

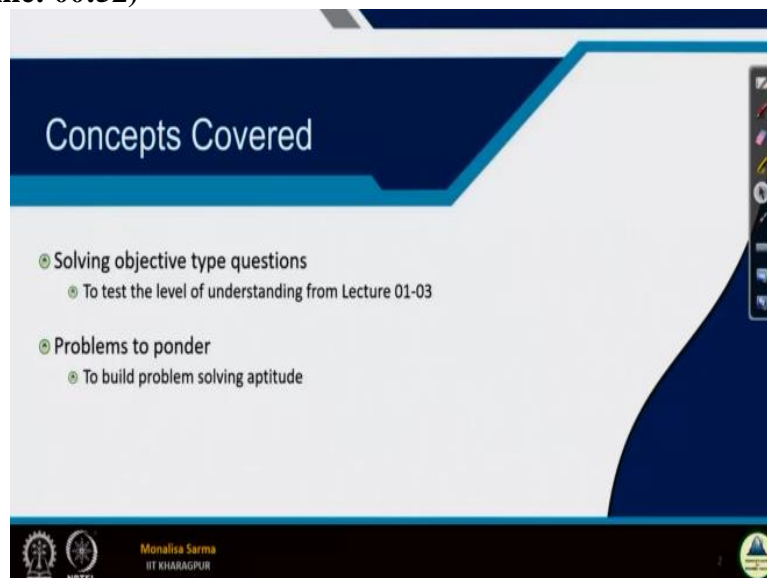
**Statistical Learning for Reliability Analysis**  
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**Lecture - 04**

**Tutorial on Introduction to RE, SL and Probability Theory (Part - I)**

Hello everyone so today is the 4th lecture today basically; we will be doing some tutorial. Till now, we have covered introduction to reliability engineering, we have covered introduction to statistical matter, and we have also discussed, we have also taken a class on probability theory. However there I will take 1 more lecture on probability theory, but before that, let us say we have a tutorial session on the last 3 classes what we have covered.

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So, in this tutorial class I will basically be solving few objective type questions and after the objective type questions, then we will go to some problems which will help your problem solving aptitude.

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**Question - 1.1**

**T 1.1: Which of the following statement(s) is (are) true?**

- a) Even seemingly identical systems, operating under similar conditions, can fail at different times. [ **TRUE** ]
- b) The failure rate versus time graph of most mechanical and electronic systems is called the *reliability curve*.
- c) Failures due to defective materials or poor quality control usually take place during the *infancy period* and are generally covered by the manufacturer in the form of warranty/guarantee.
- d) Reliability of a product can be measured quantitatively and the values are in the range of 0 to 1, both exclusive.
- e) Reliability is a non-functional requirement of a system.

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So, now, to start with the objective type questions. So, the first question is here I have given few statements, so, you have to identify whether the following statements are true or false. Now, I suggest you please do not go to the answers immediately of course I will give you the answer also here. So, but I says to please do not go to the answers immediately first you try to find out whether you yourself can answer the questions or not.

If you have understood the last 3 lectures, you will definitely be able to answer these questions. So, now, the first statement; what is the first statement? Even seemingly identical systems operating under a similar condition can fail at different times like let me take you an example, let me take an example of a washing machine again suppose 2 friends x and y, x and y both bought the same type of washing machine say Samsung washing machine x and y both bought the same washing machine.

So, same type of washing machine means from the same brand same technology. So, the reliability has to be same, but is it possible that the machine which x is bought it has failed in the first month itself and the machine which y has bought it is running for 5 years and still there is no failure is it possible? So, that is what I want to know. Yes, the answer is yes, it is possible, why it is possible?

Both the components reliability is the same, both are from the same made, so, why it is possible? First to answer this, let me take an analogy from a real life situation. Suppose there are 2 friends both are imperfect human beings, imperfect human means both tell lies, the thing is that suppose person A and person B, person A tells 90% of the time he does not tell

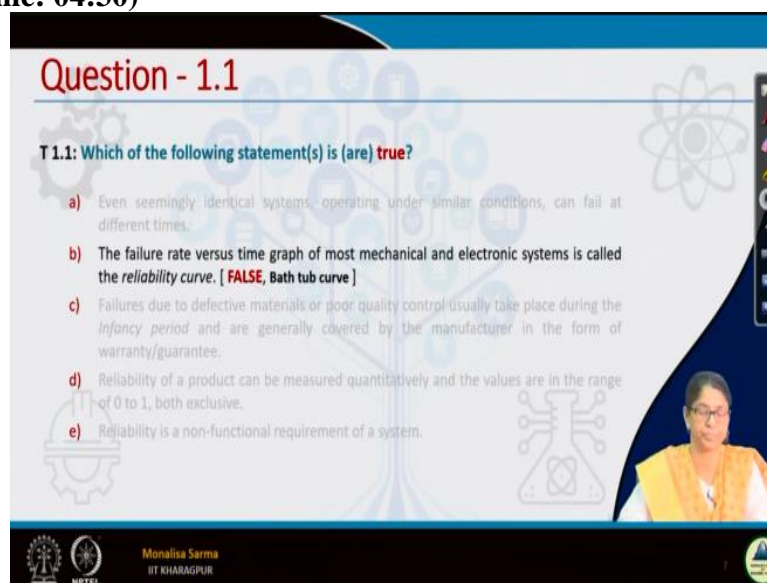
lie and a person B 50% of the time he tells lie that means person A 10% of the times he tells lie.

And person B 50% of the time he tells lie and which person is more dependable, definitely person A is more dependable. Now, I just want to analyse these 2 persons suppose these 2 persons are sitting, I am just observing these 2 persons, these 2 persons are sitting in a restaurant and they are gossiping and I have noticed this. So, it so happened that person A who just tells lie 10% of the time he happens to speak the first lie that day itself.

