Foundations of Cyber Physical Systems

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Module No # 04
Lecture No # 18
Dynamical System Modeling, Stability, Controller Design

Welcome back to this course on Foundations of Cyber Physical Systems. So, from today's lecture we will be discussing this topic of how to model dynamical systems which is very important for creating a formal model for continuous time systems. Primarily that we call as plants which are to be controlled, and also in this week we will learn about how to design some basic controllers which satisfy some control objective with respect to these plants. So, let us get on with this topic.

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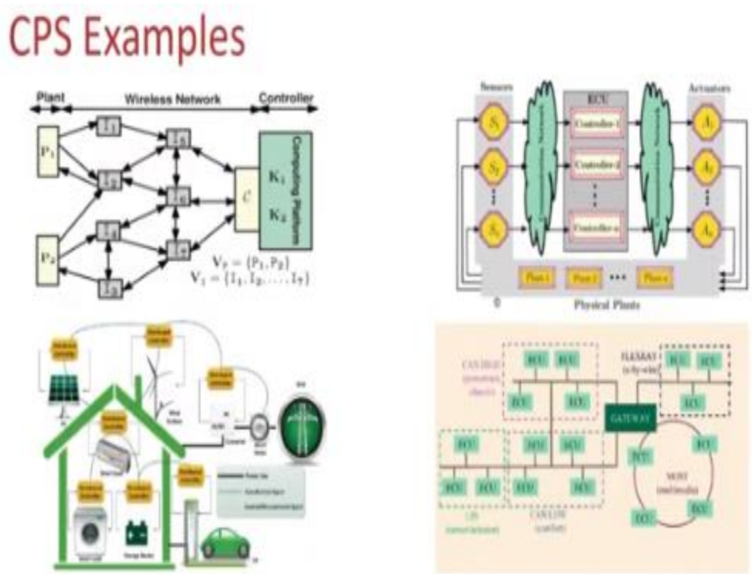

So, if you remember from our earlier lectures we have talked about several cyber physical system examples. And in all of these examples we are talking about how some controller is going to control some plants through a cyber-physical infrastructure. So, we will be touching upon these concepts of mathematical control theory

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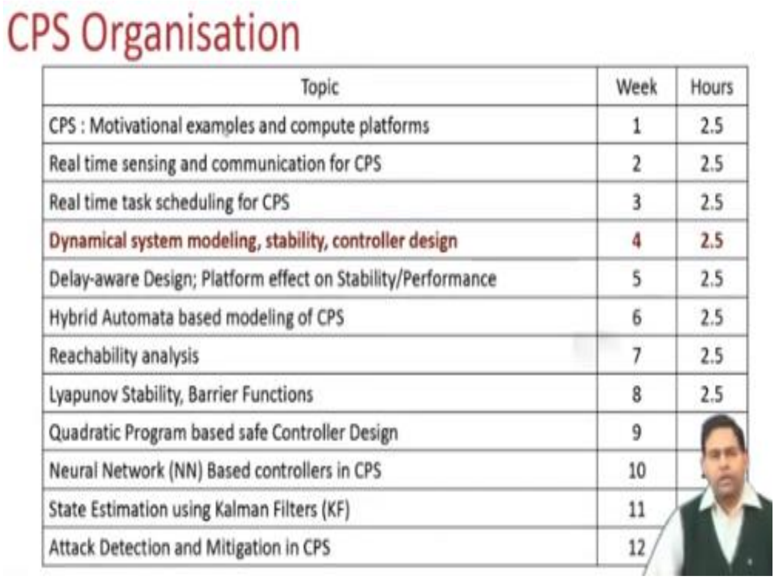

CPS Organisation

Topic	Week	Hours
CPS : Motivational examples and compute platforms	1	2.5
Real time sensing and communication for CPS	2	2.5
Real time task scheduling for CPS	3	2.5
Dynamical system modeling, stability, controller design	4	2.5
Delay-aware Design; Platform effect on Stability/Performance	5	2.5
Hybrid Automata based modeling of CPS	6	2.5
Reachability analysis	7	2.5
Lyapunov Stability, Barrier Functions	8	2.5
Quadratic Program based safe Controller Design	9	
Neural Network (NN) Based controllers in CPS	10	
State Estimation using Kalman Filters (KF)	11	
Attack Detection and Mitigation in CPS	12	

which are required, I mean, for a student to learn in case they are embarking on this idea of that they want to model such dynamical systems at a theoretical level to understand what it means by stability of systems and how to do some basic control design. So, what we will be starting with is how to model continuous dynamical systems.

