**Statistical Learning for Reliability Analysis**
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**Lecture – 03**
**Concept of Probability and Probability Theory**

Hello everyone. So, in today's lecture we will be learning probability theory. Now the question is why we will learn probability theory in this course on statistical learning for reliability analysis. Nothing is that in this while we are learning statistical methods that are in this course on statistical learning, we will be doing inferences on many applications. After that, we will have to evaluate the reliability of these inferences.

To evaluate the reliability of these inferences, we will be reading the concepts and principles of probability. And that is the reason we will be discussing some concepts and theories of probability in the coming 2 lectures.

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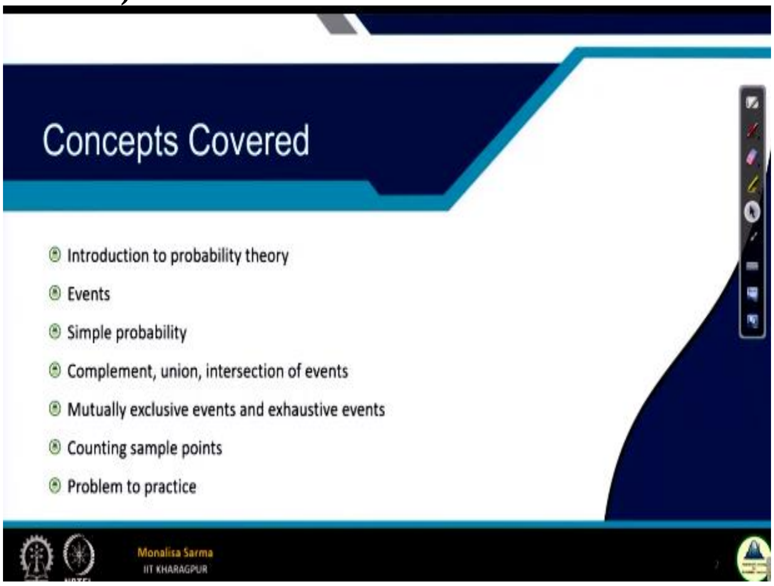


So in this class, I will cover what is an event, then I will cover different types of events like complement, union, intersection and also mutually exclusive event, exhaustive events. Then after that when of course, I will also cover when we will be discussing probably definitely we will have to talk about sample space also, now sample space how to calculate the sample space for that also we will be discussing few points that is counting sample points and then we will be discussing few of course 2, 3 problems also in this lecture.

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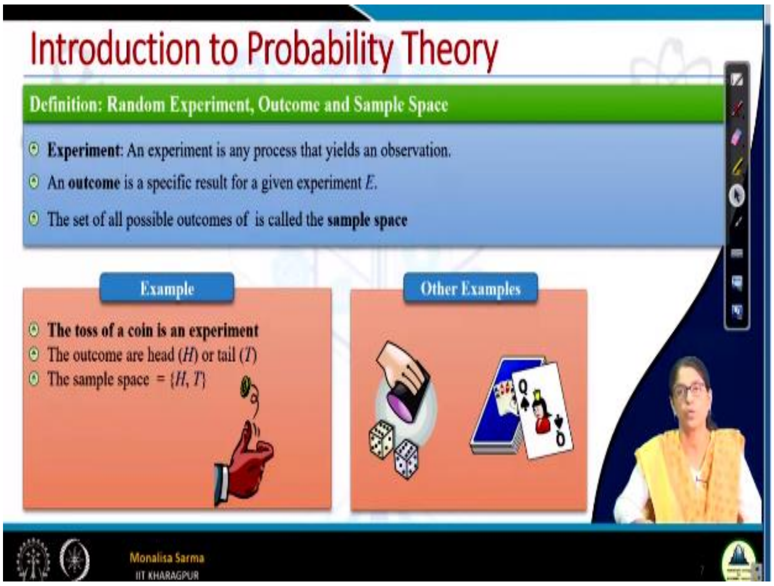


So to start with, we will go through a few definitions before learning what is the probability theory, the first definition that we need to know is and what is an experiment. So, an experiment is any process that yields observation or I can say an outcome. So, what is an outcome then? An outcome is a specific result for a given experiment E. Now, what is sample space? The set of all possible outcomes is called a sample space.

