Artificial Intelligence for Economics

Prof. Adway Mitra

Artificial Intelligence

Indian Institute of Technology Kharagpur

Week – 04

Lecture - 16

Lecture 16 : Causality in Time-Series

Hello everyone, welcome to this course on Artificial Intelligence for Economics. I am Adway Mitra, an Assistant Professor in Indian Institute of Technology, Kharagpur. Today we are starting our lecture number 16, the topic of which is Causality in Time Series. So, for the past few lectures we had been dealing with the topic of learning that is where we had been seeing how we can use past data to extract useful information which we can apply on future data to make some predictions. So, under this we this broad category of methods which can be roughly called as machine learning. We have first discover discussed unsupervised learning and then we discussed supervised learning also.

Now, from this and the next lecture we will be shifting to a slightly different topic and this the topic is that of causality. So, in today's topic we will first understand the notion of causality and we will try to understand how causality differs from correlation even though these concepts often seem to be highly like highly confusing and many people confuse the two of them. And finally, we will discuss one particular notion of causality which is known as the Granger causality. We will continue the discovered discussion of causality in the next lecture also.

So, first of all what is causality? So, let us consider any two variables x and y. So, they can be any kind of variables they can be spatio-temporal variables also that is to say. like it might be that like x is the observation of a particular quantity at a location s_1 and y is the observation of the same quantity at s_2 or it might be that x is the measurement of some quantity at a time point t_1 and y is the same thing at different time point t_2 and stuff like that. Now, in any case let us say that x and y are like any two variables in we defined in any particular way. Now, suppose someone makes a statement like this that variable x causes variable y ok.

So, what do what does it mean it roughly means that the value of x influences the value of y. So, in our daily life we often hear statements like smoking cause cancer or clouds cause rainfall and things like that. So, like how do I like so, these are statements which we may be saying loosely with like without too much of mathematical like insights into it, but suppose I want to express these things as some like these kind of relations between smoking and cancer or between clouds and rainfall I want to express is this as some sort of a mathematical relationship then how do I go for it. So, like so so one possible way is we we consider ah like like we somehow measure the value of smoking somehow we somehow quantify the amount of smoking that someone is doing by x and we also denote by y whether that person is having cancer or not. Similarly, like let us say we can somehow measure the amount of clouds in a particular location and we call that measurement as x and the rainfall at the same location is somehow measured and that is the variable y.

So, now that we have got the two variables x and y what does it mean to say that variable x causes variable y. So, one possible meaning can be that if the like like one variable changes then the other variable will also change may be by a different amount. So, suppose like that is suppose the like x and y these are the two variables that we talked about maybe the smoking and cancer or maybe cloud and rainfall and so on. Now, suppose right now both of the variables are having their own values. So, now somehow I change the value of x to x' that is somehow the value of the x variable is changed.

Then what happens to the y variable then like I mean can we can it be like does it happen that y also changes if so then what is the nature of the change and so on. So, like when we are talking about causality then these are the questions which we are interested to answer. And supposing it happens that whenever I like that change in the value of x is usually accompanied by the change in the value of y. Does it necessarily mean that x is the cause and y is the effect? So, it turns out that even that need not be the case. So, why that need not be the case we will try to understand it in a bit.

So, like but the question which we are in like trying to understand in or ask in that case is that. like if x is changed manually then will y also change or in other words we can say suppose there is a change in y now how much of that change can be attributed to the change in x. So, let it be that x is found to change from to x' and y is also found to change from to y'. Now, the change that happened in the variable y that is it changes value to y' was this change actually because of the change in x or was it for some other reasons or would it have changed anyway that is something which we do not know. So, how do we understand whether like the change in y can be attributed to the change in x that is the question.

And even if it can be attributed how much or what is the strength of the attribution is it could also be that the change of y like can partly be attributed to the change in x, but it may be it can also be attributed to some other causes maybe there is some other variable z which also changed and the change of y was influenced mainly by that. So, these are the kind of questions which we must answer if we have to answer this question that can we say like if the whether x is we can say that x is the cause and y is the effect. So, like you in many cases we call the variable x as the treatment and y as the outcome and we are the question which we are trying to answer is is there a causal relation from the treatment to the outcome. So, just to under get a better understanding of what is happening let us consider the example of the COVID pandemic. So, at that time many people were or many medical agencies were claiming to come up with a vaccine and then they were saying that if this vaccine is administered to people then they will not have COVID or then maybe they will quickly recover from COVID or something like that.