And a person B who happens to speak 50% of lies he did not he told all the truth. So, does that mean the person B is more reliable more dependable? No, the answer is not it is actually when we talk of reliability, it is basically what we do is that we take on an average assessment based on an average assessment person A is more dependable, but it so happened that he spoke is first truth on that day.

Similarly, so when we are talking of 2 components with the same reliability it may fail at different times the component maybe the component with a lower reliability has not failed for quite a long time, but component with a high reliability he has may have failed in the first stage. So it may happen, because reliability as we have seen it is an engineering uncertainty it is the concept of probabilities. So what is probability? Probability is the science of uncertainty. So, the first question answer is true.

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**Question - 1.1**

**T 1.1: Which of the following statement(s) is (are) true?**

- a) Even seemingly identical systems, operating under similar conditions, can fail at different times.
- b) The failure rate versus time graph of most mechanical and electronic systems is called the *reliability curve*. [ **FALSE**, Bath tub curve ]
- c) Failures due to defective materials or poor quality control usually take place during the *infancy period* and are generally covered by the manufacturer in the form of warranty/guarantee.
- d) Reliability of a product can be measured quantitatively and the values are in the range of 0 to 1, both exclusive.
- e) Reliability is a non-functional requirement of a system.

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Coming to the second question, the failure rate versus time graph or most mechanical and electronic system is called reliability graph. The failure rate graph what we have seen that is the different types of failure the early failure than the normal life failure wearout failure that is the failure rate curve, right? So failure rate curve, is it called the reliability curve? No, it is not called the reliability curve it is called the bathtub curve.

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**Question - 1.1**

**T 1.1: Which of the following statement(s) is (are) true?**

- a) Even seemingly identical systems, operating under similar conditions, can fail at different times.
- b) The failure rate versus time graph of most mechanical and electronic systems is called the *reliability curve*.
- c) Failures due to defective materials or poor quality control usually take place during the *infancy period* and are generally covered by the manufacturer in the form of warranty/guarantee. [ **TRUE** ]
- d) Reliability of a product can be measured quantitatively and the values are in the range of 0 to 1, both exclusive.
- e) Reliability is a non-functional requirement of a system.

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And the third question; failures due to defective materials or poor quality control usually takes place during the infancy period and are generally covered by the manufacturer in the form of warranty or guarantee. So, what is the answer? Yes, that is true.

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**Question - 1.1**

**T 1.1: Which of the following statement(s) is (are) true?**

- a) Even seemingly identical systems, operating under similar conditions, can fail at different times.
- b) The failure rate versus time graph of most mechanical and electronic systems is called the *reliability curve*.
- c) Failures due to defective materials or poor quality control usually take place during the *infancy period* and are generally covered by the manufacturer in the form of warranty/guarantee.
- d) Reliability of a product can be measured quantitatively and the values are in the range of 0 to 1, both exclusive. [ **TRUE** ]
- e) Reliability is a non-functional requirement of a system.

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Then the next question next statement reliability of a product can be measured quantitatively and the values are in the range of 0 to 1 both exclusive, this is an interesting question interesting statement, reliability can be measured quantitatively of course, that is true and the

values are in the range of 0 to 1 both exclusive, 0 and 1 are exclusive, but we know probability can take what value? Probability can take value any value from 0 and 1, 0 and 1 are inclusive.

So, but in case of the reliability, we are trying to find out the reliability of a product, definitely we will not try to find out the reliability of the product which is not working which is a failed product that means its reliability is 0, it will not work at all. So, will not try to find out the reliability of the product. Similarly, and nothing is permanent in this universe, stars also it dies and nothing is permanent in this universe. So, when star can die, what about a system a system will definitely have its own death so reliability cannot be 1. So, the statement is true.

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**Question - 1.1**

**T 1.1: Which of the following statement(s) is (are) true?**

- a) Even seemingly identical systems, operating under similar conditions, can fail at different times.
- b) The failure rate versus time graph of most mechanical and electronic systems is called the *reliability curve*.
- c) Failures due to defective materials or poor quality control usually take place during the *Infancy period* and are generally covered by the manufacturer in the form of warranty/guarantee.
- d) Reliability of a product can be measured quantitatively and the values are in the range of 0 to 1, both exclusive.
- e) Reliability is a non-functional requirement of a system. [ **TRUE** ]

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Then reliability is a non-functional requirement of a system. Yes, it is a non-functional requirement of the system, it is not a functional requirement of the system. Execute is a non-functional requirement of the system. User interface, usability. Usability is a non-functional requirement of the system so, true.

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**Question - 1.2**

T 1.2: The main reason for the failure of different light bulbs at different times is due to:

- a) **Random life of bulbs**
- b) Infant mortality of bulbs
- c) Poor quality of bulbs

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So, the second question the main reason for the failure of different light bulbs at different times what maybe the main reason? Is this random life or the infant mortality or poor quality of bulbs? Definitely it is a random life of failures a different light in the normal lifetime and a normal lifetime why it fails? It may be due to the random reasons. So, it is a random life of the bulbs is the answer.

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**Question - 1.3**

T 1.3: Failures during the normal operating period are usually due to:

- a) Poor quality control
- b) Wearing of the system
- c) **Random life of the system**

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Failures during the normal operating period are usually due to what? Normal operating create is usually due to poor quality control wearing of the system random life of the system, which one is correct? Normal operating period why it fails random life of the system it cannot be poor quality control, poor quality control means it has to be in the infant period, wearing of the system since it is the end part of the light it is the wear out light. So, in a normal operating period, it is due to the normal life of the system.

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### Question - 1.4

T 1.4: As the reliability increase, failure intensity

- a) increases
- b) decreases**
- c) no effect
- d) none of the above

As the reliability increases failure intensity, what happens? As the reliability increases failure intensity definitely it is very obvious failure intensity decreases.

