Statistical Learning for Reliability Analysis
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Lecture - 05
Conditional, Total and Reverse Probability

Hello everyone, so in continuation of our earlier lecture and probability theory to be will be again discussing from other concept of probability basically what to say total probability.

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Will basically discussing dependent events, independent events, conditional probability, total probability and then we will also discuss Bayes theorem. So now first, we will discuss what is dependent event and independent event.

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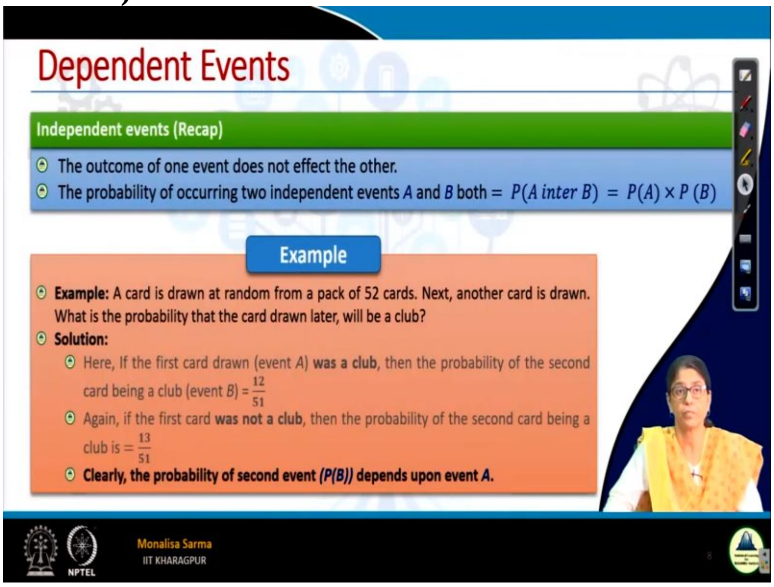

Independent event of course, we have discussed in our last class on probability that is in first class. And still we will repeat it here so that it becomes easier to correlate between dependent events. Now, when we talk of independent events, independent events are those events for which occurrence of one does not have an effect on any other, on the occurrence of the second event, like let me give you a very simple example.

Suppose to today is raining. So, probability that today is raining and what is the probability that you are attending the class. Now, if you are attending an offline class, so you have to come from the hostel to attend the class, so probability, if it rains, so this event will definitely may affect your attending the class. So, this becomes a dependent event. But however, now since all of your attending your class in the comfort of your home.

So, whether it is raining or it is not raining, so it does not make any difference. So, now, it is even that it is raining an event of your attending class, these 2 events then it becomes totally independent, that is the occurrence of one event does not have any effect on the other events then we call this as a independent event and vice versa, the other is the dependent event, if the occurrence of one has a has an effect on the occurrence of other events then we call that the dependent event.

Now, if the events are independent, so probability of occurring 2 events A and B with we have seen our last class that is probability of A intersection B. A intersection B is the probability of offering 2 events. So, that is we have seen that is probability of A cross probability of B. So, small example, like a card is drawn at random from a pack of 52 cards, from a deck of cards there total in a deck of cards total 52 cards from a deck of cards, we have drawn one card next we have drawn another one card. What is the probability that the card drawn later will be a club? So, now, there is 2 things first of all, if the first card that we have drawn, if it is a club then the second card will be a club does a different probability because it will be dependent on the first one there was number of clubs limited, right? And then it is a fixed number.

So, again, the first card that we have drawn is not a club then, the second that we have the second card, we will draw probability that it is a club that also be a different probability. So, we will see in what way it is different see here, here in the first card drawn was a club then the probability of the second card being drawn will be 12 / 51, why 51 because earlier the number of cards is 52.

Now, already we have drawn the one card, so the remaining card is 51 and already their total how many clubs in that pack of card total 13 clubs already if the first card is a club, then the probability that the second card is club is it becomes 12 / 51. So, again it the first club was card was not a club, first card is not a club then what happens there are total 13 clubs in the pack and you the first slot that you have drawn is not a club.

Then what is the probability that the second card is a club that is again that will be 13 / 51 fine, it will not be 52 here also it will be 51 because already one card we have drawn that is only total non-remaining card is 51. However, if we keep back the card, we have drawn one card then we have again put back the card in the deck we have shuffle it then we draw the card then even becomes independent.

This is an example of a dependent event. Clearly the probability of your second event depends upon event A obvious right?

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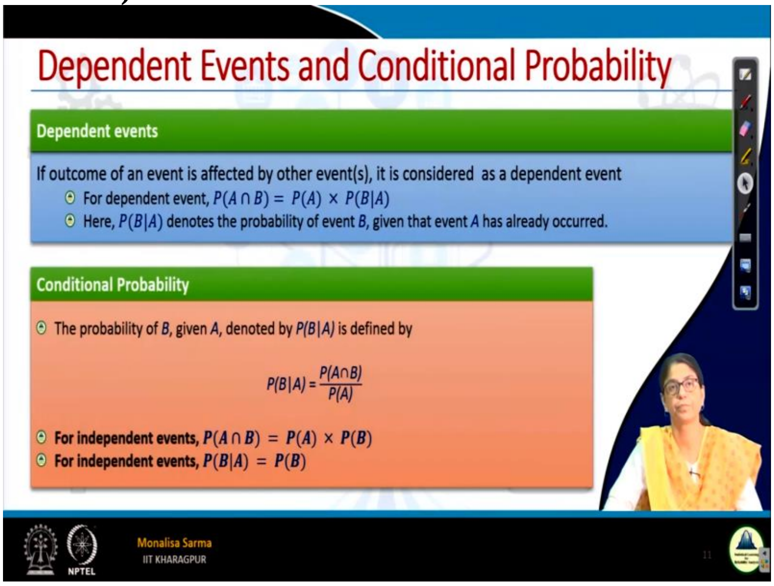

So, if the outcome of an event is affected by other events, then it is considered a dependent event. That is what I told. For dependent event, B intersection B that is probability of P probability of A and B. So, for dependent event, it is what it is not P probability of A cross probability of B, but it is probability of A cross probability of B | A, probability of A first A as occurred then probability of B | A what is the probability that B has occurred given that A has occurred.