Systems for which we expect that with time continuum there exists some evolution and how to control such systems. So, I will just repeat that this is a very deeply studied topic of basic control engineering which we will be covering at a high level and we will be touching upon the basics, so that it kind of arms you with the artifacts of control theory which are required for modeling and controlling cyber physical systems.

And as you can understand the basic idea in this course is to use such theoretical primitives in a more practical sense that how to make them mappable to processor architectures and network infrastructures and make some good use of them.

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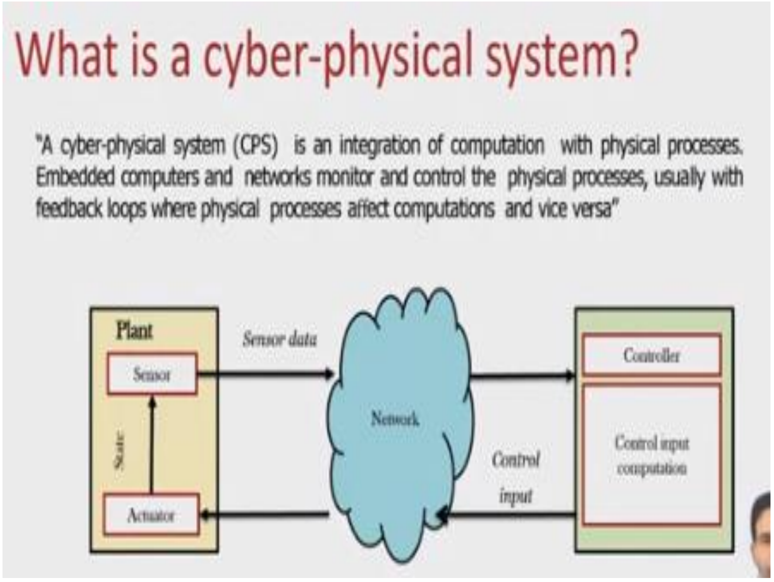

So we will again study from this introductory parts that what really we mean by a cyber-physical system. So essentially, we are talking about an integration of computation along with physical processes and that is primarily what most embedded computers do. That means computers which are very low power small form factor computing chips which may be ubiquitous embedded inside some other network fabric and which is tasked with this idea of monitoring a physical plant and providing it with suitable control messages, okay.

The idea as we have discussed earlier also is that such compute platforms will be executing this control algorithms and they are going to execute in the loop with this cyber physical plant. And what we will be talking about here is how to model this kind of plants mathematically and how to design a control algorithm for which in future we will be writing some programs and scheduling and mapping them to an onboard system.

So, we will be talking about the mathematical parts here, like modeling the plant and creating a basic controller which is going to control the plant, that is all.

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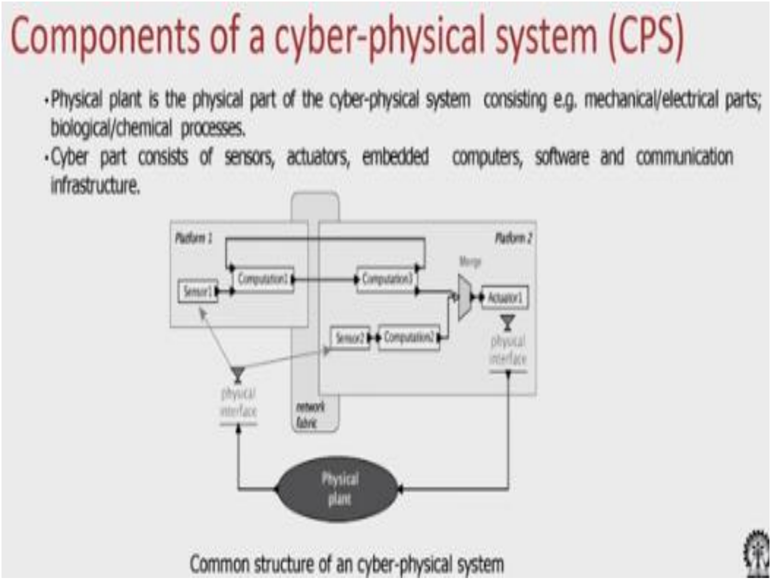

So like we discussed that a cyber-physical system has got various components and inside this big fabric where we have sensors, we have actuators, we have computation, we need to, when we are starting to do a design the first step would be that how you model this plant using a set of equations, okay.