Now, in their support they may even have in the to support their claims they may have shown sub data that they can say that they gave the medicine to so many or the vaccine to so many people and so many people recovered and things like that. So, now the question is Like can it be say like if we have to believe their claim then we have to like understand the people who recovered from covid did they recover because of that medicine that was given to them or would they have recovered anyway. So, like unless this can be satisfactorily answered we cannot say whether like we can like whether this treatment variable I mean in this case the medicine is indeed the like a cure for the outcome that is the COVID. Now, the when we are talking about causal relation between variables it is interesting to note that the causality can also be bidirectional. That is here we were mostly saying that like that is where x is the cause and y is the effect this kind of thing that is we are saying that a change of x should should also like if indeed that is the case then a change of x should also be accompanied by a change of y and so on.

but the reverse can also be true it can it can happen simultaneously that x has a causal implication on y, but y also has a causal implication on x. And the nature of this self this and the nature of this bidirectional causal relation it can be self-replacing or self-destructive. So, what do I mean by that let us consider these two examples. Now let us say that like the temperature in a place goes up it is a very it is a very hot summer. So, that I denote temperature as the intervention or the treatment variable x.

So, there is a positive change in x meaning that it becomes hotter. Now, as a result of that there is evaporation of water that is water from all the lakes and rivers they start evaporating. And now when these evaporated water they create clouds and the when some other conditions are satisfied then these clouds can cause rainfall. So, that is and the rainfall is the variable which I am causing as y. So, like what I am saying of course, like the process of evaporation and rainfall they also involve many other parameters which I am conveniently ignoring for now.

I am saying this just to give you an understanding of the notion of the causality involved here. So, like what I am saying is that an increase of x which is temperature results in an increase of y which is rainfall, but when rainfall happens then the temperature tends to come down. So, an increase of y has the impact of decrease of x. So, this is what is known as a self destructive process that is x increases as a result of that y increases and as a result of that x decreases. So, the like so, the causal relation is on both sides that is increase of x caused increase of y again increase of y caused decrease of x.

So, there is a relation from x to y a positive relation and there is another relation from y to x which is a negative relation. So, the two relations are causal relations are working against each other. So, it is called as a self destructive effect, but it can also be a self replenishing effect. What does that mean? So, let us say that the temperature is high it is a hot summer. So, x the temperature it increases.

Now, as a result of that people start in using air conditioner, but these air conditioners they release the carbon dioxide and we know that carbon dioxide is a greenhouse gas and these like an increase of carbon dioxide which we call as a variable y that causes the temperature to rise up even further. So, we basically it becomes what is known as a vicious cycle that is like as x increase the temperature x increases the CO_2 release that is y also increases which is a positive causal relation. Now, that again has the implication of increasing of x itself increasing. Of course, it is a once again the atmospheric process is not at all this simple and increase of x as a result of this greenhouse gases is much slower than the process of like like this process of people using their air conditioners and releasing CO_2 as a result of hot summer. So, I like I am greatly simplifying this whole idea, but the reason is I am just trying to give you a feel of the causal in relations involved here.

So, as you can see that in this case the two causal relations that are involved from x to y and y to x are both positive. So, this is called as a self replenishing effect. So, which basically means that x just keeps on going higher and higher and higher without any restraint. I mean if this is the these are the only two things which are involved then the like temperature will just keep on rising and in fact, like like it this is a very crude way of explaining why global warming is taking place, but that is of course, another story. Now, like if we want to understand this with the help of a graph.

So, let us say that there is a variable x which is changed from whose value is changed to x'. Now, let us say that there is another variable z which is like causally dependent on x. Now, when x change to x' z also changes to z'. And now there is a third variable y which is causally dependent on z. So, as z change to z' then y also change to just to y'.

