Statistical Learning for Reliability Analysis
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Lecture – 06
Tutorial on Conditional Probability and Total Probability

Hello all of you. So, today we will be discussing some problems which the probability theory what we have discussed in the last 2 classes, last 3 lectures in fact, one lecture on tutorial and we have discussed probabilities on 2 lectures. Now, the last lecture we have discussed probability on conditional probability and total probability today, we will be doing tutorial on those topics.

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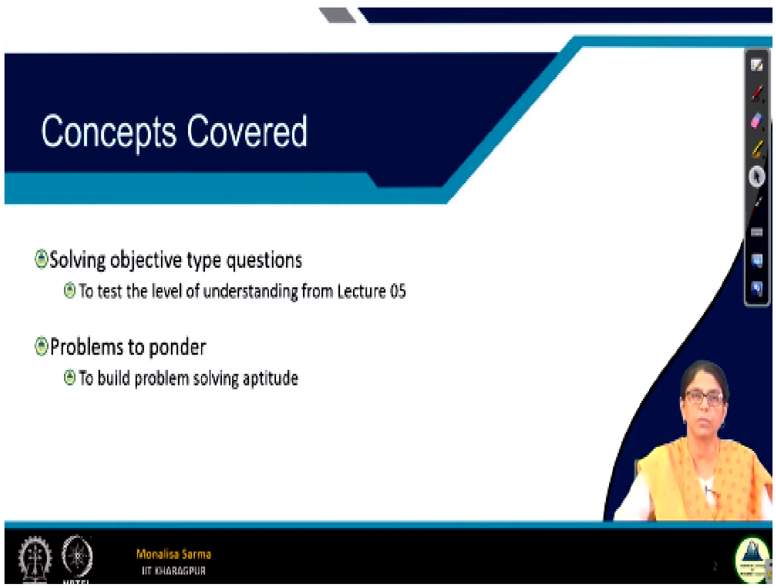

So, I will also solve we will later we will see some objective type questions, and mainly we will be doing some problems which will really help in building a solvable problem, problem solving aptitude. So, now first objective type question.

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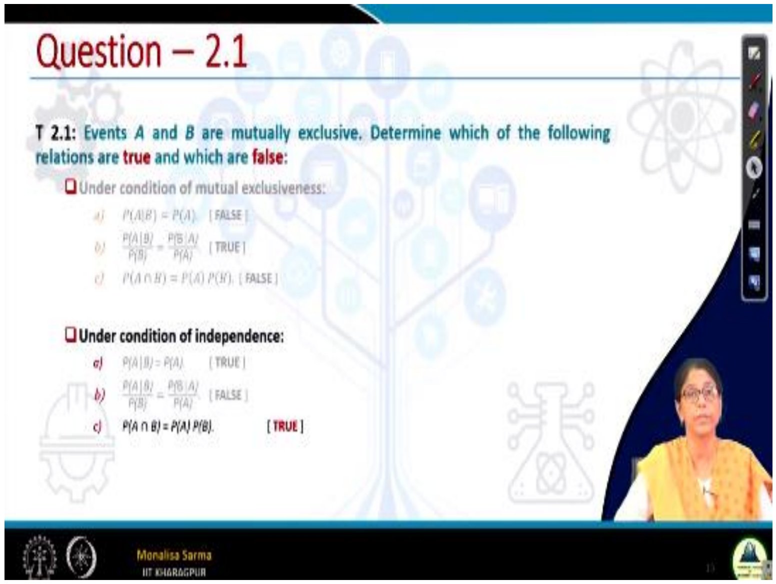

So, very simple objective type question all of you will be able to answer it just see very carefully, what it is given us events A and B are mutually exclusive determine which of the following relations are true and which are false. First one is given that A and B are mutually exclusive. So, even if A and B are mutually exclusive, so then the first question that is A, what it is given probability of A given B it is equals to probability A it is given. So, it is true or false what it is telling that event B has occurred, what is the probability that A has occurred?

Now, since A and B are mutually exclusive, what will be the probability of A given B it will be definitely it will be 0 it will not be probability A so, it is false. Now, coming to the second one what it is given? Probability of A | B / probability of B = probability of B | A / probability of A. So, I can now we will be able to answer this question what it will be because probability of A given B that is 0, 0 by PB is definitely 0, right hand side also it is 0. So, this is true.

Then a third question, third question what is probability of A intersection B is equal to probability of A into probability of B again mutually exclusive events we have seen probability A intersection B is 0. So, this is false. Now, we will see under condition of independence if the event A and B are independent, then under that condition a probability of A given B is also probability of A it is true or false. Of course, it is true.

And the condition of independence, it does not affect whether the event B has occurred or not. So, probability of A given B is probability of A then what is probability of A given B upon probability of B is it equals to probability B given A by probability of A that will be able to do it by yourself and that is obviously that is false. Next is probability of A intersection B, probability A intersection B is also probability of A into probability B and that is true in case of independent event.

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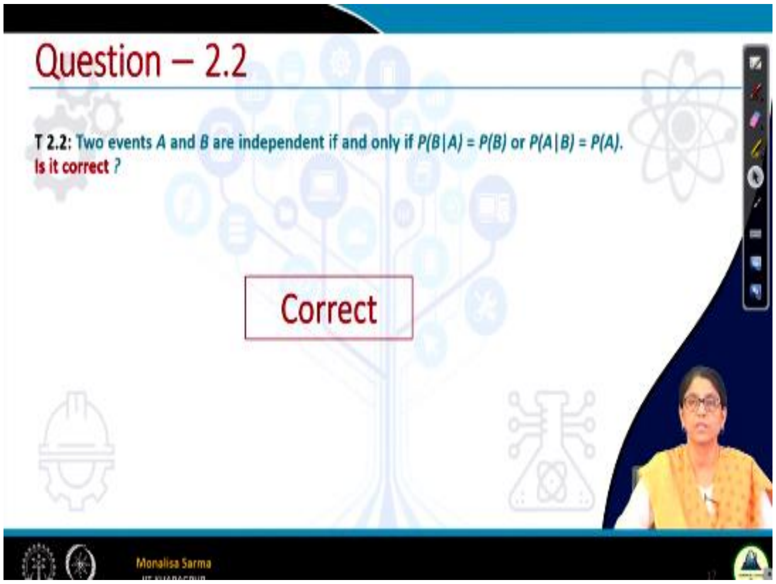

Second questions, 2 events, A and B are independent if and only if probability of B given A is P of B and probability of A given B is probability of A is it correct? Obviously that is correct. So, it is a very simple question. Now, we will be solving some big problems, which will really help you to understand the concept of conditional probability as well as the total probability.