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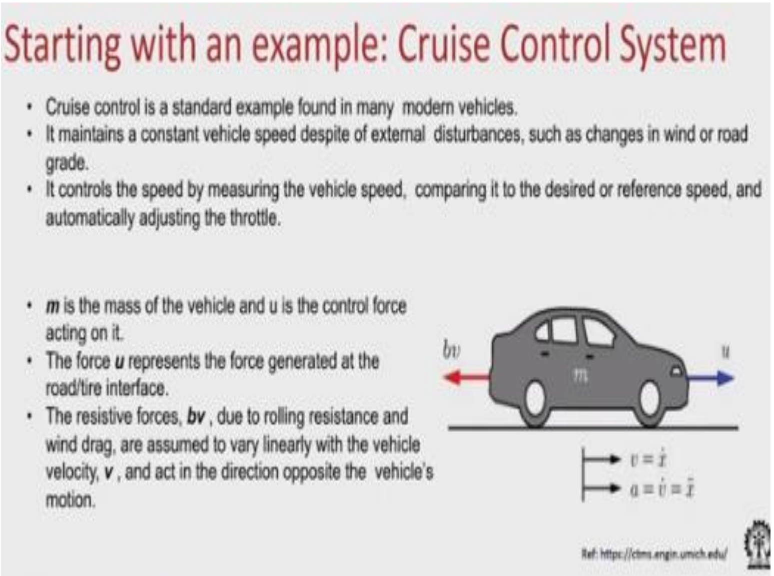

So let us start with one of the very basic examples that are of a cruise control system. So, as we all know that a cruise control is a standard example and it can be found in many modern vehicles. A cruise controller task is to maintain a constant vehicle speed and why that is a difficult task because the vehicle may have may suffer from external disturbances such as the drag resistance etc., which will happen due to changes in wind, due to changes in road condition, which will affect the effect the friction which this vehicle is about to face on the road etc.

So, the idea of a cruise controller is it is going to control the speed of the vehicle and the way it will do is it will measure the vehicle speed at the run time. And compare that with the reference speed and accordingly it is going to decide whether to increase or decrease the throttle input stuff like that, okay. So let us say here we have a simple vehicular model it is taken from this website ctms.in.umiz.edu. So, if you go to this website, you will see lot of examples of such dynamical systems and how to design basic controllers, we are just taking an example from there only.

So, let us say you have a vehicle with mass m and let us say u is the control force which is acting on it and this control force, let us think how it is going to be going to be generated. So the driver is going to press the accelerator or the throttle and that would kind of generate a torque request to the engine, right, and accordingly the engine power trend in order to meet the torque request it will do some actuations.

And due to that, the torque that is being pushed on the wheels of the vehicle that will get affected and accordingly the vehicle speed will increase or decrease. So that is how the thing works at the high level we are just saying that well let us create the force equations of this mechanical system and equate them like we do in standard mechanics. So this force u represent the force that is generated at this road tire interface. So, we are talking about this force here.

And because we need to create the equations for this free body diagram here. Now what are the different forces that are acting on the system? So of course, this forward force, forward pushing force will create acceleration and that will create a speed and change in position of the vehicle, right. So that is one thing. And another is while the vehicle is trying to move forward let us say, the vehicle will also face resistive force due to the surface friction.

Now the way we model such resistive forces is that is typically a proportional to the current speed of the vehicle or the current, so, if the current velocity is v the resistive force is bv due to this rolling resistance, which is, and the wind drag. So, there are different factors such play here and what we are doing is we are doing a simplistic modeling, we are pushing all those factors inside a scalar coefficient b, and well we are saying that well the net backward force would be nothing but it is a proportional to the forward movement velocity and that is why that force is some bv, okay.

So, that is how this force bv is kind of acting in the direction that is opposite to the vehicle's motion and bv is assumed to be varying linearly with the vehicle velocity. Now in order to analyze this vehicle, we will need to figure out which are the variables that that we need to talk about. So of course, the net force will create an acceleration which will create a velocity and that will definitely create a change in the position of the vehicle. So let the position of the vehicle be denoted by x. So that v is $\dot{x}$ and a, is the acceleration which is $\dot{v}$ equal to $\ddot{x}$ that is the second derivative of the position x, okay.

