Statistical Learning for Reliability Analysis
Prof. Monalisa Sarma
Subir Chowdhury School of Quality and Reliability
Indian Institute of Technology - Kharagpur

Lecture – 09
Discrete Probability Distribution (Part 1)

Hello guys, so continuing with our discussion on probability distribution, today we will be discussing some discrete probability distribution. So, there are many different types of discrete probability distributions. So, in today's lecture, we will be discussing few of them. And next few of them we will be discussing in our next lecture.

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So, today's lecture will cover basically uniform distribution, binomial distribution, multinomial and hypergeometric distribution. There are some other distributions as well like Poisson negative binomial geometric I will be covering that in my next lecture.

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So, before going to discuss the different probability distribution, let us take a quick recap of what we have learned till now, just a few terminologies that we have learned that is the first is probability, what is probability? The relative frequency of the occurrence of an outcome of an experiment we have seen that even while trying to calculate the mean also have a stress in this factor, how will we take the relative frequency while calculating the probability of an event, remember that.

And then what is an event it is defined as a combination of outcomes, then what is a random variable, it is a function that associates a real number with each element in the sample space. So, the random variable may be discrete, it may be continuous, so, if the random variable is discrete, the corresponding probability of the different values that a random variable will take in an action that will give us a discrete probability distribution.

In a random variable is a continuous in an experiment that different values that are random variable can take along with the probabilities that is a continuous probability distribution.

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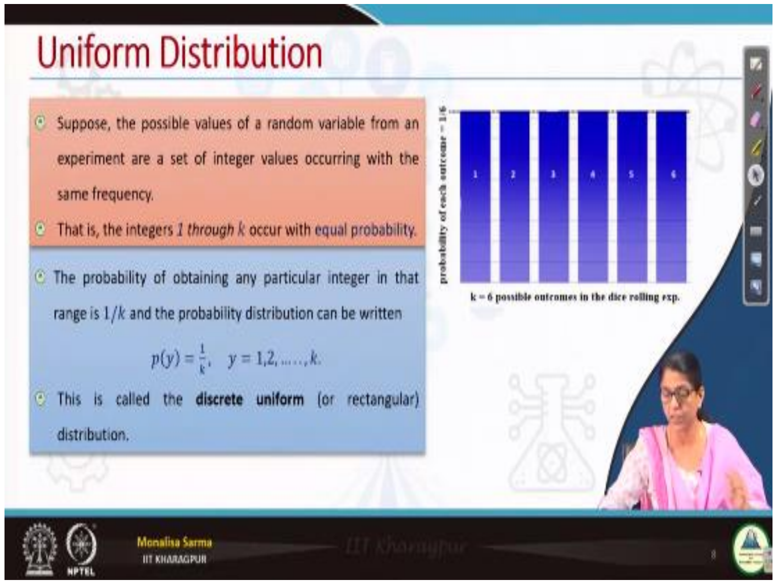

So then next, first we will start with the uniform probability distribution; this is a discrete probability distribution. However, before discussing on this uniform discrete probability distribution I would like to mention here this distribution is not very much used in any practical applicants or access. Of course, it is not that it is not used at all, but its applicability is less compared to some other distribution like when we were discussing binomial distribution, Poisson distribution, although the quite used in many applications.

Like the uniform distribution is not very much used, but for completeness, you should know this also as well. So, what is uniform distribution? So, uniform distribution suppose the possible values of a random variable from an experiment are a set of integers from an experiment the probability values that a random variable can take and we have discussed this in an experiment random variable can take some possible values. So, possible values these are a set of integers and they occur with same frequency.

Like when we toss a die what are the value the random variables may take this 1 2 3 4 5 6. So, what is and this in case of uniform probability distribution what is the condition is that this all this values occur with the same frequency. So, that is in the case of die we can say it is 1 to 6 let us tell that it is k where k is 6 that is the integers one through k occur with equal probability. So, if it equals if it occurs with equal probability, then what is the probability of getting any n number say p (n) let me tell it as p (y).

So, p (y) will be equal to 1 / k. So, this is the discrete probability distribution function p (y)or we can tell it as f(y) also. Now, when we describe any distribution any maybe continuous distribution or discrete probability distribution any distribution when we describe there are a few characteristics which you call it as parameters also which basically characterize the distributions, which having the information of these parameters will help us to find out what is the shape of the distribution.

Helps us to give us different information of the distribution so, like for different parameters that are that, we can also say that different characteristics that are one is the mean of a distribution, variance of a distribution and then symmetry, symmetry it is called and another one we can call it a kurtosis. But when we talk of a mean and a variance definitely mean and variance we will be discussing in this lecture of course, mean and variance is very much applicable when we talk of discrete in discrete probability distribution.

It is also applicable for continuous probability distribution definitely. And but when we talk of asymmetry, how symmetry the curve is. So, this is not very much relevant from the discrete probability distribution persue. So, we will not be discussing here, when I will be discussing continuous I will specifically mentioned about that, then again kurtosis, kurtosis is how flat the curve is that is also a delivered from the not very much I should not say it not very much relevant from the discrete probability distribution contexts.

So, the different parameters that we will be seeing in the discrete probability distribution are mean and variance. Now, how to calculate the mean of this distribution already we have seen how to calculate the mean of register and remember mean of the distribution is x f(x) summation of x all the values of x into f(x) what is f(x)? f(x) is the probability of x we have seen that. So, now, if we calculate them, so, like mean of a distribution we have seen it is a summation of x f(x), x is the all the values that the random variable will take. That is we call it x i basically i range from y 1 to n and then f(x), f(x) is the probability that x will take.

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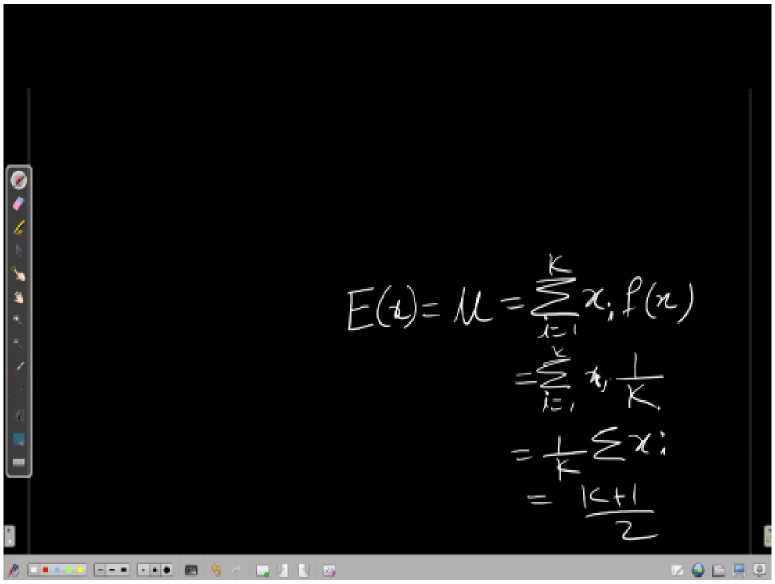

So, how let us see in case of uniform distribution what happens. So, what is we call it expected value remember we have done this expected value we could call it f of x also we can also call it μ and what is this? It is summation of x_i f (x f (x)) where i = 1 to k suppose here the values goes from 1 to k now, in this uniform distribution what is that i = 1 to k x of i and then what is f (x)? f (x) is nothing but 1 / k so, 1 by k it will just come out 1 by k so, it is summation of x i so, what it will give, what is summation of x I, what is k into k + 1 by 2?

