Artificial Intelligence for Economics

Prof. Dripto Bakshi

Humanities and Social Sciences

Indian Institute of Technology Kharagpur

Week – 04

Lecture - 18

Lecture 18 : Game Theory

Welcome to this segment, this is game theory. Before we move on, let us take a look at what lies ahead. So first we will have a brief introduction of game theory. What is game theory? Where is it applied? What is it about? Then we will talk about two kinds of games, simultaneous move games and sequential move games. Within simultaneous move games, we will talk about games with complete information and games with incomplete information which are often called Bayesian games. In sequential move games, we will touch upon Rubenstein bar gaming.

In the final segment, I am going to talk about network games which is becoming increasingly popular. Let us move on. So, what is game theory? Game theory is an analysis of strategic interaction between agents where the outcome of the interaction depends on what the agents do so my payoff is dependent on what other agents are doing and vice versa in such an interactive environment what do agents do Game theory is a study of that. So it studies rational behavior in interdependent situations.

Before we move, let us acquaint ourselves with two key concepts in game theory. One is the concept of rationality of players and the second is the idea of common knowledge. What is rationality? We'll assume that every player or every agent maximizes, selfishly maximizes his own objective or payoff given the available information present in the system or given the information available to him or her. What is common knowledge? Any fact is said to be common knowledge if each player in the game or the system knows the fact, each player knows that every other player knows the fact, every player knows that every other player knows that every other player knows that particular fact and so on and so forth. Usually the rules of the game or the rules of interaction or payoff these are common knowledge.

As I mentioned in a couple of slides before there are two kinds of games broadly speaking simultaneous move games and sequential move games. Now what you can imagine what they are right in simultaneous move games the players move simultaneously the players choose their actions simultaneously in sequential move games one player moves before another player can observe that move and act accordingly so first we will begin with simultaneous move games so let's understand what a simultaneous move game is Let's understand the structure. Let's understand the system first. So what do we have? First, we have a set of players. If we have a game, it has to be played by some players or some agents, right? So we have a set of players.

For each player, there is a possible set of actions from which he or she can choose an action. So for each player, there is a set of actions. Each player chooses an action from his possible set of actions simultaneously. So let's say you and I are playing a game. I have a set of actions.

You have a set of actions. I will choose an action from my possible set of actions. You will choose an action from your possible set of actions. This pair of actions which we have chosen, that will be called an action profile. So if we have more than two players, it's not a pair, it's called a couple of actions, right? So the couple of chosen actions by all the players is called an action profile.

Now, each player has a preference over the set of these action profiles. Now, what exactly do I mean by this? It will become clear in the next slide as I'll try to give an example. Complete information. So first we'll talk about simultaneous move games with complete information. What is complete information? Complete information is a setting where each player has all information about the set of actions of all other players and the payoffs of all other players.

Okay? So this is what simultaneous move game with complete information is. So this is the setting. Let's look at an example. A very simple game, a wait go game. So let's say we have two cars, a red car and a yellow car.

Okay? So these are my two players in the system. Now what can they do? They have come to a crossing what can they do they can either go ahead or wait so each player's set of actions so who are my players by the way to start with the red car and the yellow car each player's set of actions is given by go or wait so the red car can either go or wait the yellow car can also either go or wait So what will be the set of action profiles? So what are the all possible action profiles? Simple. Well, the red car goes and the yellow car goes. So the red car chooses an action go from his set of possible actions. The yellow car also chooses the action go from his set of possible actions.

So go go forms an action profile. Similarly, the red car choosing go and the yellow car choosing wait, that too forms an action profile. Similarly, the red car choosing wait, the yellow car choosing go or both the cars choosing wait. So, these are my all possible action profiles. Now, each player has a preference over these set of action profiles, which is usually given by a payoff matrix.

So, let us see what that is. So this is what the payoff matrix looks like. So let's understand what this means. Every entry in the matrix has two numbers, right? The first number denotes the payoff of the red car and the second number denotes the payoff of the yellow car. Now let's understand, let's interpret this matrix.

If both the red car and the yellow car choose wait, then they have a payoff of zero. Well, they are just maintaining the status quo. They both just keep waiting at the crossing. They get a payoff of zero. That's understandable, right? Let's say the red car waits and the yellow car goes.

Then what happens? Well, then the red car has a payoff of zero because he keeps waiting. The yellow car on the other hand gets a payoff of 2 because he dashes away. Now let's say the red car chooses go and the yellow car chooses wait. Then it's exactly the symmetrically opposite case. Now the red car gets a payoff of 2, the yellow car gets a payoff of 0.

If both of them choose go, what's gonna happen then? Well, then they'll crash into each other. And if they do, they get a terrible payoff. So let's, I call it minus 10 and minus 10. So both of them get a really negative payoff, which is minus 10.

