Artificial Intelligence for Economics

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Week – 06

Lecture - 27

Lecture 27 : Second Price Auction

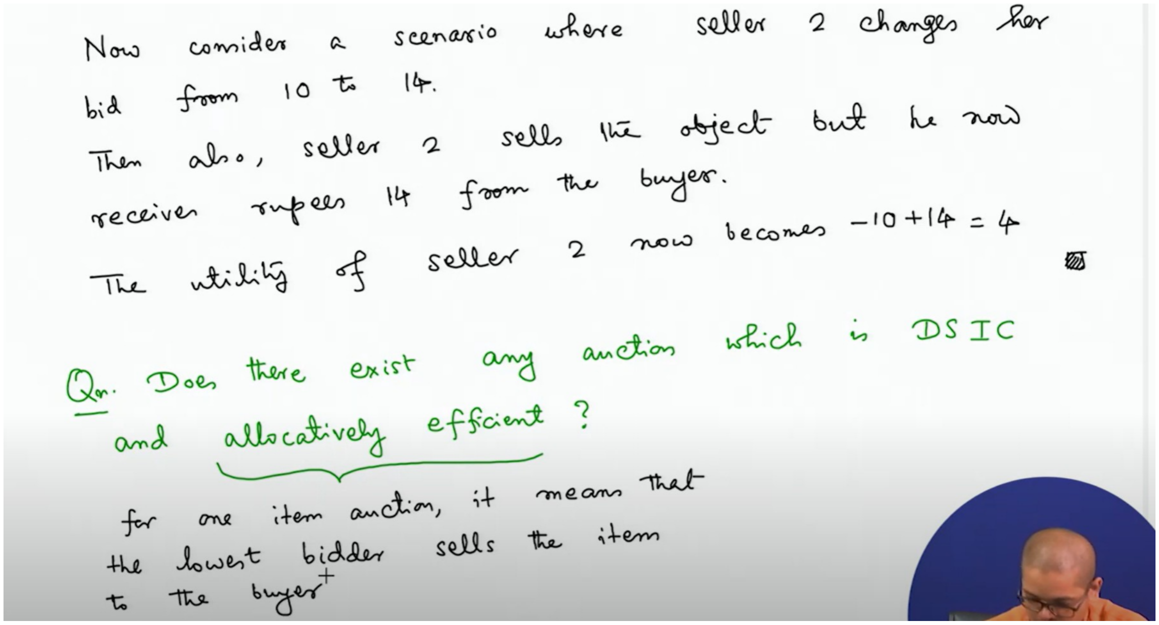
Welcome. So, in the last lecture we have we have started auction and we have seen the Bayesian game which an auction induces on the players and we have seen the example of a fast price auction. So, today we will continue our auction study of auctions. And first we make an observation that first price auction is not dominant strategy incentive compatible. That means, it is possible for players to not bid their true valuation and get better utility. So, for that we need to give an example of a scenario which is very easy.

So, let us assume we have one buyer 0 is the buyer and 2 sellers they are sellers. And, suppose their types theta 0 is say 20 that means, the buyer is willing to pay up to 20 rupees for buying the object let me write that is the buyer is willing to pay up to 20 for the item and suppose θ_1. which is the valuation of the item for the seller which is suppose 15 and θ_2 is suppose 10. Now, suppose player 2 and so, what are the truthful bits? Truthful bits $s_1 = 15$ that means, player players bidding their true valuation types is called a truthful bidding and $s_2 = 10$.

Now, an auction will be called a dominant DSIC auction dominant strategy incentive compatible if bidding your true valuation is dominant strategy is a dominant strategy. That means, for player 1 bidding 15 is the is the is best for player 1 player 1 irrespective of what player 2 does. But here we see that if player 2 say bids truthfully say 10 and player 1 bids 15. then player 1 then player 2 makes smaller bid. So, player 2 sells the item and receives rupees 10 from the buyer.

Hence, the utility of player 2 is it has it has sold the item which it values at 10. So, that is -10, but it has received a payment of 10 which is +10. So, the utility is 0 of course, the utility of the say of seller 1 is 0 because he is not selling the item ok. consider a scenario where seller 2 changes her bid. from 10 to 14.

Then then also seller 2 sells the object, but he now receives rupees 14 from the buyer ok. So, the new ok not new. The utility of seller to now to now becomes it sells the item which it values at 10. So, it is -10, but it receives a payment of 14. So, this +4.



So, we see that seller 2 is able to improve get more utility by misreporting her valuation. So, the question is does there exist. So, the important question: Does there exist any auction which is which is dominant strategy incentive compatible? So, is it possible to have an auction where reporting true valuations is best for every player irrespective of what other players are doing. Think if there is such an auction, then the job of the bidder becomes very simple. He does not have to think what or do any complex analysis, he just goes and bids his true valuation.