So, what we got is k + 1 / 2, this is the mean of the uniform distribution. Mean of the uniform distribution is k + 1 / 2. Similarly, we can find out the variance also. So, if we find out a variance similarly, remember the formula for variance, formula for variance is summation of $(x – \mu)^2$ f (x), x – μ, μ is the expected value which we got just now got $(x – \mu)^2$ f(x).

Now, similarly, if you can simplify it, I will be showing you that we did a very easy actually, if you simplify, then the variance will be getting k^2 - 1 / 12 that is the variance of the uniform probability distribution.

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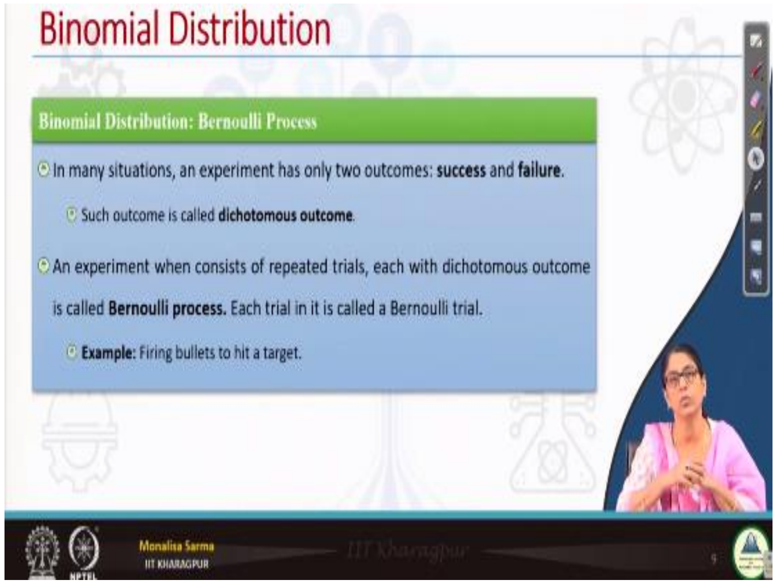

So, now, next distribution that we will be discussing is binomial distribution it is also called Bernoulli process. Now, in some situation an experiment has only 2 outcomes like we are trying to hit a target. So, hit or miss that is like we are doing an test by some say blood tests we are looking for some disease it is present or not. So, it is positive negative like now, we can support COVID positive negative. So, these are the in some experiment has just 2 outcomes.

Such outcomes is called dichotomous outcome. So, an experiment which consists of repeated trials, we do many trials each with dichotomous outcome is called a Bernoulli process, now, you are firing a target. So, we are doing it we are repeatedly doing it, there is a target fixed we are trying to hit the target repeatedly we are doing the experiment that is called repeated trials. Now, a blood test we are not suppose we are not convinced we are doing the experiment again and again. So, that is a repeat we call it repeated trials.

So, when the experiment when it is consists of repeated trials each for each of the trial we will get a dichotomous outcome that means, either success or failure success or failure are hit or miss that is in generalized form we call it as a success or failure it is not necessarily as success or failure only like for if we hit the target we call it a hit or miss. So, it is not a question of success or failure. So, but in generalized form we call it either we got a success or we call it failure.

So, this type of process is called a Bernoulli process and each such trial is called a Bernoulli trial. Example, but I have already just told firing bullets to hit a target.

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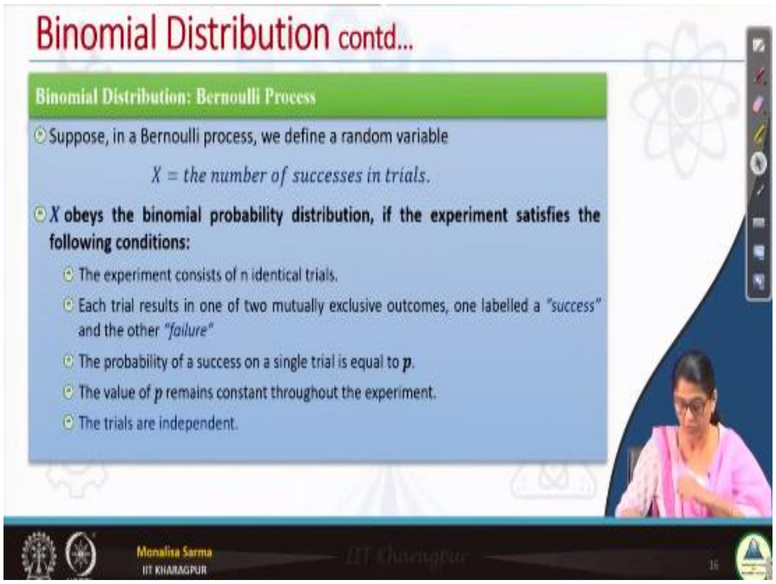

So, now, suppose in a Bernoulli process we define a random variable. Probability distribution means there has to be a random variable, then only we can find a probability decision isnt it? Means what is the value that random variables will take as well as the probabilities of those random variables. I am repeating this again and again because what is probability distribution many students find it difficult to answer this question.

So, suppose in a Bernoulli process, we define a random variable. So, what is a random variable? Random variables is X, X is the number of success in trials. So, X is the number of success in trial. So, suppose that means let us take that we are doing total n trials, we are trying to hit a target, suppose in front of me there is a target, I am trying to hit the target, I am totally I am doing this n times, out of n time, suppose my number of success maybe 4, so that is X out of n my number is success maybe 1 2 3 4 up to n listen it, it can take any value from 0 to n basically.

It can be 0 as well, there cannot be any there may be no success at all. So, X is equal to the is represents the number of success in trials. So, X definitely X obeys the binomial probability distribution, because X will take the either X will, it will either hit or it will be missed. So, it follows the binomial probability distribution. So, if the experiment set is now for a random variable, to call it as a binomial, that it obeys the binomial probability distribution, the experiment has to satisfy certain conditions.

That we are going to experiment now, this way trying to hit the target that is an experiment we are doing a blood test that is an experiment anything. So, now what to say now to tell this experiment is a binomial process, it needs to satisfy certain condition. So, what are the condition first is that the experiment will consist of n identical trials, n identical trial means I am trying to hit it from here. Next time also, I will try to hit it from here it is not that I am trying to hit it from here this time.

Next time I am going near the target and I am trying to hit it or next time I am trying to hit it from a different angle no, it should be the experiment consists of n identical trial the experiment each trial should be same. Second, each trial results in 1 or the 2 mutually exclusive outcome, definitely the outcomes are either success or failure both cannot happen success and failure but cannot happen together. So, each trial results in 1 or 2 mutually exclusive outcome, which we can label it as a success and the other as failure.

Then the probability of success on a single trial is p the probability of success out of n success n trial suppose your number of success is X, so what is the probability, probability is X / p. So, probability of success on a single trial is equals to p. And the value of p remains constant. Obviously, when the experiment consists of identical trial, the value of p will remain same, isnt it. So that is what the value of p remained constant throughout the experiment. And the trials are independent.

