Artificial Intelligence for Economics

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Lecture - 20

Lecture 20 : Game Theory (Contd.) Games with Incomplete Information

welcome to the next lecture of game theory in the first two lectures we have seen the idea of Nash equilibrium both pure strategy and mixed strategy Nash equilibrium in this lecture we'll start talking about simultaneous move games with incomplete information let's move on Let's imagine a scenario that you've been transported back to the 1960s or 1970s where there is no telephone communication, no mobile phones. And imagine there's a couple who want to go out on a date in the evening. So there are two agents or two players in the system, right? A boy and a girl. Let's call the boy player one. Let's call the girl player two.

now where can they go out on a date for let's say they can go out together to watch a cricket match or they can go out to watch a movie together but they don't know they can't communicate and go to one particular place so each player in this situation where they can't call each other and communicate where they are going they have two options they can either go to the cricket match or go to the move. And let us say this is the payoff matrix remember the strategies the pure strategies of the actions of the boy are marked in yellow and those of the girl are marked in red Every payoff you see, payoff vector you see in each cell of this matrix, the first payoff is that of the boy, the second payoff is that of the girl. Okay. So what's going on here? Let's interpret the payoff matrix a little more carefully.

If the boy goes for a cricket match and the girl too goes for a cricket match, the boy gets a payoff of 10, the girl gets a payoff of five. so imagine these payoffs as units of happiness they derive so if both of them go to the cricket match the boy is happier he gets 10 units of happiness if both of them go for a movie then the boy gets five units of happiness and the girl gets 10 units of happiness let's say what happens if they go to different places The boy goes for a cricket match, the girl goes for the movie. Well, then both of them get a payoff or utility of zero. Why? Because the idea was to go out on a date. If they go to different places, it's not a date at all.

So both of them end up getting a payoff of zero. Great. If this is the scenario, what strategy or what action profile will constitute a Nash equilibrium? What action profile will constitute a pure strategy in Ash equilibrium? That should be fairly easy to, if you recall the last lecture, it should be easy to figure out. If the boy chooses to go to a cricket match, what is the girl's best strategy, best action, best response? Well, if the girl chooses to go to cricket, she gets a payoff of 5 if she goes for the movie she gets a payoff of 0 given that the boy is choosing to go to the cricket match so what should the girl do well go to the cricket match if the boy choose to go for the movie then what is the girl's best response if she goes for the cricket match her payoff is 0 and if she goes for the movie the payoff is 10 so if the boy goes for the movie then the girl's best response is also to go for the movie right great now let's see if the girl chooses to go for the cricket match what is the boy's best response if he also goes to the cricket match he gets a payoff of 10 if he chooses movie he gets a payoff of 0 so what will he do go for the cricket match if the girl chooses to go to the movie what should the boy do he can either go for the cricket match or go for the movie if he goes for the cricket match his payoff is 0 if he goes for the movie his payoff is 5 so the boy too should go for the movie that is his best response so we can see that there are two action profiles here (cricket, cricket) this and (movie, movie) both choosing movie so both choosing to go to the cricket match or both choosing to go to the movies these two action profiles constitute a nasheq constitute nasheq philibria there are two of them okay so (cricket, cricket) and (movie, movie) constitute two Nash Equilibria, two pure strategy Nash Equilibria of this game. Great.

So, these are my two pure strategy Nash Equilibria CC and MM. But we have seen in the last lecture that we can also have mixed strategy Nash Equilibria to different games. So, should not we inspect that does this game have a mixed strategy Nash Equilibria. Let's find them first. Let's say the boy plays a mixed strategy $(p, 1-p)$ that is he chooses C, he chooses to go to the cricket match with probability p and he chooses to go to the movie with probability $1-p$.

Remember what is a mixed strategy? A mixed strategy is a probability distribution over the set of actions of a player. let's say the girl also plays another mixed strategy $(q, 1-q)$ that is she chooses to go to the cricket match with probability q and she chooses to go to the movie with probability $1-q$ fine now for what p and q will these two mixed strategies constitute a Nash equilibrium let's do it we have done it in the lecture before But just for practice, let's do it once more. Let's say if the boy chooses to go to the cricket match, sorry, the boy is playing this strategy, remember? $(p, 1-p)$. So the boy is choosing to go to the cricket match with probability p and to the movie with probability $1-p$. What is the girl's mixed strategy? She's choosing to go to the cricket match with probability q and to the movie with probability $1-q$.

