it will look like $\alpha_2 \times \theta_2$. This is $\theta_1 - b_1$ probability $\theta_2 < \frac{b_1}{\alpha_2}$. Now, here we will assume we will use our second assumption that θ_1 and θ_2 are distributed uniformly randomly in between 0 and 1 that real interval is $\theta_1 - b_1$ and then this probability this now become $\frac{b_1}{\alpha_2}$ ok. Now, this is $\frac{b_1}{\alpha_2}$ if $\frac{b_1}{\alpha_2} \leq 1$ otherwise this probability is 1 and this is $\theta_1 - b_1$ otherwise.

So, if $\frac{b_1}{\alpha_2} > 1$, then probability that theta is less than something greater than 1 is 1. So, this is this one ok. Now, player 1 wants to maximize maximize his or her utility which is $u_1(\theta_1, \theta_2, b_1, b_2)$. So, player 1 chooses b_1 which maximizes this b_1. let us the b_1 which maximizes $u_1(\theta_1, \theta_2, b_1, b_2)$.

b_1^* it should be a function of θ_1. So, if you solve if you maximize this function you look at from b player 1's perspective player 1 knows θ_1 and and he wants to compute b_1. So, player 1 say b_1^* maximizes this. Now, from elementary calculus it follows that b_1^* is $\frac{\theta_1}{2}$ if $\frac{\theta_1}{2} \leq \alpha_2$ and α_2 if $\frac{\theta_1}{2} > \alpha_2$ ok. So, this is from player 1's perspective.



So, this is for player 1. Now let us analyze player 2. So, similarly let $b_2^*(\theta_2)$ maximizes the utility $u_2(\theta_1,\theta_2,b_1,b_2)$ of player 2, then we have $b_2^*(\theta_2)$ equal to $\frac{\theta_2}{2}$ if $\frac{\theta_2}{2} \le \alpha_1$ and this α_1 if $\frac{\theta_2}{2} > \alpha_1$. So, this is player 2's. Now, to solve these two can I does there exist a choice of α_1, α_2 so, that this satisfies both the things.

So, question is does there exist α_1, α_2 such that $b_1^*(\theta_1)$ which at one hand it should be $\alpha_1 \times \theta_1$ on the other hand it should be this $\frac{\theta_1}{2}$. if $\frac{\theta_1}{2} \le \alpha_2$ and this α_2 if $\frac{\theta_1}{2} > \alpha_2$ that is 1 and $b_2^*(\theta_2)$ which is $\alpha_2 \theta_2$ on the one hand and in the other hand it is supposed to be $\frac{\theta_2}{2}$ if $\frac{\theta_2}{2} \le \alpha_1$ and α_1 if $\frac{\theta_2}{2} > \alpha_1$. ok and if you look at it if you plot it the answer is very easy is choose $\alpha_1 = \alpha_2 = 1/2$ satisfies both the constraints both the in equation both the equations satisfies the above equations. And, hence any choice of α_1, α_2 which satisfies this is the best choice for play both player 1 and player 2 simultaneously.

Hence, $(\frac{\theta_1}{2}, \frac{\theta_2}{2})$ is a Bayesian Nash equilibrium ok. And this is this exercise this thing you can extend easily to n players. So, let me just write the theorem for n players. $(\frac{n-1}{n} \times \theta_i)_{i \in [n]}$ is a Bayesian Nash equilibrium of first price auction assuming there are n buyers second. buyers are risk neutral, buyers are risk neutral and third the valuation of every buyer is distributed uniformly randomly in $[0,1]$ ok.

So, let us stop here. So, we will continue with our auctions and we will see more sophisticated auctions in coming lectures ok. Thank