So, first is, we need to know what is an experiment. So, once we know what is an experiment, so experiment is output outcome, then outcome then an experiment they can be different possible outcomes, all the outcomes put together we call it as a sample space. So, there is an example suppose tossing of a coin simple tossing of a coin is an experiment. So, what if we toss a coin what output we can get one observation that can bring come head or tail?

So, these are the 2 outcomes so then what is the sample space is? Sample space is a set H and T. Similarly, another example of a tossing of a dice, that is an experiment, so what are the different outcomes? Outcome is getting different values from 1 to 6 and so what is the sample space? Sample space is 1, 2, 3, 4, 5 and 6 this is the sample space.

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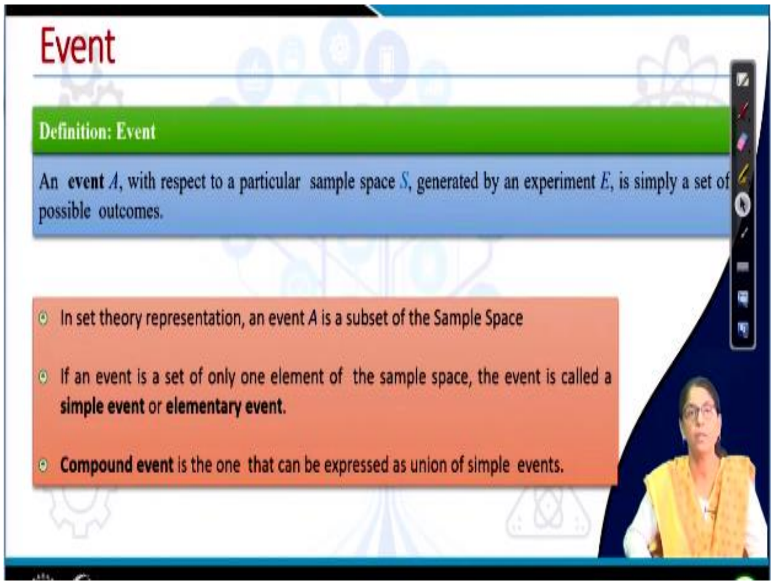


Now, let us see what is an event? So, an event A with respect to a particular sample space S generated by an experiment E is simply a set of possible outcome. So, an experiment for an experiment there can be different events, so for any event A, it will be basically a subset of the sample space. So, an experiment can have different outcomes or different events. And all events have some order other value in a sample space. So, for an event, what we get? It is simply a subset of the sample space.

So, set theory representation and event A is nothing but a subset of a sample space. Now, if the event is a set of only one element, then we call it a simple event. And if it is a union of many simple events, then we call it a compound event.

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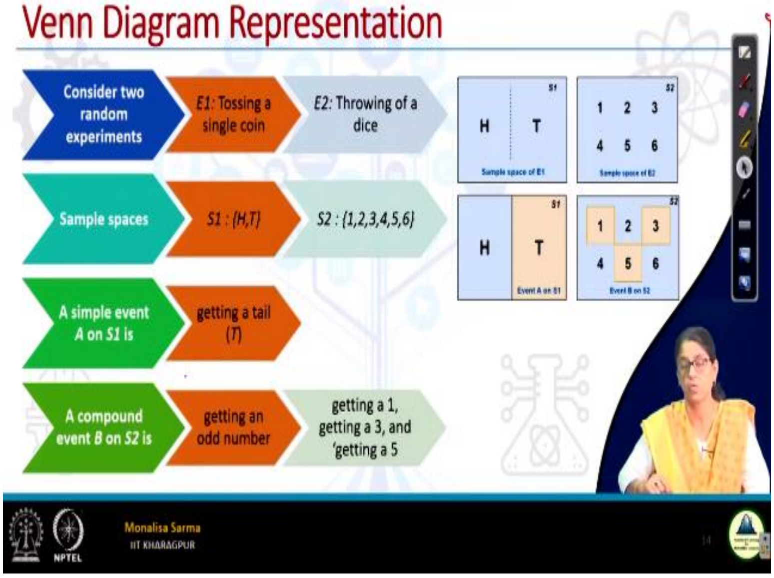


So, we can see the Venn diagram representation of this, consider 2 random experiment like tossing a coin or throwing a single dice, if we toss a coin so what is the sample space, we have already seen sample spaces H and T. A sample space contains 2 elements and similarly tossing a dice let us I should say throwing a dice, throwing a dice there is a sample space sample space consists of 6 elements, so 1, 2, 3, 4, 5 and 6.