But for this question the answer is trivial yes, you can think of an auction which always gives an item or which always buys the item the buyer buys the item from one particular seller it completely ignores all the bits, but that auction is not is not desirable. So, what we want is some more desirable property not only DSIC. does there exist any auction which is DSIC and the and allocatively efficient. Now what does allocatively efficient means? It means for one item auctions for for one item auction it means. that the lowest bidder sells the item to the buyer.

So, it seems that there are many such auctions and we will study them elaborately and one such prominent one is called second price auction. What is second price auction? Again recall that to define auction we have to define the function g which selects from a strategy profile to outcome. An outcome is a tuple it has two parts one is allocation part and another is payment part. And, so that function g can be thought of as two functions one is allocation rule another is payment rule. So, to define second price auction I have to define what is the allocation rule for second price auction, what is the payment rule for the second price auction.

So, what is the allocation rule? Allocation rule is the same as the fast price auction. That means, whichever bidder bids the lowest that bidder sells the item to the buyer. If there are multiple lowest bidder any bidder any lowest bidder any one lowest bidder sells the item to the buyer. that is what we mean that that was the allocation rule for first price auction and that will be the allocation for the second price auction also. It is the payment rule where this auction differs from first price auction.

So, as usual the losers sellers That means, losers are those sellers which are who are not chosen by the auction to sell the item to the buyer. The losers do not pay anything, this part is also same as the first price auction. the winner seller receives the lowest of all other sellers bids that is because the seller is the lowest bidder the amount of money that he or she receives is the second lowest bid. ok and thus the name second price auction. Now, an important an interesting theorem for this is this that bidding valuations is a weakly dominant strategy equilibrium for the second price auction that is bidding valuation.

is the weakly dominant strategy for every player, player means we mean seller. So, let us assume the scenario that buyer is not bidding that means, he will buy for example, this is the case for example, when an organization say like an IIT is buying an buying an item. Now, weekly doing for every is a is the weekly dominant strategy for every player in the second price auction. So, let us prove it ok. So, to prove it let us first introduce some notation.

So, let us first concretely write what exactly we need to show. We need to show that for $i \in \{1,\ldots,n\}$, the strategy $s_i^* = \theta_i$ is his weakly dominant strategy. Again some notation s_{-i} by this we denote $j \in \{1,\ldots,n\}$ except i, s_j ok. And and if you have a s_{-i} is a tuple $(s_1,\ldots,s_{i-1},s_{i+1},\ldots,s_n)$.

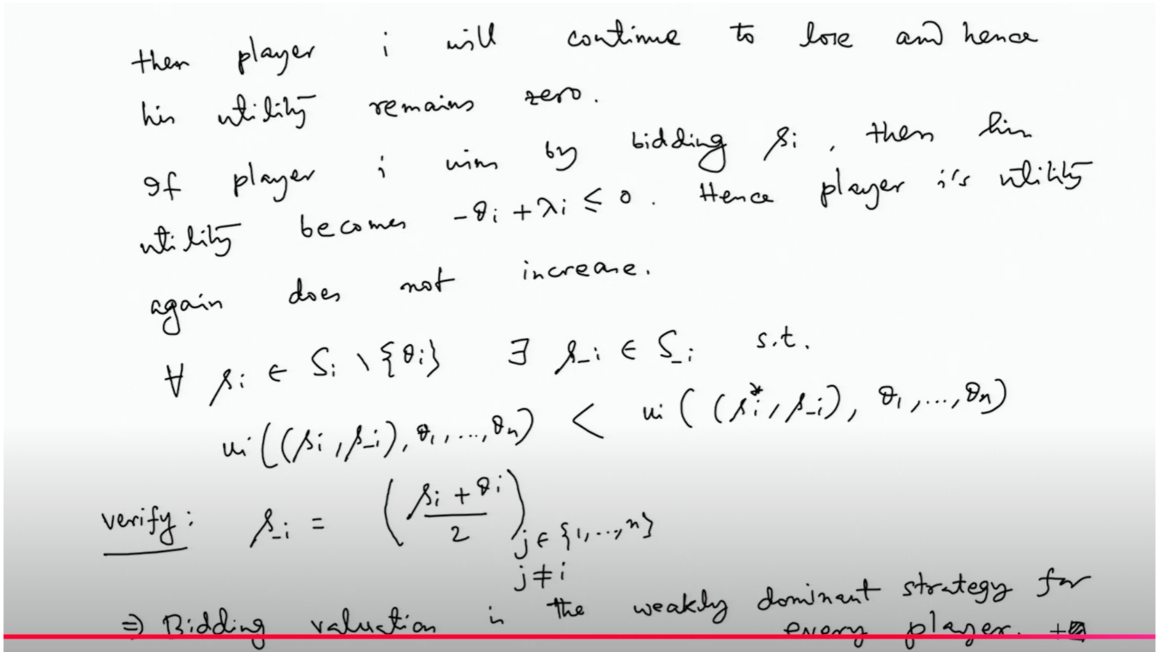
So, this is a tuple in s_{-i}. So, we need to show that this is a weakly dominant strategy that means, irrespective of what other players beat this beating s_i^* is best for player i. So, let s_{-i} be an arbitrary strategy profile in S_{-i}. So, let first ask say λ_i equal to minimum of minimum of s_j $j \in \{1,\ldots,n\}, j \neq i$. That means, in the strategy profile s_{-i} which is here you first ask what who is the minimum bidder, because if player i bids below it or equal to it or below it say then player i can win otherwise it cannot win.