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### Question - 1.5

T 1.5: if the number of component that fail in time  $t$  is  $N_f$  out of  $N$ , the reliability of the component is:

- a)  $1 - \frac{N_f}{N}$**
- b)  $\frac{N_f}{N}$
- c)  $\frac{N}{N_f}$

Now, here, if the number of component that fails in time  $t$  is  $N_f$  out of  $N$  then what is the reliability of the component? There are 10 components out of 10 components 6 component had failed at time  $t$ . So, what is the reliability always I forgot to mention maybe reliability is always with the perspective of the time when you talk about reliability it is with the perspective of a time. So, what is the reliability of time  $t$ ?

So, at time  $t$  out of 10 components 4 component has failed. So, what is the remaining component? Remaining component is 6 so, what is the reliability at time  $t$ ? Reliability at time  $t$  is  $6 / 10$ . So, here, so, what will be the reliability  $1 - N_f / N$ .

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## Question - 1.6

T 1.6: Which of the following represents "Five point summary"?

- a) Histogram
- b) Bar chart
- c) Box plot
- d) Scatter plot.

1 min
2 lower quartile Q1
3 median
4 upper quartile Q3
5 max

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Which of the following represents 5 points summary? I already told this mentioned this when I was discussing statistical matter, introductory class on statistical matter, that is boxplot. There is 5 points summary why 5 point summary? The first vertical line, that what it gives is the minimum value and the top vertical line it gives the maximum value of a data set, then the blue line that gives the 25th percentile.

That is the lower quartile, than this upper blue upper red line, that is the upper quartile that is the 75th percentile. I am talking percentile not percentage mind it, then the middle red line is the median so, it is a 5 point summary.

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## Question - 1.7

T 1.7: If we have data on the yearly average temperature of a particular geographical area for last 10 years. You are interested in change over time, which is most effective graphical display:

- a) Histogram
- b) Bar chart
- c) Pie chart
- d) Scatter plot

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So if you have data on a yearly average temperature a particular geographical area for last 10 years you are interested in change over time, which is the most effective graphical display? If you are interested in change over time for a year definitely when we talk of year definitely it



is not an quantitative data that is a categorical data. So per year, that means it is not what type of data? It is a nominal data. So we will be using bar chart here.

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**Question - 1.8**

**T 1.8: Identify the following events whether "independent", "dependent" or "mutually exclusive" ?**

- a) Tossing two coins and observe as HH, HT, TH or TT. [ **Independent** ]
- b) Drawing four cards from a deck of 52 cards one after another to see if all the card drawn belong to Diamond.
- c) Drawing four cards from a deck of 52 cards in such a way that each time a drawn card will be returned to the deck and to see all four cards are Aces.
- d) Drawing a red card or drawing a club from a deck of 52 cards

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So now this is the question on probability identify the following events whether it is an independent event, dependent event or mutual exclusive, I think you remember what is an independent event? What is a dependent event? What is a mutually exclusive event? Tossing 2 coins we are tossing 2 coins and we have observed either as head head, head, tail, tail, tail or tail, head. So what is it? What is an independent event, dependent or mutually exclusive? You are tossing 2 coins so definitely, this is an very much an independent event.

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**Question - 1.8**

**T 1.8: Identify the following events whether "independent", "dependent" or "mutually exclusive" ?**

- a) Tossing two coins and observe as HH, HT, TH or TT.
- b) Drawing four cards from a deck of 52 cards one after another to see if all the card drawn belong to Diamond. [ **Dependent** ]
- c) Drawing four cards from a deck of 52 cards in such a way that each time a drawn card will be returned to the deck and to see all four cards are Aces.
- d) Drawing a red card or drawing a club from a deck of 52 cards

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Second, drawing 4 cards from a deck of 52 cards one after another to see if all the cards drawn belong to diamond, what I have done? Deck of 52 cards, I have drawn the first card. I have seen whether this is diamond or not, I have kept the card aside, then I can pick the

second card, I have seen whatever it is diamond or not, I kept it aside, I did not mix it with this pack of cards. Similarly, I have drawn total 4 cards.

And if all the cards do not belong to diamond or not, this is an independent event or dependent event? Definitely, it is not a mutually exclusive event. This is very much a dependent event. Because when we are keeping it aside, we are not mixing our number of diamonds changes number of cards in total number of cards changes if and moreover, if you have already picked a diamond and then the number of diamond as well changes in dependent population, is not it? So it is very much dependent event.

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**Question - 1.8**

**T 1.8: Identify the following events whether "independent", "dependent" or "mutually exclusive" ?**

- a) Tossing two coins and observe as HH, HT, TH or TT.
- b) Drawing four cards from a deck of 52 cards one after another to see if all the card drawn belong to Diamond.
- c) Drawing four cards from a deck of 52 cards in such a way that each time a drawn card will be returned to the deck and to see all four cards are Aces. [ **Independent** ]
- d) Drawing a red card or drawing a club from a deck of 52 cards

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Third, drawing 4 cards from a deck of 52 cards in such a way that each time a drawn card will be returned to the deck and see all the 4 cards are aces each time I am returning it. So this is an independent event.

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### Question - 1.8

T 1.8: Identify the following events whether "independent", "dependent" or "mutually exclusive" ?

- Tossing two coins and observe as HH, HT, TH or TT.
- Drawing four cards from a deck of 52 cards one after another to see if all the card drawn belong to Diamond.
- Drawing four cards from a deck of 52 cards in such a way that each time a drawn card will be returned to the deck and to see all four cards are Aces.
- Drawing a red card or drawing a club from a deck of 52 cards [ **Mutually exclusive** ]



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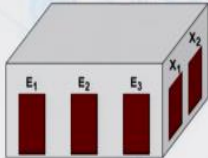

4th, drawing a red card or drawing a club from a deck of 52 cards club, can club be red it has to be black, right? So red card and a club that means a red card and black card where if I am drawing 1 red card, I cannot draw a black card at the same time. I am drawing 1 card is not it? So these 2 are mutually exclusive event like drawing getting a head or tail that is a mutually exclusive and so that is why our 4th is the mutually exclusive event.