So now, what is an event? Now we will see that event is a simple event A1 S1. Suppose I am talking about simple event on the sample space S1. So, events say my one event maybe getting a tail. And of course, an experiment that is tossing a single coin. They can be just 2 events, either getting a tail or getting a head there can be just 2 events. So, if I talk about getting a tail, so my event is my sample spaces this subset of this, this is this T.

Let me take the pen so I will be able to point it properly so this is T ok, so once again this is not writing properly, so this is the sample events for getting a tail now, if I considered a second sample space this is S2 there may be different event for this the event maybe getting a particular number getting 1 getting 2 getting 3 like that event maybe getting a number or even maybe getting an even number, even maybe getting an odd number, even maybe getting as number combination of these numbers like what is ?

If I am interested in finding event of getting a total of 7 if I total of 7 that will be discussing, what is the event of getting a total of 7 in a sample space S2. I will come to that what is that event? So, 7 is there in the sample space it is not there. So, we will see what is that. Now, we will talk about compound event. So, in a compound event on B I will talk of compound event on B so getting an odd number, what is getting an odd number in sample space B that is a how many odd numbers are there that is 1, 5 and 3.

So that is the set of events that is propagating an odd number, this is getting 1, 5 and 3, that is the subset of that set as to which corresponds to this event.

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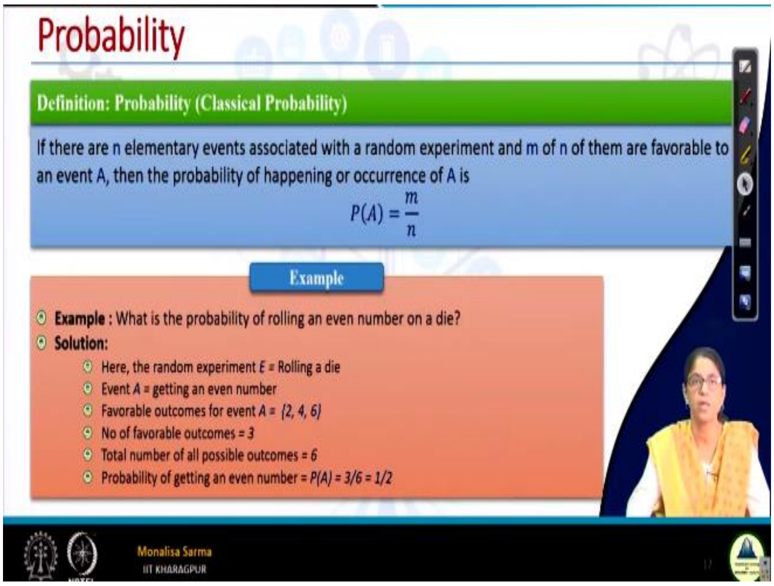


Now we will define probability. So, if there are n elementary events associated with a random experiment, like as I told you in a first sample spaces S1 which we have discussed in this previous slide, sample space S1, how many probable events are there probably event 1 event of getting a tail another event of getting a head. So similar, that is what I am telling if there are n elementary or events associated with n random experiment and m of the n of them are favourable to an event A.

Any event out of this n events, we are just interested in even a so if we are interested in finding out the probability of A so to find out the probability of A, then we have to find out the out of this sample space which favours the event A, so then if there is n elementary event associated with random experiment and m of n of them are favourable to an event A. Then the probability of happening or occurrence of A is nothing but m by n.

Where n is the size of the sample space and m is the size corresponds to an event A, m is basically the number of elements that corresponds to the event A number of elements in the sample space. So, like, what is the probability of rolling an even number on a die? This is an event. So, what is the event? Event is finding an even number. So, how many elements correspond to this in a sample space of rolling a die there it is 6 sample space size of the sample space is 6.