Okay. Now let's understand. Given that the girl is playing this mixed strategy, I repeat once more, given that the girl is playing this mixed strategy $(q, 1-q)$, what is the expected payoff? So I'm talking about the boy's payoff now. what is the boy's expected payoff if he plays c if he plays c what is the boy's expected payoff if the girl is playing this mixed strategy $(q, 1-q)$ let's understand so if the boy plays c with probability q the girl will also play c then the boy will meet the girl in the cricket match and his payoff is going to be ten With probability $1-q$, the girl will go to the movie. So with probability $1-q$, the boy's payoff is going to be 0. So what is the expected payoff? With probability q, the boy ends up getting a payoff of 10.

And with probability $1-q$, he gets a payoff of 0. So this is the expected payoff of the boy if he plays C. Given that the girl is playing the mixed strategy $(q, 1-q)$. correct, great. Similarly, given that the girl is playing this mixed strategy $(q, 1-q)$, what is the boys expected payoff from playing M? If he plays M with probability Q the girl will go to the cricket match, he will get a payoff of 0 and with probability $1-q$ the girl will go to the movie as well then the guy gets a payoff of 5.

So, this is 5 $1-q$ right very good. Now, when will the boy play this mixed strategy $(p, 1-p)$ let us understand we have seen this in the lecture before that any player will play a proper mixed strategy that is it will choose every action with or it will choose it won't choose a pure strategy that is it won't choose one particular action is when his expected payoff from choosing all the actions is equal given the mixed strategy of the other player. right so this is let's say this is 10q this is 5-5q now if 10q is greater than 5-5q then the boy will or this is simply q greater than 1 third then the boy will simply play c so this is the boy playing if this happens in the boy will simply play c and if q is less than one third then the boy will play m but then we are in the paradigm of pure strategies. So, when will the boy play a pure a proper mixed strategy where p is strictly positive only when these two expected payoffs are equal that is when $q=1/3$, okay. We had also seen in the last lecture that if $q=1/3$ if that is the mixed strategy played by the girl then no matter what p the boy chooses his expected payoff is going to be the same, right.

so q equal to one third that is the girl choosing cricket with one third and movie with two third that is a mixed strategy which will make it rational for the boy to play a proper mixed strategy great let's do the same analysis for the girl if the girl sorry if the boy is playing a mixed strategy $(p, 1-p)$ then now this is the girl's expected payoff right. So, if the boy is playing this mixed strategy $(p, 1-p)$ what is the girls expected payoff if she plays C? Well, if she chooses to go to the cricket match with probability p she will meet the boy and get a payoff of 5 and with probability $1-p$ she will not meet the boy because the boy will go to the movie plus $0\times(1-p)$. What if she chooses m? What is her expected payoff? Well, the boy is choosing cricket with probability p. So, with

probability p the girl will not meet the boy and get a payoff of 0 and with probability 1 minus p she will meet the boy because the boy chooses to go to the movie with probability 1 minus p.

So, her payoff is going to be $0\times p+10\times(1-p)$ if they meet in the movie the girls payoff is 10 you can see from the payoff matrix. Now, the girl will play a proper mixed strategy where q is strictly positive only if these 2 payoffs are equal. Now, when will they be equal if $5p=10(1-p)$ or in other words when $p=2/3$ okay so when only when $p=2/3$ only then the girl will play a proper mixed strategy with a strictly positive p sorry with a strictly positive q great. So, $p=2/3$ and $q=1/3$ if these two things happen then only both the players the boy and the girl will play proper mixed strategies otherwise both of them will play pure strategies and we will be in this pure strategy in these two pure strategy Nash equilibrium we land up with either of them right. So, for the players to or the players will play both the players will play mixed strategies proper mixed strategies only when this happens.

So, what is the only possible mixed strategygy Nash equilibrium? Well, it is q equal to one-third, $1-q=2/3$, $p=2/3$, $1-p=1/3$. So, the boy, so this is my mixed strategygy Nash equilibrium. So, the mixed strategygy Nash equilibrium is the boy playing cricket with two-third and movie with one-third and the girl playing cricket with one-third and movie with two-third this is the boys probability distribution a mixed strategy that is this is the girls mixed strategy okay so this this constitutes a mixed strategy Nash equilibrium of this game anyway Now, let us make things a little more interesting. So, these are my two mixed strategy Nash equilibrium. Boy plays this and the girl plays the other one.

