Artificial Intelligence for Economics

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Lecture - 21

Lecture 21 : Game Theory (Contd.) Games with Incomplete Information (Contd.)

Welcome to lecture 21. We will continue with games simultaneous move games with incomplete information. In the last lecture we saw we were looking at one particular kind of game one particular game the battle of sexes game where we had a boy and we had a girl they wanted to go out on a date they could either go for a cricket match or they can go for a movie the girl can be of two types either interested or uninterested the boy just has one type interested if the girl is uninterested then this was the payoff matrix if the girl was sorry the girl is interested this is the payoff matrix if she is untested this was the payoff matrix and then we found out the Bayesian Nash equilibrium of that game what is the Bayesian Nash equilibrium that is what we defined at the end of last lecture A Bayesian Nash equilibrium is a set of strategies one for each type of each player in a game such that it is not rational for any type of any player in the game to deviate from what he or she is doing given the beliefs which exist and the beliefs are common knowledge. so if these are the two payoff matrices of the girl is interested or uninterested we solved it in the last lecture and we figured out that the only pure strategy Bayes Nash equilibrium of this game is given by CCM that is the boy choosing to go to the cricket match the interested girl this is the interested girl I am sorry this is the interested girl choosing to go to this is the boy the interested girl going to the cricket match and the uninterested girl going to the movie.

If this happens this constitutes a Nash equilibrium a pure strategy in base Nash equilibrium. We have argued about it in the and this is a unique Bayes Nash equilibrium pure strategy Bayes Nash equilibrium of this game. In this lecture we will try to compute or we will try to see if there is a mixed strategy equilibrium of this game with incomplete information. Again remember the definition of the Bayes Nash equilibrium, there is a strategy for each type of each player.

The boy has only one type, so let us say the mixed strategy the boy plays is $(p,1-p)$, let us say the mixed strategy which the girl plays is $(q_1,1-q_1)$ and the mixed strategy which the uninterested type girl plays is $(q_2,1-q_2)$. Can these constitute a Nash equilibrium?

Remember the belief of the boy remains the same. So, we are trying to find the mixed strategy profile, we are trying to find which mixed strategy profile will constitute a Bayes Nash equilibrium given the beliefs. What is the belief in this game? The boy believes that the girl is interested is of type interested with probability half and is of type uninterested with probability half. great.

So, again what we how will we solve this problem? We will again step into the shoes of the boy and try to see the world through his lens, let us try that. Given that the girl is of interested type plays $(q_1, 1-q_1)$. this is a mixed strategy remember what does this mean it means that the interested girl plays c with q_1 and m with $1-q_1$ and the uninterested girl plays a mixed strategy $(q_2, 1-q_2)$ ok. If that is the case what is the expected payoff of the boy if he plays c understand this carefully let us look at this the interested girl is playing $(q_1, 1-q_1)$ so let us say we are in a world where the girl is interested so the girl chooses cricket with probability q_1 this is cricket and movie with probability $1-q_1$ and the boy is playing cricket so with probability q_1 both will meet in the cricket match and the boys payoff will be 10 and with probability $1-q_1$ the boy will play will go to cricket match and the girl will go to the movie so in that case the boy's payoff is 0 right you can go back to the payoff matrix and verify then the boy's payoff is 0 so what is the boy's expected payoff in a world in a world where the girl is interested it is simply $10q_1+0\times(1-q_1)$ okay so this is the expected payoff of the boy in a world where girl is of type interested ok now let us look at when the girl is of type uninterested if the girl is uninterested the uninterested type girl plays this mixed strategy she chooses c with q_2 and m with $1-q_2$ that is her mixed strategy right great now if the boy plays c then what is the expected payoff if the girl is of uninterested type well the girl goes to the cricket match with probability c sorry goes to goes to the cricket match with probability q_2 then with probability q_2 the boy will bump into the uninterested girl in the cricket match and get a payoff of 10 and with probability $1-q_2$ the boy will go to the cricket match the girl will go to the movie they won't meet and the boy will end up with a payoff of 0 so this blue within the braces this is the payoff of the boy the expected payoff of the boy from playing C in a world where the girl is of type uninterested okay great. Now the boy does not know whether the girl is interested or uninterested the boy does not know what kind of a world it is it is it a world with the girl being interested or is it a world with the girl being uninterested so what is the boys expected payoff given the belief that he believes that the girl is interested with probability half so what is the expected payoff of the boy well it is the red payoff multiplied by probability half remember this is the payoff this is the payoff when the girl is interested and the girl is interested with probability half.

