Foundations of Cyber Physical System

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Lecture No # 18

Module No # 04

Dynamic System Modeling, Stability, Controller Design(Contd.)

Hello and welcome back to this lecture series on Foundation of Cyber Physical Systems. So, we will just continue from where we left.

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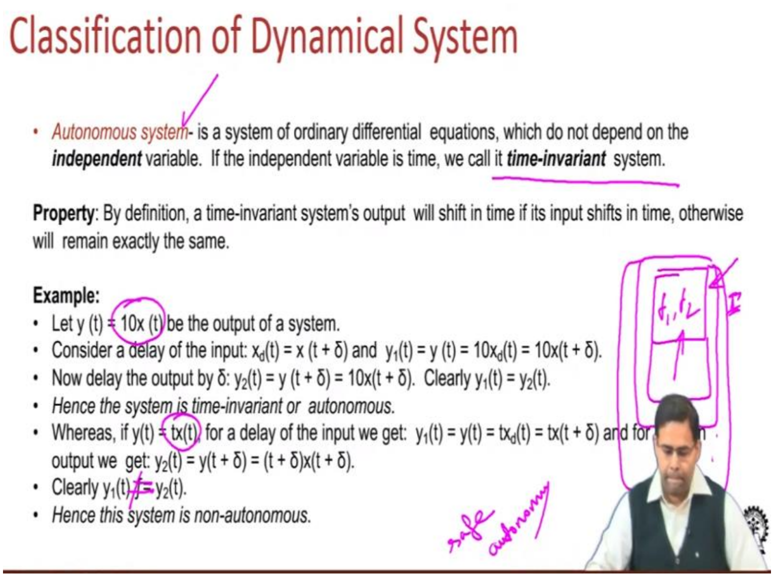

So, we have been discussing about (refer time: 00:34) the classification of dynamical systems. So, we have talked about what is an autonomous system and we said that it is something which does not depend on any independent variable, right. And if it does not depend on the definition of the system does not depend on time then we have a time invariant system. We gave some examples, for example this is a time invariant definition whereas if we take this is time variant definition.

And as we saw that, since this is a time variation variant definition when we are changing the delay in both the input and output the systems, they are not really equal here, right. And I, as I also mentioned that when I say autonomous it means that well you have an input here and this is let us say your system model. The system model itself knows how to behave with respect to the change in the input and it is not that well at different points of time it depends, depending on various values of the input how the system will behave, that is kind of externally controlled.

Let us say you have a system which can either follow a dynamics f1 or follow dynamics f2. And there is an external agent which tells that when this system will follow f1 and when this system will follow f2, that is a non-autonomous example. But let us say if this logic of when to follow f1 and when to follow f2 is already known to this system that means it does not have to depend on any such thing to react any environmental disturbance I, right.

So that would be an autonomous system. Now, in modern days when we always talk about autonomous systems in a very general way and because with the advent of data driven control and stuff like that, we have a significant amount of autonomy everywhere around us, it is also in software operations. Modern software's are so much intelligent they have control components themselves where basically by that, I mean, the decision logic which is so much AI driven, that increasingly more and more of this kind of layerings are going inside the systems definition and increasing their autonomy, okay.

So that is how it is. But of course, you also want the system to maintain certain safety requirements and other stuff. Where if, in case you want to apply such autonomy ideas for safety critical systems and that leads to this definition of what we call as safe autonomy.

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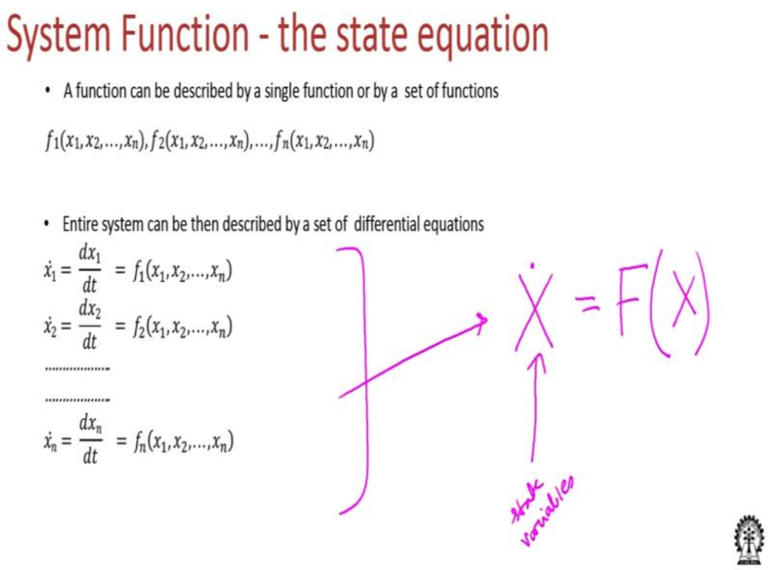