So, it is probability A intersection B is probability of A cross probability of B | A. So, here probability of B | A denotes the probability of event B given that event A has already occurred. So now if you are interested in finding out; so, this is the expression. Probability of A intersection B is also probability of A x probability of B | A now, if we are interested in finding out what is the probability of B | A.

Just from this expression only we can find a probability of B | A is nothing but probability of A intersection B / probability of A. So, for independent events, we have seen probability of A intersection B is follow a cross probability of B. So, what will be probability of B | A for independent event from the B | A will be probability of B it is not dependent on A so, what is the probability of B | that A has occurred, it is same because it is not dependent on A so, it will be probability of B only. Now, we will see some examples.

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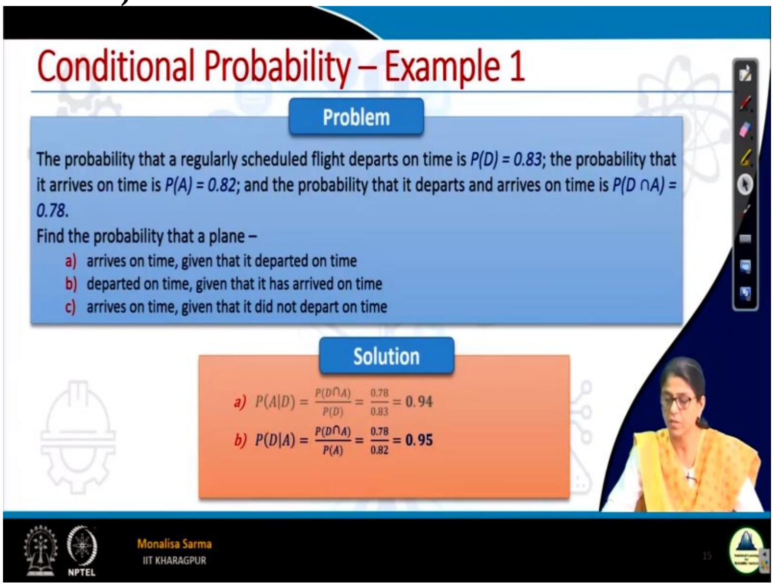

So, these examples you try to do it yourself of course, I will be explaining it. So, if you do it yourself your concept of conditional probability will be quite clear. So, the first question the probability that a regularly scheduled flight departs on time is given that is probability of D is 0.83 and the probability that it arrives on time that is probability of A is given a 0.82 and the probability that it departs and arrive on time.

That is departs on time and arrives on time that is probability of D intersection A it is given as 0.78. So, we have to find the probability that a plane arrives on time given that it has departed on time, given that it has departed on time what is the probability that it has arrived on time that means, we need to find out probability of A | D. So, what is given to us ? We need to find probability of A | D.

How what is somewhat to say equation for probability of A | D what is the expression that is probability of D intersection A / probability of D and so, probability of D intersection A is given 0.78 probability of D is given 0.83 so, we could compute probability of A | D. Now, similarly, departed on time given that it has arrived on time that means probability of D | A given that it has arrived on time what is the probability that it has departed on time.

So, same probability of D intersection A / probability of A simple formula you do not have to remember the formula if you understand the concept you will you do not need to remember the formula. So, now, see the next one that what it is asking is the probability that the plane arrives on time given that it did not depart on time. So, what is I am telling that has arrived on time given that it has not it did not depart on time we have to find that probability.

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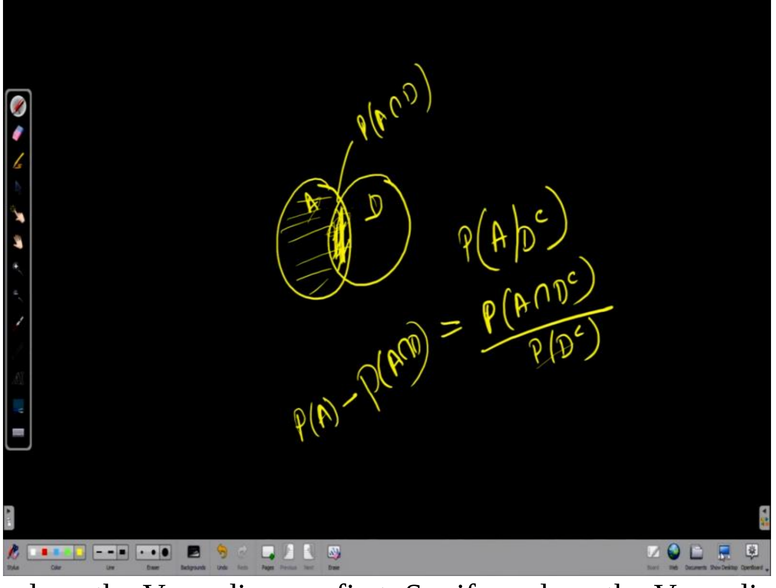

So, for that let us draw the Venn diagram first. So, if we draw the Venn diagram, see, if this is the event for the that plane arrives on time and if this is the event D that the plane has departed on time, now, this portion is probability of A intersection D that means, it has arrived on time and departed on time. Now, what it is asking probability that it has arrived on time given that it did not depart on time.

So, for that what we have to find, it is ask that probability that it has arrived on time given that it did not depart on time. So, what is this? This is equal to probability of A intersection D complement by probability of D complement. So, this is so, now, what is probability of A intersection D complement. So, that means probability of A intersection D complement is that means this whole area, this area is A this whole area. This whole area is A from this portion basically; we have to remove this portion, this portion, right?

So, it will be what is probability of A - probability of A intersection D then we will be getting probability of A intersection D complement. So, from the Venn diagram, it will be able to visualise it so, it is easier if we go from the Venn diagram.