Each trial is independent means 1 trial, the results of 1 trial will not affect the other trial. So, the trials are independent. Now, let me give you a simple example of this drawing a car now we have in a deck of cards, we have 52 cards out of this 52 cards, what is the probability of in this pack of cards now my experiment is getting say black king. So, getting a black king, so out of this, I am picking it what is the probability of getting a Black King in my deck of card? What is the probability? How many black kings are there?

So, what do you find the probability? So, now suppose, what you do, you found a particular card, whatever car and you kept it, you do not put it back into us sub, you do not put it back in the deck of cards. And now again, when you try to pick one than, then it done my experiment will not be identical, why it will not be identical sorry, then my experiment will not be independent, why it will not be independent? Because I have now the number of cards is reduced. Early it is 52. Now, I have picked 1 card and I kept it aside, I did not mix it together.

So, now my trials are not independent. So, this is the concept of dependent independent according to the different situations different subjects.

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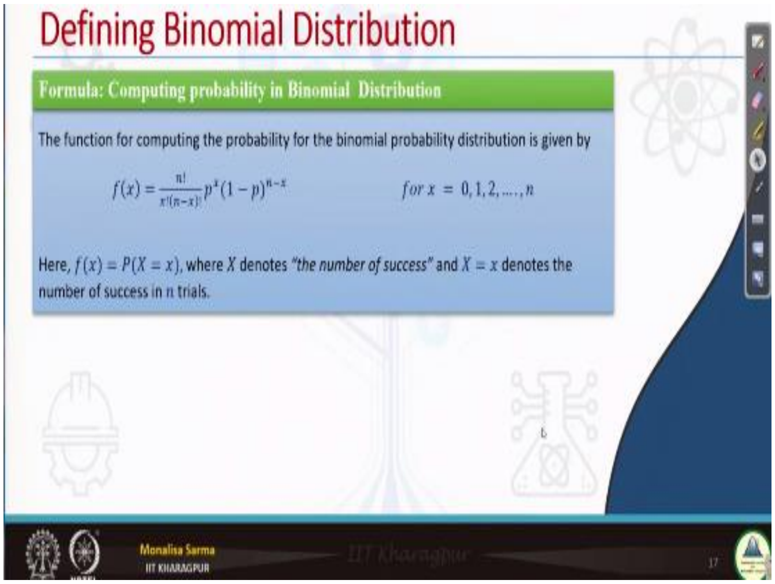

So, now, what is the, so any probability distribution we know we need to know what is f(x)? First we need to know what is a random variable, what the random variable represents in an experiment, then once we know what is the random variable, what the random variable represents, then we need to know what is the probability of f(x), what is the probability that is f(x). So, what is the in a binomial distributions what is f(x), for uniform distribution we have seen what is f(x)? f(x) = 1 / k in uniform distribution, now in binomial distribution, what is f(x)?

So, this is the formula basically nCx p ^ x 1 - p ^ n - x. That means probability of x success in n trials total the n trials out of n trials what is the probability of n success is n C x p ^x 1 - p ^ n - x. Let us see how we got this formula suppose we have total 5 trials suppose 5 trials out of 5 trials suppose we are interested in finding out what is the probability of step 3 success, what is the probability of 3 success and 5 trails.

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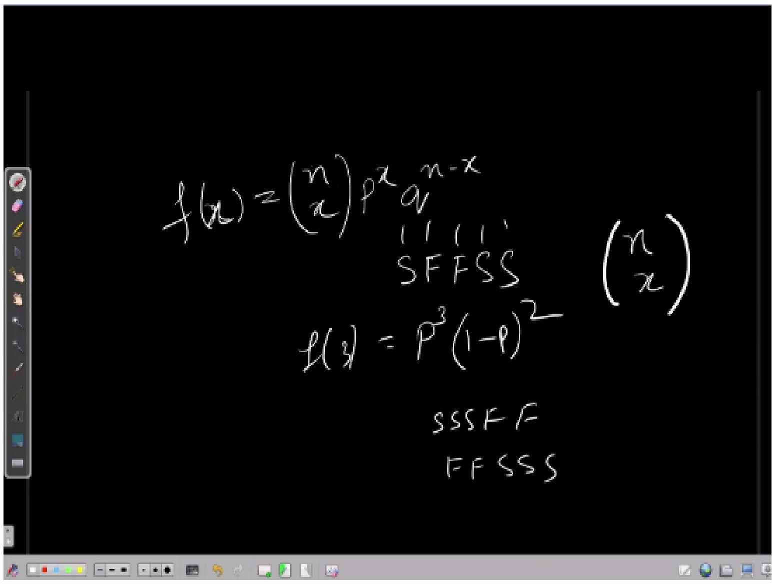

3 success means, suppose I got success, failure, failure, success, success, 5 trails this is 1 2 3 4 5. So, this suppose this is my sequence, so, then what will be my probability? Probability is independent so, my probability is p how many p is there? So, p^3 into how many failure that is 1 - p x 2 that is 5 - 3 this is 2. So, this is my f(x), f (3). Now, this is one sequence there may be another sequence like I may get S S S F F this may be 1 sequence, I may get F F S S S.

So, similarly, there may be different types of patterns where I may get 3 success. So, that is why when there are 3, when we have to select the x object from n objects, where we are not this distinguishing between the objects, what is the probability we have seen listen that is n C x. So, that is what. So, that is why my probability now f (x) = 2 n C x p ^ x q 1 - p let me write it as q q ^ n - x this is how we got the formula. So, that is the formula for probability binomial probability distribution random variables representing a binomial probability distribution.

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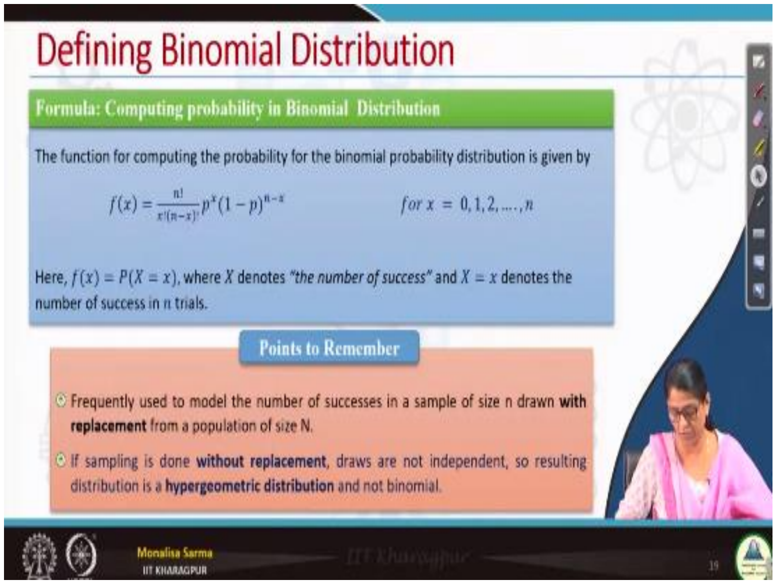

So, now, some one important point to remember, this binomial distribution is frequently used to model the number of successes in a sample of size n drawn with replacement from a population of size N I have given the example of card where we need to replace it then only it will be independent and industry or weather in from a reliability perspective or quality perspective where this sort of thing where we use suppose in a production line in a pipeline, there are some products are coming out, some products are coming out in a pipeline.