This is the payoff when the girl is uninterested and this is the probability of the girl being uninterested according to the boy's belief. So, according to the boy's belief this is the expected payoff from playing C. Similarly I can find the expected payoff of the boy if he plays M it is simply given by so this is if he plays C this is if he plays M okay very good. Now when will we have seen this before? when will the boy play a proper mix strategy that is when will he choose a p greater than 0 when the expected payoff from playing c and m are equal that is only when the boy will choose to randomize or randomly choose between c and m with some positive probability. So, in order for the boy to play a proper mixed strategy these two expected payoffs this and this they must be equal, right.

So, that is exactly what I am what the slide says. So, these two payoffs should be equal this is the payoff. the expected payoff from playing C and this is the expected payoff from playing M remember that's what we saw in the slide before these two should be equal well and I call this the girl connect equation and I'm just naming this equation you can call it something else I just want to name this equation so this equation must this equality must hold for the boy to play a proper mixed strategy otherwise the boy will simply play a pure strategy C or M and once he does that the girl will also do either C or M depending on what type she is and we will go back to the pure strategy Bayes Nash equilibrium which we saw before. So this equality is a must for the boy for it to be rational for the boy to play a proper mixed strategy okay great let's move on now let's step into the show of the girl let's step into the shoes of the girl and let's say that the girl is interested if she is interested okay now let's say she plays a mixed strategy $(q_1, 1-q_1)$ what is the boy playing the boy is playing a mixed strategy $(p, 1-p)$ okay then what is the expected payoff of the interested girl if she chooses c let us understand this let us go back to the payoff matrix of the interested girl we had this before so the boy is playing this with p this with $1-p$. So, this is the interested girl what is the interested girls payoff if she plays c well if she plays c then she will meet the boy in the cricket match with probability p and get a payoff of 5 and with probability $1-p$ the boy will go to the movie and she will not meet the boy and get a payoff of 0.

So the expected payoff is simply $5p+0(1-p)$. Similarly if she chooses to play M then what is the expected payoff of the interested girl? She goes to the movie with probability p the boy goes to the cricket match so they won't meet the interested girl gets a payoff of 0. With probability $1-p$, the boy also goes to the movie. The interested girl gets a payoff of 10. So if she plays M, the expected payoff is $0\times p+10\times(1-p)$.

Okay? Great. So that's what we have here. Now, for the interested girl, to play a proper mixed strategy these two should be equal otherwise she will play a pure strategy ok right. So, for that to happen for the interested girl to play a proper mixed strategy it is necessary that the that her expected payoff from playing c and m are equal. So this is equal to this and that leads to $p=2/3$.

Okay? Fine. Let us keep it at that. Let us move ahead. We will come back to this in a second. Now let us turn our attention to the uninterested girl. Okay? Let us see.

what is the or when will the uninterested girl play a proper mixed strategy given that the boy is playing the mixed strategy $(p, 1-p)$ let's see let's go back to the payoff matrix of the uninterested girl sorry uninterested girl, let us see. So, the boy is playing $(p, 1-p)$, right, the boy is playing $(p, 1-p)$. Now, if the girl plays c, the uninterested girl plays c, what is her expected payoff? With probability p, she will bump into the boy which she does not want, if she bumps into the boy, then she gets a payoff of 0. With probability 1 minus p she will not meet the boy and she will get a payoff of 5. So, her expected payoff from playing c is $0 \times p + 5 \times (1-p)$.