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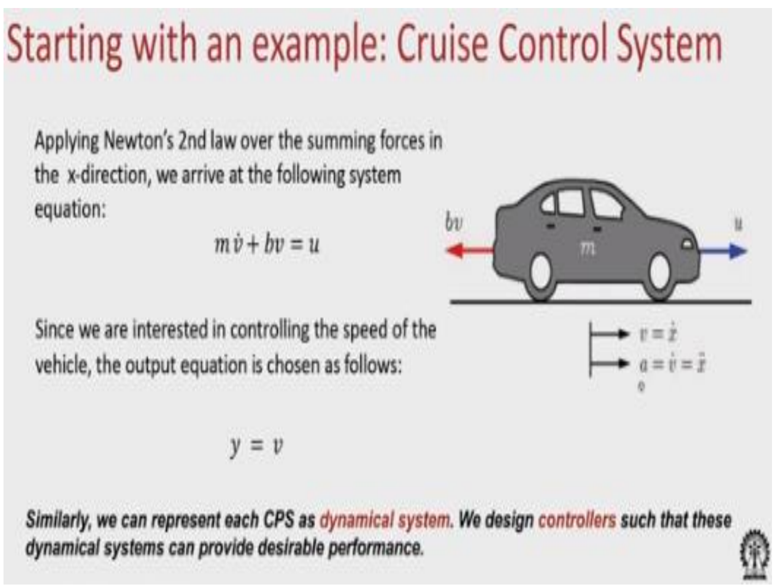

So now if we apply standard equations of motion and sum them up in the x direction, we will arrive at something like this.

$$m\dot{v} + bv = u$$

So that is the net force acting on the vehicle in a forward direction which is $m\dot{v}$ should be equal to u-bv, right. So, you have this equation here and what we are interested in is to control the speed of the vehicle but let us understand that what the controller is going to measure.

So, typically for a vehicle like we have discussed earlier during our lectures on sensors and actuators the way that we measure a vehicle's dynamics is we measure its velocity using some hall effect sensors, right. So, the output equation in this case should be some y which is the measurement variable to be equal to v, right. So, these are the 2 equations which are going to model the dynamics of the system.

And, typically for most CPS systems we can create such a set of equations, right, and we have to create such a set of equations which may be very complex based on the complexity of the CPS itself, right. And what we are tasked here to do is we need to design controllers such that this dynamical system can provide us with some desired performance in this case.

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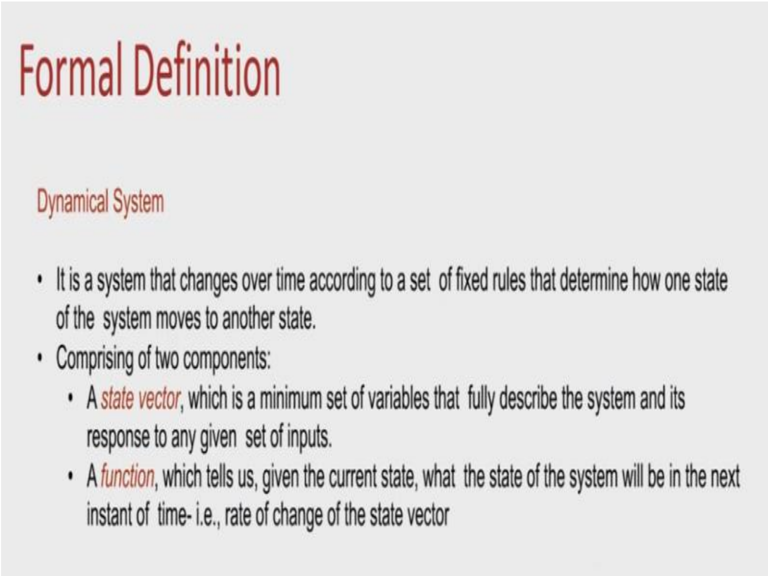

So if we try to give a formal definition here that well what is a dynamical system. It is the system that will change over time according to a set of fixed rules. Now those some of those rules may also be time variant in some cases we will come to those things. And what this dynamical system model determines is how the state of the system changes with respect to time. So, suppose if we have a dynamical systems model what is important to understand is that given this dynamical systems model, if we know some of its state variable values at some state, using this model we should be able to evaluate the state variables values at some future time point, okay.