So, we can say that because of the change of x the change of y also happened though not directly, but indirectly through this other variable called z. Of course, instead of one intermediate variable there could have been more intermediate variables also. Now, when y changes let us say there is a fourth variable w which is causally dependent on y. Now, this change of y causes a change of w and furthermore x itself is causally dependent on w and as w changes in reaction to that x further changes. So, this whole thing goes on in a cycle.

Now, once again this cycle can either be like that is it might be that some parts of this cycle are positive the other parts of which are negative which is basically the self destructive process or it could be that all of them are positive or all of them are negative which would be something like a self replenishing process. So, that is the bidirectional causality. So, now the question is how do we understand which variables are causally related and which or whether they are or not. So, like so, let us say that we have somehow focused on two particular variables x and y as I said earlier x we call it as the treatment variable and y is the target or outcome variable. Now, I want to understand whether there exists a causal relation from x to y.

So, now there are many ways of doing of understand of trying to answer this question of let us start from the simplest approach. So, let us just say just consider that we have many past observations of both x and y and from that we are trying to answer this question. So, this it turns out that this approach is laden with lots of risks, but let us try to understand what is the nature of such risks. So, like so, primafacie it seems that if they are in should exists a causal relation, then the those two variables they should be there should they should exhibit some kind of correlation and like. So, especially if we let us say that we have a time series of both the variables.

We have already discussed the concept of time series and time series forecasting in the previous lecture and we have also made a comment that time series is usually like associated with a quantity called auto-correlation that is the previous values are correlated with the strong value with the with the past values and that is the property which helps us to forecast the coming upcoming values of time series. right, but now here we are talking about cross correlation that is we have two different time series one for x and another for y and try we are trying to understand whether the past values of x are correlated with the present or future values of y not x ok. So, let us just see an example just to understand the matter. So, here you can see like a time series of x and a time series of y. So, like each of these rows of the this of this matrix we can consider as time points.

So, we can see that at every time point x whenever x increases that is from t_1 to t_2 we see an increase of x and we also see an increase of y. Again from t_2 to t_3, I see a decrease of x and a decrease of y. From t_3 to t_4, increase of x, increase of y and so on and so forth. So, we are seeing that whenever x increases, y also increases and whenever x decreases, y also decreases. So, it is clear that in this case, x and y, they exhibit a high correlation.

That is if we can calculate the Pearson correlation coefficient between x and y, it will turn out to be quite high. Now, consider this table. So, here we will see that here from t_1 to t_2 I see an increase of x, but a decrease of y. So, I may think that in this case they are anti correlated that is the Pearson correlation coefficient is high, but with a negative value. but like is that necessarily true let us see.

So, from again that supported here also we see that from t_2 to t_3 x decreases, but y increases, but from t_3 to t_4 this increases x increases y also decreases. So, far so good, but now we see that x is increasing y is also increasing. Again I see that x is increasing and y is also increasing and so on and so forth. So, it is like our claim that they are anti-correlated seems to be have fallen apart, but we can see make an interesting observation here that whenever x increases y increases in the next step and whenever x decreases then y decreases in the next step. That is see from 12 to 25 there is an increase of x, but in y there is an increase from t_2 to t_3.

There is from t_2 to t_3 there is a decrease of x again from t_3 to t_4 there is a decrease of y. Again here we are saying that from t_5 to t_6 there is an increase of x from t_6 to t_7 there is also an increase of x. From t_7 to t_8 there is an increase of x, t_8 to t_9 there is an increase of y. So, here it seems that like if we just there seems to be a time lag between x and y that is whatever is happening to x the same thing is happening to y, but with a lag.

with with lag of one time point ok. So, like we can see that if we can like correct for that lag then x and y will have high correlation. So, we can say that x and y they have high correlation where y seems to be lagging x by one step by one time step. Now, in this case like as a similar thing happens like I will like I am not going line by line here, but you can observe that whenever x increases then y decreases in and whenever x decreases then y increases in, but this happens in the next step. So, in this case it is it is a case of like we see two different behaviors on one side there is high lagged correlation And, there is also high anti correlation that is the first part that is whenever x increases then y decreases x decreases y increases. So, this indicates anti correlation, but this thing that when y increases then x decreases in the next increases in the next step this indicates lagged positive correlation.