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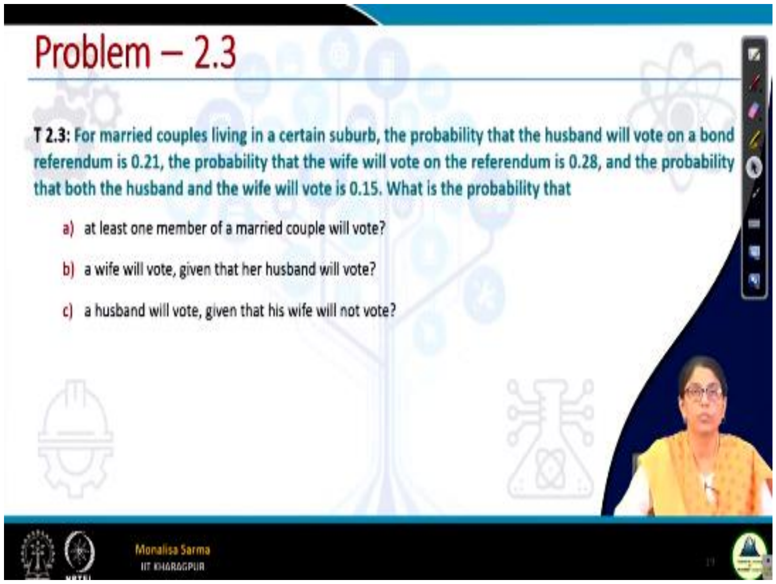

First question, for married couples living in a certain suburbs, the probability that the husband will vote on a bond referendum is 0.21. The probability that a wife will vote on the referendum is 0.28. And the probability that both the husband and wife will vote is 0.15. So, what is the probability that a at least 1 member of a married couple will vote? Now the question is that here in my last tutorial, I have also mentioned that whenever any probability problem always first try to write down the events. If you write down the events and solving it will be become very easier. It will be able to picture it actually clearly.

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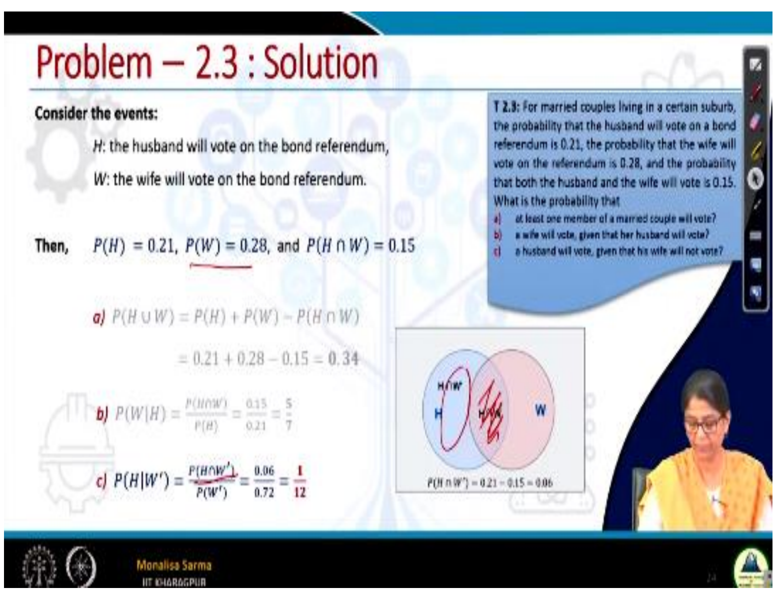

So, now what is it? So, see here the Venn diagram, from the Venn diagram it will be clear because what it is given the probability that the husband will vote that is okay, let me show it for a pen, this portion, this whole portion, this whole portion is the probability that the husband will vote and the probability that the wife will vote is the other portion is the W this circle.

Now and what it is given probability that both the husband and wife will vote. Both of them will vote means that is the intersection part. So, which one is the intersection part the slight pink this one, this one is the intersection part. So, all these probabilities are given. So, and this is given to us probability of H is given, probability of W is given, probability of H intersection W both husband and wife will vote that is H intersection W and both is independent.

So, now, what it is given, what is the probability that at least 1 member of married couple will vote at least 1 member means what we need to find out at least 1 member means either husband will vote or wife will vote or both will vote at least 1. At least 1 means it is union so, it is probability of H union W. So, what is probability of H union W we know the formula probability of H plus holder W and we will have to subtract the H intersection W part remember why we have subtracted it.

Because we have calculated that part twice once while we have calculated it for H we have taken that part again when we have done for W we have taken that part. So, we have taken it taking that part twice. So, we have to subtract it once. So, this problem H intersection W H union W yes we will be doing by using this formula. Then I guess next question a wife will vote given that the husband will vote? A wife will vote given that the husband will vote so, what is that means what given that has been that means probability that probability of W given H.

So, probability of W given H that is probability of W given H probability of H intersection W probability of H probability of H intersection W is given to us probability of H is given so, we will be able to solve it. So now, then similarly the next question a husband will vote given that his wife will not vote see, the third is a bit complicated not very complicated, if you draw the Venn diagram, so, always draw the write down the events and then draw the Venn diagram then things will become very clear.

Now, from the picture you can understand it has given a husband will vote given that the wife will not vote. So, husband will vote in this picture is a circle of H. This is the circle for H where husband will vote and this portion, this light pink portion, this is the portion where both the husband and wife will vote. Now, it is given that a wife will not vote that means from the husband part from this whole circle, we will have to subtract this portion.

Whole circle is probability of H from this whole circle, we will have to subtract this portion listen it? So, what is this probability of H given W bar that is given the wife will not vote is a probability of H intersection W bar by probability of W bar, W bar is from P of W we can easily find from P of W we can easily find P of W bar. Now, what is H intersection? H intersection W is from H we will minus this part then we will get H intersection W.

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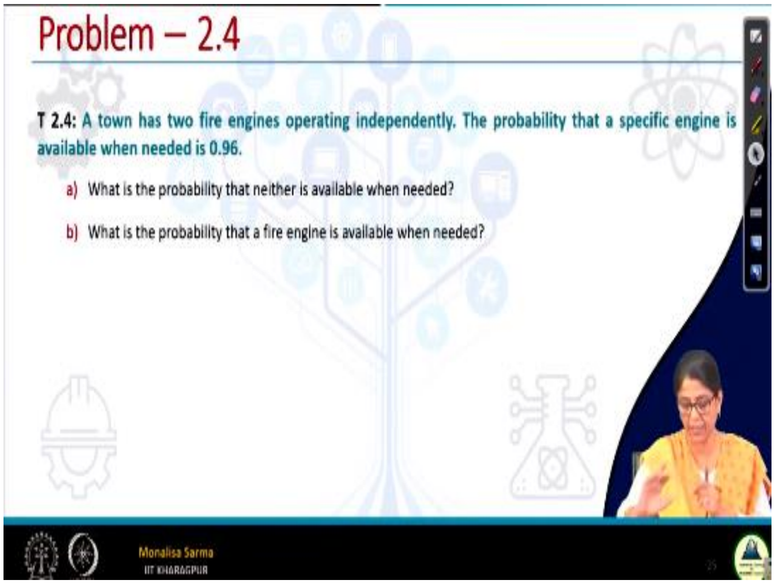

Now next sum, a town has 2 fire engines operating independently. That is independent events. The probability that a specific engine is available when needed is 0.96. Both the fire engines are what independently there are no dependency between them. So, what is the probability that neither is available when needed? So, first we will have to find out what is the probability that lable, then what is the probability of that the other one is not available?