So, in the previous lecture we talked about, I mean, well (refer time: 03:24) how to model system like this and when we define that how we are trying to create a dynamical system. We said that eventually what we need is, we need to identify this set of state variables for the system and we need to identify well these states of the system, how do they change with respect to their current state. So, we need to identify this flow function. So, once we have this, we say that well we have dynamical system model for the system under question here.

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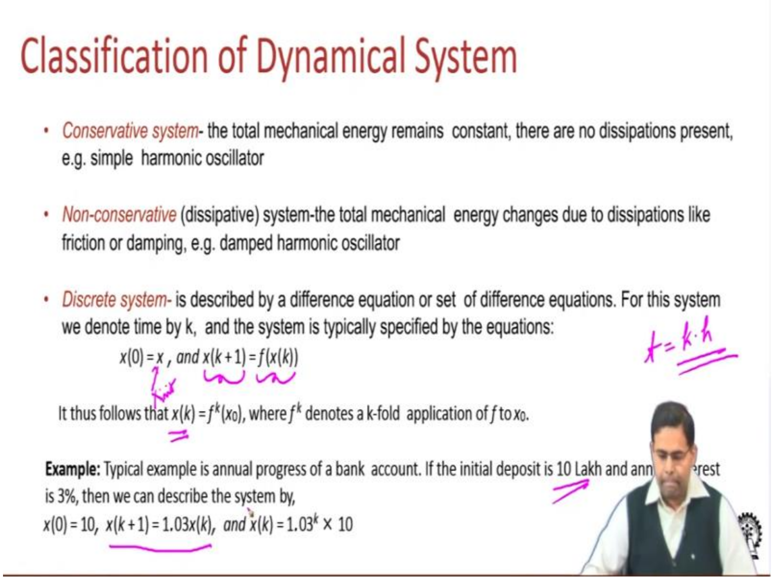

So continuing from this classification of dynamical systems we talked about conservative and non-conservative or dissipative systems. So, conservative system means the total mechanical energy would be constant and there is no dissipation that is present. For example, a simple harmonic oscillator but the more practical one's are the non-conservative systems where the total mechanical energy will definitely change due to dissipations due to practical things like friction and damping.

For example, in real life what you essentially have is a damped harmonic oscillator. And then we talked about another classification which was discrete and continuous systems. So, when we talked about discrete systems it is like instead of having a set of differential equations, a set of continuous differential equations to describe the system state values at every real time point. We can have a discretized system where we do not have this notion of continuous time.

So, when I say continuous system what we are really talking about is well I have this knowledge of x(t) where t is something that is a member of the positive real's. And when I say that well, I do not work like that and what I am more interested in is let us say if this is your time axis and then you are not interested in all the infinitesimal number of points that you have on the real time axis.

But rather you are interested on some specific time points let us say with a repeated periodicity of h. And you are interested in these time points, 1 times h, 2 times h, 3 times h, so and so forth. Then instead of having continuous dynamical equations, what you have a difference equations. Soa a difference equation means at this point the value of the state would be given by some X(1). Here it would be given by some X(2), here it is given by some X(3) and so on and so forth.

So, when you write it like that, what you essentially have is let us say some initial state so this is some x in it which is known and otherwise in the next step what x(k + 1) will be known as the function of x(k). So, if you work like this, suppose I want to know that some time t = k times h or the kth times instant what is the value of the state, so it is given by k and the way I will evaluate it is just, I will say that it is k times application of f on initial state x naught.

$$x(k) = f^k(x_0)$$

A simple example of dynamics the annual progress of bank account so; if you have this deposit of 10 Lakhs and you have an annual interest of 3%, then you can describe this system with a period of 1 year as something like this, right.