Now, we will be doing some solving some problems. I suggest again immediately do not see the solution. First, see the equations, try to answer it yourself then go to the solution if you do not if you cannot answer it.

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### Problem - 1.9

T 1.9: There is a community hall with three doors E1, E2 and E3 for entry only and either of the two doors X1 and X2 for exit only. A visitor visited the hall. What is the probability that he entered through E2 and exited through X2?

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The first question so there is a community hall with 3 doors. The 3 doors are E1, E2, E3, for entry these 3 doors are for entry. And for exit we have 2 doors X1 and X2 people can enter from any of the door and people can exit from any of the door X1 and X2. A visitor visited

the hall what is the probability that he entered through E2 and exited through X2, so now he can enter through any of the doors E1, E2, E3. And he can exit through any of the doors X1 and X2. What is the probability? Probability of entering to any one of the doors?

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**Problem - 1.9: Solution**

Given, number of entry doors = 3 and number of exit doors = 2

Therefore,

Probability of entering through  $E_1$ ,  $E_2$ , and  $E_3$  doors are

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

Probability of exit through  $X_1$  and  $X_2$  doors are

$$P(X_1) = P(X_2) = \frac{1}{2}$$

Now,

By the principle of mathematical induction

Probability of entering through  $E_2$  and exit through  $X_2$  is

$$P(E_2) \times P(X_2) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$$

**T 1.9:** There is a community hall with three doors  $E_1$ ,  $E_2$  and  $E_3$  for entry only and either of the two doors  $X_1$  and  $X_2$  for exit only. A visitor visited the hall. What is the probability that he entered through  $E_2$  and exited through  $X_2$ ?

The diagram shows a rectangular hall with three entry doors labeled  $E_1$ ,  $E_2$ , and  $E_3$  on the left side, and two exit doors labeled  $X_1$  and  $X_2$  on the right side.

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First answer that was the probability of entering to any one of the doors the equally probable so probably of entering any one  $E_1$ ,  $E_2$ ,  $E_3$  is  $1/3$ . Similarly, the probability of exiting any of the doors that is also equally probable. So it is  $1/2$  there are 2 doors for exiting and both the events are independent events. It is not if I am entering through 1 I cannot I have to enter exit rather there is no sort of dependency is there, it is totally independent events.

So since this is independent event. If 2 events are independent  $X$  and  $Y$  are 2 independent events. What is the probability of occurring of  $X$  and  $Y$ ? Probability of occurring of  $X$  and  $Y$  is probability of  $X \times Y$ . So, by the principle of mathematical induction I can write that probability of entering through  $E_2$  and exit through  $X_2$  is  $1/3 \times 1/2$ .

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### Problem-1.10

T 1.10: A series-parallel system of a supply line is shown below. In the system,  $R_1$ ,  $R_2$  and  $R_3$  are the reliability of three components. What is the reliability of the whole system?

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So, now coming to the next question, the next question is also a question on probability, but here we will be seeing the application of reliability as well. So, like this question, what we have seen is there is given a series parallel system of a supply line. So, in the system  $R_1$ ,  $R_2$ ,  $R_3$  are reliable, they are the 3 components we can see there are 3 components this is 1 component and this is the second component.

So, then this is the first component and then this is the second component and this is the third component. So, reliability of the first component is  $R_1$ , reliability of the second component is  $R_2$ , reliability of the third component  $R_3$ . Now, what is this reliability? Basically, reliability is the probability of failure for your operation. So, that means, this is the probability that this component will not fail that means, this is the probability of this non failure of the component.

So, essentially  $R_1$  is the probability that the component one will not fail. Similarly,  $R_2$  and  $R_3$ . So, now, this just comes to a simple probability problem. So, now what is the here just we need to use the concept of series parallel structure actually, we have learned in physics in class 8, 9 I suppose and even first year engineering also they learn it. So, I will not go to those details, but I am sure all of you know when the 2 components are in parallel that means, for the system to work if the 2 components are parallel.

If one of the components works the system will work however, if both the components failed and the system will not work. So, and for a series if 2 components are in series, both the component has to work then the system will work. So, like for here, let me consider this as

this whole thing, this whole thing as one or sorry, this one, this is one component and the other one is another component.

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**Problem-1.10 : Solution**

$$R_{SYSTEM} = R_A \times R_B$$

$$Q_A = (1 - R_1) \times (1 - R_2)$$

$$R_A = 1 - Q_A$$

$$R_B = R_3$$

Therefore,

$$R_{SYSTEM} = R_A \times R_B$$

**T 1.10:** A series-parallel system of a supply line is shown below. In the system, R<sub>1</sub>, R<sub>2</sub> and R<sub>3</sub> are the reliability of three components. What is the reliability of the whole system?

So, I have a figure here so, here this is the 1 component, I am just considering this as 1 component, this sub component basically I can say, this component is a combination of 2 parallel components. And so, again, this is 1 component of course, now, if I am interested in finding the reliability of this system reliability means that the system will not fail. So, I will have to find out the reliability of each sub component.

So, first let me find out the reliability of first component that is R A I am naming it as R A. So, what is R A? For finding an R A what we will do is first we will see when the system will not work that is let us, calculate the unreliable the probability of failure. So, when the system will not work, since 2 components are in parallel, so, this system will not work and both the components will not work.

So, if the probability of working is R 1 the probability of not working is 1 - R 1. So, probability reliability of the system is definitely R A cross R B is not it? Because it is a series system this is series and this first component is in series with the second component. So, that is R A cross R B then we are interested in finding out the reliability of the R A to find out R A first time finding out Q A because that will be easier.

So, what is Q A? Q A is that both the components are not working but the components are not working means 1 - R 1 into 1 - R 2. So, what will be my R A? R A is nothing but 1 - Q of



A reliability is one minus unreliability and R B we know it is relevant is given R 3. So, what is my system reliability is R A x R B and so, simple.