Because there is independent events on when we try to find out when both is not available it is just a multiplication of both the events. That is all, is not it? When it is independent event what is probability A intersection B is probability A into probability of B.

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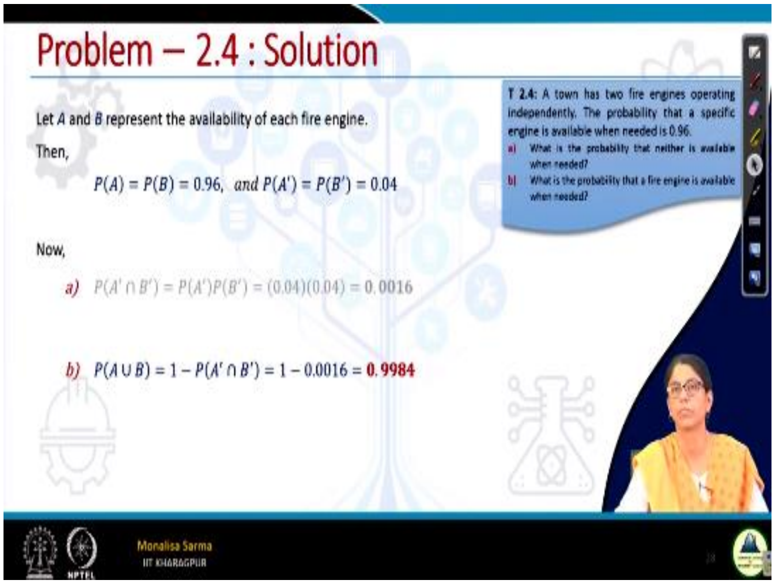

So, we have what is P of A, we have what is P of B and from that we can find out that ~A ~B it is just 1 - P (A) will give us one ~P(A). Similarly ~P (B). So, once we got that to find out value A intersection ~A intersection ~B, it is just a product of disturbance and then what is the probability that the fire engine is available when needed?

See here what is the probability that the fire engine is available when needed that means fire engine is available when needed that is can be 1 engine may be available, 2 engine may be available means either 1 is available or both are available that means when I need fire engine that is 1 that means either 1 is available or both are available. So that is A union B probability of A union B, then just put it in the formula.

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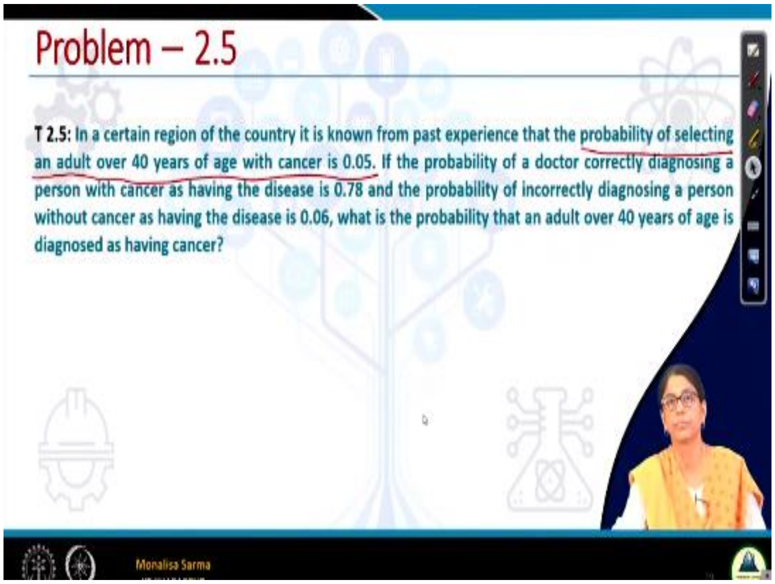

So, draw the Venn diagram, write down the events. For all the case, you do not need to write down the event or draw the Venn diagram of course, but events you will write for all the cases and even if you draw a Venn diagram, then things becomes more clear. So, next in a certain region of the country; it is known that from past experience, that the probability of selecting an adult over 40 years of age with cancer is 0.05, so what it is given the probability of selecting an adult over 40 years of age with cancer is 0.05.

If the probability of a doctor correctly diagnosing a person with cancer as having the disease is 0.78 that means the person has cancer and the doctor has diagnosed that is he has cancer that means given that the person has cancer, the doctor has diagnosed that he has cancer, conditional probability. And the other one is and the probability of incorrectly diagnosing a person without cancer is having the disease that means given that the person does not have cancer, but a doctor has diagnosed that he has cancer.

So now, what is the question what is the probability that an adult over 40 years of age is diagnosed as having cancer. So, this is clearly a question of what can you understand.

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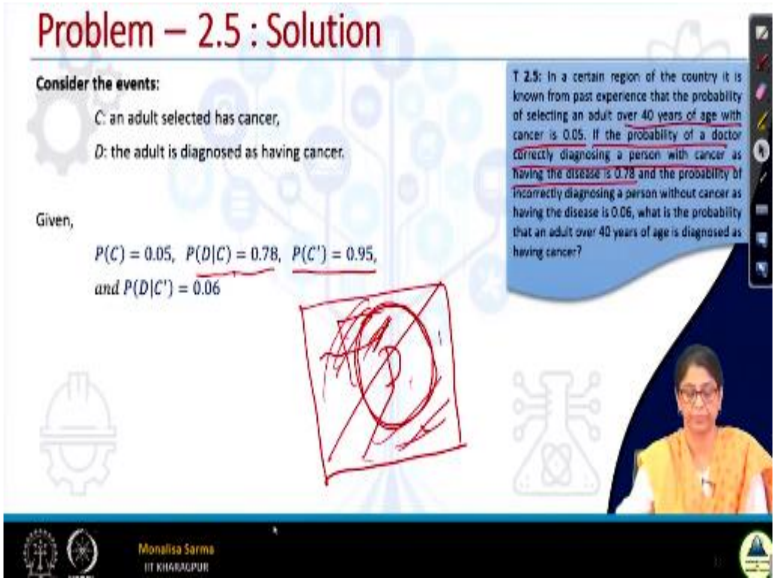

So, it is a question of total probability. So, first we have drawn, we have written down the events first season and adult selected has cancer that in their adult over 40 years of age, I have just removed age, because age is just a redundant information here, this is not very important information here sometimes in some question, you will see some very redundant information means which are not very much required. So unnecessary, we do not need to drawn those while writing down the events or sometimes we totally need to ignore some information.