Now, from there, we want to check how many defective products are there. So, what we do? The products are coming out in the pipeline, we are just picking up 1 product, and we are checking whether it is defective or not, we are putting it back. Again, we are picking another way of putting it back. So, this is we are sampling with replacement, we are checking it on we are putting it back. So, this is in this sort of case, we use binomial distribution.

Again, there are some situations where, like, if you are interested in finding out the life of a bulb, so again, from the given a population of bulbs, we are interested in finding out the life of a bulb. So, definitely to find the life of a bulb, what we will do? We will try to do we will take some bulbs and we will see how long it works and from that basically, we will try to infer that what is the life of the bulk of the whole population.

Definitely we will not check for the whole population, we will take a smaller sample from the smaller samples, we will use those products till it becomes bad. So that means in that case, we cannot replace it back we have taken it, we will use it till it becomes bad. And then only we can find out the life of the bulk till when it lost. So, we could not replace it back. So, that is not a case of binomial distribution that is a different distribution we have for that that is called hypergeometric distribution.

When sampling is done without replacement, so that is hypergeometric distribution, it is not binomial difference mean there is a difference between both this is the only difference between the binomial and hypergeometric actually.

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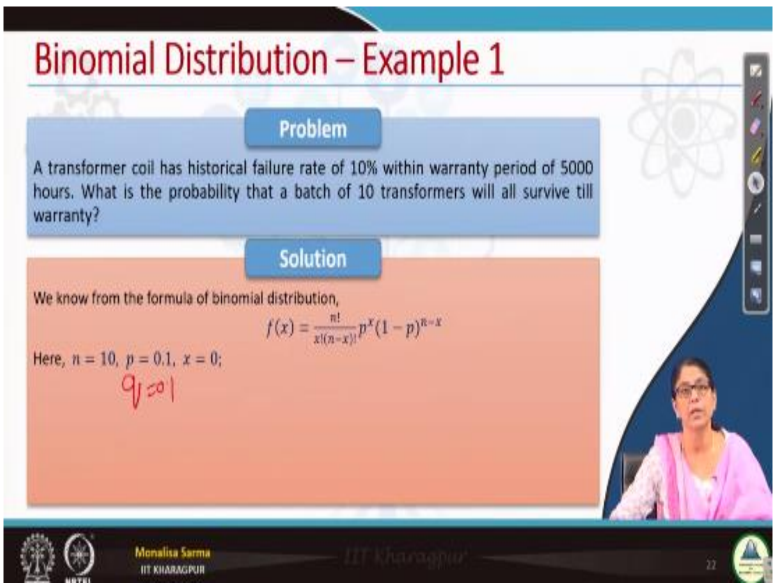

So, now, a small example before proceeding further. A transformer coil has historical failure rate of 10% within warranty period of 5000 hours, what is the probability that a batch of 10 transformer will all survive till warranty? So, for a failure rate of 10% within warranty period of 5000 hours what is the probability that a batch of 10 will all survive till warranty will all survive remains there is no failure, what is the failure rate is given failure probability is given 0.1 if the 10% means 0.1 failure probability is 0.1 so, what is the success probability successful is 1 – 0.1.

And what it is given what is the probability that a batch of 10 transform will all survive till warranty means there is no failure what is the probability? Probability of no failure in 10 transformers that basically we have to find out that means we have to find out f of 0. So, your n = 10 p = 0.1 and x = 0 sorry here p is not equals to 0 rather I should say a p is because q = 0.1. So, there is a slight mistakes here this is this one can you see this is not p this is but q, this is not p this is q or sorry, this is not a question. Yeah, I am sorry for the confusion. So, this is not p this is q, q = 0.1. So, if q = 0.1 that means, p = 1 - 0.1 so, it is 0.9.

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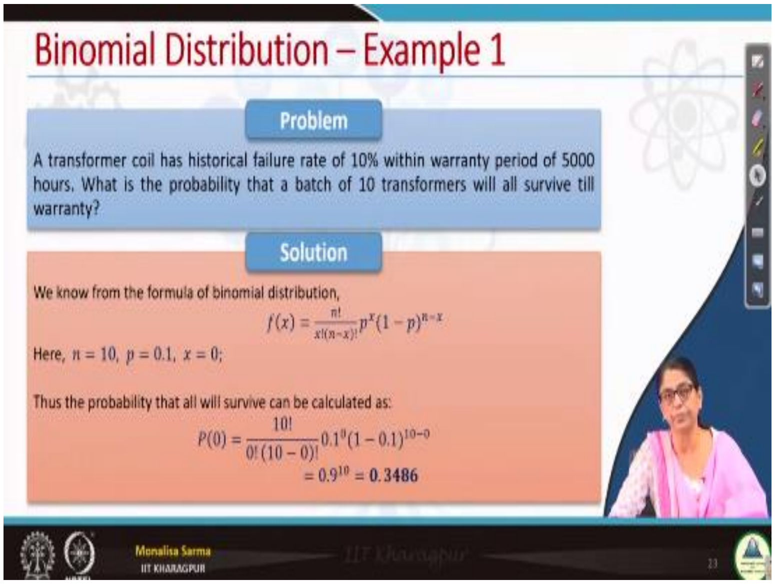

So, here the problem I have solved with taking p = 0.1. So, there is a mistake here accordingly you guys please change it. So, this is P is equal. So, according to equation p is equals to not 0.1, but 0.1 But here I have solved using p = 0.9. So, that is not an issue anyway whatever is p accordingly p value we will be putting here and whatever is then q will be 1 minus of p that is all.

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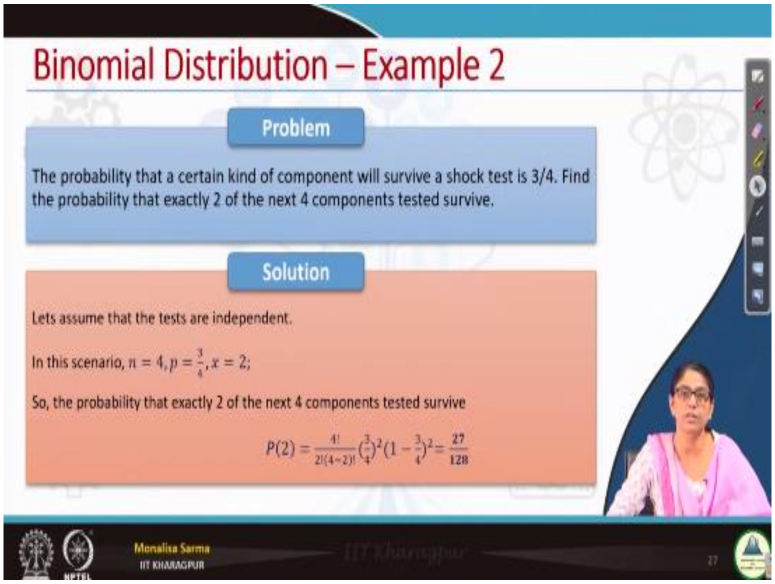

So, next the probability that a certain kind of component will survive a shock test is 3 4th find the probability that exactly 2 of the next 4 component is that survive. So, here probability is 3 by 4 and we have to find out of 4 components 2 will survive. So, it is again how we will be using binomial test thing distribution that means, here my random variable is a problem of survivability the number of component that has survived so, here I want to find out f of 2. So, f of 2 so, just put it in the formula that is all, nothing.