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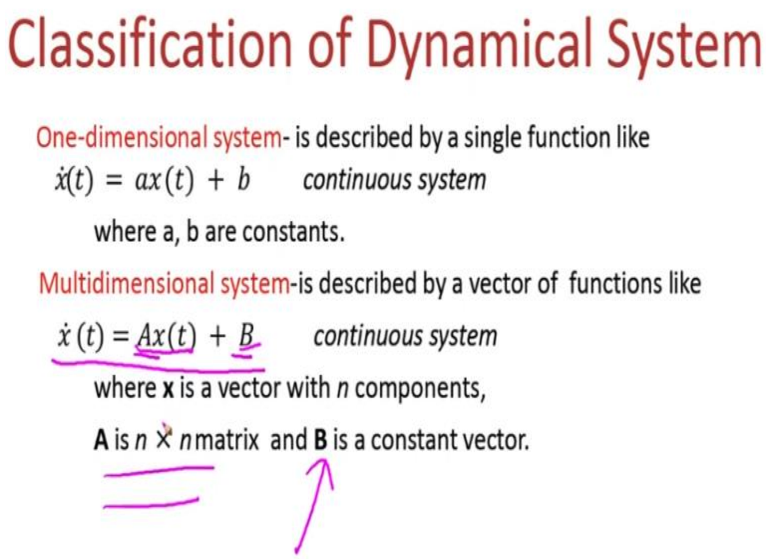

So, the other classification we have was suppose you have a very simple system there is only one variable of only one scalar variable of interest. So, it is a one-dimensional continuous system and if you have typically what we have is a complex system with multiple continuous variables, right. And then you will have a multiple dimensional system where if there are n components then x is vector of size n and the way you will talk about the flow is this where you have constant part and a variable part here, right.

So this B is a constant vector and A is like the dynamics vector of the system, right and A definitely is an N cross N matrix because it is going to be getting multiplied with an n cross 1 vector to give you n + 1 vector and b is n cross 1 vector. I mean B is a n sized vector to be precise.

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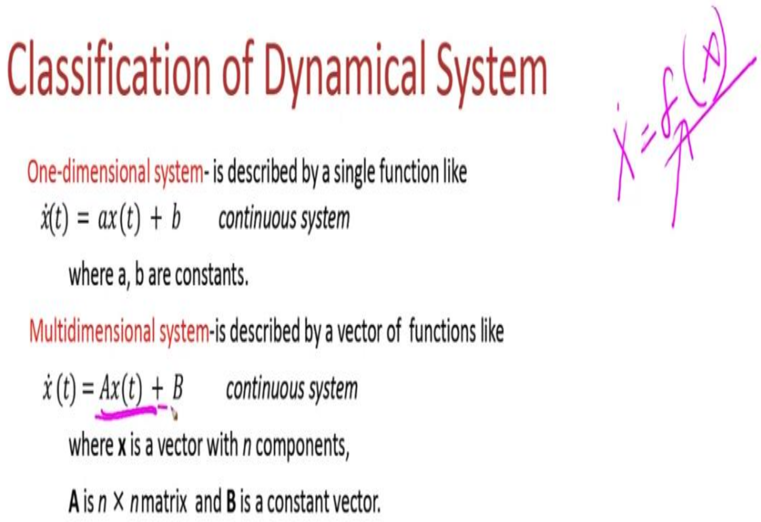

So here we have given you an idea that how a continuous times system is going to be modeled, right. So, you will need that x dot = f of form. And so, if it is as linear system then so in general when we say like it is f dot, where f is any function linear or non-linear. When this is linear what we will have is the representation like this.

$$\dot{x}(t) = Ax(t) + B$$

So this gives you a line in the n dimension and there is a constant shift of B.