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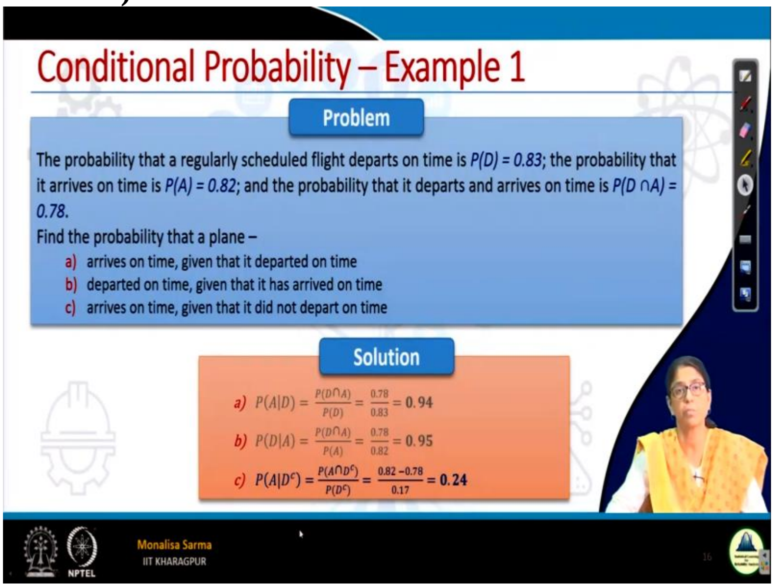

So, this is a probability of A that it has a probability that it has arrived on time given that it did not depart on time. So, this is probability of A intersection D complement / probability of D complement. So, putting the values we will get the result.

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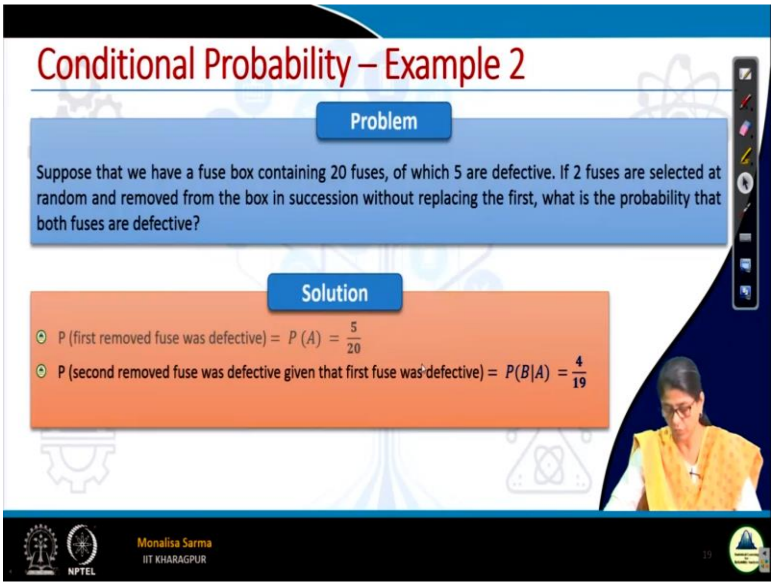

So, next example; suppose that we have a fuse box containing 20 fuses of which 5 are defective. So, my sample fuse is 20 of each 5 of defective if 2 fuses are selected at random and remove from the box in succession without replacing the first. We have selected one fuse and we did not replace it again we are taking the second fuse what is the probability that both fuses are defective.

When we are taking the first case first event probability that the first fuse is defective what is the probability the first fuses defected total there are 20 fuses and after them 5 are defective what will be the probability that the first use is defective it will be 5 upon 20. Now, when we are taking the second fuse second fuse what happened already we have taken 1 fuse that sample space has reduced from 22 it has become 19 and as well as our defective has also reduced if the first one is defective. That was from 5 it has come down to 4. So, what second probability that second remove fuse was defective given that the first fuse was defective is 4 / 19. So, then what is the probability that both fuse are defective 5 / 20 x 4 / 19. So, this is the answer.

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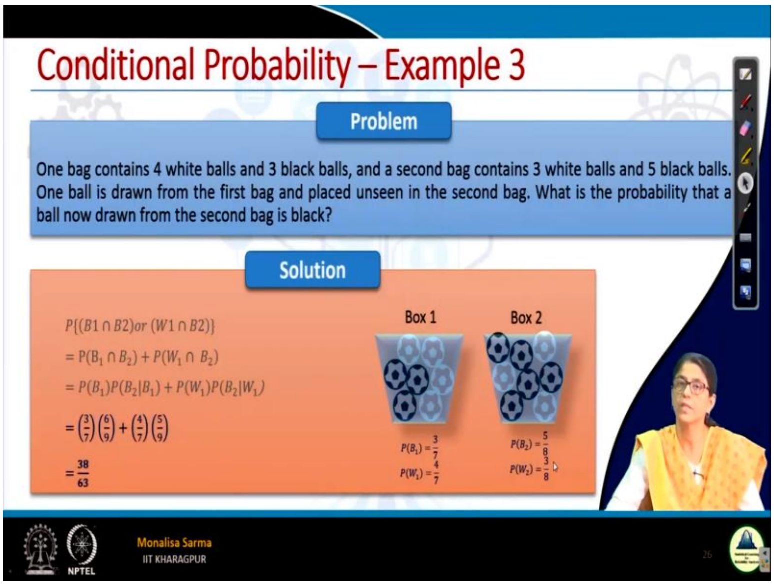

So, next question. A bag contains 4 white and 3 black balls and a second bag contains 3 white balls and 5 black balls. This is the first box which contains 4 white and 3 black, this contains 5 black and 3 white 1 ball is drawn from the first bag and place and change in the second bag from here I am drawing 1 ball and I am not seeing what is the colour that is white or black I am just putting it here on the box 2.

Initially in a box to how many balls are there 1 2 3 4 5 6 7 8 balls there it is 8 balls here it is 7 balls. So, what is the probability that a ball now drawn from the second bag is black. What is the question I have first I have drawn 1 ball from the box 1 I did not see the colour what it is and I have put it in the box 2. Now, what is the probability that the ball now drawn from the second bag now I have drawn one ball from the second bag.