Not rocket science is here just putting into a formula and solving that. So, when you have the value of n, you have the value of p you have the value of x I am just put it in that formula.

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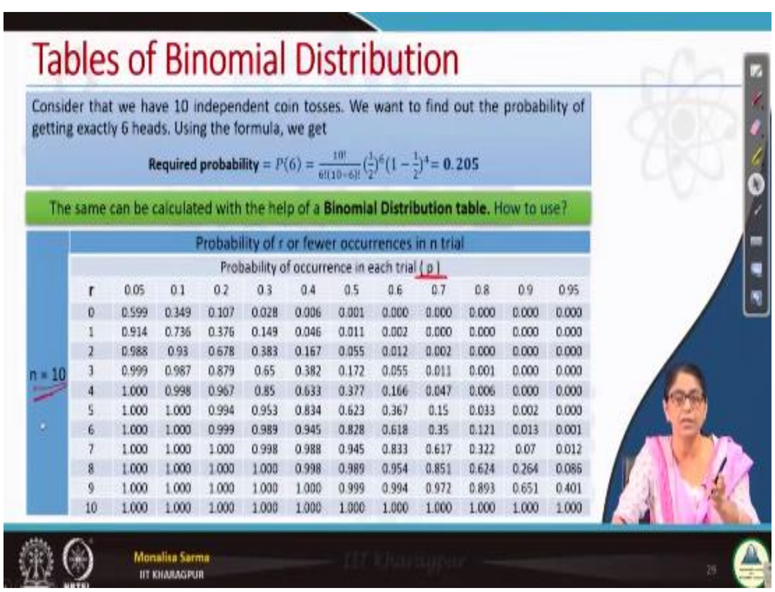

r	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
0	0.599	0.349	0.107	0.028	0.006	0.001	0.000	0.000	0.000	0.000	0.000
1	0.914	0.736	0.376	0.149	0.046	0.011	0.002	0.000	0.000	0.000	0.000
2	0.988	0.93	0.678	0.383	0.167	0.055	0.012	0.002	0.000	0.000	0.000
3	0.999	0.987	0.879	0.65	0.382	0.172	0.055	0.011	0.001	0.000	0.000
4	1.000	0.998	0.967	0.85	0.633	0.377	0.166	0.047	0.006	0.000	0.000
5	1.000	1.000	0.994	0.953	0.834	0.623	0.367	0.15	0.033	0.002	0.000
6	1.000	1.000	0.999	0.989	0.945	0.828	0.618	0.35	0.121	0.013	0.001
7	1.000	1.000	1.000	0.998	0.988	0.945	0.833	0.617	0.322	0.07	0.012
8	1.000	1.000	1.000	1.000	0.998	0.989	0.954	0.851	0.624	0.264	0.086
9	1.000	1.000	1.000	1.000	1.000	0.999	0.994	0.972	0.893	0.651	0.401
10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Now, here I will show you one more, I do not want to discuss one more example, but then I want to show you one concept like sometimes binomial distribution, we can also instead of calculating we can also use a table. So, but in the table we do not get a binomial distribution value, but what we get is a cumulative distribution remember, when we are discussing distribution probability distribution, we have also discussed cumulative distribution.

Cumulative distribution means we want to find out suppose in case of binomial distribution, suppose we are interested in finding out what is the probability of at least at most 2 success say at most to success means, so, there will be 0 success 1 success 2 success. So, it is cumulative, cumulative means we need to find a probability of 0 success + probability of 1 success + probability of 2 success. So, it will be finding many standard many tables standard tables where it gives the cumulative distribution.

So, suppose some from the cumulative distribution table also you can use for finding out distribution at 1 point probability at 1 point you can use the table how you can use so, see this question consider that we have 10 independent coin tosses, we want to find out the probability of exactly 6 heads. So, here we want to find out a probability of exactly 6 heads. So, what is the probability? Probability of getting a head is half. So, 6 that means 10 C 6 half ^ 6 1 - 1 / 2 ^ 4.

Now, instead of solving this we can see this from the table how we can see this from the table brought exactly 6 heads how do we get exactly 6 at that table I told you it is cumulative. So, in the table that means I will get the value at most 6, then again I will get the value at most 5 heads. So, from at most 6 head if I subtract at most 5 heads, I will be getting the value exactly 6. That is what instead of calculating I can use the table. So, I will show you how we have so, this sort of probability tables are there.

It will find in any standard textbook in the appendix this sort of tables are given. So, this is a binomial thing here in this n = 10, n = 10 is the number of trials, this is the number of trials. And here we get this, this side you get the probability you get the probability of an r is the number of successes, where what we have used here x number of success, 0 success 1 success 2 success. So, r is the number of success and this is for this I have this table is only for n = 10.

Similarly, there are different tables will find tables for n is equal to 11 12 many numbers and for r also for probability values also, today only after going through this lecture I request please go through this table you will see this table available in the net. So, how do we find out here for this question. So, for this question, we want to find out exactly 6 heads. So, how do we get exactly 6 head at most 6 whatever value we get, we will minus it from at most 5.

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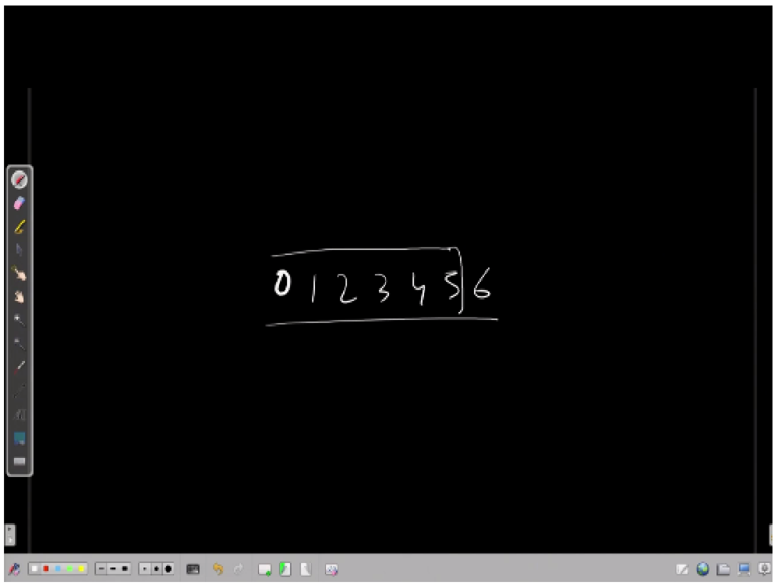

So, I will just show you suppose it is 0 1 2 3 4 5 6 at no 6 is, till this portion is in probability of 0 probability of one, 0 + 1 + 2 + 3 + 4 + 5 + 6. Now, if I did at most 5, so this so at most 6, if I subtract at most 5 from at most 6, I will be getting exactly 6.