Which is some information are just given so as to make the common problems complicated, like here that adult is whatever age it is immaterial in the questions basically. So, here event C is an adult selected has cancer and the D is the adult is diagnosed as having cancer. Now, what is given say, first thing is given probability of C is 0.05 adult lower 40 years of age, if cancer is 0.05 here it is given. Next it is given a probability of a doctor correctly diagnosis a person with cancer is having the disease is 0.78 this is the thing.

Given that the person has cancer and the doctor has diagnosed that he has cancer and the other one C is 0.05 than probability of C bar is 0.95, 1 – 0.05. And the other one, the next part, you see and the probability that incorrectly the person does not have cancer, but the doctor has diagnosed he has cancer. So, he is not a very good doctor maybe or maybe he was not very much attentive at the time of reviewing the reports. So, this is the thing so we have all this information.

Now, what we have to find out what is the probability that an adult is diagnosed as having cancer, this is basically a question of total probability, but now, if I want to draw the diagram basically, I need to find out this is my D I need to find out what is the probability of this D and what information I have say this is my inverse of sample space, I have how many disjointed now disjoint events yet 2 disjoint events. I have 2 disjoint events here. One is this portion that is the person has cancer and one is this portion the person does not have cancer.

This is the portion where a person does have cancer, maybe this is the person that person does not have cancer and this D is the circle in the D circle all the person are diagnosed with cancer. So, some portion of the people who has cancer are diagnosed with cancer, some portion of this thing who does not have cancer, but still diagnosed as having us cancer. So, now that means, what we will do to find out probability of D is basically intersection of this part and this part intersection of what intersection of this with D intersection of this with D what we have seen yesterday.

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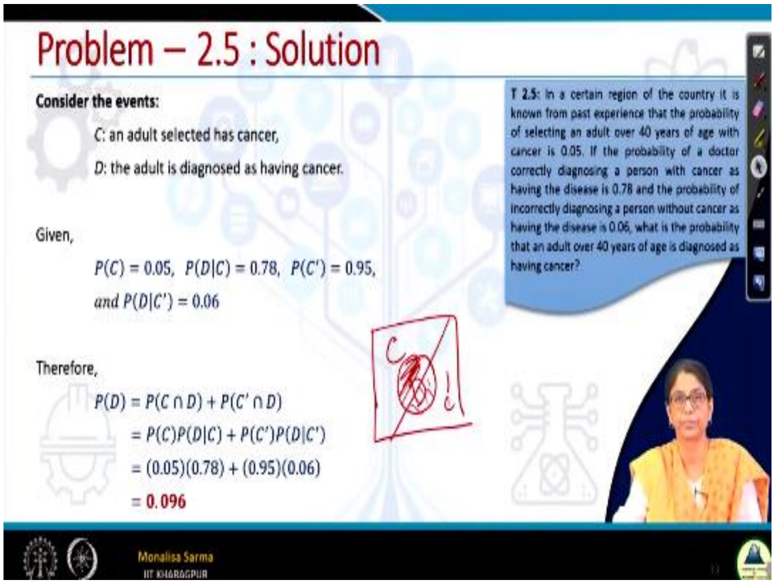

So, what is probability of D? The probability of D is the people that person has cancer C intersection D + C bar intersection D that is the second part. So, this is C, this is C bar and this is D. So, this portion is C intersection D and this portion, this portion is C bar intersection D. So, and we know what is this we have the formula we have the value for all this just put it in a value and we will get a result.

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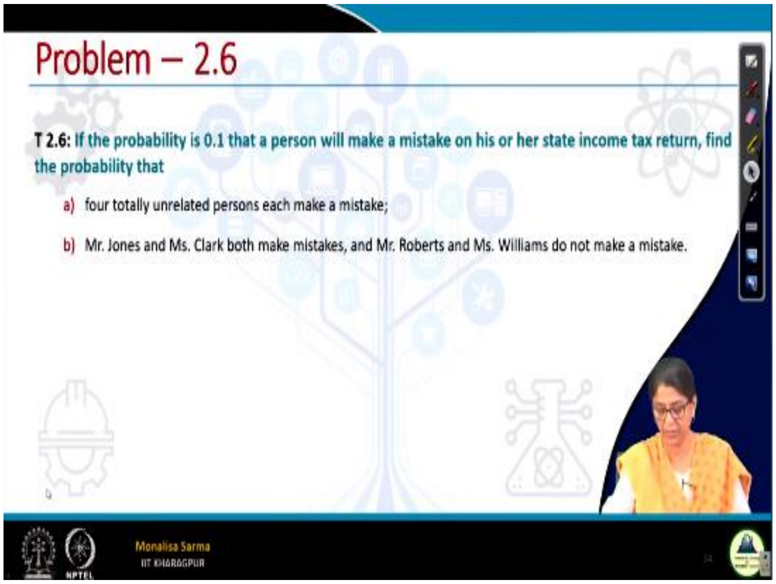

So now, next question if the probability is 0.1 that a person will make a mistake on his or her state income tax return find the probability of that 4 totally unrelated person each make a mistake. Now, is this a question of independent events, dependent events? Definitely, it is a question of independent events unrelated people. So, there cannot be any dependency if one is making this mistake means that does not mean the other will make any mistake totally unrelated people.

So, for totally unrelated person, it just unrelated people means it is just a multiplication of all the things.

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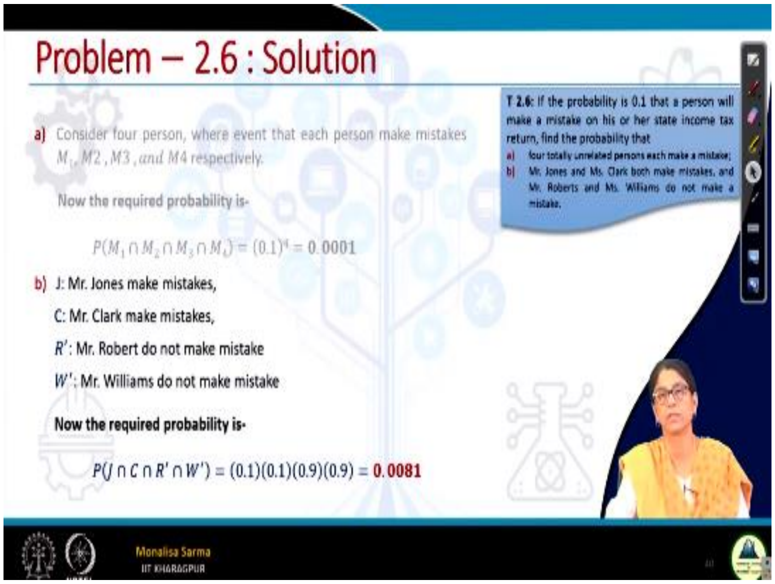

So, this is that goal then second is Mr. Jones and Miss Clark both makes mistakes and Mr. Roberts and Mr. William do not make a mistake. 2 person make mistake 2 person does not make mistake out of the 4 person may be and there 4 person out of this 4 person 2 person make mistake 2 person does not make mistake what is the probability of the not making mistake, it is just 1 - of probability of making mistake complimentary event.