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**Problem - 1.11**

T 1.11: The probability that an American industry will locate in Shanghai, China, is 0.7, the probability that it will locate in Beijing, China, is 0.4, and the probability that it will locate in either Shanghai or Beijing or both is 0.8. What is the probability that the industry will locate

- a) in both cities?
- b) in neither city?

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Now, coming to the third problem, what is given first let us go through the problem, the probability that an American industry will locate in Shanghai as 0.7. The probability that it will look at in Beijing is 0.4. And the probability that it will locate in either Shanghai or Beijing or both means it may locate in either Sangha or in Beijing or in both the cities that it has given 0.8. So, what is the probability that the industry will locate in both cities and industry will locate in neither cities?

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**Problem - 1.11 : Solution**

Consider the events  
S: industry will locate in Shanghai  
B: industry will locate in Beijing

Given,

$P(S) = 0.7$        $P(B) = 0.4$        $P(S \cup B) = 0.8$

T 1.11: The probability that an American industry will locate in Shanghai, China, is 0.7, the probability that it will locate in Beijing, China, is 0.4, and the probability that it will locate in either Shanghai or Beijing or both is 0.8. What is the probability that the industry will locate

- a) in both cities?
- b) in neither city?

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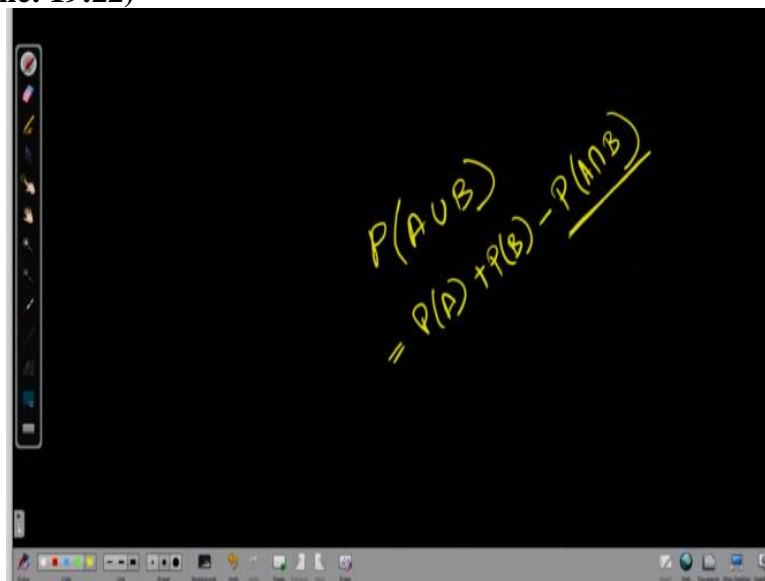
So, first what we will do is that we will consider the events the concept of events we have discussed in our last lecture. So, first we will consider the events what are the 2 events? One event I am calling it as an S, that is the industry we will locate in Shanghai the second event,

let me call it as B that the industry will locate in Beijing. So now since I am interested in finding out the probability that the industry will locate in both the cities.

So, industry will locate in border citizens it has to be located in Shanghai as well as it has to locate in Beijing. So, basically, what I need to find out? I need to find a probability of S intersection B means it needs to basically probability of S into probability of B probability S into probability of B is as told earlier S intersection B. So, I need to find out probability of S intersection B.

So how do I find probability of S intersection B I have this information PS is 0.7, PB is 0.4 and P S or B it is given that it is either located in Shanghai or Beijing or both. So it is S union D so, this is we have 0.8. Now, we know the formula what is the formula for probability of S union B, so we can go to the let me take a piece of paper and then explain it to you.

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$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

So, like what is probability of A union B? It is probability of A + probability of B - probability of A intersection B is it not? We have learned in the last class. So, now, what we are interested we need to find out this probability of A intersection B. So, what is the property of A intersection B? Just simple this probability of A intersection B will be probability of A + probability of B - probability of A union B.

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## Problem - 1.11 : Solution


Consider the events  
**S:** industry will locate in Shanghai  
**B:** industry will locate in Beijing



Given,  
 $P(S) = 0.7$        $P(B) = 0.4$        $P(S \cup B) = 0.8$

Now,  
 (a)  $P(S \cap B) = P(S) + P(B) - P(S \cup B) = 0.7 + 0.4 - 0.8 = \mathbf{0.3}$

(b)  $P((S \cup B)') = 1 - P(S \cup B) = 1 - 0.8 = \mathbf{0.2}$

**T 1.11:** The probability that an American industry will locate in Shanghai, China, is 0.7, the probability that it will locate in Beijing, China, is 0.4, and the probability that it will locate in either Shanghai or Beijing or both is 0.8. What is the probability that the industry will locate:  
 a) in both cities?  
 b) in neither city?





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So, this is what all the probability of S intersection B; as this put it in a formula and we just get it. Now, the second is what is the probability that industry will locate in neither city means it will neither to locate in Beijing nor to locate in Shanghai it will not locate in any of the cities. So, what is the probability of that? Basically, we know what is the probability of locating in either Shanghai or Beijing or both.

So, if we know the probability and just finding up in either city is just the complement of that. So, just a complement of that  $1 -$  probability of S union B, so, any problem when you try solving any probability problem first what you do is it first always try to write the events when you write the events, then things becomes very easier to calculate. If you do not write the events if you immediately go on try to calculate the probability then chances are there that you will go wrong.

First try to write down the events then write down what or what probabilities are given which probability is a given.

**(Refer Slide Time: 21:10)**

## Problem-1.12

**T 1.12:** In a high school graduating class of 100 students, 54 studied mathematics, 69 studied history, and 35 studied both mathematics and history. If one of these students is selected at random, find the probability that

- the student took mathematics or history;
- the student did not take either of these subjects;
- the student took history but not mathematics.