So, now what are the; how many elements correspond to this even rolling an even number that is 3. So, probability will be 3 / 6, see so your random experiment E is rolling a die, event A is getting an even number, favourable outcomes for even A is 2, 4, 6, number of favourable items total 2, 4, 6 total number is 3. So, total number of possible outcomes sample space that is the size of the sample size is 6, so probably getting an even number is 3 by 6.

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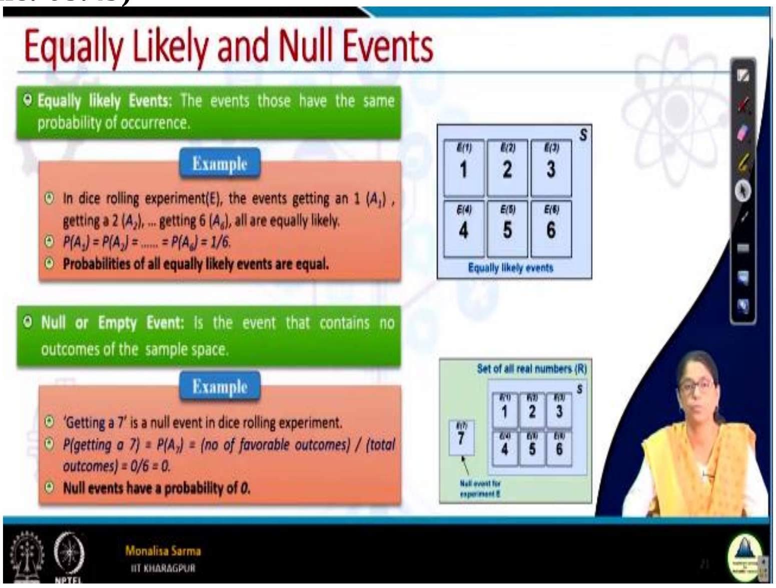


Now what is an equally likely event? The events, those are the same probability of occurrence then we call it equally likely events. Like when we toss a coin, the probability of getting a head and probability getting a tail it is the same probability. So, it is called equally likely events. Similarly, tossing a die probability of getting any number is equally likely. So, in a die rolling the events of getting 1, 2 or 3 anything 1, 2 up to 6 are all equally likely.

So, what is this probability? Probability is equal to 1 by 6 so this is the probability of all like equally likely events are equal. Now null or empty event remember, when I asked in a sample space S2 what is the probability of getting 7, so that was a I was telling that is an event when probability of getting a 7, so 7 available in the sample space it is not there. So that is an empty event or null event It is the event that contains no outcomes of the sample space So, getting a 7 is a null event in a dice rolling experiment. So, null event have a probability of 0.

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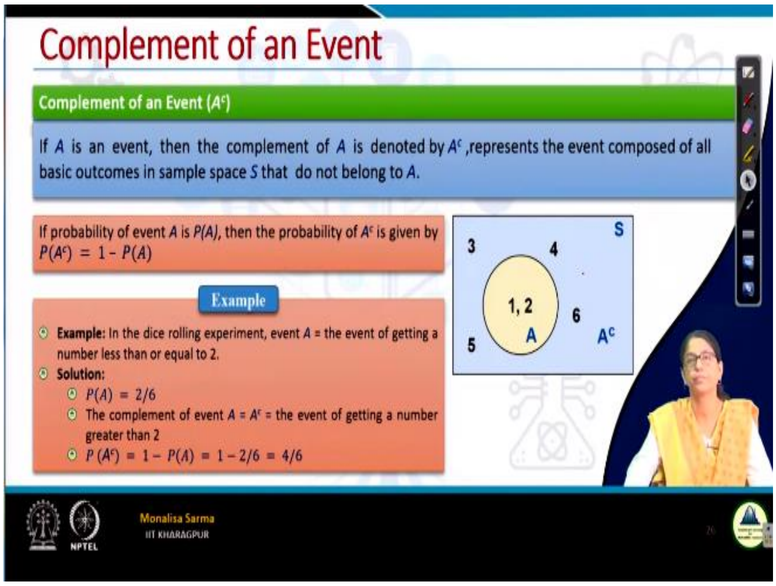


So, compliment of the event, it is easier if A is an event compliment of A is just 1 minus because probability will always range from 0 to 1. So, if one we know the probability of a certain event happening, and then what is the probability of that event not happening is just 1 minus of that event that is the complement. So, if A is an event then a complement of A is denoted by A complement represent the events composed of all basic outcomes in a sample space that do not belong to A.