So, both are of these are happening simultaneously. So, basically whenever we are having two time series x and y and we try to understand whether there is a causal relation between them or not, we may start off by calculating these kinds of correlation coefficients or not. But it turns out that the like while that is if they are indeed exists a causal relation then true they should be that is we should get to see some kind of correlation, but the reverse is not true. That is if like even if you have correlation that does not necessarily mean that they are causally related. Now, we can say we can use the correlation this kind of correlation test to rule out the presence of causality.

If it turns out that there is no correlation or no cross correlation being observed between these two time series, then we can safely say that there is probably no causal relation between them. But even if we do find that there exists a correlation between them that does not necessarily mean that they are there is a causal relation also. So, why is that the case? So, for that we have to understand this notion of spurious correlation where the correlation coefficient is artificially high even though this is not happening as a result of any actual causal relationship between the two variables. So, let just take this famous example. So, if we consider the amount of spending in the behind science space and technology in the United States and we compare it by the number of suicides taking place in the same country over the years, we see two the time series look like this.

So, both of them are continuous seem to be continuously increasing. So, it seems that they are indeed is a positive correlation between these two things. So, does it mean that there is a causal relation between these two variables that is can be, but can we actually hope to say that if the country spends more behind signs then more people will commit suicide and or like if more people commit suicide then the country spends more money behind signs. Both of the claims seem to be quite absurd and far fetched. So, then what is what can be the reason? Why are we seeing this kind of correlation? Now, it may happen that they are does exist a causal relation between them.

or it may happen that this is a classical case of spurious correlation. Now, what can be a possible causal relation between them? I am not saying that this is the case, I am just hypothesizing this may be the case. It may be so when the country increases its funding on space technology etcetera, it may be that it is doing so by decreasing the spending on let us say healthcare and social security and so on. which in turn has an negative impact on the people that is people cannot afford much health care they they cannot afford loans for education and so on. And as a result there is depression in the many among many people and hence there is an increase of I mean there is an increase in the rate of suicide.

So, this is a possibility again I am saying this is just a hypothesis. So, if these kind of this is the indeed the explanation then we can say that there is a positive correlation between these two variables that is sign spending and suicide. So, yeah, but it can also happen that there is simply some other confounding variable which we are which we do not know what it is, but both of them are impel driving I mean are I mean both these two variables x and y are being impacted by that confounding variable. That confounding variable whatever it is, is influencing the sign spending to increase gradually, it is also somehow influencing the suicide rate to increase gradually right. So, like this is another possible represent I mean one another possible explanation of the situation.

So, I can in this case I do not know what is the actual thing I mean whether they are actually is the causal relation or it is a case of spurious correlation due to a confounding variable. This example however, is easier to explain. So, let us say that like in a or it has been observed that in some beaches the ice cream sales tends to be strongly correlated with the number of shark attacks taking place in a beach. So, does it mean that there is a causal relation between people eating ice cream and they getting attacked by shark? No, obviously that cannot be true that seems quite absurd.

So, this must be a case of spurious correlation. Now, like in this case the variable involved it might be temperature. So, it might happen that as the temperature rises more and more people buy ice creams and at the same time more people venture out to the sea and as a result of which shark attacks happen. So, this is an example of this spurious correlation. Now like we see plenty of examples of these kind of questions of causality in economics where the notion of confounders can also come in. Say for example, how does fertilizer affect crop yields? So, it like it may there may be some data which shows the amount of fertilizer which a farmer has used and the crop yield which they have obtained.

So, and they can also fit a linear model which may suggest that if you increase x then the y will also increase that is there they may say that this is a I mean there is a causal relation between x and y the fertilizer spend I mean the quantity of fertilizer and the crop yield. But that need not necessarily be the case it is may happen that the crop yield increase not because of the fertilizer, but because of other factor may be the weather also improved during that period when they applied the fertilizer. Or like if we are considering different regions where like it might happen that the places where more fertilizer was applied also where somehow the same places where the soil was fertile anyway. So, like these could be possible confounders. So, unless these I mean these are examined and their impacts are somehow removed we cannot necessarily say that like there is a causal relation between x and y even though there is a definitely a positive correlation between them at least according to the data which we have.

