

Artificial Intelligence for Economics

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Lecture - 26

Lecture 26 : Introduction to Auction Theory

Welcome to the third part of AI for Economics course. I am Professor Palash Dey. I will be teaching game theory and how it is used in AI related applications. So, I will start mainly focus on the application of auction. auctions we are aware of and there are two kinds of auctions one can think of what is one option is there is one buyer and multiple seller. and the other option is multiple buyers and one seller.

In the first case when we have multiple seller each seller bids for selling the object. And in the other case when we have multiple buyers and one seller here each buyer bids for buying the object ok. So, let us see examples of both kind of auctions. So, for the first case when we have one buyer and multiple seller think of an organization IIT Kharagpur for example, wants to buy.

a product which could be say machine or service etcetera releases a tender. And anyone any company or any other organization who wants to sell this product they can bid for selling this product to IIT or to this organization. The other case when we have multiple buyers and one seller when you have one seller we have one seller has one object to sell. So, here an important example is government auctioning spectrum or any public resource for some particular time period for lease period or something is for example, some coal mines or some mines government want to give it to some company for some particular time period and then government can can form or call for an auction and ask for bids for potential buyers.

Another important example is say. IPL auctions players. know we in IPL Indian Premier League we have seen that that individual teams bid for various players and that is also auction. As you can see there is only once one object or one player that is to be sold and multiple teams place auction place bids for that. So, it turns out that the theory of auction for both kind of auction is fairly similar.

And so, we whenever we are studying one auction similar kind of analysis we can

perform in the other kind of auction and we can get a similar kind of result. So, for studying auction we will fix one kind of auction and study that kind of auction. And, the similar results hold for other kinds of auctions also ok. So, when we have an auction what are the important questions in an auction. What are the important questions? There are many important questions, but specifically we will study we will focus on two important questions.

First as a bidder in an auction what should be your bid. So, here we need to again need to make standard game theoretic assumptions. So, we make standard game theoretic assumptions that every player is rational. That means, each player has a utility function and every player wants to maximize the utility that he or she derives from the outcome rational intelligent. which means that each player has a computational resource and other intelligence and other required intelligence or reasoning power to compute what is the best thing for or what action would maximize his or her utility rational and intelligent and the game is common knowledge ok.

So, we will soon see that each auction induces a game among the bidders. So, we will just elaborate on what game we mean. The second question is as an organizer of the auction often called auctioneer or mechanism designer. we would like to know whether the auction is incentive compatible in dominant strategies. This property is called DSIC dominant strategy incentive compatible or Bayesian Nash equilibrium, this is called Bayesian Nash Bayesian incentive compatibility BIC ok.

So, now let us come back and highlight what which game we are meaning what is the game of an auction. So, Bayesian game, so each auction induces a Bayesian game on the bidders. So, Bayesian induced by an auction ok. So, what is what is a Bayesian game? So, for concreteness let us assume that we have one buyer, who wants to buy one item from n potential sellers ok. So, we have the set of players.

which is $n + 1$ where 0 is the buyer $\{1, \dots, n\}$ where 0 is the buyer and $1, \dots, n$ are the sellers. Typically in mechanism design we want money generated to be distributed among the players itself and to ensure that we include both buyers and sellers in the game even if the buyer will buy and does not buy even in that case. So, that is why we are including buyer also in the game. So, 0 player 0 is the buyer and players 1 to n are the sellers ok. Now, what is set of outcomes? The set of outcomes denoted by X which is a tuple the first part is the allocation $a_0, a_1, \dots, a_n$.

So, a_i is 1 it that means, that i -th player sells the item to the buyer and p_0 is the payment of player 0. So, $p_0, p_1, \dots, p_n$, p_i is the payment of player i . This is in R^{n+2} such that a_i is either 0 or ± 1 . ok ah. So, -1 means the object is taken from player i , $+1$ means the

object is given to player i ok.

This is this for all $i \in \{0, 1, \dots, n\}$ ok $\sum a_i = 0$ that means, if a player sells then another player has to buy. So, that means, nothing is supplied from outside the system and nothing goes from the system. and similarly $\sum p_i = 0$. So, think of $a_i = 1$ if player i receives the item 0, if player i neither receives nor gives the item and -1 if the player if player i gives the item. Similarly, p_i is is payment made by player i.

if p_i is negative that means, player i receives payment receives payment of $|p_i|$ if p_i is positive that means. player i makes payment of p_i ok. So, the condition $\sum p_i = 0$ guarantees that the auction neither generates any surplus money nor receives any money from outside . Such an auction is called strongly budget balanced ok. So, it neither receives any money or generates any money.

Then the most very important thing the set Θ_i . is the set of all possible valuations of the item by player i. this is also called the typeset of player i. So, if you do not know anything then we can assume that Θ_i is set of all positive or non-negative real numbers. Then the set of strategies or bids for player i is S_i for $i \in \{0\}, i \in [n]$ So, we are assuming that in the most general setting, setting which we are assuming the also submits a bid, it is like the maximum price that the buyer is willing to pay for that item to any seller.

And then the function g or a function G that takes all the bids as input and outputs and outcome from X ok. and what is the utility of a player say player i what is the utility function of player i for an output this a 0 to output recall the first I have allocation tuple $(a_0, \dots, a_n)$ and I have the payment tuple $(p_0, \dots, p_n)$. ok and the utility of player i depends on in most general case it depends on the output and the type profile of all the players ok. So, θ_1 is in capital $\Theta_1, \dots, \theta_n$ is in Θ_n this is Now you can say that you know if player i gets the object and makes a payment, if player i gets the object. then its utility is you see player i values the object at θ_i .

So, its utility should be $\theta_i - p_i$, p_i is the payment it has made. On other hand if player i does not get the object and it is making a payment of p_i . So, these two we can succinctly write as $a_i \times \theta_i - p_i$ ok. Now, two prominent examples of auctions. The first example is the standard auction that we see which is called first price auction. In first price auction everything remains same we only need to decide this function g .

So, let me write this function this function g varies from auction to auction. What is this function? It takes all bids as input and outputs the outcome. Now, for first price auction this function g has two parts. The function g has two parts, what are the parts? The first

part is allocation rule, it decides what is the it decides from the bid profile to who gets the object which is $\{0, \pm 1\}^{n+1}$ and the other part is payment rule. Again it is from bid profile to payments who makes how much payment.

For first price auction the seller who bids the let me write a seller because the lowest bid there could be multiple sellers bidding the lowest bid a first price auction can pick any seller any such seller. A seller who bids the lowest sells the item and this is one and the buyer pays the seller her bid value. So, in the next class in the next lecture we will see another auction and we will see and we will contrast this with this first price auction and we will argue why that auction which is called second price auction or Vickrey auction is superior to first price auction ok. So, let us stop here.