So, what is the probability that it is black. There are 2 scenarios here if the first time first draw in my first draw, if I have drawn a white ball from here and I have put it here of course my number of balls has reduced increased, but then my white balls remain same isn't it, but in the first but however if I have taken a picked a black ball in my first draw and I have put it in here, then number of balls increase along with that number of black balls also increase.

So, the accordingly probability will change. So, what I want you to this is probability that my first draw is black and second draw is also black probability that I have in the first draw I have drawn a black which I put it here without seeing it what is the probability that it is that black and second one is also black or probability the first time I have drawn a white ball and second time I have drawn a black ball.

So, what is this probability? So, this is once you know this once you can write this now it will be very easy for you to so what is probability a B 1 intersection B 2 is nothing the same it is or both it cannot be it is a mutually exclusive event both cannot happen together. So, it is just + probability or B 1 intersection B 2 + P of W 1 intersection B 2. Now, you just need to find out what is probability of B 1.

Now, here see probability of B 1 is I am drawing it first time, so that is independent. So, it will be there what is the probability that the first ball is black? So, what is the probability of the first ball is black here how many balls there are 7 balls out of the 3 black balls probability that it is black is 3 / 7. So, probability of B 1 is 3 / 7. Now given that the first one is black. What is the probability with that second one is black? Given that the first 1 is black.

So, now here my black balls earlier it was 5 now my black box has become 6 and total how many balls it was earlier it was 8 now it has become 9. So, it is 6 / 9. So, my probability of B 2 | B 1 has become 6 / 9. Similarly what is the probability of W 1, probability of W 1 is 4 / 7 and what is a probability of B 2 | W 1 initially if I picked a white then my black balls remain same. So, what is how many black balls? 5 black balls and total black balls is now 9 that is 5 / 9. So that is it.

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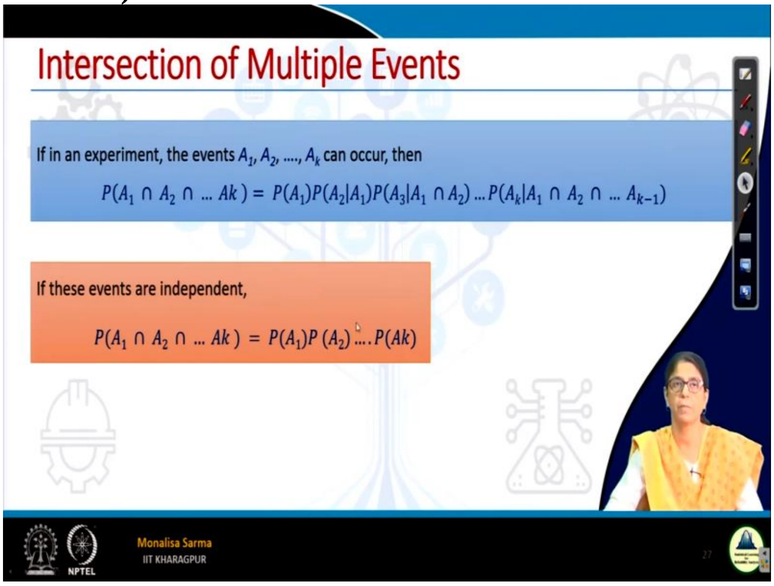

So, now if your intersections are interested in the intersection of multiple events, even experiment the events that A 1 A 2 A k can occur, then what is the occurrences of it is multiple events like probability of A 1 and A 2 and A 3 and A 4. So, if it is an independent event then it is just a multiplication of all the events. If it is an dependent event then of course, we have to go for dependent probability that is a conditional probability.

Probability of A 1 probability of A 2 | A 1 probability of A 3 | A 1 intersection A 2 and likewise, but if it is independent simply multiplication of all the probabilities.

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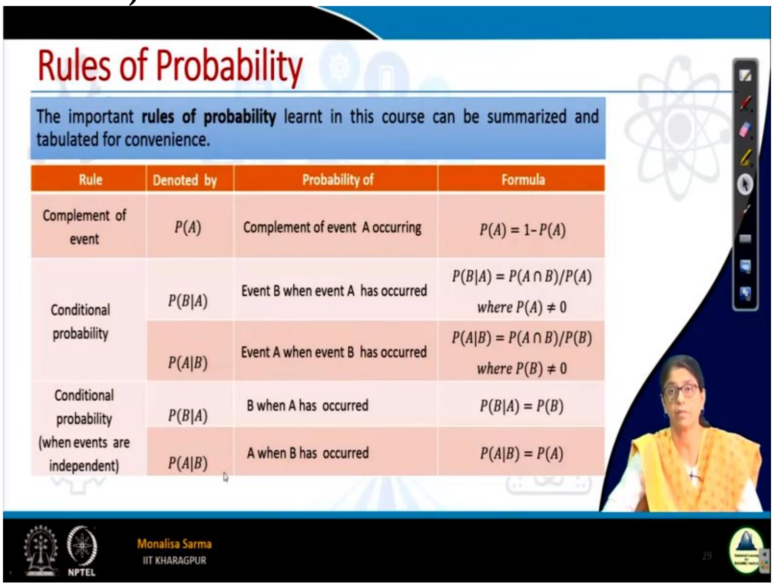

Now, here, I will just summarise the probability rule still know what we have learned I am not going to introduce anything new, but follow whatever we have learned till now, that is complement of the probability if it is P(A) is the probability its complement is ~P(A) that is 1 - P (A) conditional probability of B | A what is probability of B | A? Probability or B | A is probability of A intersection B divided by probability of A what we have already seen.

But where, where probability condition is a probability of A is not equals to 0. Similarly, probability of A | B same thing it will be probability of A intersection B upon probability of B here where P of B is not equals to 0. Now, when does events are independent, what is the probability of B | A it is probability of B. What is a probability of A | B that is probability of A if it is independent events.