So, again to compute B again since, all are independent events just a multiplication of all these events. So; now, the required probability this just the multiplication of this 0.1 person making mistake, 0.9 person do not making mistakes.

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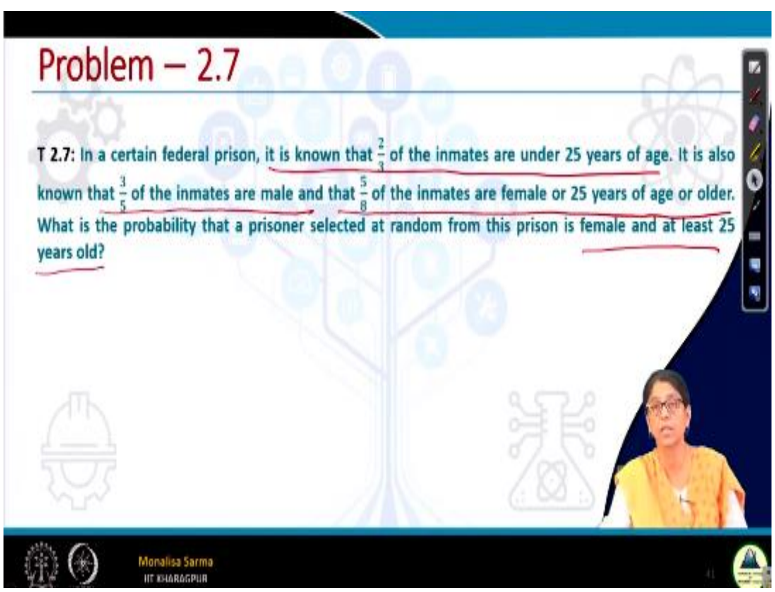

Now, in a federal prison, it is known that 2/3rd of the inmates are under 25 years of age. It is also known that 3/5th of the inmates are male and then 5 / 8 of the inmates are female or 25 years of age or older. What is the probability that a prisoner selected at random from this prison is female and at least 25 years old. Now, here age is not a redundant information here we age is necessary like in the last question for detecting cancer, the person is 40 years old and all that was like unnecessary information, but see here age is an important information.

So, we just cannot ignore it. So first, what is given we will see one by one it is known that 2/3rd of the inmates are under 25 years of age. So, this is one important information 2/3rd of the people are under 25 years of age. So, we will write it even corresponding to this next, it is also known that 3 / 5th of the inmates are male. So, male is 3 / 5 then what is female 1 - 3 / 5 because it is just a complimentary event. And 2 /3rd of the inmates are under 25 years. So, what is the percentage of above 25 years, so it will be 1 minus that right?

That is under 25 years, it is less than 25 not equal to 25, then what is another information is it a 5 by 8 of the inmates are female or 25 years of age or older? Here 5 by 8 are female or 25 years or older. Now, what we need to find out probability that the prisoner selected at random from this prison is female and at least 25 years old. Now write down the events first when you write down the events things you will see it is very easy.

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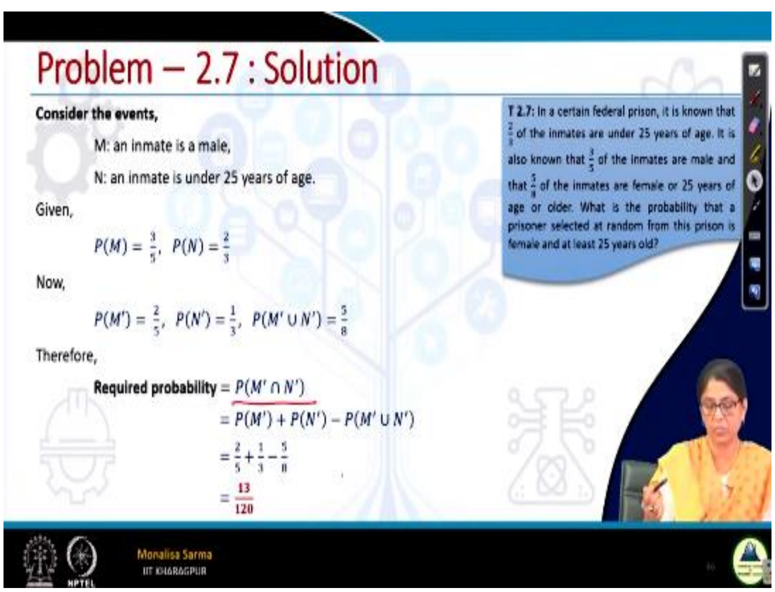

First is we will write that an inmate is a male and inmate is under 25 years of age that is the first information that we had and this was male is 3 / 5 and was 2 / 3, then again we had one more information is 5 / 8, 5 / 8 of the inmates are female or 25 years of age. So, what is that? Female or 25; if male we under writing by M than female we can write the ~M. Male we are the denoting the male event we are denoting by M than female event denoted by ~M.

And is event I am denoting that event inmate is under 25. So, N bar is an inmate which is 25 or above just a complimentary so my probability of M bar union N bar it is given it is 5 / 8. So, now what is this, what is the probability that a prisoner selected at random from this prison is female and at least 25 years, is female and at least 25 years or older. So, that means I have to find intersection. So, what do I need to find?

Probability of M intersection ~N, ~M denotes female and ~N denotes inmate is 25 or above at least 25 years old at least 25 means 25 or above. So, what is this formula corresponds to we have seen it before just put it a value and we will get it so, any probability problem always write events properly, clearly if you write events you will be able to solve it that I guarantee.