Let us try another 1 problem so, in a high school graduating class of 100 students, so, there are total 100 students. So, 54 studied mathematics, 69 studied history and 35 studied both mathematics and history. So, it is that means, if I consider 1 event as M another event as H. So, probability of M intersection is or I can get probability of M x probability of H. So, what will be that probability of M x probability of H will be 34 / 100 our total sample space is 100 is not it?

So, now, what is the question there is not a question. Question is if one of the students is selected at random find a probability that the students took mathematics or history.

**(Refer Slide Time: 22:03)**

## Problem-1.12 : Solution

Given,

$$P(H) = \frac{69}{100}$$

$$P(M) = \frac{54}{100}$$

$$P(H \cap M) = \frac{35}{100}$$

Now,

- $P(H \cup M) = P(H) + P(M) - P(H \cap M) = \frac{69}{100} + \frac{54}{100} - \frac{35}{100} = \frac{88}{100} = \frac{22}{25}$
- $P((M \cup H)') = P(M' \cap H') = 1 - P(H \cup M) = 1 - \frac{22}{25} = \frac{3}{25}$
- $P(H \cap M') = P(H) - P(H \cap M) = \frac{69}{100} - \frac{35}{100} = \frac{34}{100} = \frac{17}{50}$

So, what is the question if one of these students is selected at random find the probability that the student took mathematic or history what is a probability student of mathematics or history? How will you find that out? Is not it? It is H union M mathematics or history. So,

then this is what  $H \cup M$  the formula a probability of  $H$  is given  $M$  is given,  $H \cap M$  is given. So, we could find this out, next is the student did not take either of these subjects what is that this is just the complement of the previous one is not it?

It is just like the previous question what we have done the company located in neither of the cities so the student did not take either of these subjects. So, it is just the complement of the first case. Then second case, second case you have to notice it the student took history, but not mathematics. See, here the student took history but not mathematics. So, if you see the tests what to say this diagram what is this?

This is whole circle is for history and this whole circle this green circle is for math and this between the slight pink we can say this is  $H \cap M$  means the students who took both history and mathematics. So, now what I am interested in is the students took history, but the student did not take mathematics that means student history is this circle and did not take mathematics is this portion, this light pink portion, I will have to delete this portion because this portion student is taking both history and mathematics.

So, but I am interested only where the student took only history. So, if I find that out then this means history if I subtract this then I will get this value. So, from probability of history, I am subtracting this part. So, now of all putting the values I get the result.

**(Refer Slide Time: 24:24)**

**Problem-1.13**

T 1.13: Interest centers around the life of an electronic component. Suppose it is known that the probability that the component survives for more than 6000 hours is 0.42. Suppose also that the probability that the component survives no longer than 4000 hours is 0.04.

a) What is the probability that the life of the component is less than or equal to 6000 hours?  
b) What is the probability that the life is greater than 4000 hours?

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Now, the next question interest centres around the life of an electronic component. Suppose it is known that the probability that the components arise for more than 6000 hours is 0.42 to

that the component that survives more than 6000 hours that probability 0.4 again it is given that suppose also that the probability that a component survives no longer than 4000 hours is 0.04. And sometimes I will also want to mention here something sometimes in some question you see.

But if you see the question, the answer may be very simple but some extra data are given. So, do not get confused with those extra data like here you may get confused with components having no longer than 4000 hours is 0.4 and more than 6000 hours is 0.42 might be do we need to find out some intersection of 6000 and 4000 just for that, you will have to read the question properly. So, do not get confused with the information. So, here see what we have to find out?

What is the probability that the life of the component is less than or equal to 6000 hours? So you might get confused, it is given that no longer than 4000 hours is 0.4 then again for 6000 hours is some below. So do I need to consider these 4000 this value also somewhere? No, and we do not have just it is very simple beyond 6000 is the 0.04 less than or equal to 6000 is just minus of this, this is the complement of that.

(Refer Slide Time: 26:00)

**Problem-1.13 : Solution**

Given,  
Probability of survives more than 6000 hours = 0.42  
Probability of survives up to 4000 hours = 0.04

So,  
a) Probability that the life of the component is less than or equal to 6000 hours  
 $= 1 - 0.42$   
 $= 0.58$

b) Probability that the life is greater than 4000 hours  
 $= 1 - 0.04$   
 $= 0.96$

**T 1.13:** Interest centers around the life of an electronic component. Suppose it is known that the probability that the component survives for more than 6000 hours is 0.42. Suppose also that the probability that the component survives no longer than 4000 hours is 0.04.  
a) What is the probability that the life of the component is less than or equal to 6000 hours?  
b) What is the probability that the life is greater than 4000 hours?

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It is just a complement of more than 6000 hours. Similarly, what is the next question? What is the probability that a life is greater than 4000 hours? Similarly, again, in the question only is given the component survives not longer than 4000 hours means the component has survived till 4000 hours is 0.4. Now, what I need to find out that the life is greater than 4000,



greater than 4000 the component of the death that life is better than 4000 hours is just the complement of that  $1 - 0.04$  so it is 0.96.

(Refer Slide Time: 26:41)

**Problem - 1.14**

T 1.14: A bag contains 6 white and 4 black balls. 2 balls are drawn at random. Find the probability that they are of same color.

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The bag contains 6 white and 4 black balls, 2 balls are drawn at random find a policy that they are of the same colour say the bag contains how many balls? 6 white and 4 total 10 balls, 2 balls are drawn at random here the position is not important. So, when we want to find out what is the total sample space, we have discussed in our last lecture probability lecture. So, how to find out a sample space for the sample space will be just from 10 we have to select 2 total sample space will be  $10 C 2$ , is not it?

So, that is the total sample space now, what is the probability that they are of the same colour? That is the event we need to find out that event.

