

Foundations of Cyber Physical Systems

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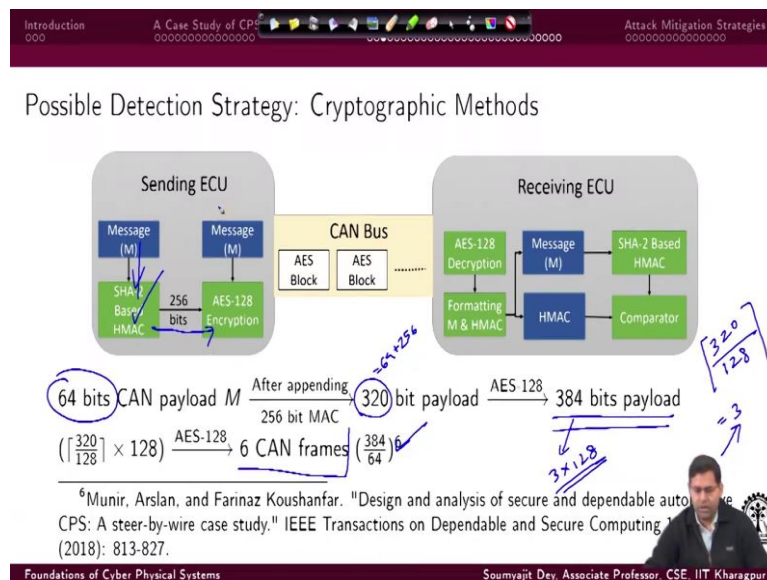
Indian Institute of Technology – Kharagpur

Lecture – 55

Attack Detection and Mitigation in CPS (Continued)

Hello and welcome back to this lecture series on Foundations of Cyber Physical System. So, we have been talking about several attack detection techniques for for a CPS implementation.

(Refer Slide Time: 00:41)



So, let us carry on ah from that point. So, in the previous lecture, ah we we had covered ah this cryptographic method. And we were trying to show that well ah why really you have ah CAN bus overload if you are trying to encrypt packets and followed followed by this Ah message authentication code computation. So, if you just look into this process, so, you have a 64 bit, CAN payload, let us say and when you are trying to send it, you see in this pipeline, you first have a message authentication code which gets appended so that is 64 plus 256. right So, this should give you 320. Now, ah what we are saying is? In the next phase of the pipeline ah you have ah this AES engine. So, typically the way, ah the AES engine is going to work is, ah it is trying to encrypt some data. So, it will do the encryption in blocks of 128. right. So, if you are doing this in blocks of 128.

So that is why it is 3 times 128. So, this is the total number of bits which are contained in these three packets. right So, eventually that is why we are saying that well when you start with 64 bits, you will end up with an increased load of this 384. right So, and and that that is essentially I mean I mean when you are trying to transmit it here. This will actually result in 6 CAN frames. So, this has been taken from this reference. Ok

(Refer Slide Time: 03:18)

And if you recall after this, we went further into our discussions on lightweight detectors.
(Refer Slide Time: 03:23)

Introduction
000
A Case Study of CPS
0000000000000000
Attack Mitigation Strategies
0000000000000000

Stateless Detector

II. χ^2 -based detector:

- ▶ Let, the covariance of estimation error $e_k = x_k - \hat{x}_k$ is Σ_e .
- ▶ Covariance of residue r_k be

$$\Sigma_r = E[r_k r_k^T] - E[r_k]E[r_k]^T = E[(Ce_k)(Ce_k)^T] + E[v_k v_k^T] = C \Sigma_e C^T + \Sigma_v$$
- ▶ χ^2 statistics $g_k^a = (r_k^a)^T \Sigma_r^{-1} (r_k^a)$.
- ▶ χ^2 -based detector signals an error if $g_k^a \geq Th$

$\Sigma_r = C \Sigma_e C^T + \Sigma_v$
 $= E[(r - E(r))(r - E(r))^T] + E[v v^T]$
 $= E[r r^T - r E(r)^T - E(r) r^T + E(r) E(r)^T] + E[v v^T]$
 $= E[r r^T] - E(r) E(r)^T + E[v v^T]$
 $= E[r r^T] - E(r) E(r)^T + \Sigma_v$