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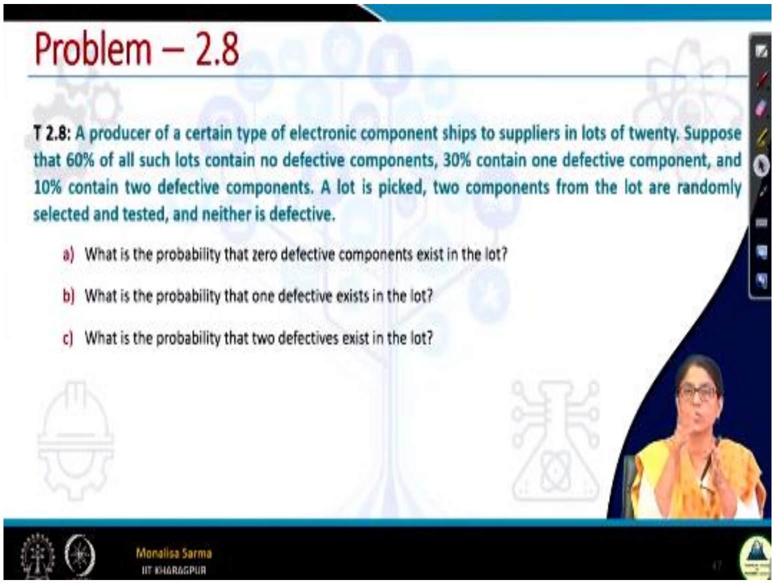

And for total probability always draw the Venn diagram which portion you have to find out, which total probability of what and what are the disjoint events always specify the disjoint events which disjoint in total probability about that different disjoint events cover up the required probability, but is not it so, now, this required probability is nothing but a summation of all these disjoint events. So, once you do that and whatever value is given you will be able to calculate it.

So, problem is if you could find draw the Venn diagram properly then it is done. Next a producer of a certain type of electronic component ships to supplier in lots of 20, suppose that 60% of all lots contain no defective components, 30% contains 1 defective component and 10% contains 2 defective components. 60% of the lots content no defective, 30% contain 1 defective component and 10% contained 2 defective component means out of 20, 60% is no defective again other different lot.

Lot means different categories and again in another lot 30% content 1 defective and another lots 10% contains 2 defective. A lot is picked 2 components from the lot are randomly selected and tested and neither is defective, 2 components from the lot are randomly selected and tested and neither is defective, what is the probability that 0 defective components exist in a lot? 2 components are picked and both are found to be not defective.

Now, what is the probability that it has come from that lot, which has 0 defective. So, this is a question of Bayes theorem if I would have asked what is the probability that getting no defective components. Then it is a question of total probability. Now, I have asked 0 defects we have 2 components from the lot we have selected and found that it is not defective what is the probability that it is come from a particular one disjoint area? So, it is a question of Bayes theorem.

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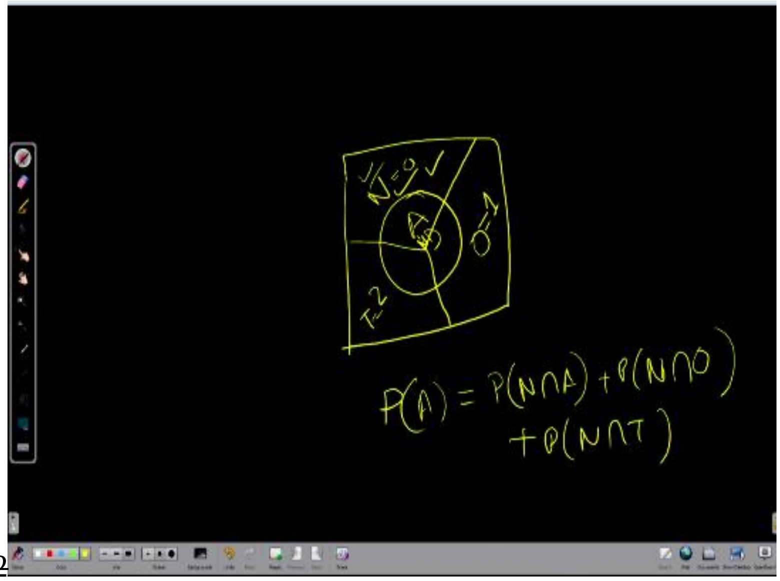

So, now this is the law which no defective component no defective and so, there are 3 different categories here, 3 different disjoint categories here, what are the 3 disjoint component? One with 0 defective say this is one category which is where there is 0 defective another say it is what we have say whatever event I have named I forgot here 0 is it is say with 1 defective another say it is T say it is 2 defective, here it is 2 defective. Now, here what I found? I have picked 2 component from a lot I have picked 2 component I found both the components are not defective.

So, what is the probability that I have picked from this lot or I means I need to find out when I found that both the components are not defective what is the probability that I have picked this lot from this one where there is no defective component. So, first we will for that first we will find out the total probability. Total probability this one, this whole circle, this whole circle maybe let me consider this whole circle as this okay this I am one second.

This I am writing is N then it is easier to differentiate N is no defective, 0 is 1 defective, T 2 T for 2 is 2 defective and this one with 2 defective components with no defective I am writing as A suppose this circle. So, in my what will be my probability of A? Probability of A will be what probability of N intersection A probability of N intersection O and probability of N intersection T this is my probability of A. That is the total probability.

Now, I am interested in finding out that my defect if it is some and I found that it is non defective what is the probability that I have picked it from this no defective component?

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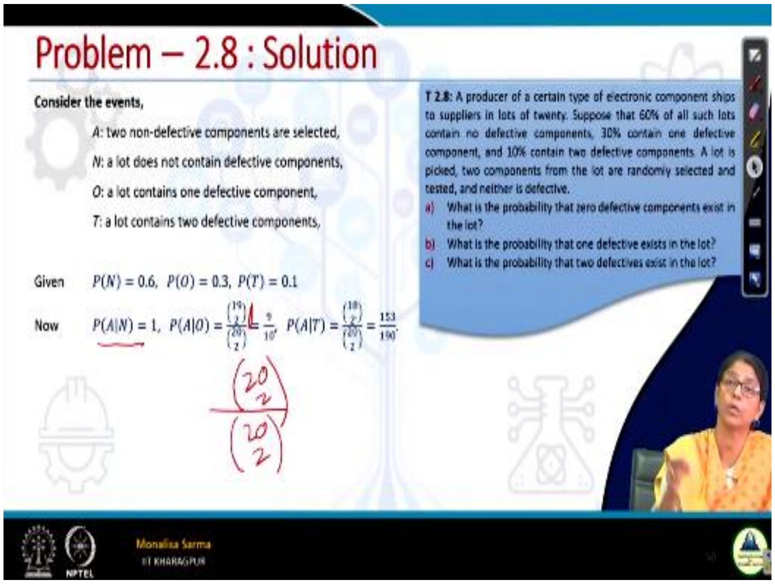

Let me come back to this. So, these are the things which are given probability of N is 0.6, 0 is 0.3 and probability of T that is contains 2 defective component 0.1. So, these are the things given what is see here. Probability of A given N what is this? Probability of A given N means what is this means given that the lot contains no defective component what is the probability that I have picked up 2 defective components.