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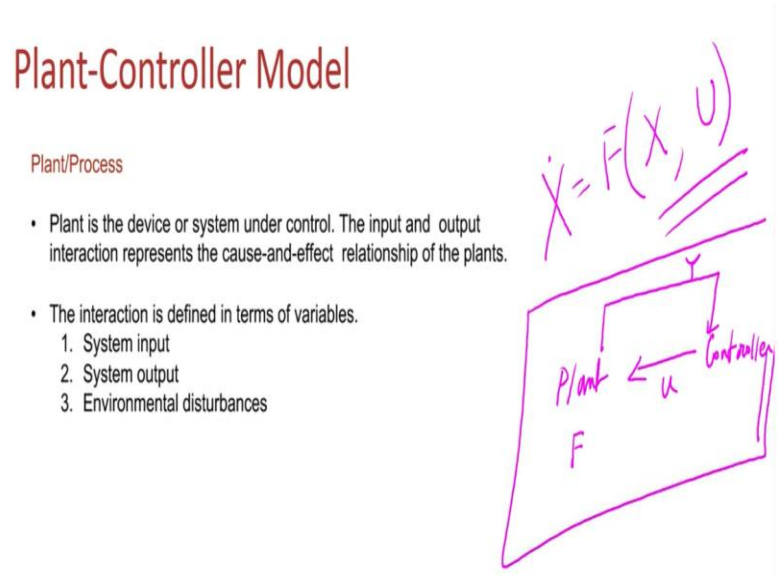

Then next thing that we will do is we will now bring the controller into the plane. So, we have talked about modeling plants now of course our interest here is in controlling such plants, right. So, you have the input and output relation. That means the plants will have its state and the state will be the input for the controller and the control will have some output u and this based on this u the plants will change its trajectory, right.

So, what we are now going to do is, we are going to talk about x dot as some function of x as well as some external control u, right. So, you see you have a plant which is following this function f and you have a controller which is generating this u based on this feedback measurement y, right. So, this plant is not autonomous, but if it looks at the plant and the controller is the combination and that is your world view, it’s an autonomous system here.

So, what we need to talk about is for such autonomous systems what are the system input? What are the system outputs? And what are the external disturbances which I am unable to model using whatever artifacts we are described till now, right.

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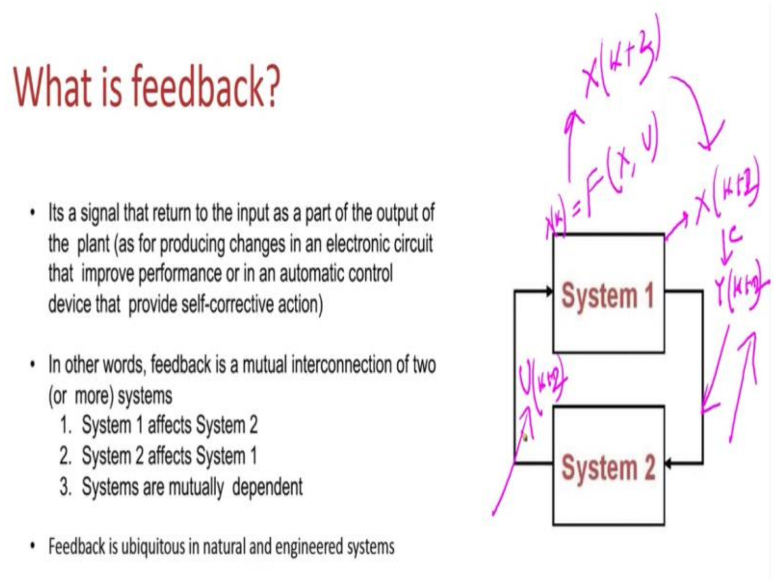

Now in this situation, the one important thing we must introduce is the concept of feedback, right. Because if you remember the previous picture I was just drawing, I said that the plants measurements are going to the controller and the control input is again coming back to the plant, right. So it is like a signal that returns to the input as part of the output of the plant, right. And also I mean so this is typically the idea of where we bring feedback. So, if I generalize it if you look at the right-hand side diagram we are showing that there are 2 systems.

System 1's output effect system 2. System 1 can be plant, whose measurement will be used by the controller to compute a control output. And this control output again if you remember the equation I just wrote, we said it is F of X, U. Here, you have U and here the system changes from some x(k) to x(k+ 1). I am writing in a discrete sense here.

And, this is being acted upon by some C and that gives you a y(k + 1) which is used by this system 2, which is the controller to generate a U(k + 1) and then this U(k + 1) and the current X(k+ 1) would be used here in this system to generate. So earlier it was X(k) using this equation I got X(k + 1) and the measurement I got was C times X(k + 1) which is Y(k + 1) there is the output variable.