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So, for for simplicity, this is for the kth instance right. So that is why r_k ah, let us just write r for any instance in general. So, this is ah by definition the covariance of the residue in the kth instance. So, we will try to see that well how we can write it directly as this expression. Because if you can write this expression then it becomes simple. Right Because we will just write r_k is C of e_k ah as you can see. Right

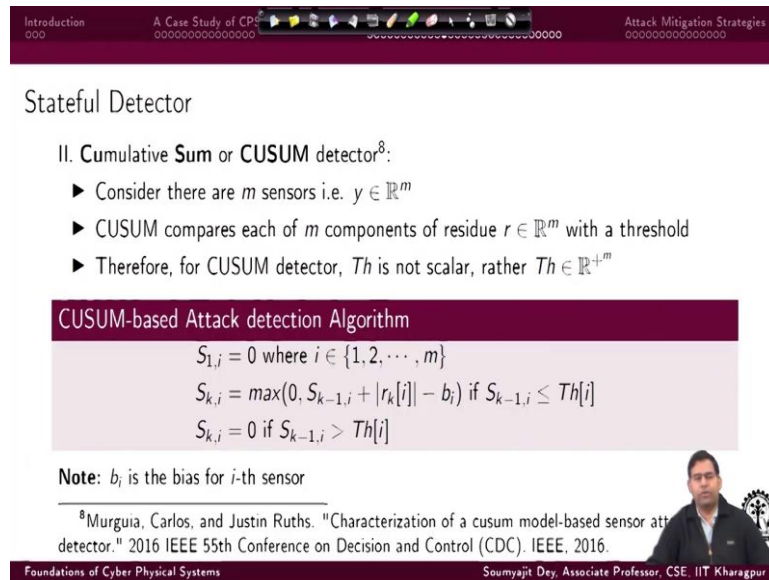
Because ah yeah and then when you talk about this expectation here, ah it is let us, if you just go back. ok First, let us do this derivation and then we can go back to the previous things here. So, sigma r is covariance of r so, if you just apply the covariance formula. Now, if you you just bring in the expectation here, this again expectation over two expectations. So that that it is an invariant, so, it will remain whatever it is.

So, you get something like this. So then you can just strike up any two of this, so, you get. yeah So that is what we have here. So, ah once you are able to compute this sigma r here. ah Then you can just apply this chi square statistics formula ah which uses this sigma r inverse here. Ah So that is basically doing a scalar computation, ah where it is using the residue under attack and it is transpose.

So, this is the r_k^a that is residue under attack in the kth instant. ah It is transpose and the original value here and they are being weighted by sigma r inverse. So, this is just chi standard, chi square statistics formula and you are applying that here to check that whether it will be greater than the threshold or not. So, you can just use the constant threshold base detector which just compares directly the value of the residue with respect to the threshold.

A standard norm base detector or you can use this kind of chi square statistic space detector which is this one. Fine. ok ah

(Refer Slide Time: 10:18)



Introduction A Case Study of CPS Attack Mitigation Strategies

Stateful Detector

II. Cumulative Sum or CUSUM detector⁸:

- Consider there are m sensors i.e. $y \in \mathbb{R}^m$
- CUSUM compares each of m components of residue $r \in \mathbb{R}^m$ with a threshold
- Therefore, for CUSUM detector, Th is not scalar, rather $Th \in \mathbb{R}^{+m}$

CUSUM-based Attack detection Algorithm

$$S_{1,i} = 0 \text{ where } i \in \{1, 2, \dots, m\}$$

$$S_{k,i} = \max(0, S_{k-1,i} + |r_k[i]| - b_i) \text{ if } S_{k-1,i} \leq Th[i]$$