What is the probability that I have picked up 2 non defective components what is my A? A is to non defective component this how will it find out what is the total sample space? Total sample spaces is from I am picking up from 20, 20 C 2 is my total sample space and out of these 20 when there is no defective components, how many defects it is no defective components. So, all are non defective components. So, that is from 20 again from non defective components I am picking 2. So, this is my probability of A | N is 1.

Probability of A | O; that both the components are not defective when I have picked it from the lot which has 1 defective component. So, if it is 1 defective component you can see here it has given this one 19 from 19 I am picking 2 because 1 defective components that I am removing it, so from 19 if I am picking 2. So, this is my probability of probability of A | O similarly, what is a probability of A | T that is there 2 defective components. So, I am picking 2 from the rest 18. So, from 18 I am picking 2 that is 18 C 2 divided by 20 C 2.

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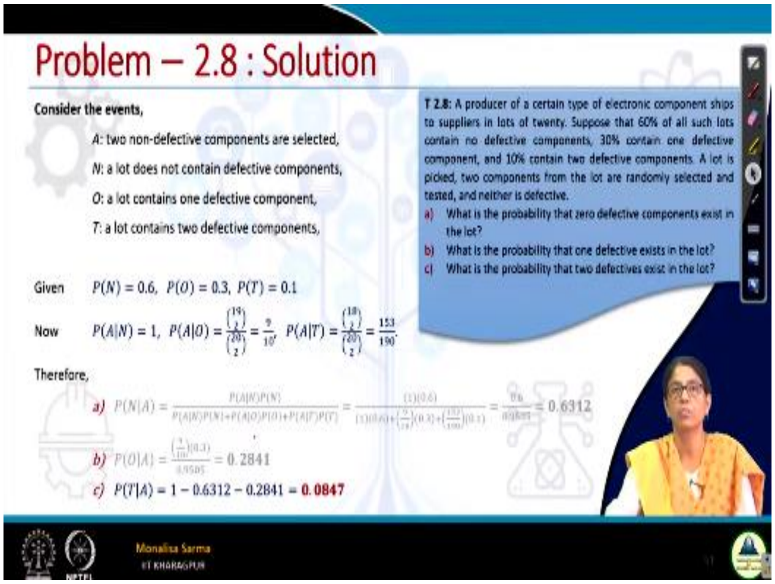

Now, what I need to find out is probability of N | A given that 2 non defective components are pick what is the probability that I have picked from N that is no defective components, this denominator is the total probability which I have just now I have shown into a whiteboard that a black board which I have shown in the blackboard how we have calculated the total probability this is the total probability, probability of N given A what is this probability A given N.

Probability of A that both the components are not defective given that we have picked from lot which are not defective into probability of N. So, by this the total probability that is probability of A this denominator is nothing but probability of A. So, now putting the formula we will be getting. Similarly, I can next question what is the probability that 1 defective exists in a lot same way, then again what is the probability that 2 defective exist in a lot same way just probability of T given A.

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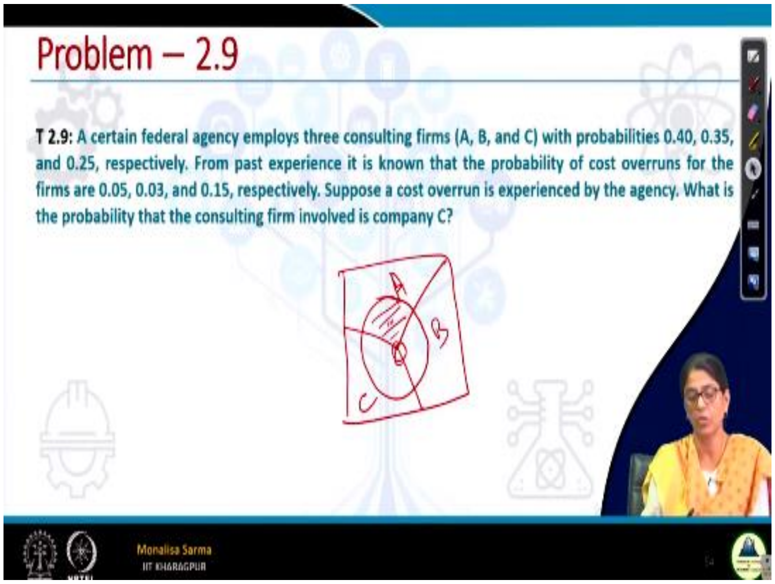

Next this question, if a certain federal agency employs 3 consultants in consulting firms A, B and C with probabilities of 0.40, 0.35 and 0.25 respectively, it employs 3 consulting firms each firm's the probabilities of each firm is given and from past experience it is known that probability of cost overrun for these firms are point 0.25, 0.03 and 0.15 each firm has a cost overrun. So, each firm is used some first one is used 40% of the time B is 35% of the time C is 25% of the time.

However, there is a cost overrun for A is 5%, cost overrun for B is 3%, cost overrun for C is how much 1.5% sorry 15%. Now, then, suppose a cost overrun is experienced by the agency what is the probability that a consulting firm involves in company see again this is a question of Bayes theorem cost overrun is involved. So, this is maybe the cost overrun. So, maybe overrun I can say it is let me take it as O, let me take this event as O.

Then, the universal sample space, there are 3 that is A, B and C. A is 0.40, B is 0.35, C is 0.25 and this portion this intersection, intersection of A and O disjoint diameters O intersection of A and O is 0.05, intersection of B and O is 0.03 intersection of C and O is 0.15. Now, we have to we know the things now, what is the probability of consulting a firm involves can you see just same as previous question.

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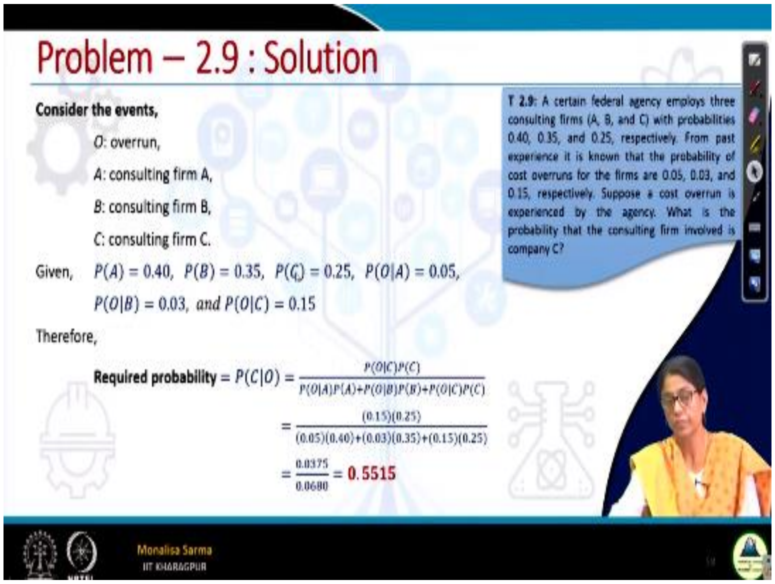

So, these information are given. So, required probability is this denominator is the total probability first calculate the total probability and what is, it is given what is the probability that a consulting firm involves company C. So, we have calculated probability of O that is the total probability now, what is given O what is the probability that it has cost overrun has happened because of the company C. That is all.