Or similar questions might also be asked about how does education affect the income. Can we say that necessarily say that like if I like some that is someone is gain making more income because they are better educated. So, that is not necessarily the case there could be other confounding variables may be they were anyway born into rich families. So, anyway they had their they inherited their parents business and so they are easily able to make income and so on and so forth.

Similarly with the impact of advertising on a sales of an item. So, there are lots of examples in economics where we are looking where we are there is some phenomena and we are trying to understand the root cause of it, but we are like we are distracted by these confounders. So, confound in this case the word confounder basically you can understand it by confuser a variable which causes confusion. So, now anyway so so these are problems are there, but still somehow we have to estimate the like causality. So, how do we estimate the causality that is how do we say whether like two two variables like is there a causal relation between them or not. So, there are a number of approaches the first approach thus simplest and the coarsest approach let us start with.

So, this is known as Granger causality for time series. So, this was actually proposed by an economist in the 1960s. So, like here the idea is quite simple. where like the idea is that like if x that is I will try to express y like in terms of the past values of x and if I am able to do so, then I will say that there exists a causal relation between them. So, basically I will I have two models for predicting y the first model. So, this is the simple time series forecasting which we had discussed earlier also this is the simple autoregressive model that is why where we are trying to express y_t as a linear combination of past values of y such as $y_{t-1}, y_{t-2}, y_{t-3}$, etcetera.

So, these coefficients a_1, a_2 etcetera can easily be estimated by linear regression by minimizing a loss function over a over some training data set. Now, this is the this model we call as M_1. Now, I consider another model M_2 where I am trying to regress y the present value of y not only on the past values of y, but also on the past values of x. So, this is what the new equation looks likes and we have more coefficients to estimate in this case which also we can do using the same approach, but probably we will need more data to do for a better estimating for getting stronger more robust estimates. So, in this case let us say that I have estimated both of the parameters in both of these model M_1 and M_2.

Now, I will compare them on some held out or validation data and see that which of these models do a better job of forecasting the values of the time series. Now, if it turns out that error of the model M_2 is less than the error of the model M Then that means, that including these additional predictors helped us to make better predictions which can be interpreted in as that there exists a causal relations from x to y. So, like specifically we say that a in if this happens then we can say that x Granger causes y. The it is it might be too strong a statement to say that x causes y, but at least we will say that x Granger causes y.

Now, we may also check whether the reverse happens. So, we can build another model M_3 where I am trying to estimate x or predict the future values of x based on past values of x and another model M_4 where I am trying to predict the future values of x based on not only on the past values of x, but also on the past values of y. And in this case also the these parameters can also be estimated using the least square regression as already discussed. And we can check the whether the we can compare the errors of these two models on some held out data. So, it if it happens that error of M_4 is lesser than error of M_3 that is then we can say that.

like estimates of x improves if we are considering pass values of y. So, we can say that y Granger causes x. So, if both of these two conditions hold together then we can say that x Granger causes y and y Granger causes x that is we have a case of bidirectional Granger causality. So, this idea of Granger causality has been used extensively in economics. I am just showing you these two papers just to give you an idea of what kind of analysis is done. So, in this paper it causes like it discusses a Granger causality analysis between GDP and CO_2 emissions of major emitters and implications for international climate governance.

That is to say like in general it is like for different countries they have one of the variables is the time series of GDP of that country the strength of their economy and the other time series is y that is the amount of carbon dioxide emitted by them. So, can we say that x causes y that is like as the economy increases the unnecessary impact of that is increase of carbon this CO_2. Can we say this or is that not necessarily the case necessarily the case that is there are countries whose GDP has increased which is considered a good thing, but their CO_2 has not increased which is also a good thing. like is that possible or does it mean that if you that there does exist a causal relation between the two that is it is like saying that if x increases that is if you grow then you will also be emitting a lot of carbon dioxide that is everything comes at a cost. Another application of Granger causality here is the impact of FDI on GDP per capita in India using Granger causality.