$$S_{k,i} = 0 \text{ if } S_{k-1,i} > Th[i]$$

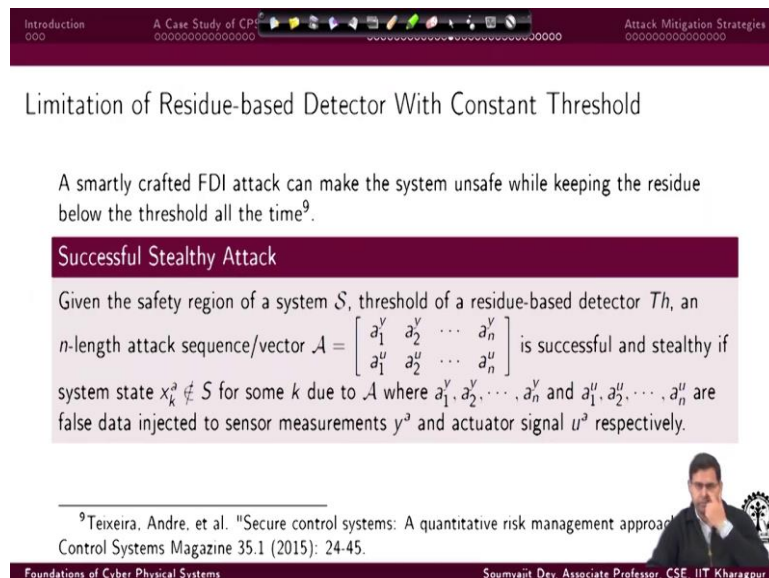
Note: b_i is the bias for i -th sensor

⁸Murguia, Carlos, and Justin Ruths. "Characterization of a cusum model-based sensor attack detector." 2016 IEEE 55th Conference on Decision and Control (CDC). IEEE, 2016.

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So, maybe we can progress further from here because we also discuss the state full detector.

(Refer Slide Time: 10:23)



Introduction A Case Study of CPS Attack Mitigation Strategies

Limitation of Residue-based Detector With Constant Threshold

A smartly crafted FDI attack can make the system unsafe while keeping the residue below the threshold all the time⁹.

Successful Stealthy Attack

Given the safety region of a system S , threshold of a residue-based detector Th , an n -length attack sequence/vector $\mathcal{A} = \begin{bmatrix} a_1^v & a_2^v & \dots & a_n^v \\ a_1^u & a_2^u & \dots & a_n^u \end{bmatrix}$ is successful and stealthy if system state $x_k^a \notin S$ for some k due to $\mathcal{A}$ where $a_1^v, a_2^v, \dots, a_n^v$ and $a_1^u, a_2^u, \dots, a_n^u$ are false data injected to sensor measurements y^a and actuator signal u^a respectively.

⁹Teixeira, Andre, et al. "Secure control systems: A quantitative risk management approach." Control Systems Magazine 35.1 (2015): 24-45.

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And what are the limitations that are going to be there if we have this residue base detectors with Contin constant thresholds. right ah

(Refer Slide Time: 10:35)

Introduction A Case Study of CPS Attack Mitigation Strategies

Limitation of Residue-based Detector With Constant Threshold

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Because we said that well that there can be a successful stealthy attack because somebody can stay below the attacker. If the attacker has ah knowledge about the systems model, the threshold threshold value that has been set for the detector. Then he can choose to inject the system ah with so, small amount of attacks in both the measurement values and the control values in such a nice way that every time you calculate the residue. Ah

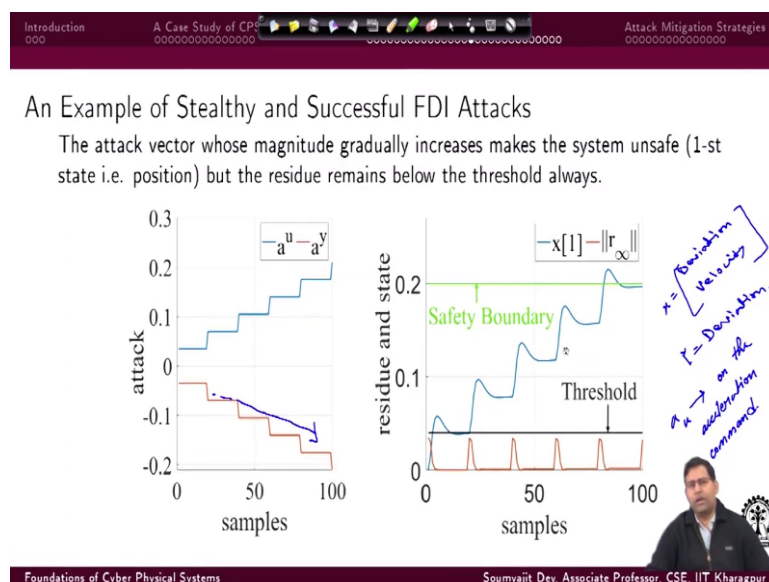
So, every time you will calculate the residue ah here. ah it will be It will remain below the threshold. ah But ah eventually the systems trajectory will we will get out of some safety boundary. And the thing is, you are unable to actually measure it also because the measurements you are getting of the system. ah Those measurements are also being pastured by the a y component of the attack. And due to this, and due to that measurement perturbation, you have an issue there. Right And do to that measurement perturbation you are unable to kind of figure out that well the system is really going out of the boundary. So that can be a situation.