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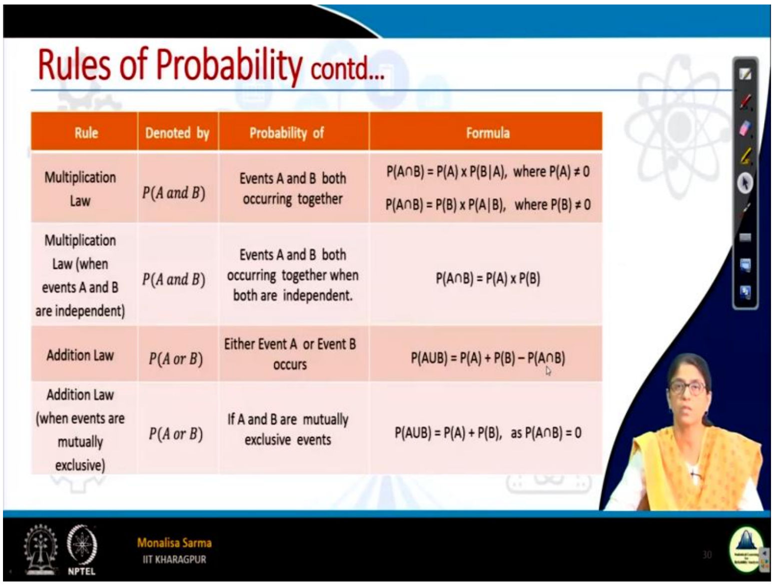

So, then multiplication rules what we have seen that is probability of A and B. Probability of A and B is nothing that probability of A x what is B probability of B | A if it is dependent, if it is independent simple probability of A cross probability of B what we have just seen it is just an I have summarised it in this table, we have even see probability of A or B also that is A union B probability of A + probability of B – probability of A intersection B and if it is mutually exclusive event, do you remember what is in case a mutually exclusive event. So, probability of A intersection B this portion will be 0. So, problem A or B will be simply probability of A + probability of B.

Now, total probability, it is something many people find it very complicated. If you understand it properly, then you will see it as very easier if you can conceptualize it, if you can understand it, what is actually total probability where we are and applying it. So first, so I am requesting you please pay attention and try to understand it properly.

So, when we talk of total probability first, before we going to the slide, let me first explain you with the help of this small example.

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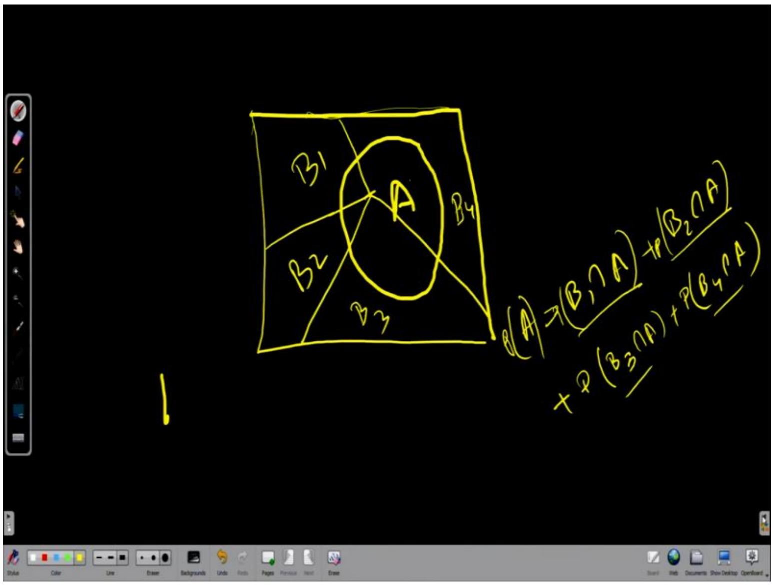

So, suppose I am interested in finding an event A, I do not know the occurrence of this event, what is the probability of occurrence of this event see what I know is that A is covered by some disjoint events. So, A is covered by some disjoint events so let me take the pen A is says suppose this is the universe sample space universal sample space. So, A is covered by some disjoint event.

So, some disjoint event say suppose this is B 1, this is B 2 B 3 B 4. So, A is I do not know that occurrence of probability of A but I know that some disjoint events collectively cover A these are the disjoint and this is the universal sample space, this is the outer one is a universal sample space. So, what are the disjoint events B 1 B 2 B 3 B 4 these areb the disjoint events. Disjoin event means there is intersection between the time is 0.

Now, what is that, I know these disjoint events collectively forms A and this scenario if I know the probability of this scenario, how much there is some probability of this how this A is formed by what? A will be formed by summation of all this. So, basically A is B 1 intersection A + B 2 intersection A + B 3 intersection A likewise all the + B 4 intersection A, I know this probability, this is probability fine let me write it a bit clearly plus probability of B 3 intersection A + probability of B 4 intersection A.

So, A is formed by this disjoint element. A formed by this disjoin events B 1 B 2 B 3 and B 4 and I know the occurrence of this probability of occurrence of this, probability of this scenario. So, if I know this then I can find a probability of event A using this is not it, is it just a summation of all these disjoint events. So, that is only total probability nothing else nothing else. So, it is that easy.