Here we are trying to understand that like whenever there is a like increase of foreign direct investment on the economy does it necessarily mean that the GDP of the country will also increase that is the economy will grow or is that not necessary the case. So, these are the type of questions which we try to answer using Granger causality in economics. Now, this the concept of Granger causality it is a quite simplistic that is it just involves solving some regression problems, but it is as its own problems. So, the first of the first problem is the assumption of linearity that is we are trying to express x t plus 1 as a linear function of x_t, x_{t-1} and also y_t, y_{t-1} that is as we saw earlier in those formulae we are formulating them as linear regression problems. Now, what if, but I mean in that case are not we restricting ourselves to a very specific assumption of linearity.

So, it turns out that even if we do not make the assumption of linearity, we can still adopt the concept of Granger causality and like we can bring in a non-linear Granger causality. So, we can this function we can just put in any non-linear function f that is I am considering x_{t+1} as any non-linear function of the pass values of x and also of y. So, this function f can be approximated by a neural network as we have seen neural network is a function approximator. So, we can strain a suitable neural network. So, this problem even if though if it it is one criticism of Granger causality it can be dealt with.

The other problems are more serious. One of them is that if there are confounders involved in the system then Granger causality generally will not be able to detect them. We it is possible to still detect them, but it will involve like identifying causal relations between like all possible pairs of variables and see if there is some kind of conflict arising in which case we will understand that there is a confounder involved, but that is again very tedious. So, this is one problem of Granger causality. Another problem is that it cannot handle contemporaneous causal relations that is like it may happen that x_t and y_t they have a causal relation, but Granger causality will not be able to resolve it. The Granger causality will work only if there is a causal relation between x_t and y_{t-1} or x_{t-1} and y_t and why is that the case that is because in this case Granger causality is like that is if the Granger causality is based on regression.

Now suppose there exists a contemporaneous causal relation between them. So, if regression of x_t improves the if the regression of x_t improves by considering y_t as among its predictors, then the converse will also be true that is the regression of y_t will also improve if we are considering x_t as one of the predictors. So, we will not know that whether it is the case of x causing y or the cause of y causing x or both simultaneously. the claim of causality in Granger causality is entirely centered on the premise that the future cannot be that the future cannot be cause of the past. So, if I say that the x_t regression of x_t is improving because of including y_{t-1} then only y_{t-1} can be the cause and x_t can be the effect because something the reverse is not possible because x_t happened is the future and y_{t-1} is in the past. So, the only the it is only possible that the past is the cause of the future the reverse cannot be said, but if I am dealing with x_t and y_t which both are at the same time then even if the regression is works.

we still will not be able to like make any causal statement out of it. So, another approach to deal with some of the problems of Granger causality especially the problems with this contemporaneous relation is like is the Peter Clark algorithm this is based on the concept of conditional independence. So, here we have like we consider a probabilistic graphical model in where you like we have many nodes where each node represents one of the variables and we try to find a edges between them or rather we assume initially that all of them are connected by an edges, but then we take pairs of them and see if we can say that one is conditionally independent of the others by carrying out certain statistical test if that is the case then we drop the edges and only the those edges which remain after carrying out the algorithm they indicate which variables are the causes of which other variable. So, in this case as you can see that the possible causes of the variable x_t are y_t, y_{t-1} and y_{t-3} that is y at 3 different lag values. So, this is like this this allows us to handle the problem of contemporaneous correlations and there have also been some improvements to the PC algorithm to help identify the confounders. So, to conclude causality is the question if a treatment variable can causally influence a target variable or not.

Causal influence relations across time series can hold at different lags thus like a correlation in I mean if there is causal relation there should be correlation may be a lagged correlation, but the presence of correlation does not mean that there is actual causal relation because there can be spurious correlations also which are which may are induced by confounders. Now, Granger causality is a simple regression based approach of understanding of seeing whether one time series or the past values of one time series can be used to predict future values of another time series. If that happens then we say that one time series Granger causes the other, but it has multiple drawbacks. Now, PC algorithm is another algorithm based on where the notion of causality is related to the statistical notion of conditional independence and it constructs it proceeds by constructing a causal graph using conditional independence test. So, with this we come to the end of this lecture which is lecture 16 in the next lecture also we will continue our discussion on causality. So, till then all of you please stay well we will see you soon bye.