(Refer Slide Time: 11:52)

And g is a constant value. ok So, you are ensuring that this overall attack value which is slowly increasing ah or with every k th instant. Because λ is increasing in exponentially and k is also increasing in a linear manner. So, ah over an N length of N may be a significantly large value. ah you are, you are injecting different attack values and the way you will like to do this kind of attack attack is because if we have this kind of a generator ah enclosed in an algebraic expression, then it is very easy to implement in a computer that well these are the attack values which will go and get injected to that variable very fast continuously. right Because the attack values have to go in sync, with the periods of the system, the sampling sampling period of a system. Because you want the attack values to part of the measurements here. So, what you are doing is for this system you have ah your states that is displacement from the reference trajectory and the speed of the velocity and the speed of the vehicle.

This will not be velocity, of course. And whatever I mean you are also you are you also you are also measuring this deviation from the trajectory. So, x is essentially the deviation and the velocity and your y is basically one of the x components itself which is the deviation component. Ok So that is why, when you are applying the attack, it is basically, you are choosing one of the components because you are trying to attack one quantity here. Ah

(Refer Slide Time: 14:01)



So and based on that this is our plot of what values of attacks we are really doing ah on y and what values of what attack we are really doing on u . So, you can see that this is my ah a k right and based on that I am generating a sequence of values of attack for the x variable. ah sorry the sorry, the y measurement variable which is basically the first state in x . right ah And also we are generating an attack on the ah on the acceleration.

And accordingly, you have a u and a y . So, if I just write down the states here. So, x is deviation from the trajectory intended trajectory. And this is velocity and the measurement y that I am taking is the deviation itself. And that is what I am perturbing. right ah So that is my perturbation here. And I am doing the attack a u on the control input which is ah which this setup on the control input is basically happening on the acceleration command.

So, based on that as you can see, ah you you have this systems deviation slowly. This is the actual deviations value not the attack value. And that value is slowly wearing out of the safety boundary.

(Refer Slide Time: 15:57)

Introduction A Case Study of CPS Attack Mitigation Strategies

Variable Threshold-based Attack Detector

How can we compute the threshold of such a lightweight residue-based attack detector?

1. The attack should be detected as early as possible.
2. False alarm rate should be minimized as much as possible.

Possible Solution: The detection parameters (threshold, window length for stateful detectors) can be changed adaptively by observing the system states under FDI attack¹⁰.

Handwritten notes:
 $x = \begin{bmatrix} \text{Deviation} \\ \text{Velocity} \end{bmatrix}$
 $y = \text{Deviation}$
 $u \rightarrow \text{on the acceleration command}$

¹⁰Koley, Ipsita, Sunandan Adhikary, and Soumyajit Dey. "Catch Me If You Learn: Real-Time Detection and Mitigation in Learning Enabled CPS." 2021 IEEE Real-Time Systems Symposium (RTSS). IEEE, 2021.

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So, the next thing we talked about is well is it possible to have a variable threshold-based detector and we discussed about an RL based strategy.

(Refer Slide Time: 16:11)

Introduction
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A Case Study of CPS
0000000000000000
Attack Mitigation Strategies
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Variable Threshold-based Attack Detector

Let us consider a windowed χ^2 detector in place whose detection threshold and window length are to be changed dynamically.

- ▶ The χ^2 statistics g_k of residue r_k is $g_k = \sum_{i=k-l_k+1}^k r_i^T \Sigma_r^{-1} r_i$ where Σ_r is covariance of residue and l_k is χ^2 test window length at k -th iteration.
- ▶ Probability density function PDF of g_k is $P(g_k) = \frac{g_k^{\frac{ml_k}{2}-1} e^{-\frac{g_k}{2}}}{2^{\frac{ml_k}{2}} \Gamma(\frac{ml_k}{2})}$ with mean ml_k where m is number of sensors.
- ▶ The cumulative distribution function w.r.t. Th_k is defined as $P(g_k \leq Th_k) = \frac{\gamma(\frac{ml_k}{2}, \frac{Th_k}{2})}{\Gamma(\frac{ml_k}{2})}$.



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We will talk about that again in the next lecture. Thank you for your attention.