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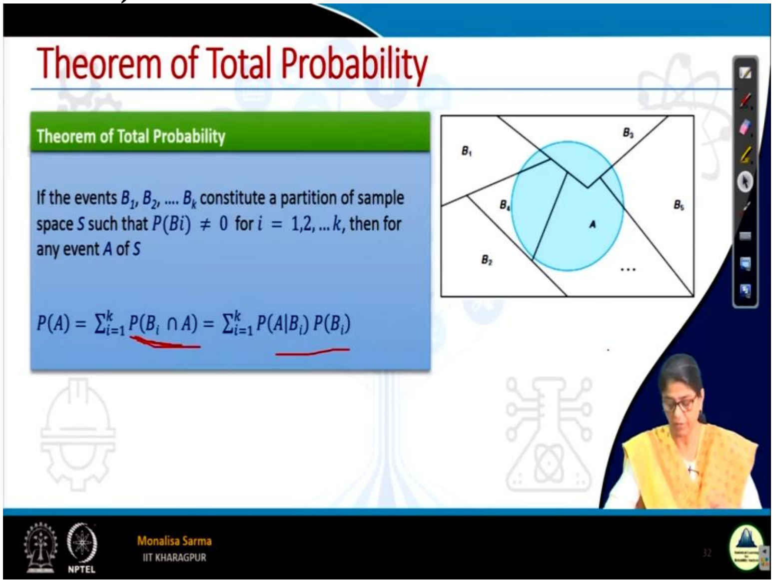

So, so, we are interested in this finding out what is this event A and we know A is formed by this disjoint event B 1 B 2 B 3 and up to B 6 it is there. So, what is probability of A? Probability of A is summation of all this. So, what is this intersection probability of B 1 intersection A what is that it is nothing but probability of A | B 1 x probability of B 1 simple conditional probability what we have seen for in this is when we are interested in finding out the A intersection B.

So, this is the formula for total probability. So, probability of A is summation of simple summation of this fellow, this summation of all these disjoint events so, now I know what is the probability of A intersection B. Probability of A intersection B is what? Probability A intersection B is probability of A | B x probability B so, that is what so, this instead of A intersection B I have written it in this form that is all that is total probability.

Now, this here again definitely we will be doing some example, but total when we talk about total probability again other extension is the Bayes theorem. So, what is Bayes theorem now, we know that some even A has occurred, what is the probability that this event has occurred because of set event B 1. A is occurred what is the probability that it has occurred because a B 1 or what is the probability that it is occurred because of B 2 that is Bayes theorem that is all I will see it more.

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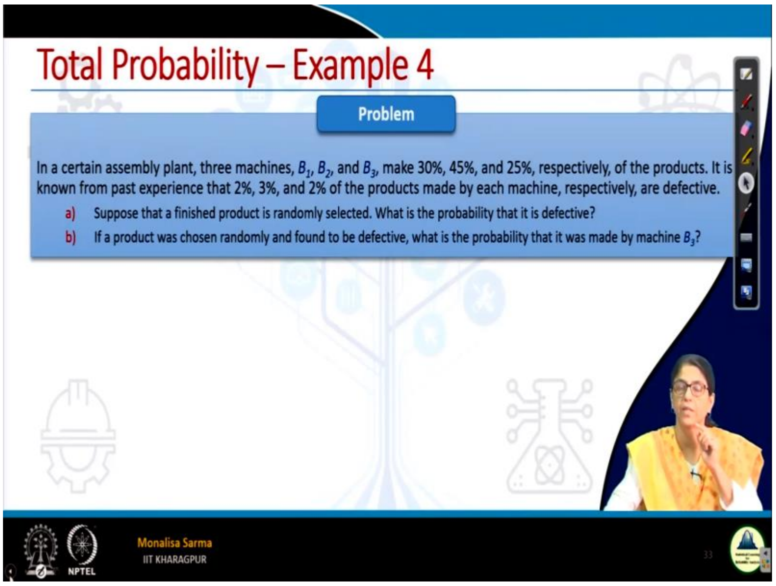

So, first let us do an example on total probability then we will go to Bayes theorem see here in a certain assembly plant 3 machines B 1, B 2, B 3 make 30%, 45% and 25% respectively of the product. So, in a universe of space total 3 years 100%, so, we have I am telling it is a universal sample space. So, it is B 1, B 2, B 3 3 machines that makes a whole all type of products like so, it is B 1 makes 30% a product, 45% is made by B 2 and 20% is made by B 3.

The 3 machines made in that assembly plant the 3 machines make all the product so, that is the universal samples was basically, it is known from the past experience that 2%, 3% and 2% of the product made by each machine respectively are defective. So, 2% of 30% is defective, 3% of 45% is defective, 2% of 25% is defective fine. So, I am interested in finding out so, what is suppose a finished product is randomly selected. What is the probability that it is defective, a finished product is randomly selected what is the probability that is defective.

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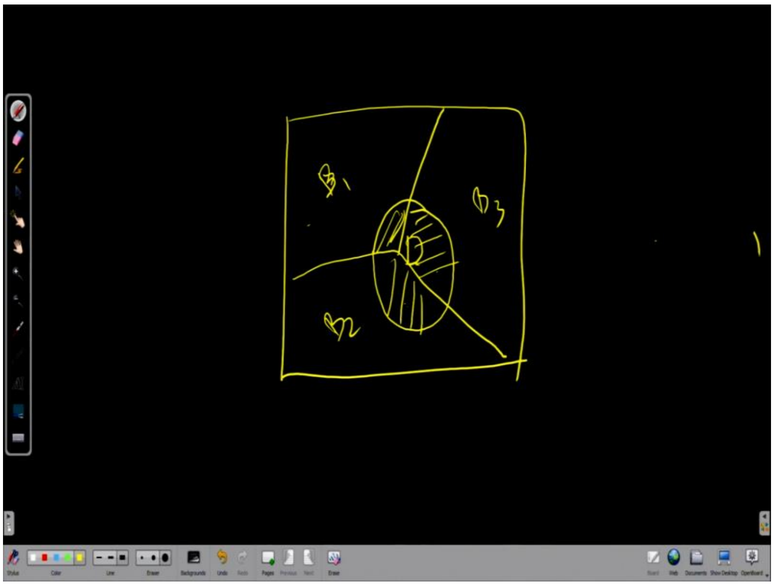

Here also basically is finished probability is randomly selected what is the probability say this is the sample space for defective product. Now finish, what are the different product and a universe of sample space, we have total 3 machines B 1, B 2 and B 3. So, portion of B 1 is defective means this portion we have seen is not a 2 portions some 3 portions and this figure is not okay not accurate some 2 portion and this is defective because of B 2 this portion is defective because of B 3.

So, we know that in this the disjoint areas which forms D so, then what is this whole D this nothing but summations of all this will bring give me D fine.