So primarily when we are trying to model a dynamical system, we will say that it comprises 2 components. One is a state vector that is a minimum set of variables that fully describe the system and its response to a given set of inputs. Now when I say fully describe a system and we talk about a car, a car is a very complex system there are so many variables which are changing their values as time passes, right.

Air to fuel ratio and several other things in the mechanical part and maybe several other things, let us say the temperature inside the car, that is also changing, right. The point is we are trying to focus on a specific functionality of the car. Let us say we are talking about just the cruise controller. So, in this case we will be only concerned about the variables of interest, which are velocity and acceleration and thereby the position, right. And we want to see how this controller affects that velocity those variables and accordingly we will choose that set of variables only.

And if those variables are related by certain equations, then we will try to minimize to a set of variables that fully describe the system, and then we will call this set of variables as a state vector, okay. And the other important part is using this set of dynamical equations or differential equations that I can write which express the inter relation of these state variables and the output variable, we will try to create a functional form which represents how these state variables change their values over time.

So, once I do that what I have is a function that tells me that given the current state and what will be the state of the system in a future instant of time. So that is basically a function which gives me the rate of change of the state vector. So this is the most important part of dynamic or system modeling. We have to identify what is the state vector what are the variables of interest and this is the minimum set of variables that I must model, because if I do not use that; set of variables and kind of eliminate any one of them then maybe I will be missing some property of interest for this dynamical system.

Or maybe I will not be able to model how some of the other variables which I am already considering they are going to change their values, their functional relationships. So, in this is why we will try to figure out what is my state vector and then we will try to figure out what is this function that tells me that given this state vector at t equal to t. Let us say t, what is the time what is the value of the state vector at some any future time point t prime. So that that is what this function which we call as a flow function will actually tell me.

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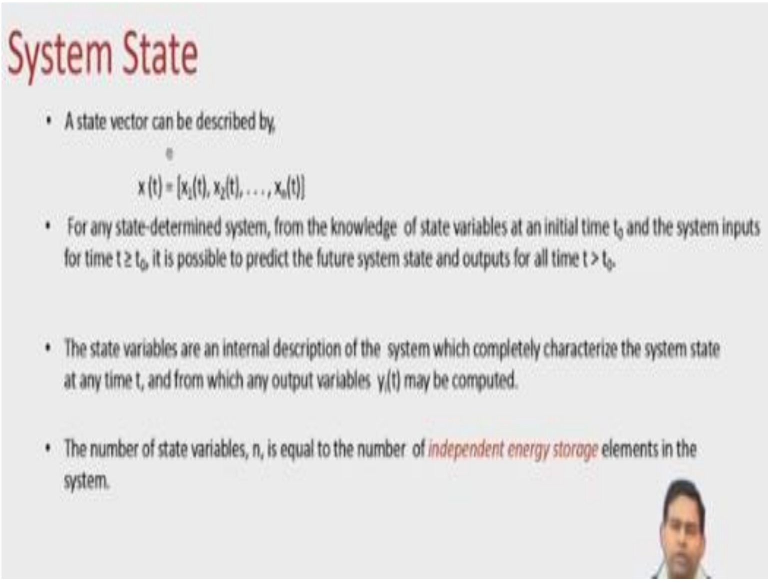

So, let us say my variables typically when we are talking about a dynamical system modeling, we will be saying that let us say the state vector is given by some x(t). And let us say there are n variables of interest which are these vector components and they are $x_1(t)$, $x_2(t)$, up to $x_n(t)$. So, for any state determined system from the knowledge of the state variables, like I said, at initial time t0, and the system inputs that means what are the system inputs like the values of u here right. Now what is what is the value of u that I am giving if these are provided to u?

You should be able to figure out that well in future what is the value of the state variables at some future time point, right. So, the state variables are often internal description of the system and they completely characterize the system state at any time like we are saying so this is the minimum set of variables and they represent and which we must need to consider. And they are good enough to characterize the system state of interest, okay. And typically using the state variables we will also model what is the output variable y.

So, for such systems where we have full state feedback, we will think that well this entire state variable set or the state vector is observable. And then the y will be equal to x and otherwise may be a subset of them may be observable or maybe some transformation some c times the x vector x

= y is what is observable, okay, and that is my set of output variables. Now this number of state variables that is the size of this vector is basically equal to the number of independent energy storage elements in the system. Because if we eliminate any one of them, we will be missing on in the description of the potential of the system.