Okay? Great. If this is the payoff matrix, what should the cars do? Let's understand. In order to understand what they'll do, or how they should behave or how they are likely to behave, let's understand the concept of a Nash equilibrium. What is a Nash equilibrium? Any action profile, remember what an action profile is? It's a couple of actions which have been chosen by all the players. An action profile is called a Nash equilibrium if no player has an incentive to do anything else given what the other players are doing that is no unilateral deviation is rational or profitable. So if you and I are playing a game an action profile is called a Nash equilibrium if given what I am doing you just do that and given what you are doing I just do that.

None of us have an incentive to deviate from the actions chosen in that action profile given what the other person is doing. So each player is playing his or her best response given the actions of the other players. So that's a Nash equilibrium. Now let's try to find what the Nash equilibrium of this particular game is going to be like. Let's look at the best responses.

if the red car chooses wait then what is the best response of the yellow car well it's to go because if the yellow car chooses wait he gets a payoff of 0 if he chooses go he gets a payoff of 2 so clearly the yellow cars should choose go if the red car chooses go then what should the yellow car choose well if the red car is choosing go if the yellow car 2 chooses go then well they will crash into each other and the yellow car is going to get a payoff of minus 10 so but if he waits his payoff is 0 so what is the yellow car's best response it is to wait so these are the best responses which are marked with a star these are the best responses of the yellow car given the different actions of the red car. If the red car chooses wait, the yellow car's best response is to choose go. If the red car chooses go, the yellow car's best response is to choose wait. Now, let's look at the best responses of the red car. Well, it's exactly what the yellow car does.

It's very similar to what the yellow car's best responses are. If the yellow car chooses wait, the red cars best response will be to go if the yellow car chooses go the red cars best response is going to be wait so now we have two action profiles go wait the red car choosing go and the yellow car choosing wait and the red car choosing wait and the yellow car choosing go these are two Nash equilibria of this simultaneous move game why? Inspect it a little more carefully consider the action profile where the red car chooses go and the yellow car chooses wait if the red car chooses go the yellow cars best response is to choose wait so the yellow car has no incentive to deviate from wait and if the yellow car chooses wait the red cars best response is go the red car has no incentive to deviate from go which means in the action profile where the red car chooses go and the yellow car chooses wait no player has an incentive to deviate given what the other players other player is doing similarly when the red car chooses wait and the yellow car chooses go both the players don't have an incentive to deviate from what they are doing given what the other players are doing Hence these two action profiles constitute a Nash equilibrium or constitute Nash equilibria. There are two of them. Let's look at one more popular simultaneous move game which you will encounter in almost all game theory textbooks. It's called the prisoner's dilemma.

So imagine there are two prisoners who have been incarcerated and they are being interrogated in different cells. They have been kept separate and they can't communicate with each other. Now they have two strategies. Either they can confess or they can not confess. By cooperate we mean they are not confessing.

they are cooperating between each other or with each other ok so they are not confessing cooperate is not confess and defect is to betray the other person ok so let's see so if both the prisoners choose to cooperate that is if they choose to respond in a way such that the crime won't be caught then both of them get a 1 year jail. If both of them choose to defect or not cooperate, then both of them end up getting 5 years in jail. Now, if player A or prisoner A, if prisoner A cooperates and if prisoner B defects, then something interesting happens. Then the defector is is is benefited the defectors actually let Scott free to the defector get 0 years in jail and prisoner A who was cooperating gets 10 years in jail and vice versa if prisoner A defects and prisoner B cooperates then prisoner A gets 0 years in jail and prisoner B gets 10 years in jail now if this is the scenario what do you think will be the Nash equilibrium So this is Nash Equilibrium I guess, a little correction here, this is not Nash Equilibrium R, we just have 1, okay. Now let us see, if prisoner A chooses cooperate, what is the best response of prisoner B well if prisoner A chooses cooperate then if prisoner B chooses cooperate as well prisoner B ends up getting one year in jail but if he defects he gets 0 years in jail so clearly defecting is a better strategy given that prisoner A is cooperating So if prisoner A cooperates, prisoner B will necessarily defect.

What will happen if prisoner A defects? What is prisoner B's best response? If prisoner A defects, prisoner B's best response is again to defect. See, if prisoner A is defecting, if prisoner B cooperates, his payoff is 10. If he defects, his payoff is 5. Which is better? 5 years in jail is better than 10 years in jail. So prisoner B again will choose to defect.

So we can conclude that no matter what prisoner A does, prisoner B's best response is to defect. Okay? If prisoner B necessarily chooses to defect, what is prisoner A's best response? Well, if prisoner B is defecting that is what we have seen no matter what prisoner A does prisoner B will defect, but if prisoner B defects what should prisoner A do? If he cooperates he gets 10 years in jail and if he defects he will get 5 years in jail and 5 years in jail is better than 10 years in jail. So, prisoner A will also defect. which means both prisoners defecting is a Nash equilibrium in fact it is the unique Nash equilibrium of this game now notice something interesting in this particular game it is rational for both the players or both the prisoners to choose defect and what is it that they end up getting five years in jail but had they cooperated had both of them cooperated they could have got you can see here I am sorry they could have got 1 year in jail and here they are getting 5 years in jail in the equilibrium so the outcome of the Nash equilibrium is not quite desirable. In fact, we can say that the outcome when both of them cooperate Pareto dominates the outcome when they both defect.