And this is being measured I need this controller takes this is the input and generates U(k + 1) using U(k + 1) and the current X(k + 1) this equation would generate X(k + 2) here and then again this is 2 this is Y changes to 2 this will change to 2 this will be used here again to generate X(k + 3) and that is how the evaluation will continue. I hope that is clear here right. So, in general what we have is feedback which is the mutual connection between 2 systems, right. A classic example is plant and controller.

And as we understand in nature feedback is ubiquitous, right. So, how your climate change occurs is based on a feedback loop where there are several variables. For example, how much contribution your vehicle does in terms of nitrogen and carbon content to the environment. How much carbon contribution is provided by the factories which are producing the electricity etc. etc., right. So, if you look at your nature biological systems the climate system, they are all feed they are all continuous dynamical systems with very complex feedback loops.

And, the same principle follows into all our engineering systems most of them are implemented to react to something. So, there will be an external input based on which the system will do something and based on what it does, it will observe what happen. And then based on its observations why it will again do something, U and again your system will change the world will change. And this changes in the world will again update Y and based on those changes you will again generate U and that is how this feedback loop will act, right.

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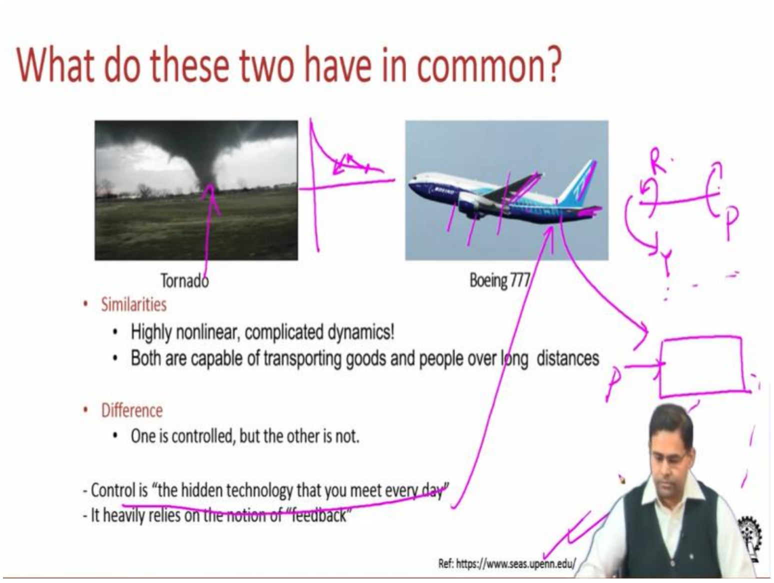

So when we talk about dynamics and we talk about control as a simple example we have the picture of a tornado and a Boeing 777. These are some example pictures taken from here. And you see that both are dynamics systems and we are talking about control here, right. So the tornado is something which is high in non-linear and having a continuous dynamics and the difference is this Boeing 777 is also non-linear trajectory it may have and there are several complex control loops internally, I mean, which are continuous time, and they are deciding that how the vehicle this aircraft will move with it will change this trajectory with time etc.

But the difference is this one has internal control signals. It can observe the environment and based on that the pilot as well as the pilot will give some input to the control systems here. And based on this control systems these guy’s engine speed will change, its flaps will be deployed or not deployed, its yaw, roll and pitch, the primary controls for the pilot.

If I look at it, you have roll here, you have yaw here and you have pitch here. So based on its intended of where it wants to go, the pilot will suitable comments that will control these values. Based on these comments this all these control surfaces here in the tail and in the wings, they are going to change their positions. When the plane wants to land, the pilot will activate again suitable control surfaces to engage the landing gears here. The flaps here, the engine breaks here and all the stuff.

So, this is the controlled system which is kind of autonomous because it kind of exactly knows how to be. If I am modeling the pilot behaviour as a part of the system then the system knows how to exactly behave based on changes in the input, right. This is a system where I mean apart from nature there is no control, right, because it is changing there is nobody which is going to control these dynamics, because well like I am saying it is all about the frame of reference from which you are looking at it.