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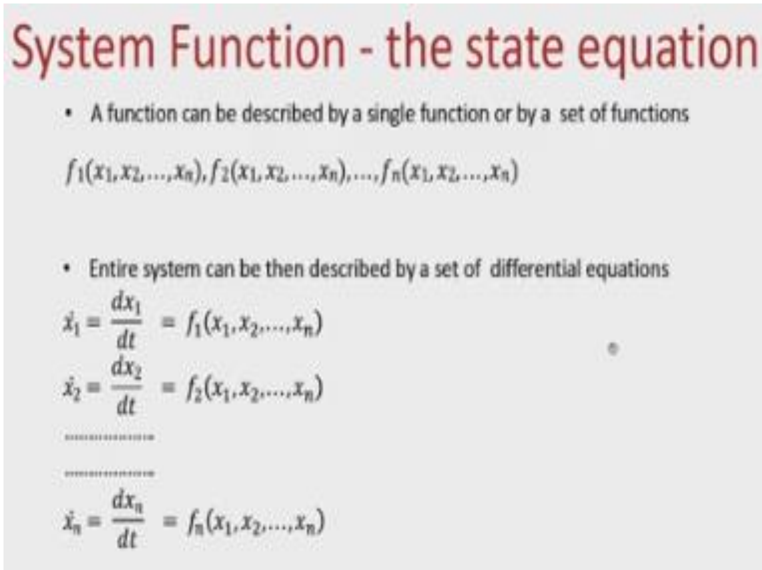

So we have also not only talked about the state variables we have also said there must be a function which tells me the rate of change this of these variables. Because I must know the rate of change with these variables in case, I want to see that well what is the value of those variables in future, right. So, we are saying that there must be a function that describes the flow of these variables, okay. So, typically the way we will write it can be a single function or it can be a set of function components, okay.

So that each of these components give you the rate of change of each of these individual components of the state vector. So, it may be like this. So, if you see the way we are writing is we are considering $\dot{x_1}$, $\dot{x_2}$, up to $\dot{x_n}$. So, the rate that those are the derivatives of x_1, x_2, x_n like that and we are saying that well they may be expressed as f_1 of x_1 to x_n, f_2 of x_1 to x_n and finally like this f_n from x_1 to x_n. So, when we are writing it like this it is like a big function f which comprises this member functions f_1, f_2, up to f_n. And they together give me the dynamics of the state, that means the dynamics essentially means how the states are changing with respect to the flow of time.

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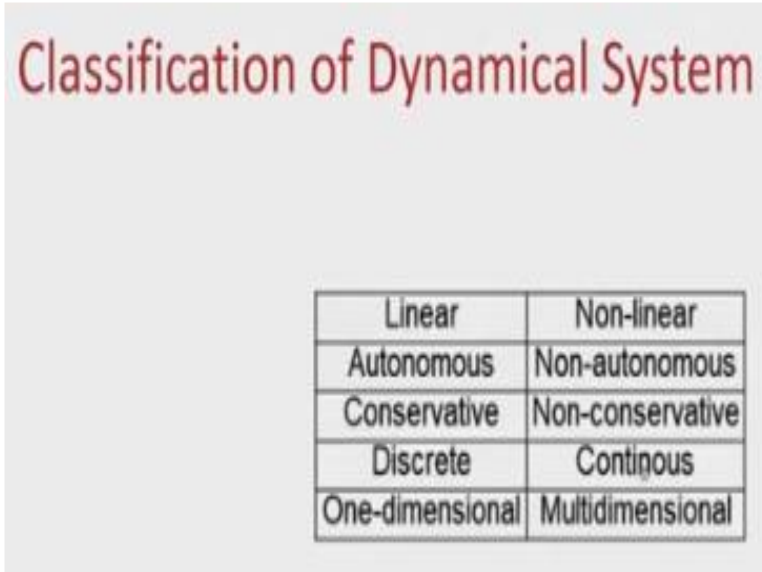

So now once we understand that well we need to identify a set of state variables and we need to compute a set of functions. So, as you can see from these vehicular equations here it is clear right which should be my state variables and what should be the functions, we will again come back to that through some real examples. So, at a high level we can understand we need to figure out what are these x's, x_1 to x_n for any CPS system that is given to you and you need to identify what are these functions f_1, f_2, up to f_n.