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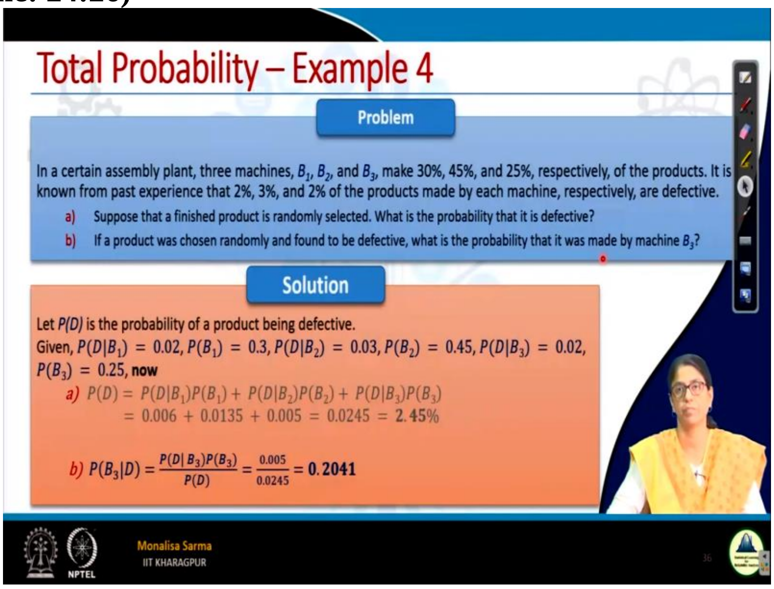

So, probability a P D is probability of probability being defective. So, what is given to us probability that is defective given that it is from B 1 is 2%. So, 0.02 probability of B 1 what is the probability of B 1? Probability of B 1 is 30% and probability that it is defective given that it is from B 2 is 3% and what is the probability that B 2 is 0.45 45%, similarly what is the probability that it is defective given that it is from B 3 again 2%.

And probability of B 3 is 0.25 now, we need to find out what is probability of D, probability of D is nothing but the summation of all the intersection part probability of B 1 intersection D + probability of B 2 intersection D + probability B 3 intersection D. So, probability of B 1 intersection D is this portion. Then probability of B 2 intersection D is this portion, for B 3 intersection D is this portion then put the values are given and you will get the result.

Now, next either product is chosen randomly and found to be defective what is the probability that it was made by we should not be this is based your at chosen a product and it is found to be defective now, what is the probability that it is from a machine B 3 or this from machine B 1 or from machine B 2 if you understand that is nothing but that is the Bayes theorem. So, first let me go to the next slide.

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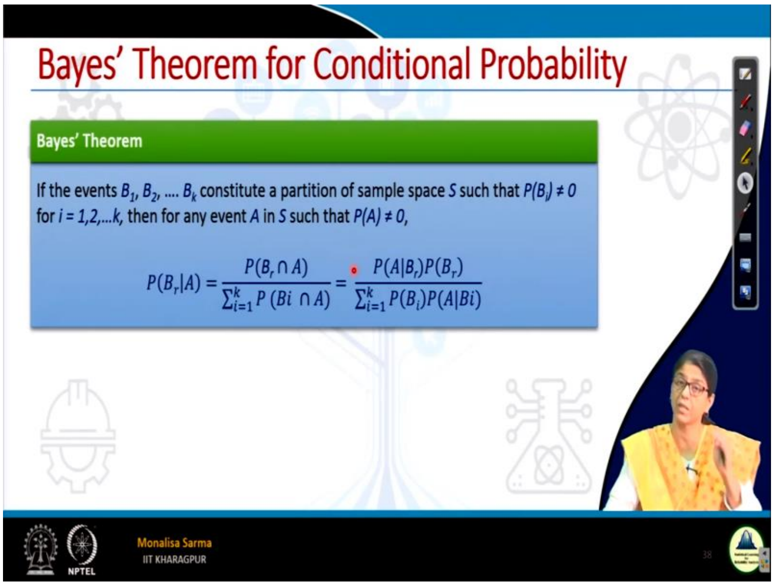

Then we will come back to this question again. So, what is Bayes theorem if the event B 1 B 2 be a constitute a partition of a sample space as such that probability B i is not equals to 0 and then for any event A in S such that P(A) ≠ 0 what is the probability that given that A has occurred what is the probability of B_r. So, it is nothing but probability of A | B_r x probability of B_r and divided by whole sample space what is whole sample space means the total probability.

Because we have found that it is defective so, what is the sample space? Sample space means that the whole defective portion which is the whole defective portion here in our question, the whole effective portion is the big circle D that we have drawn. So, that is the sample space that was the total probability. So, in the denominator is our whole sample space that is a total probability and a numerator is probability of $B_r \mid A$ how we can find that so, that means probability that it was defective given that it was B_r x probability of B_r.

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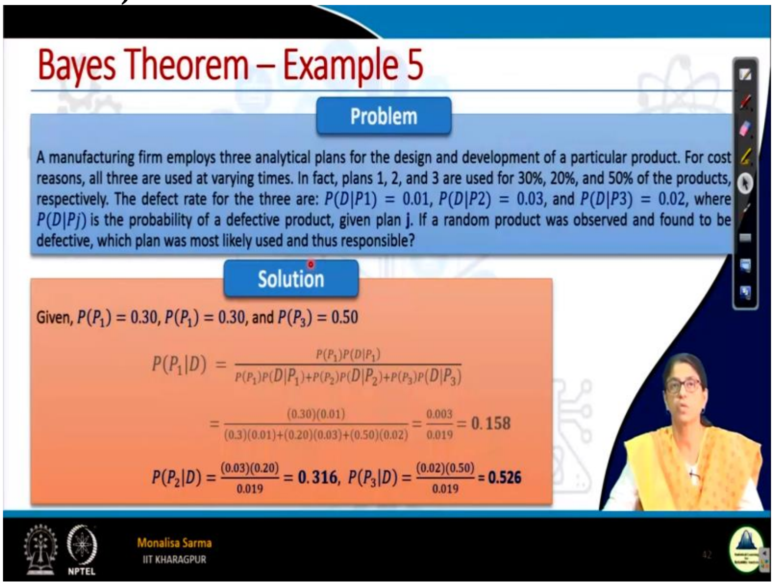

So, now so, we can we are coming back to the same question again if the product was chosen randomly and found to be defective, what is the probability that it was made by machine B 3. So, simple this was probability of B 3 given D so, this is nothing but the uses of Bayes theorem. If you have still doubt and total quality and Bayes theorem I suggest you please go through different examples and while trying to solve the total probability question on total probability a question and based on always try to draw the Venn diagram.