From out frame of reference, at least we are not the people who are controlling the dynamics of this thing. But in this case the people sitting inside the pilot is, right. So, in general what we can say that this control is the hidden technology that we meet every day, right. So that is the important quote we have from these references here, right.

And this idea of control is heavily relies on the notion of feedback. Like we just said that well the pilot will I mean the plane wants to follow some intended trajectory related let us say like this. And based on the pilot’s idea he is giving some of this Y, R and P commands and then again at this points he wants to go here and he gives some commands but he saw that well it is coming like this so that is like a feedback for the pilot.

So, the pilot now can change all these comment values and again it will course correct and come back here, right. So, there is a feedback inside this control loops which are here there is a feedback of the pilot looking at the environment and then acting on the control surfaces, right.

Now if I look into any of these control surfaces, they are also implementing low level control loops, right. So for example, the pitch controller is the pitch command from the pilot and accordingly there will be certain subsystems which will be doing something to achieve that pitch, right.

After one iteration it sees that it is still not achieved it will again do something like that, right. So, control works at various levels that is the point I am trying to make there are high level control loops there are low level control loops there are even lower-level control loops.

Every high level control loops is going to give a reference to a lower level control. Based on that the lower-level control loop can give a reference to the even lower-level control loop. And that is how works in nature and that is how it works in every complex engineering system that we have around us.

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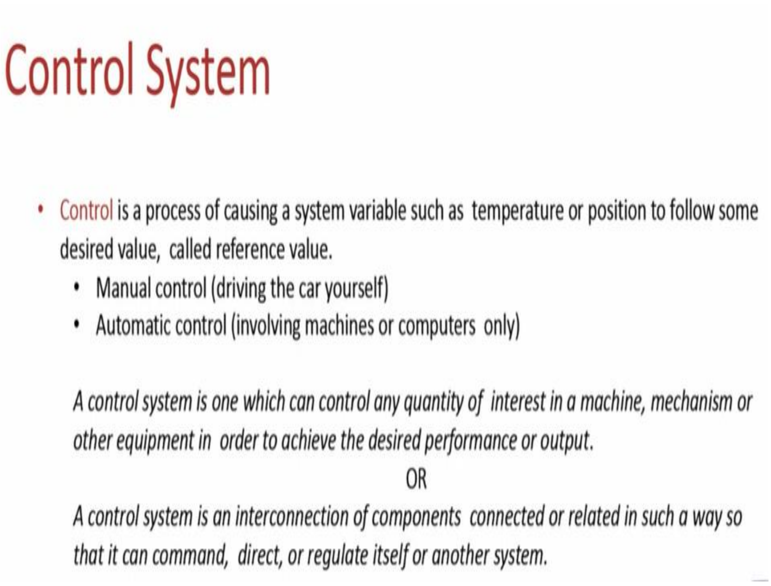

So overall if we try to come up with some philosophical definitions when we talk about control it is a process of causing a system variable let us say it is temperature or its position to follow some desired value or reference. If you are doing a manual driving of a car, you are giving reference to some lower level control loops. If you are riding in an autonomous vehicle the vehicle perception system is generating those reference values for the lower-level control loops.

And, like I said those lower levels control loops may be generating references for even lower-level control loops. So, it is entirely about who is giving the references to whom. Even if I consider and fully autonomous vehicle, the question is, I say it is autonomous because if I just model the vehicle it is autonomous. But that would also require some control inputs from the user which is his desire to go from place A to place B through place C. So, that is like a control input, right.

So again I will repeat, it when we talk about autonomy it is all about what frame of reference. If inside my frame of reference there is nobody taking a decision which is manual or which is not already implemented the decision logic is not already implemented, it is an autonomous, it is a non-autonomous system. But if inside my reference frame of reference, every decision logic is already there, I do not have to take the decision, then I am inside a autonomous system, okay.

So, in general I can say that a control system is one which can control any quantity of interest in a machine a mechanism or rather equipment in order to achieve some desired performance or desired output value. Or it is an interconnection of components connected to or related in a way so that it can command, actuate, direct, regulate whatever you call it any, I mean, itself any other subsystem which is a part of it.