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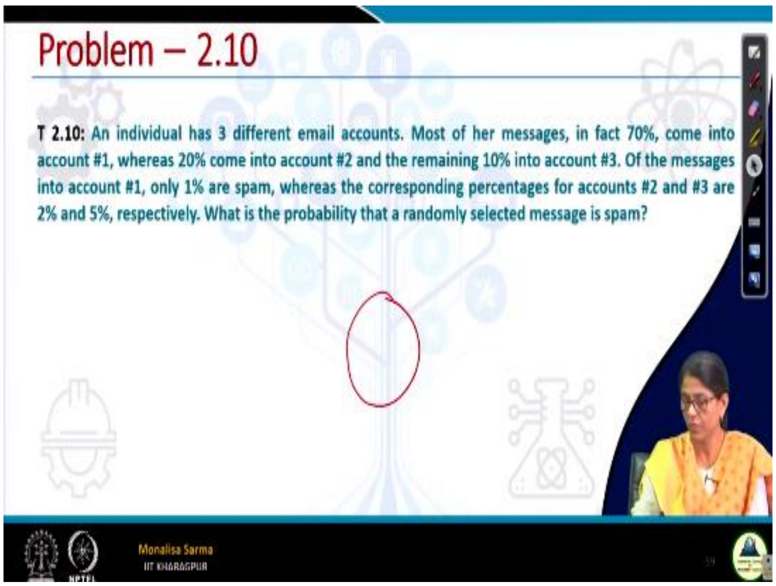

There is one more problem; I think the last problem in this tutorial. So, an individual has 3 different email accounts most of our messages in fact 70% come into account 1 whereas 20% come into account 2 and remaining 10% into account 3 the particular person has 3 accounts, all the messages into account 1 person only 1 person has spam account 1, one person on the mail as spam mail whereas the corresponding person is for account 2 and 3 are 2 person and 5 person.

What is the probability the randomly selected messages just spam? Again what is the probability the randomly selected message you send spam. So, this is a question on what total probability not Bayes theorem we just want to find out the circle probability of the circle.

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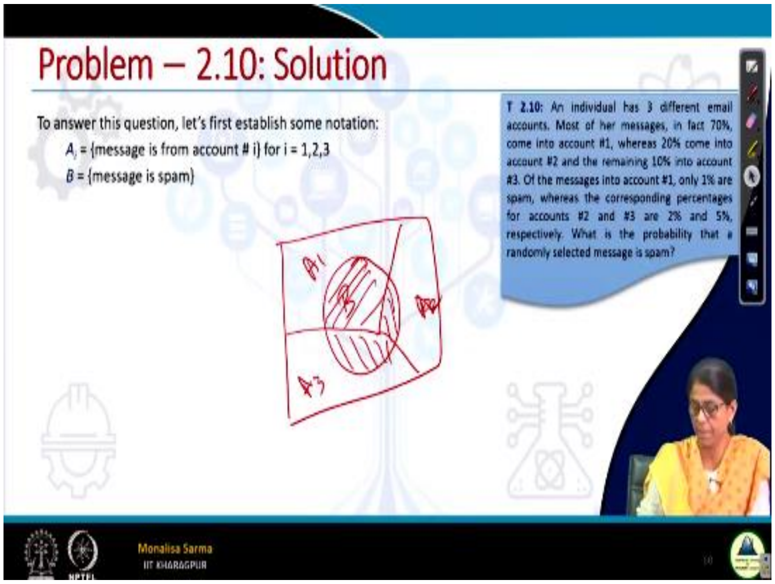

So, first let me write down the thing what to say event so, I have written as A 1 is messages from account 1, A2 is message from account 2, A 3 is message from account 3. So, this is now, this is my circle is B again this, this is A 1, this is A 2, this is A 3. A 1, A 2, A 3 and what is this? This intersection of A 1, intersection of A 1 and B what is given intersection of A 1 and B is 1 person, intersection of A 2 and B how much it is given A 2 and B is given 2 persons.

A 3 and B is given 5 person this intersection is given, this whole A 1 is how much given, whole A 2 is given how much whole A 3 is given how much to finding the total probability is easy, is not it?

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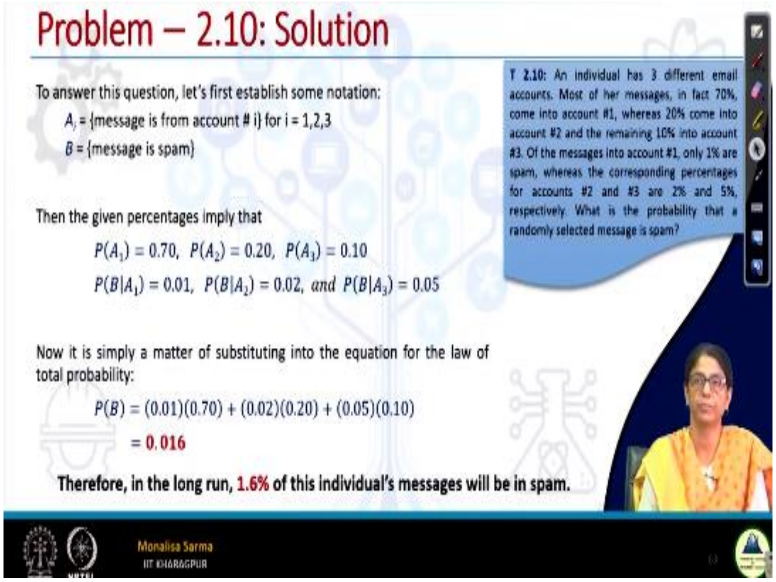

So, that is it. So, therefore in the long run what we have got 1.6% of this individual message will be in spam. So, what again like In last tutorial I have suggested this again I am suggesting you solve as many problems as you can try to find out any book on probability you will just Google will be getting many free online books. Just go and browse those books you will get many problems, solve those problems in any books the answers are also given.

Do not directly see the answers for solving than try to see where it ends such a character not the solving this, it will be more which will give you more confident with that I end this tutorial. So, these are the references and thank you guys.

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