So, case 1. that player i wins, suppose player i wins. that means, what that means, s_i^* player i wins by bidding S i star so that means, S i star is less than equal to λ_i. So, in this case what is the utility of player i? Utility of player i player i is playing s_i^* other players are playing s_{-i} this is the strategy profile and I have type profile $(\theta_1,\ldots,\theta_n)$. In this case if because player i wins then we have the payment of player i is λ_i player i sells the item a which values it at $s_i - \theta_i$ which is its valuation, but it receives a payment of λ_i. Now, theta i is same as s_i^*.

So, it is $-s_i^* + \lambda_i$ but we have seen we know that because player i has own the own the auction s_i^* must be less than equal to λ_i. So, this is greater than equal to 0. Now, think of player i deviating from s_i^*. Suppose player i plays or bids s_i instead s_i^*. Now, 2 things can happen that player i continues to win if player i continues to win then if player i continues to win then its utility then his utility remains is again it gives the object which it values at θ_i and it receives the payment which is λ_i which is same as $u_i\left(s_i^*, s_{-i}, \theta_1, \ldots, \theta_n\right)$.

So, if player i continues to win then by deviating from s_i^* to s_i player i's utility does not increase. Hence by deviating from s_i^* to s_i the utility of player i does not increase. On the other hand, if player i now loses that means, player i does not sell by deviating from s_i^* to s_i. If player i loses by deviating from s_i^* to s_i, then his utility becomes 0. right because it is not selling the item and the utility of the losers are 0.

Hence, in this case also in this case also the utility of player i does not increase ok. The other case case 2 is player i loses player i loses by bidding s_i^* what does that mean? That means, s_i^* is greater than equal to λ_i that is the only way it can lose. So, the utility of player i is 0 here. i is 0. Here also let us ask if player i deviates from s_i^* to some other strategy s_i, then what will it is his utility be? If player i bids s_i which is greater than s_i^*, then player i will continue to lose.



Then player i will continue to lose and hence, remains 0 again it does not increase ok. On the other hand if player i wins by bidding then his utility becomes it minus theta i this is the object he gives, but he receives a payment of plus λ_i. But in this case you see s_i^* is greater than λ_i that means, but s_i^* is θ_i. So, θ_i is greater than equal to λ_i that means, $\lambda_i - \theta_i$ is less than equal to 0 this is less than equal to 0. Hence, player i is utility again does not increase ok.

This shows that bidding valuations is a weakly dominant strategy a very weakly dominant strategy equilibrium. To show that it is a weakly dominant strategy equilibrium, we need to show that for all $s_i \in S_i \setminus \theta_i$ there exists an $s_{-i} \in S_{-i}$ such that there exists a strategy profile of other players where if you play s_i when other players are playing $(s_{-i}, \theta_1, \ldots, \theta_n)$ this is strictly less than $u_i(s_i^*, s_{-i}, \theta_1, \ldots, \theta_n)$. That means, for every other strategy s_i other than s_i^* which is θ_i there exist a strategy profile of other players which will lead less utility for player i if player i plays s_i and what is that it is easy to check or let me write it verify it. it is easy to verify take it as a homework verify that s_{-i} if you look at this profile which is $\frac{s_i + \theta_i}{2}$ this profile this is $j \in \{1, \ldots, n\}, j \neq i$.

If every player is bidding this $\frac{s_i + \theta_i}{2}$, in this case playing s_i will lead will give less utility to player i compared to playing s_i^* for player i. So, this shows that bidding valuation is the and it is a well known result in game theory that if there is a weakly dominant strategy for any player it is unique. So, bidding valuation is the weakly dominant strategy for every player. So, you see for second price auction it is so simple for from the bidders point of view that they can simply bid their valuation and that is the best for them and from the auctioneer point of view also this is a good auction because the he or she the auctioneer gets to know what are the true valuations of each of the seller.

So, let us stop here. So, in next couple of lectures we will see more and more auctions ok. Thank you.