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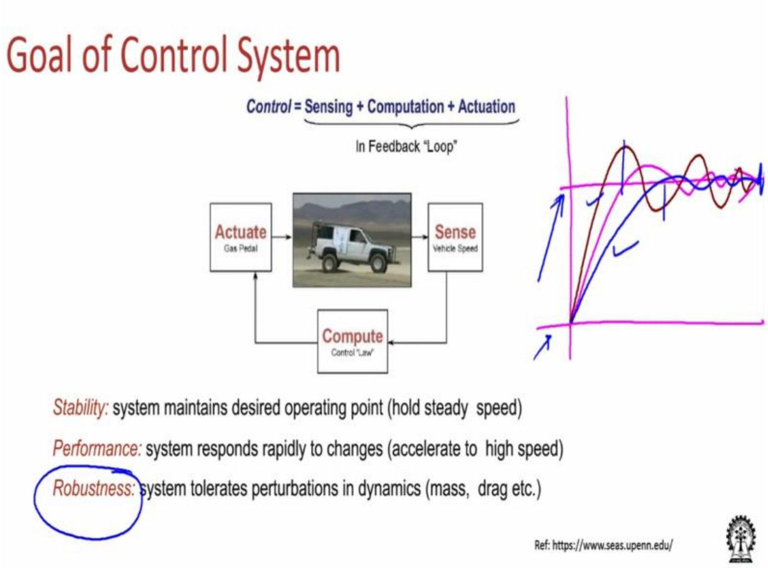

So for example if we come to this vehicular system you have this control loop, right, like a classic control loop where you have sensing then computation and actuation which is happening in the feedback loop. And this feedback loop is designed around the plant we use the vehicle, okay. So actuator will actuate the gas pedals angle right and then the gas pedals angle will decide that how much of gas will be what is the amount of fuel to be injected into the engine and that will control the engine subsystems, right.

The sensing part is sensing the vehicle speed and accordingly the control law is deciding how much should be the gas pedal pressed, right. So, when we do all these things, we need to have some ways to measure that well my design is a good design or a bad design. I need a way to measure that right. So, in this way what we need is some performance measures and some metrics of what we call as stability.

So let us understand these measures which tell me about how good or bad a control system design is, so first of all stability. Stability means that in this case if I interpret the meaning of stability, it means that how nicely is the system able to measure is able to maintain a desired operating speed, okay. Now so that is about stability. And then the question is performance. It is about rapidly the system response to request of changes.

So, if I just try to give an idea here. So let us say this is my desired speed, okay. And then if I say that will is the system stable? The question of stability is that well, this is my desired reference. How fast, I mean, how nicely do I get to this reference? So that means am I able to maintain this desired speed or not, right. If I am able to maintain this, well I am a stable system. If I am able to achieve this thing.

Now the question is well I can achieve this thing in various possible ways, right. So, I can achieve it like this, just I have drawn, or I can achieve it like this, or I can achieve it like this, right. What is good that is the question. I mean when do I say that this is good or this is good how do I decide that. So, this brings us to the question of performance so let us say this was a rapid command initially I was here and then a command came to go here and responded back.

So, this is like a fast response right but also this is the fast response but the numbers of perturbations are big. These are like slow responses for the number of perturbations are less, right. So, this is like a fast response, so this brings in the question of performance. Now the other thing is the question of robustness. So, suppose I have reached this goal and then there are perturbation due to external noise and other stuff, right.

So, which are disturbing me from attending the goal completely. Now do my system reject those perturbation well? That means does my system does my system have the capability to identify well this is a noise and this is really a control command and it will react nicely to the control command. And this will reject the noise so that is what we call as robustness, okay. So, these are the typical metrics on which my system depends. As long as, whenever there is a input and I am able to achieve that input I say that my system is stable.

As long as I have achieved the input and I am able to reject any noises I would say that noise system is robust, okay. And I mean these are also very much interrelated definitions. I can say that its robust is stable because well I am at my operating point here and there are some disturbances coming in. In spite of these disturbances, I am able to reject them and I am finally able to again go back to my operating point, so that is also a notion of stability here, right.

And then we say about performance it means that how fast does my system response is it very slow to react to an input or is it quite fast to react to the input. So that is about performance. So as you can understand, a control engineers primary goal would be design controllers, design the entire loop, the timing and the delay in the loop in a way that all these metrics whatever they have been specified those are kind of attained. So, we will look into those aspects in future, and with this we will end this lecture, and thank you for your attention.