Once we have identified this set of functions and this set of variables on which the functions is defined, we have a dynamical model of the system. So now if I want to classify these dynamical systems into, I mean, it all means that well what are the different functional forms I am assuming here, okay. So, the system may be linear or non-linear based on the functions being linear or non-linear. The system may be autonomous or non-autonomous based on whether the system requires an explicit input or not.

The system may be conservative or non-conservative, it can be even a dissipative system which continuously dissipates energy. The system may be discrete or continuous by the model. That means, the model is such that it only tells you the value of the variables at certain time periods, at certain time instance let us say some t = kh, k being an integer and h being a period of the system. So, then I am only interested in the values of the variable at those discrete time points. Also, the system may have multiple state variables it is multi-dimensional or it may have a single dimension it may be a 1-dimensional simple system.

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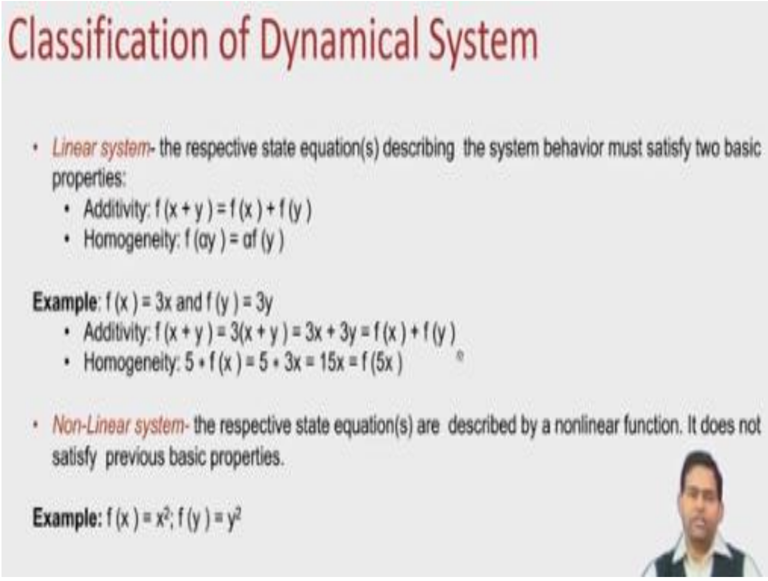

So just to understand when we call them a linear system and non-linear system. So it is basically the whether the functional form we talked about is linear or non-linear. And by definitions we know that if a function is linear, it should satisfy this kind of relations, any linear transformations like additively and homogeneity like this so either alpha is a constant, okay. For example, f(x) equal to 3x, f(y) equal to 3y, these are linear functions, right. And when the given functional form is by the mathematical definition and non-linear function. That, means I cannot express it as a line in a multi-dimensional coordinate system, then I will say that well it is a non-linear function. For example, these are of course if you, if you get their geometric interpretations, these are not linear functions here.

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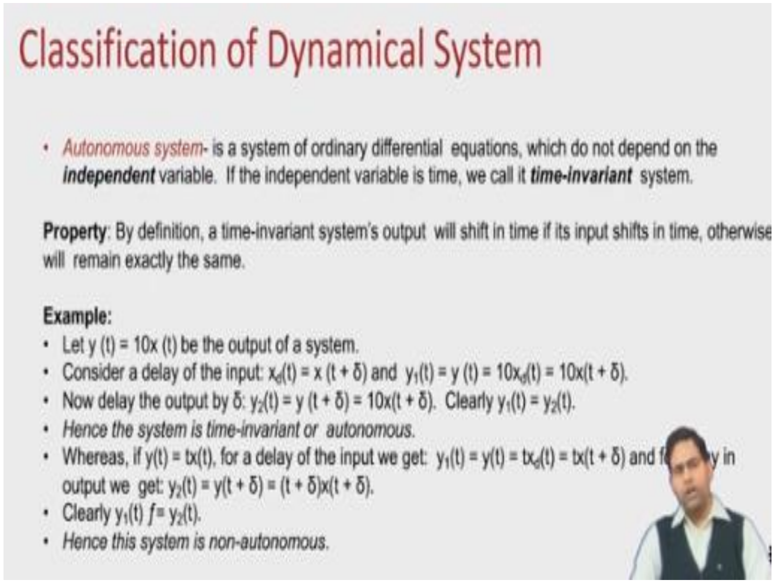

Now coming to this autonomous systems an autonomous system is again a system of ODEs or Ordinary Differential Equations, and they do not depend on the independent variables. There is no extraneous independent input. All the inputs that are there that are already part of the model and if the independent variable in this case is time, then we call it a time invariant system. So, by definition when I say that a system is time invariant, that only means that well if I shift the input's time, the system's output will also only shift in time.