Whatever event you are interested in finding out find to make a circle for that and make it universal sample space give the what to say designed partition, and then try to find out the required probability of the particular event which are interested to find out. So, always draw out draw the figure or a Venn diagram and try to do it. If still, if you have doubt, we can clear this in the doubt clearing session.

But I do not think you will be having any doubt solve as many practice as many problems as you can. So, we will solve one more problem here. So, a manufacturing firm employs 3 analytical plans for the design and development of a particular product for cost region all 3 are used at varying times, plan 1, 2 and 3 are used for 30% 20% and 50% of the product respectively.

The defect rate of the P_r are it is given probability that it is defective given that it is P1 is 1% probability that it is defective given that it is from P2 it is 3% and probability that it is defective given from P3 is 2% where probability of D | P_j is probability of a defective product | Plan j. If a random product was observed and found it to be defective, which plan was most likely used and thus responsible?

So, I have selected an entire product and I found it to be defective. Now, I need to find out which plan has most likely to be chosen, so for this what I will have, it is definitely a question Bayes theorem and what is what I will try to do, I will try to find out the probability of all the 3 options probability that it is the defective given that it is defective probability that it is from plan 1.

Since then, I will find out given that it is defective what is the problem is that it is from plan P 2 third I again I will find out given that it is defective, what is the probability that it is from plan 3 P 3. So, when I will calculate all these 3, whichever probability that is highest that is the most likely and thus responsible. So, these are given probability of P 1 probability of P 2 probability of P 3 given the 3 plans.

And so, what is the probability P 1 given the simple use of Bayes theorem denominator is a total theorem. So, we got probability of P 1 given the similarly we will find a probability P 2 given the probability P 3 given the and we found a probability P 3 given the is the highest so that means it is the which plan was most likely use so plan 3 was most likely use and that is responsible.

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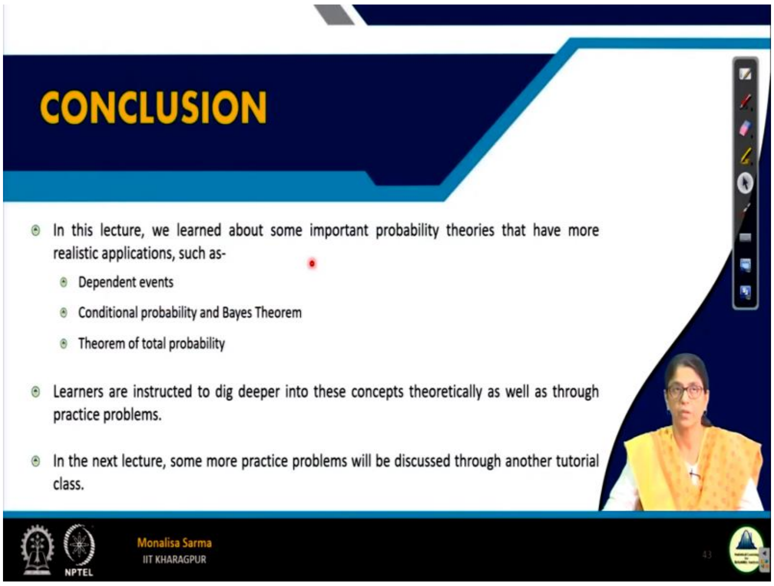

So to conclude this lecture, in this lecture we learned basically some important probability theories that have more realistic applications like based theorems total probability has many realistic application in many research studies in many industrial applications, you will find various application of Bayes theorem then we have this what to say we have learned as conditional probability as well dependent what is dependent events.

And learners are also, request to instruct to dig deeper into the concept theoretically as well as through practice problems, then your concepts will be more in clear. So, in the next lecture, definitely, I will be doing a tutorial class we will be practising some of the problems.

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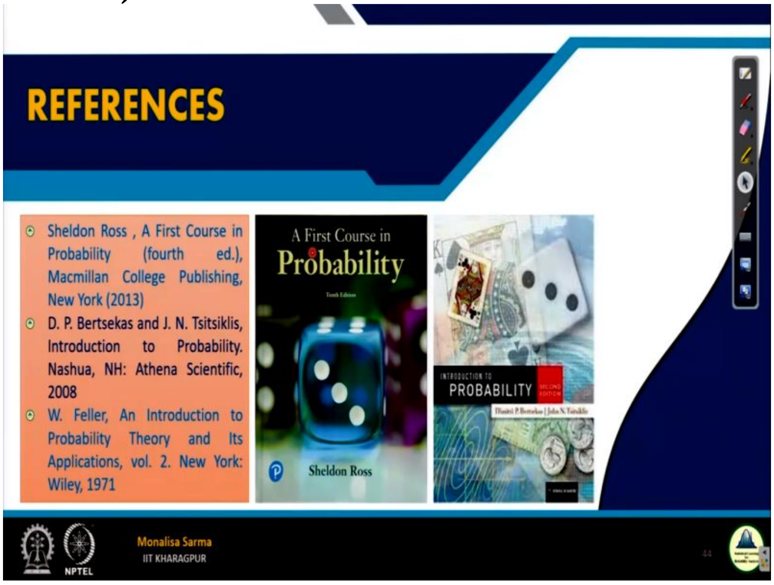

Then, these are the references which you can refer for some practising a problem, but these are not the only ones you can come to refer any book on probability and thank you.