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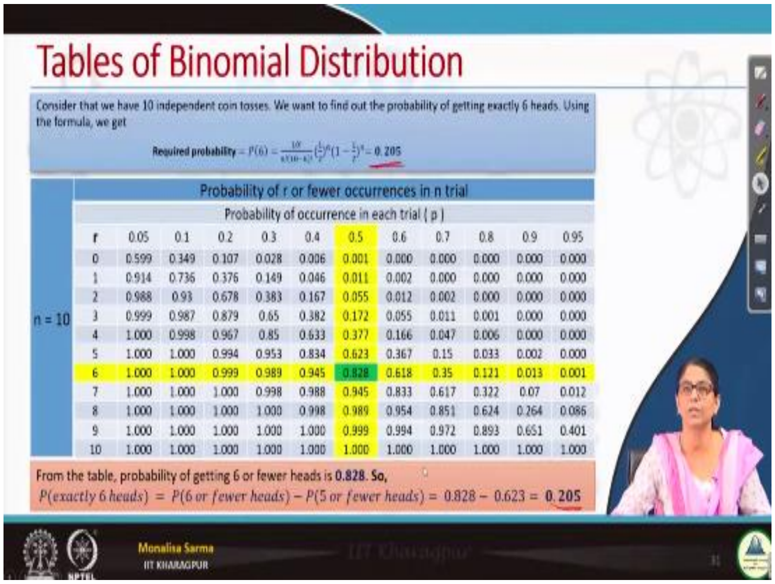

So, this is the value this is how this is the way we check. So, this is this yellow colour line you can say for n = 10 for r value is equals to 6 and probability is equals to half that is 0.5. So, where it matches so probability value is 0.828. Now, what is the value for at most 5? At most 5, you see, so for just above the horizontal line above the 6, so, at most 5 probability of 0.5 is 0.623 just above 0.28. So, we subtract this 0.623 from a 0.828 we will get the value see this in the top thing here, see you got 0.05.

So, we subtract this 0.623 from a 0.828 we will get the value see this in the top thing here, see you got 0.05. We calculated it manually here we calculate it from the table we got the same value. So, why unnecessarily will calculate it by hand we will consult the table the tables available but just you need to know how to see the table.

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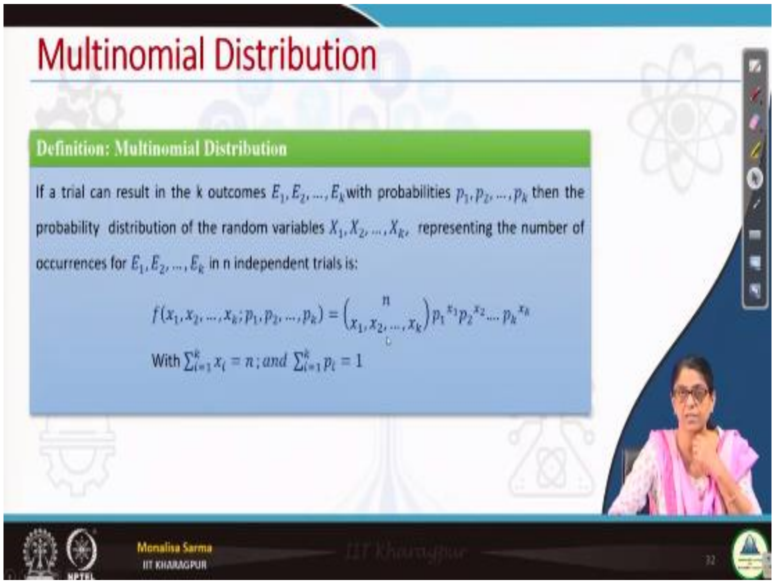

So, that is binomial distribution. Similarly, in the same line, we have another one type of distribution that is called multinomial distribution, multinomial distribution is also exactly same the properties are exactly same what we have seen the properties that the probability is same for all the trials, the trials are independent the trials are identical. So, here also the same but the only thing is that and they are with each trial, we get any one of the 2 outcomes.

So, here each trial can result in k outcomes mutually exclusive of course, they are also mutually exclusive 2 outcomes here are mutually exclusive k outcomes. So, from E 1 E 2 E k and each with different probability E 1 with probably p 1 E 2 probability P 2. However, the probability is remains same in all the trials. So, this is called multinomial distribution same quality characteristics exactly the same, but in binomial we have 2 outcomes in multinomial we have more than 2 outcomes, it may be 3 maybe 4 it may be 5 and even same exactly same.

So, exploration is also same there it was n C x via this n C x 1 x 2 up to x k how many different outcomes you have.

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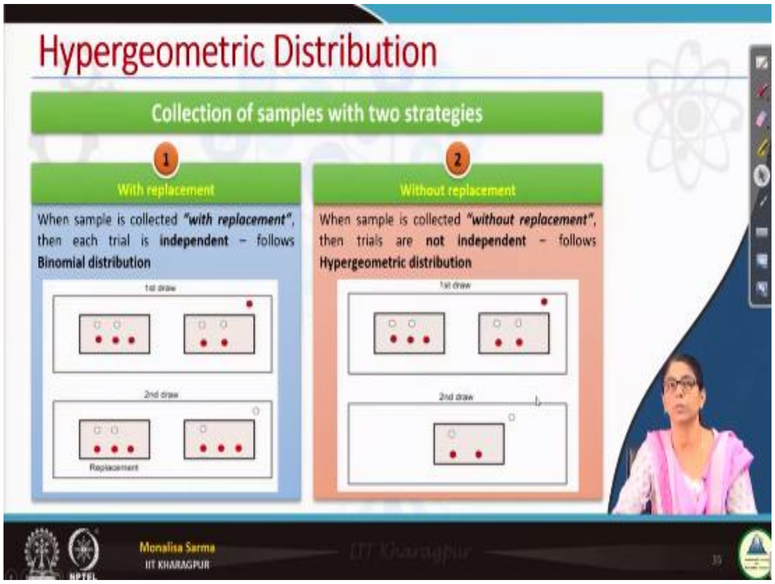

So, now hypergeometric distribution. So, what is hypergeometric already I have specimens and what is hypergeometric distribution when I was talking about binomial distribution to so, we replace without replacement what we get. So, binomial distribution is with replace, the trials are identity independent, but in hypergeometric distributions, so, what we did we do it without replacement from a sample of cards, we pick a card we do not replace it back. So, from in our next draw our probability changes because the number of card changes.

So, you can see this example. So, this week replacement suppose in a box there are some 5 balls, so, what we have done we have just picked 1 then the number of remaining ball is 4. So, now, we have replaced it again the number of balls became 5 then again we picked if we try to pick 1, so, our probability will not change because the number of the original number of balls remains same when we try to pick 1. But here we do not replace it initially suppose if we change them identify the color also so, suppose there are 3 red and 2 white balls.

And repeat 111 red ball. So, initially, here what will be the probability of picking a red ball picking a red ball what will be the probability, anyway I will not tell you should find it yourself. I have discussed this lots. So, now again now in the second row if I want to pick a white balls what happens earlier there was how many there was totally 5 balls now my number of balls is 4 from 4 I am picking 1 ball. So, according the my probability also changes. So, this there is dependency that characteristics not satisfied in hypergeometric distribution.