And otherwise, it will be remain exactly, it will just remain the same. For example, let us say you take some examples here. Let us say you take y =10xt so that is a system equation and let us say you delay the output ,okay. So that means you are saying that there is a delay with which the output really appears and then let us say you are applying this thing to the function you are computing $y_1(t)$ equal to this. So as you can see that the output is also kind of getting delayed here, right.

Now if you delay the output here, okay, so this is your $y_1(t)$ here, okay. So, as you can see at time t, the output is 10 times xdt because that is how you have changed your definition here, and then what will happen is well since x at t and x at t + delta are same, what you get is something like this. Now if you delay the output here, you will get well y(t) is y(t + delta). So that that simply means well the output time I am as per this equation it will just also shift by delta, x will shift by delta.

And you come to the same value so clearly here y_1 and y_2, that mean the time shifted values are just same here, right. So, if I take $y_1(t)$ as this, and $y_2(t)$ as y at t + delta, so these are the 2 things we are trying to compute and they are coming out to be same, right, so it is a time invariant system right. And if you take another system like this y(t) equal to time times x at time t right, so if you see that if you delay the input what do you really get is $y_1(t)$ is this, right.

So, you see here your delay is only inside this, I mean, we are saying that the system's output is delayed like this, right. So that is what for the delay in the input. And now if I delay the output, it is going to look like something like this. So clearly here they are different, and these are not this is the time variant system example actually. Now of course this is an example with respect to time but autonomy also means several other things. Because if you have a system which is like I said, that if I try to give a general definition of what an autonomous system is it simply means that it can see, take all its decisions by itself without any kind of external input.

I mean, so does it mean that autonomous system does not have any external input? Well not really. It depends on whether i am modeling that input as part of my system also. So, if I am doing that then my than that input is included in my definition of the system. So, I will try to call it as an autonomous system. But if I consider that input as I mean as something which is not modeled inside the dynamics of the system, then it is non-autonomous.

Because every time point it is waiting, I have to if I have to evaluate the system I have to wait and see that well what is the input that I have, whose effect I am not model and then accordingly I will keep on varying myself, I mean keep on keep on evolving the system like that, right. So, when I say that it is a system of ordinary differential equations which do not depend on independent variables it’s really about all kinds of variables. It is not only about not only about time.

Now, so that is a simple classification or subset here that when we say that the independent variable is time, then we mean that well, the system's definition is not changing with time and that is why we call it a time invariant system. So, this is a classic example I am repeating that when I have $y(t) = 10x(t)$ if you just try to define it physically you see the definition of the system is given by this equation and this definition is not changing with time, right. So because if I just write I mean you do not need to do all these things, right, you just write y(t + delta) what you have is 10x(t +delta).

So, the definition is not changing with time but if we have t here, then that means the definition of the system is a function of the time variable itself, right. That means the definition at some time value t_1 is different from the definition when I am writing it for some time value t_2, rights. So, at t_1, the definition is t_1 times x of t_1, and t_2 the time definition is t_2 times x of t_2. So, if t1 is not equal to t_2, then the values are definitely different, right. So, it’s not only about the value it’s about how whether the definition of the system is changing.

So, in this case definitely its changing so this is not a time invariant system but this one really is so the is also how you can just talk about it and let me just repeat what we discussed here. So, if I give if you see that you give a delay here, in the definition, that you give a delay in the input itself and you say that well this is, this is how x is coming in a delayed manner. And then for y, I had this equation. And similarly, if I put in the delay here then also I get a similar equation here, right.

Now and in the other case if you consider the delay in the input your output is coming like this but if you consider the delay in the output here, right, and y you model that entire time itself as delayed then you definitely have something different here. So, this system is non-autonomous. So, that is why we will try to conclude this lecture and we will see what the other classifications and how it works forward from the next lecture. Thank you for your attention.