By Pareto dominance we mean that both the players are better off when both of them are cooperating. So, the Nash equilibrium is not necessarily a desirable outcome, but unfortunately the desirable outcome here is cooperate cooperate, but that cannot be sustained because once any player cooperates the other player has an incentive to choose defect. So, no player will choose cooperate. Okay, let's move on.

Let's look at another game. Let's look at penalty kick. Of course, I'm going to simplify this game a little bit. So imagine we have two players. It's a penalty shootout scenario in a World Cup game, let's say. And we have two players, right? The striker, who's going to shoot, and the goalkeeper, or the goalie.

Now what can they do? Well, the striker has two possible actions which he can take.

What are they? He can either kick left or he can kick right. the goalkeeper on the other hand has two possible actions he can dive left or dive right of course this is a oversimplification of the scenario because they can the striker can also kick straight the goalie can stay still but anyway I'm not taking that into consideration I'm just simplifying the game a little bit so let's say this is what the scenario is the striker will either choose left or right The goalkeeper will also choose to dive to the left or to the right. So what are my possible action profiles? What are the possible action profiles in this game? Well, it is the striker kicking left and the goalie diving left. The striker kicking left and the goalie diving right.

The striker kicking right and the goalie diving left and the striker kicking right and the goalie diving right. So these are my set of action profiles. Once we know, we have seen it before, we have a set of players, a set of action for each player, from that we can derive the set of action profiles. What do we have after that? The payoff matrix. so this is how the payoff matrix looks like ok so if the striker kicks left and the goalie dives left then what are the payoff well then the goal will be stopped and each player gets a payoff of 0 right if the striker chooses to kick right by the way in this matrix in every entry the first number denotes the payoff of the striker the second number denotes the payoff of the goalkeeper if the striker chooses to kick left and the goalie chooses to dive left both of them get a payoff of 0 If the striker chooses to kick right and the goalie chooses to dive left, what's going to happen? It will be a goal.

The striker will score. So, the striker's payoff is 1 and the goalkeeper has conceded a goal. So, his payoff is -1. Great. Just like (KL, DL), if the striker chooses KR that is kick right and the goalkeeper chooses DR that is dive right. then both of them end up getting 0 right the goal will be stopped.

So, we are where we were. So, what happens when the striker kicks left and the goalie dives right when again there will be a goal scored, but in this case it is not $1-1$, but it is v where v is a fraction ok. Now, why this V? Well, I am just making an assumption here. Let us say the striker is right footed. So, when he kicks towards the left, V is the probability that the striker will actually end up netting the ball, okay.

So, $v<1$. Great. So, this is the payoff matrix of the penalty shootout game. If that is the payoff matrix what do you think the striker and the goalkeeper will do? Can we find the Nash equilibrium of this game? Can we find the action profiles which constitute the Nash equilibrium of this game? Remember what is a Nash equilibrium? It is an action profile where no player has an incentive to deviate from what he is doing given what the other player is doing. Let's see. If the striker chooses KL, that is if the striker chooses to kick to the right, what is the goalkeeper's best response? It is to dive to the left. If the striker chooses KL, that is kick to the left, the goalkeeper's best response is to dive to the left.

If he chooses to dive to the right, his payoff is $-v$. $-v<0$. So the goalkeeper will choose DL. Similarly, if the striker chooses to kick right, what is the goalkeeper's best response? It is to dive right.

Correct? Okay. Similarly, we can find out the best responses of the striker given what the goalkeeper is doing. That is denoted by the red numbers with a star. If the goalkeeper chooses to dive left, What is the striker's best response? It is to kick right. If the goalkeeper chooses to dive to the right, what is the striker's best response? Well, it is to kick to the left. Okay, so now you can see from this payoff matrix, unlike in the prisoner's dilemma or in the car go-weight game, which we saw before, This particular game seems to have no Nash equilibrium.

So this game does not have any pure strategy Nash equilibrium. Okay? Now what is a pure strategy? I have not mentioned this word pure strategy before. Well, a pure strategy is nothing but an action. It is just another word for the action which a player is choosing.

Great. So we don't have an equilibrium, but we don't have a pure strategy Nash equilibrium. What if the players decide to randomly choose an action from his set of actions? Can there be a rational move of that kind? What does that even mean? We are going to talk about that in the next lecture. We are going to talk about mixed strategies. And we'll see, we'll try to inspect if this penalty shootout game has a mixed strategy Nash equilibrium. But we'll do that in the next lecture. See you.