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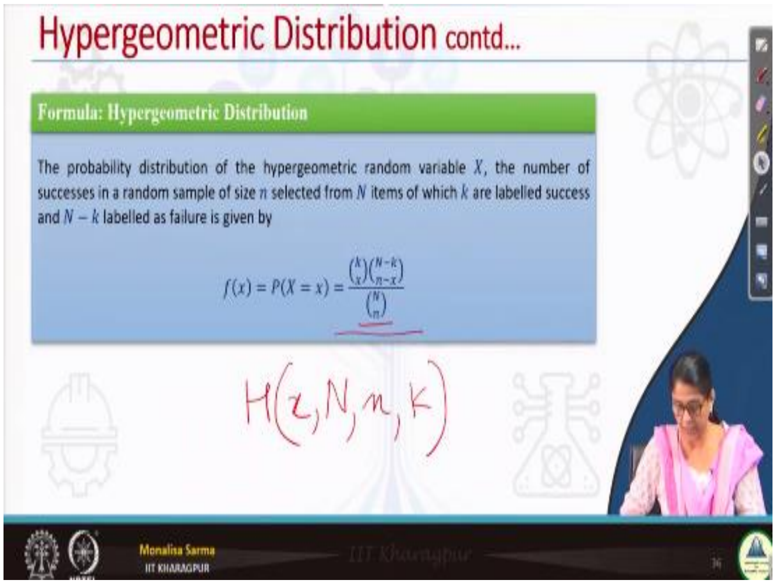

So, what is the expression for it affects so similarly, how we have done for binomial we can do it for my hypergeometric also, but I will not be solving it you do not need to solve that also, if you can, there is a very simple way by doing it again and again it will be in your brain or you do not have to it just in the tip of your tongue I should say like, so what is the hypergeometric distribution the formula for hypergeometric distribution as first here, one more thing I think we need to write it hypergeometric distribution.

How we write it basically H, H from number of successes x is number of successes here also we can tell it is a concept of success and failure, x than total number of elements, and is the total number of elements. And is this n is the sample that we have picked out number of elements that we picked out. And out of this n elements, how many successes are they, out of n, total n, that is N, and a N, how many successes were there, that is their total k success out of this.

You are trying to pick out from N you are you are trying to sample n number of items in n, x is the number of success. So, this is the formula for this h of x and k is this; what is given here this is my sample space is, this is my sample space. This is my sample space. Then from N, I am bringing n and our how many successes are there in N total k success there from k, my interest is picking out x axes. So, how many total item remains if I take out k, this n - k remains.

And from n - k how many I am picking because I am picking total n from n my success is x. So, how many is remaining and minus x is remaining. In fact, this sort problem we have done the 2 3 problems we have solved while solving probability problems that time we did not know about hypergeometric distribution, but we have done it in this way. Now, when you since you know hypergeometric distribution, solving that type of problem will be very easier, I suppose, listen it.

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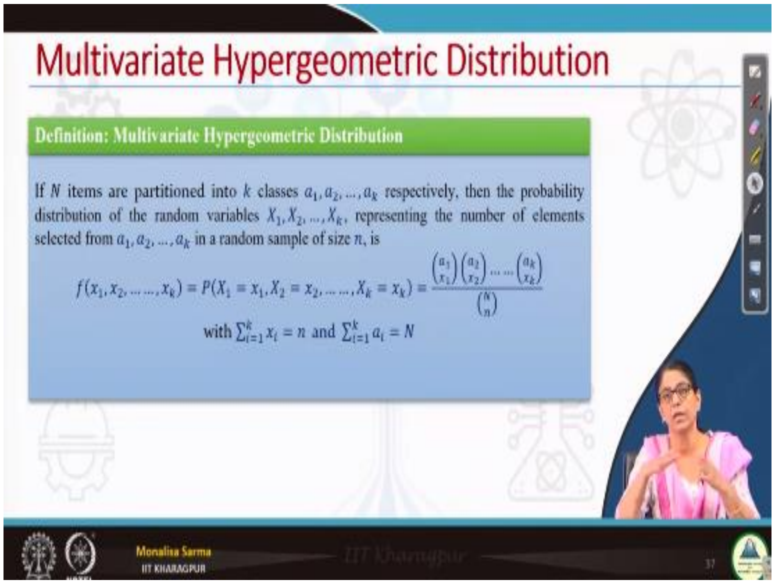

So, similarly multivariate hypergeometric distribution like what we have for multinomial binomial this multinomial distribution similarly, multivariate hypergeometric distribution, if we are interested in finding out suppose in N, there are different classes a 1 a 2 up to a n there are different classes and we are interested in picking up the x 1 item from a 1 class x 2 from a 2 class x 3 from a 3 class and that way. So, similarly to what is the formula for hypergeometric distribution same case that a 1 C x 1 a 2 C x 2 out of a k C x k. And upon what is the total number sample size? Total number of sample sizes N C r.

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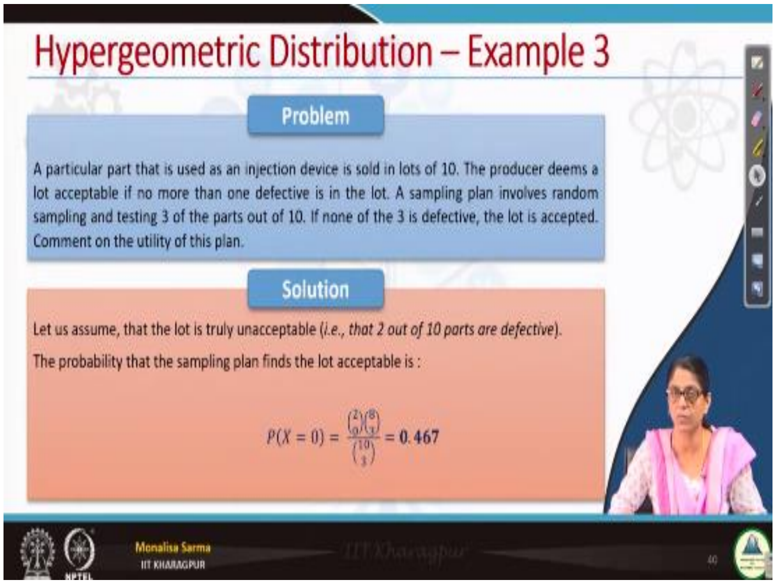

So, just you can see this as an example, a particular part that is used as an injection device is so, sold in a lots of 10. That is particular part that is used in injection device we are the it is a total around 10 10 parts are put together and in a packet that means in a packet we can say there are total 10 parts and the producer deems a lot acceptable if no more than one defective is not a lot the producer he can say that a lot is acceptable.

If there is no more than 1 if there is 1 defective the producer will accept it Chullora 1 defective we can accept it, but there should be no more than 1 if there are 2 defectives, 3 defected that means a lot is not acceptable and 1 defective lot we can accept a lot. That is the view of the producers. Now, what he will do he cannot check if form the 10 per lot in a packet there 10 he cannot check all this all the items are all the 10 items and find out whether how many are defective.

So, what he use, so, he has devised a sampling plan. So, what is sampling plan and the sampling plan involves random sampling and testing 3 of the parts out of 10 from 10 he will pick up 3 items and if none of the 3 is defective, the lot is accepted. So, what he thought this guy does guy is an intelligent guy, he at least he taught himself to be intelligent, he thought, I will not check this 10 items, let me check out of his 10 item let me just pick 3 items.

If in this 10 items at the most 1 defective is there, then if I pick 3 items, that means I should not get any defective that was his intention. So, if in 3 item if I do not get any defective, then I can be convinced that a lot is acceptable lot. Acceptable lot means that it is no more than 1 defective. So, comment on the utility of this plan. So, we have to find out whether this plan is whatever plan this person, this person has found whatever whether this plan is really good, let us find out the utility of this plan.