Now, let us introduce incomplete information. Now, what is that? Let us say the boy does not know whether the girl is interested or uninterested. So now the girl can either be interested or uninterested. Okay. If the girl is interested, this is the payoff matrix.

It's exactly what it was. So if both of them land up in the cricket match, it's 10-5. If both of them land up in the movies, it's 5-10. If the girl is uninterested, this is how the payoff matrix looks like. if both the boy and the girl end up in the cricket match the boy gets a payoff of ten the girl gets a payoff of zero so in this case the girl being uninterested she does not want to meet the boy so if both of them land up in the same place the girls payoff is zero so see if both of them land up in the cricket match (cricket, cricket) the girls payoff is zero if both of them land in the movie again the girls payoff is zero okay but the boy is interested so in this incomplete information situation the boy is interested as he was before but he does not know if the girl is interested or not if the girl is uninterested then this is the payoff matrix again if the boy chooses cricket and the girl chooses movie which is what which we are here now Then, the girl gets a payoff of 10,

but the boy gets a payoff of 0, because the boy wants to meet the girl.

Similarly, when the boy chooses movie and the girl chooses cricket, the girl gets a payoff of 5, but the boy gets a payoff of 0, because they are not meeting. Great. Fine. Now, yes, the boy does not know if the girl is interested or not. So, the boy does not know the type of the girl interested or uninterested, but he has a belief what is his belief? Belief is a probability distribution over the set of types ok.

So, for the girl what are the two types which are there interested uninterested. if the girl and the boy believes that the girl is interested with probability half and uninterested with probability half. This belief is common knowledge that is the girl knows that this is the belief the boy has and the boy knows that the girl knows about his belief so and so on and so forth. If this is the situation, let's see how we can analyze this. So there are two types of girls.

If we want to compute the payoff matrix, we can't do it at once. So this is what we are doing. We are trying to find the boy's expected payoffs. So we are looking at it from the perspective of the boy. So let's step into the shoes of the boy and analyze the situation.

Let's see. So, what are these strategies C-C, C-M, M-C, M-M, what do they mean? They simply mean the following. Let us say if the interested girl and the uninterested girl, if the girl whether she is interested or not chooses C. that is this that is this strategy CC let's say so if both types of girls interested and uninterested they choose to go to a cricket match and the boy also choose to go to the cricket match what is his expected payoff well it is 10 why because remember the payoff matrices whether the girl is interested then also the boy gets 10 if both go for the cricket match if the girl is uninterested even then the boy gets 10 if both of them go to the cricket match he is he the boy is interested and he is only interested in meeting the girl so if both of them go to the cricket match irrespective of the type of the girl the boy's payoff is 10 okay very good what if both types of girls choose C and the boy chooses M needless to say the boy's payoff is going to be 0 right you can figure that out from the two payoff matrices now the more interesting one if the interested girl chooses C and the untrusted girl chooses M which is this interested girl chooses C uninterested girl chooses M and the boy chooses M what is his expected payoff well if well with probability half the girl is interested right. So, with probability half this will happen MC that is the boy goes for the movie the girl goes for cricket and with. So, this is and with probability half this will happen this will happen with probability half, this will happen with probability half.

Remember the interested girl is going to cricket and the uninterested girl is going to movie. So with probability half and the boy is playing M, so we will end up with MC or MM. If MC happens what is the boy's expected payoff or what is the boy's payoff? It's 0.

If MM happens what is the boy's payoff? so 5/2 is the expected payoff of the boy if this happens what about this the interested girl plays C the uninterested girl plays M and the boy plays C then what happens so a girl if she is interested the boy does not know if she is remember the girl if she is interested she will go to and the boy is also playing c. So, c c will happen and that will happen with probability half because the boy believes that the girl is interested with probability half.

With another half probability c n will happen, the boy will choose c and the girl will choose n because remember the uninterested girl chooses m. So, with another half probability c n will happen. What is the expected payoff when cm happens? It is 0, the expected payoff of, sorry the payoff of the boy when cm happens is 0, the payoff of the boy when cc happens is 10. So the expected payoff of the boy here is half into 10 plus half into 0 equal to 5. In this way we can compute all the entries in this table, okay.