(Refer Slide Time: 27:25)

**Problem - 1.14 : Solution**

Let,  
S be the sample space  
E be the event of getting both balls of same color

Then,  
 $n(S)$  = no of ways of drawing 2 balls out of (6+4)  
 $= {}^{10}C_2 = \frac{10 \times 9}{2 \times 1} = 45$

$n(E)$  = no of ways (2 balls out of six) or (2 balls out of 4)  
 $= {}^6C_2 + {}^4C_2 = \frac{6 \times 5}{2 \times 1} + \frac{4 \times 3}{2 \times 1} = 15 + 6 = 21$

Now,  
Required probability,  $P = \frac{21}{45} = \frac{7}{15}$

T 1.14: A bag contains 6 white and 4 black balls. 2 balls are drawn at random. Find the probability that they are of same color.

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So,  $S$  is the sample space,  $E$  be the event of getting both the balls of same colour. So, now, what is the how do we calculate the sample space size of the sample space is as I told you, there are total 10 from 10 we have to select 2 where ordering is not important. So it will be simple combination. So it is  $10 C 2$  is the sample space. Now, how to find both the event of getting both the balls of the same colour? How do I get both the balls are the same colour?

Contains 6 white and 4 black balls. How do I get a ball? The ball so the same the same colour that means to get both the ball so, the same colour either have I have to get 2 from the 6 white balls 2 white balls from the 6 white balls or 2 black balls from the 4 black balls. So, that will be number of ways 2 balls out of 6 or 2 balls out of 4 and both are mutually exclusive again it can both have both cannot happen together.

So, it is this simple or means it is simple plus we have seen a or b is if it is mutually exclusive event so, it is probability of A + probability of B. So, it is what is the probability of A getting 2 balls out of 6,  $6 C 2$  when 2 balls out of 4 is  $4 C 2$ , so  $n$  of  $E$   $6 C 2 + 4 C 2$  this is the number now finding the probability this  $n$  of  $E$  upon  $n$  of  $S$  and total number of events is  $n$  of  $E$  and sample size is  $n$  of  $S$  probability is  $n$  of  $E$  upon  $n$  of  $S$ .

**(Refer Slide Time: 29:05)**

The image shows a presentation slide titled "Problem - 1.15". The text on the slide reads: "T 1.15: A man and his wife appear in an interview for two vacancies in the same post. The probability of husband's selection is  $(1/7)$  and the probability of wife's selection is  $(1/5)$ . What is the probability that only one of them is selected?". In the bottom right corner of the slide, there is a small video inset showing a woman in a yellow vest speaking. The slide also features a background graphic of a tree with various icons and logos at the bottom, including NPTEL and IIT KHARAGPUR.

A man and his wife appeared in an interview for 2 vacancies in the same post the probability of husband selection is 1 by 7 probability of wife selection is 1 / 5 what is the probability that only one of them is selected? We have to find out what is the probability that one of them is selected and appears in an interview for 2 vacancies in the same post. So for that first

probability of husband selection is given probability of wife selection is given first we will write down the events then we will find out what is the probability of husband not selected.

What is the probability of wife not selected? So what is the probability that only one of them is selected? Only one of them is selected means husband selected wife not selected or wife selected husband not selected. These are the only 2 ways.

**(Refer Slide Time: 29:53)**

**Problem - 1.15 : Solution**

Let,  
 $A$  = Event that husband is selected  
 $B$  = Event that wife is selected

Then,  
 $P(A) = \frac{1}{7}$  and  $P(B) = \frac{1}{5}$

Therefore,  
 $P(A') = (1 - \frac{1}{7}) = \frac{6}{7}$  and  $P(B') = (1 - \frac{1}{5}) = \frac{4}{5}$

Now,  
 Required probability =  $P[(A \text{ and not } B) \text{ or } (B \text{ and not } A)]$   
 $= P[(A \text{ and } B') \text{ or } (B \text{ and } A')]$   
 $= P(A) \times P(B') + P(B) \times P(A')$   
 $= (\frac{1}{7} \times \frac{4}{5}) + (\frac{1}{5} \times \frac{6}{7}) = \frac{10}{35} = \frac{2}{7}$

**T 1.15:** A man and his wife appear in an interview for two vacancies in the same post. The probability of husband's selection is  $(\frac{1}{7})$  and the probability of wife's selection is  $(\frac{1}{5})$ . What is the probability that only one of them is selected?

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 IIT KHARAGPUR

So A be the event that husband is selected B be the event that wife is selected. So, this is already given probability of A is given probability B is given. So, and then from there we found a probability of A complement that is probability of husband not getting selected and probability of wife not getting selected that is probability of B complement. Now, what is the probability that only one of them they selected.

What are the only one of them selected maybe 2 cases either husband selected or wife selected husband selected then wife is not selected while selected husband is not selected. So, required properties this probability of A and not B or B and not A, not A means complement. So, just put it in the formula all the values you have just put it in a formula and you get the result.

**(Refer Slide Time: 30:47)**

**Problem - 1.16**

T 1.16: A real estate agent has 8 master keys to open several new homes. Only 1 master key will open any given house. If 40% of these homes are usually left unlocked, what is the probability that the real estate agent can get into a specific home if the agent selects 3 master keys at random before leaving the office?

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The slide features a blue header with the title 'Problem - 1.16' in red. Below the title is the problem text. The background has a faint tree-like diagram with circular nodes. In the bottom right corner, there is a small video inset of a woman in a yellow top. The bottom of the slide contains logos for IIT Khargapur and NPTEL.

So, this is the last question in this tutorial session. So, a real estate agent has 8 master keys to open several new homes. He has 8 master keys there are not many new homes there are total 8 master keys to open those new homes, only 1 master key will open any given house. A given house can be opened by only 1 master key, though there are 8 master keys we cannot use other master keys. So if 40% of these homes are usually left unlocked what is the probability that a real estate as it can get into a specific home?