So, what we will do is that in this problem first let us assume that a lot is not acceptable, when the lot is not acceptable. There are more than 1 listen it. So, let us actually assume that there are 2 defective in this lot. And a lot of times suppose there are 2 defects in the lot and what he has done, he will pick 3 items. And what is the probability of getting 0 defective in the 3 item when the lot has 2 defective that is what we will find.

So, probability of x = 0 using how will from 10 he has picked 3 there are 2 defective from these 2 defective he did not pick anyone because 0 defective. What is the probability you are getting 0 defective that means 0 defective means we have not picked any defective. So, from the 8, he is picking the rest. So, what is the probability? Probability is 46% see. So, that means does even if the lot is unacceptable, it has 2 defective items that means a lot is unacceptable.

Even if the lot is unacceptable, but his sampling plan what he has done that if I pick 3 and if I do not get any defective then I can assume that a lot is acceptable, but his plan is not a good plan. His plan is a very poor plan. And because it is unacceptable lot also with his plan he will accept it 46% of the time, if this percentage is would have been very less than we can tell, his plan is good. Now, this is 46% is not a very less small number, it is a big number. So, we can tell this plan is not good. That is the common on this infinity of this plan.

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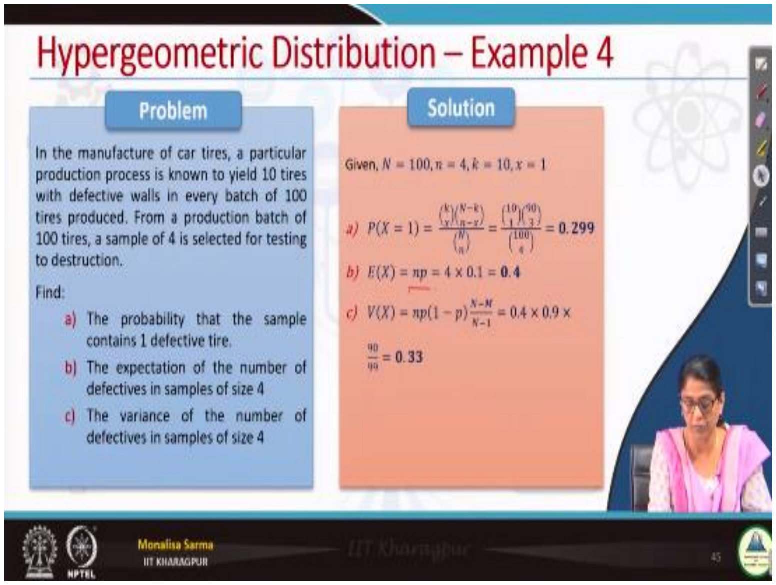

In a manufacture of car tires, a particular production process is known to yield 10 tires with defective walls in every batch of 100 tires produced from a production batch of 100 tires sample of 4 is selected for testing to destruction, this is very simple whatever we have done hypergeometric distribution same type of problem I will not be discussing it you guys will be able to solve it yourself. If you cannot solve it the answer is given here you can check it also it is the same similar type of problem, those solutions are given.

Now, here once the only one thing is that I forgot to mention the in case of binomial distribution, what is the mean what is the variance binomial distribution mean is p into q variance is root over and p and q I mean sorry, standard deviation is root over npq similarly for hypergeometric distribution when we talk about mean and variance. So, similar for hypergeometric distribution also what is mean and variance it is actually given in this formula only here.

Hypergeometric distribution mean is np, and variances np into 1 – p n – m, n - m means n you can take divided by n - 1 that is the formula for variance, you can calculate it by the formula x - mu whole square f(x). If you calculate it, you will get this or if you can remember than that is well and good. You do not have to calculate it again and again for each and every distribution. That is you understood how to calculate the mean how to calculate the variance mean is x f(x), variance is x - mu squared x - mu whole square f(x).

So, that is the way to calculate now, you can calculate it or you can remember what is whatever it is. So, this is the formula for mean this is the formula for here this is the formula for mean of hypergeometric distribution, this is the formula for variance of hypergeometric distribution.

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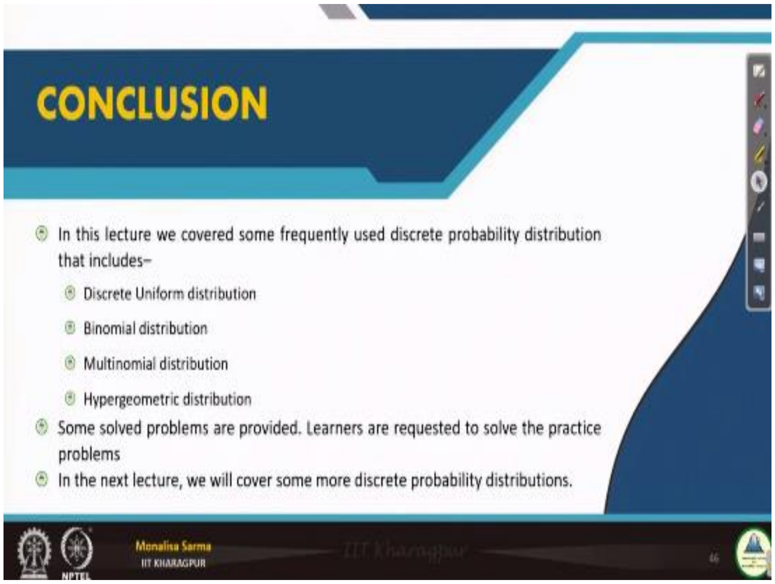

So, with this I conclude this lecture, so, here we have discussed discrete uniform distribution, binomial distribution, multinomial and hypergeometric distribution. We have also solved few problems. Few more problems we will be solving in one of our 10 tutorial classes which I will be taking after completion of all the problems the distributions.

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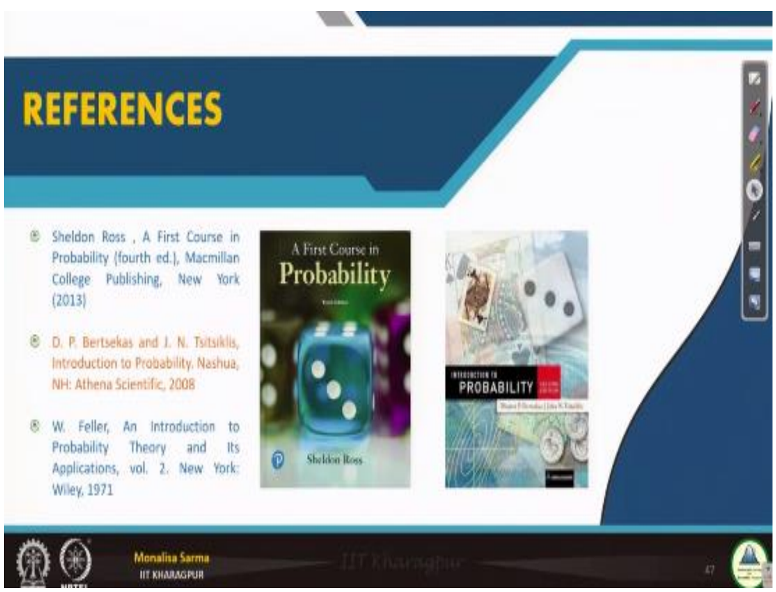

And again I request you solve as many problems as possible. These are the references which I had mentioned in my earlier lectures also and thank you.