What is her expected payoff from playing m? if she plays M, given that the boy is playing this mixed strategy $(p, 1-p)$. Let's see, if she plays M, then the expected payoff, then her expected payoff from playing M is equal to, with probability p, she will, she won't meet the boy, the boy will go to the cricket match, she won't meet the boy and she will get a payoff of 10. Remember, this is the uninterested girl, she does not want to meet the boy, she gets a payoff of 10 and with probability $1-p$, she will meet the boy at the movie and she will get a payoff of 0, which means her expected payoff is $10 \times p + 0 \times (1-p)$, great. so given that given this mixed strategy played by the boy these are the expected payoffs of the uninterested girl very good sorry sorry for slides yeah so these are the payoffs of the uninterested girl for playing c and m these two actions but for the uninterested girl to play a proper mix strategy that is with the positive that is q_2 being positive and not equal to 1. It should be these 2 should be equal right when will it be a proper mix strategy only when q_2 is a proper fraction it is neither 0 or 1.

then so the uninterested girl will play a proper mixed strategy if these two are equal this is the expected payoff from playing C expected payoff from playing M are equal and it will only happen this if $p = 1/3$. Now we are in a bit of a fix we see that if the boy plays a mixed strategy the boy will play a proper mixed strategy if this happens if this happens q_1 and q_2 satisfy this equation which we call the girl connect equation the interested girl will play a proper mixed strategy if $p = 2/3$ and the uninterested girl will play a proper mixed strategy if $p = 1/3$ okay. Let us try to, let us try to disentangle this, this, this problem we are in. Let us work our way out. Let us say if $p = 2/3$.

What happens when $p = 2/3$? The expected payoff of the interested girl is the same for playing both C and M, right. That is what we have seen. It actually turns out to be 10 by 3. and in this situation the interested girl plays a proper mixed strategy play or can play a proper mixed strategy. But what happens to the payoff of the uninterested girl when $p = 2/3$ when $p = 2/3$ For the uninterested girl, the expected payoff from playing C is

5/3 and the expected payoff from playing M is 20/3.

Which one is higher? 20/3. So the expected payoff from playing M is higher, which means that the uninterested girl simply chooses a pure strategy M. Now, the we assume that the uninterested girl was having this mixed strategy $(q_2, 1-q_2)$ that is she was choosing cricket with q_2 and movie with $1-q_2$. So, choosing a pure strategy M is synonymous with choosing q_2 equal to 0 right. Okay, so what's the bottom line? If $p=2/3$, then the interested girl gets equal payoff from playing both C and M, and she is in a position to play a proper mixed strategy.

The uninterested girl, on the other hand, has a higher expected payoff from playing M, and it is optimal for her to play M. The pure strategy M. Great. what happens when if $p=1/3$ but let's hold on for a second great if $q_2=0$ and $p=2/3$ if $p=2/3$ what does that mean it means that the boy is playing a mixed strategy $(2/3, 1/3)$ right the interested girl can opt for a mixed strategy the uninterested girl is playing this pure strategy now recall the girl connect equation it is optimal for the uninterested girl to play a pure strategy which is $q_2=0$ right that is she is choosing m but if $q_2=0$ what is q_1 according to the girl connect equation remember the girl connect equation is necessary for is a necessary condition under which the boy will play a mixed strategy otherwise the boy will also play a pure strategy. So, if $q_2=0$ and if we plug $q_2=0$ in this equation which we call the girl connect equation it turns out that $q_1=1/3$.

The interested girl the interested girl is getting equal expected payoff from playing both c and n right so no matter what q_1 she chooses her expected payoff will remain the same okay so if she chooses q_1 equal to two-third that will satisfy the girl connect equation which in turn will make it rational for the boy to play a proper mixed strategy right fine. So, now everything seems to add up right. So, if $q_2=0$ if $p=2/3$ that is if the boy plays a mixed strategy $(2/3, 1/3)$ then it is optimal for the uninterested type girl to choose $q_2=0$. and if she chooses $q_2=0$ the only q_1 which the interested type girl can choose in order to satisfy the girl connect equation is $q_1=2/3$. So, this constitutes so the boy plays this mixed strategy the interested type girl plays this mixed strategy and the uninterested type girl plays this pure strategy.

This will constitute a mixed strategy Bayes Nash equilibrium given that the boys belief is half-half about the girls type. Why is this a Bayes Nash equilibrium? What was my definition of a Bayes Nash equilibrium? Recall it. It is a profile of strategies where one strategy chosen by each type of every player such that given what the other players are doing, no type of any player has an incentive to deviate, isn't that what we are seeing here? Given that the boy is playing this mixed strategy, two-third, one-third, it is optimal for the uninterested type girl to play 0, 1, she has no incentive to deviate from 0, 1. if the boy plays $(2/3, 1/3)$ the uninterested type girl gets equal payoff from playing c and m so no matter what mixed strategy she chooses her payoff is the same so she will do no better if she chooses any other mixed strategy other than $(2/3, 1/3)$ so she has no incentive to deviate either on the other hand if both types of girls choose these two strategies this and this The girl connect equation is satisfied which means that the expected payoff of the boy from playing C and M are equal. So no matter what mixed strategy he plays he will do no better.