Why am I doing this? What is the rationale? I will come to that in a second. but for the time being let's focus on the boy let's stay in the shoes of the boy and try to see. So these are the payoffs these are the best responses of the boy if the girl irrespective of her type chooses C then the best response is to place C and get a payoff of 10. the girl if interested chooses C and uninterested chooses M then the best response of the boy is to play 5 given his belief for MC again 5 for MM its M great clear ok so these are my candidates which we need to inspect both types of girls choosing C and the boy choosing C The interested girl choosing C, uninterested girl choosing M and the boy choosing C. The other case both types of girls choosing M and the boy choosing M.

So, these are the cases which we need to inspect a little more carefully. Let us do that. Let us look at CCC. is everybody behaving rationally here remember the belief of the boy is common knowledge so the girl knows what the boy is going to do and the girl knows what the belief of the boy is about her type now let's see is CCC rational behavior for all parties in this game if the boy plays C is it optimal for the uninterested girl to play c let's go back to the payoff matrix of the uninterested girl for a second if the boy plays c what is the optimal response of the uninterested girl if she plays c she will get 0 if she plays m she will get 10 so she should play m not c which means this is not what we call a Bayesian Nash equilibrium this is not rational behavior ignore the Bayesian Nash equilibrium which I am writing here the BNE I am going to define it formally in a couple of slides from now but the CCC is not rational behavior if the girl is uninterested the uninterested girl knows that given the belief the boy has if he chooses C then she should not choose C. What about CCM? Well this seems fine, this seems like a pure strategy based Nash equilibrium that is what we call it and again I am going to define it in a couple of slides from now.

So please ignore the jargon, please ignore the term. just imagine or try to see if this is

rational behavior from everybody if the boy plays C well we have seen that if the interested girl plays C and the uninterested girl plays M then given the belief of the boy the boy's optimal response is to play C right that's what we have seen in the previous calculations but if the boy plays C is it the optimal response of the interested girl to play C yes it is go back to the payoff matrix and verify if the boy plays C is it the optimal response for the uninterested girl to place M yes it is so here everybody is behaving rationally so C C M this is where everybody seems to be behaving rationally just like the first case we can argue that cmc and mmm they are again not rational behavior why if the boy plays c is it optimal response of the interested girl to play m no the interested girl should play c so this is wrong this is irrational behavior if the boy plays M should the uninterested girl play M no this is the uninterested girls action rate should she play M no she should be playing C she wants to avoid the boy so again this is also not rational behavior from the uninterested girl so both of these have logical flaws so only CCM where the boy chooses C the interested girl chooses C and the uninterested girl chooses N only this seems to constitute rational behavior from everybody and this is what we call a pure strategy Bayes Nash equilibrium okay now let me formally define what do I mean by it so Bayes Nash equilibrium is a set of strategies one for each type of each player such that no type of any player has any incentive to change his or her strategy given the belief given the beliefs which are there about other players of given the belief any player has about other player or given all the beliefs present in the system the example which we dealt with only the girl has two types the boy has just one type so the girl does not really have a belief about the boy right the boy is interested and his type is known to the girl great. So, again repeating what is the Bayesian Nash equilibrium or a Nash equilibrium in presence of incomplete information it is a set of strategies one for each type of each player such that no player no type of any player has an incentive to deviate. See, isn't that what we are getting here? Look at CCM. This is a pure strategy Bayes Nash equilibrium.

We have a strategy or an action, a pure strategy or an action for each type of each player, don't we? The boy has just one type. So we have an action for the boy which is C. The girl has two types, interested and uninterested. For the type interested, we have C, an action. For the uninterested girl, uninterested type girl, the action is M.

So this constitutes a pure strategy Bayes Nash equilibrium. An action or a pure strategy for each type of every player. Great, now that we have understood what a Bayes Nash equilibrium constitutes and we have seen that this particular game where the boy does not know the type of the girl, we have a pure strategy Bayes Nash equilibrium, we are all set to inspect if this game has a mixed strategy Bayes Nash equilibrium. right we have talked about pure strategy Nash equilibrium we have also talked about mixed strategy Nash equilibrium now when it comes to incomplete information we just computed the pure strategy based Nash equilibrium in the next lecture we will try to see if this particular game has a mixed strategy based Nash equilibrium see you in the next lecture thank you