If agent select 3 master key at random before leaving the office? So what happened? What is the probability that he can enter a specific home the question is basically what is the property that a real estate agent can enter the specific home out of the 8 master keys he has selected this 3 master keys 3 keys and now and it is also given 40% of the homes are left unlocked. So, if it is the home which you want to go if it is unlocked and he does not need the key at all.

Next is suppose if the key home is locked then he has to have that particular key which will open the door whether that particular key because he has he did not take all the 8 keys here he has selected 3 master keys. So it has to be if the key falls within that 3 then only he will be able to enter that house.

**(Refer Slide Time: 32:18)**


## Problem - 1.16 : Solution

Let,  
 A = Event that the house is open  
 B = Event that correct key is selected

Then,  
 $P(A) = 0.4$ ,  $P(A') = 0.6$ , and  $P(B) = \frac{\binom{1}{1}\binom{2}{2}}{\binom{8}{3}} = \frac{3}{8} = 0.375$

Therefore,  
 $P[A \cup (A' \cap B)] = P(A) + P(A')P(B)$   
 $= 0.4 + (0.6)(0.375)$   
 $= \mathbf{0.625}$

**T.1.16:** A real estate agent has 8 master keys to open several new homes. Only 1 master key will open any given house. If 40% of these homes are usually left unlocked, what is the probability that the real estate agent can get into a specific home if the agent selects 3 master keys at random before leaving the office?



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So, what now first even is even though the house is open. So under what condition he can enter the house? He can enter the house under 2 situations one is on the houses open another is when the house is locked and he has the proper key. These are the 2 situations and these 2 situations are mutually exclusive. So basically if the probability of entering his home it will be just basically a probability of 2 mutually exclusive events. So, what are the 2 events?

One event is that a house is unlocked so houses what is the probability the houses unlock? It is 0.4 what is the; another event and house is locked and he has taken the proper master key. These are 2 independent events houses lock and he has taken the proper master key. So, what is the probability of taking the proper master key first is what is the size of the sample one? For finding probability always we need to find a sample space that we have seen.

So, what will be the sample space out of 8 here selected 3 to sample space will be  $8C3$  and what is the probability of taking that particular key which will open the house which he wants to enter? So that is that particular house will be opened by only 1 key. So from 1 he has to select 1 and from the rest 7, he will be selecting 2. So basically probability of B is for that if he select 1 particular key from that because each house has 1 key is not it is house can be opened by just 1 key that 1 he has selected he has taken total he has taken 3 keys.

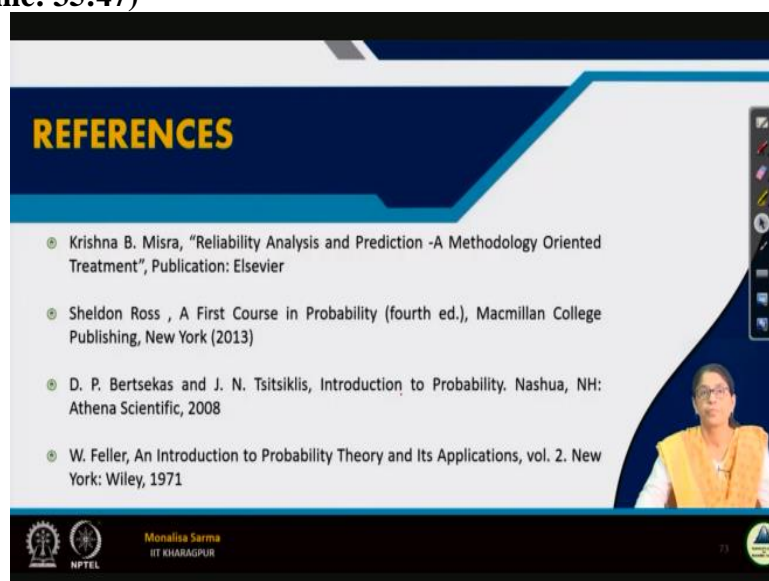
What are the 3 keys? 1, he has taken the key for that house and another 2 any 2 from the rest 7 basically. So, this is the probability for this even property that the correct key is selected. So, probability of B we can write it this way that  $1C1$  into  $7C2$  upon  $8C3$ . So now what is the probability that he can enter the house? Probability that he can enter the house is that a

probability that the house is open, plus, probability that the house is closed and he has taken the proper key. So proper key means probability of B.

So that is it therefore probability A union probability A intersection B, so probability is 0.4 and probability A complement x probability B. So that is all in today's tutorial class and what I will suggest, it is really not feasible to discuss many, many problems in one tutorial class in this sort, of course, but as to get more hands on your probability by seeing any problem you will be able to solve it. Like it is only possible if you practice more and more problems.

So it will get probability question in many books and statistics and probably there is not a single book there are many books available in the net, you can browse and in a book, you do not have to buy the books there are many free versions available in the net you can just download the books practice the problem. So essentially the more problems you solve more confidence you will get.

**(Refer Slide Time: 35:47)**



The image shows a presentation slide with a dark blue header and a light blue footer. The header contains the word "REFERENCES" in bold yellow text. Below the header, there is a list of four references, each preceded by a small circular icon. The references are:

- Krishna B. Misra, "Reliability Analysis and Prediction -A Methodology Oriented Treatment", Publication: Elsevier
- Sheldon Ross , A First Course in Probability (fourth ed.), Macmillan College Publishing, New York (2013)
- D. P. Bertsekas and J. N. Tsitsiklis, Introduction to Probability. Nashua, NH: Athena Scientific, 2008
- W. Feller, An Introduction to Probability Theory and Its Applications, vol. 2. New York: Wiley, 1971

In the bottom right corner of the slide, there is a small video inset showing a woman with glasses wearing a yellow vest over a white shirt. The footer of the slide contains the NPTEL logo on the left, the name "Monalisa Sarma" and "IIT BHARAGPUR" in the center, and a small circular logo on the right.

So, these are the references I have used. Thank you.