So he also has no incentive to deviate from the mixed strategy he is playing right now. Any other mixed strategy won't yield any better outcome for him, right. So this thus constitutes a mixed strategy Bayes Nash equilibrium. But is it the only one? No. Remember, we have just talked about $p=2/3$.

There was another p which we encountered, remember? $p=1/3$. Let's see what's going on here. If $p=1/3$, then what happens? If $p=1/3$, then the interested girl, look at the interested girl. her expected payoff from playing C is equal to five by three, right? Okay. Her expected payoff from playing M is going to be 20 by three, which is greater than five by three.

So in this case, it is optimal for her to play M, that is choose $q_1=0$. And we had seen this before, that if $p=1/3$, then the expected payoff from playing C and M are equal for the uninterested girl. So the uninterested girl can play a proper mixed strategy if she wants. Great, so if $p=1/3$ then the interested girl necessarily chooses $q_1=0$. But if $p=1/3$ that is if the boy is playing a proper mixed strategy one-third, it is rational for the boy to play a proper mixed strategy only if the girl connect equation is satisfied but if the boy plays this mixed strategy $(1/3, 2/3)$ the interested girl plays $q_1=0$ and if $q_1=0$ the only q_2 which satisfies the girl connect equation is $q_2=2/3$ okay just like we proceeded in the example in the situation before when $p=2/3$.

So, if $p=1/3$ it is optimal for the interested girl to choose $q_1=0$ and the only q_2 which satisfies the girl connect equation is $q_2=2/3$. that is fine why is it fine because the expected for the uninterested girl the expected payoff from playing c and n is the expected payoffs are equal which means no matter what mixed strategy the uninterested girl plays her payoff is going to be the same. So, why not $q_2=2/3$ she can play this it is not irrational for her to play $q_2=2/3$. So, it seems $p_1=1/3, q_1=0$ and $q_2=2/3$ do the trick just like we saw before. So, just like we argued before the boy playing this mixed strategy $(1/3, 2/3)$ the interested girl now playing a pure strategy 0 1 and the uninterested girl now playing a proper mixed strategy $(2/3, 1/3)$.

this constitute a mixed strategy Bayes Nash equilibrium the argument is exactly what it was before if the boy plays this mixed strategy $(1/3, 2/3)$ it is optimal for the interested girl to play this pure strategy 0 1 if the interested girl plays this pure strategy 0 1 the only mixed strategy of the uninterested girl which satisfies the girl connect equation is this Also if the boy plays this mixed strategy, the uninterested girl is indifferent between all possible mixed strategies. So she will do no better if she chooses any other mixed strategy apart from $(2/3, 1/3)$. So she has no incentive to deviate and choose any other mixed strategy. So this person has no, the uninterested girl has no incentive to deviate. The interested girl of course chose what is optimal.

if both of them if the interested and the uninterested girl are choosing these two strategies then the girl connect equation is satisfied which means that the expected payoff from choosing C and M are equal for the boy so the boy will get an equal payoff from choosing whatever mixed strategy he chooses so he will do no better by choosing any other mixed strategy apart from this okay So, thus we see no type of any player the boy is just one type there are two types of girls nobody neither the boy nor the girl if she is interested or the girl if she is uninterested have an incentive to deviate from what they are doing in this particular strategy profile given the belief of the boy about the girls type. So, this is my second mixed strategy Bayes Nash equilibrium. So we have two mixed strategy Bayes Nash equilibrium of this game, of the battle of sexes game which we have just seen. Mixed strategy of the boy, mixed strategy of the interested girl, mixed strategy of the uninterested girl. Mixed strategy of the boy Okay, these two are my possible mixed strategy Bayes Nash equilibria of this game.

Great, that is where we end. In the next lecture, we will start talking about sequential move games. Thank you, see you in the next lecture.