So, 1 - probability of A, probability of A compliment is 1 - probability of A so example, in the dice rolling of when event A is the event of getting a number less than or equal to 2. So, if I am interested in the event of getting a number less than or equals to 2, how many numbers corresponds to this less than or equal to 2 that is 1 and 2. So, size of this event is 2 and what is the size of the sample space? Size of the sample space is 6.

So, what is the probability of that probability that is 2 / 6. So, this is, this light yellow coloured circle, what you can see this is the this corresponds to the even that event of getting a number less than or equal to 6, then in this the whole sample space which is shown in a light blue colour, this whole sample space and this is the whole sample space, this is S and this circle, this small circle corresponds to event A then what is A complement? A complement is this 1 minus of this A that means the out of portion of this circle. The complement of event A is the event of getting a number greater than 2. So, what is 1 - P (A).

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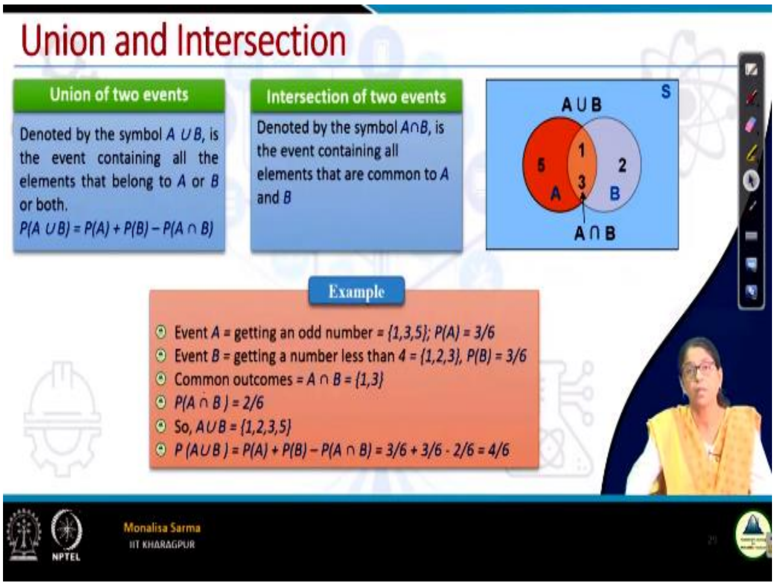


Now union of 2 events denoted by the symbol A union B is the event containing all the elements that belongs to A or B are both. Union of 2 events means all the elements containing to event A event B or both. So, A union B is P(A) + P(B) and why we are doing -P(A intersection B), because there are some elements which belongs to both A and B, those elements will be counted twice. So that is why we are just subtracting it -P(A intersection B).

So, intersection of 2 events, intersection of 2 events is the event containing all the elements that are common to A and B. So, if you see the Venn diagram, so here say red colour corresponds to event A and this violet colour it corresponds to event B, then this orange colour, this portion it belongs to both A and B. So, when we are trying to find a probability A or B, so this orange portion we counting it again, you are counting it twice, we have counted it for when we have counted for A.

We have counting it again when you have counting for B, so we are counting it twice, but now we need to count it once. That is why we are again subtracting once that is minus of A intersection B that is why we have this minus of A intersection B in probability of A union B. So, simple example, even A maybe getting an odd number, what is the n rolling of a die? So, what is the odd number enrolling of a die is 1, 3, 5. So, what is property of A is 3 by 6.

Similarly event B means getting a number less than 4, what is the number less than 4 is 1, 2, 3. So, probability B is again 3 by 6. Now I am interested in finding out the common outcomes that is probability A intersection B, probability A intersection means that is which is common to both A and B, if we can see which are the common to both A and B, A and B 1 and 3 is common to both A and B. So, it will be 2 by 6.

And now if you are interested in finding A union B, so union name is added up if you see 1 and 3 will be added twice so we are again subtracting that, so probability of A union B is probability of A + probability of B – probability of A intersection B that is why we got 4 by 6.

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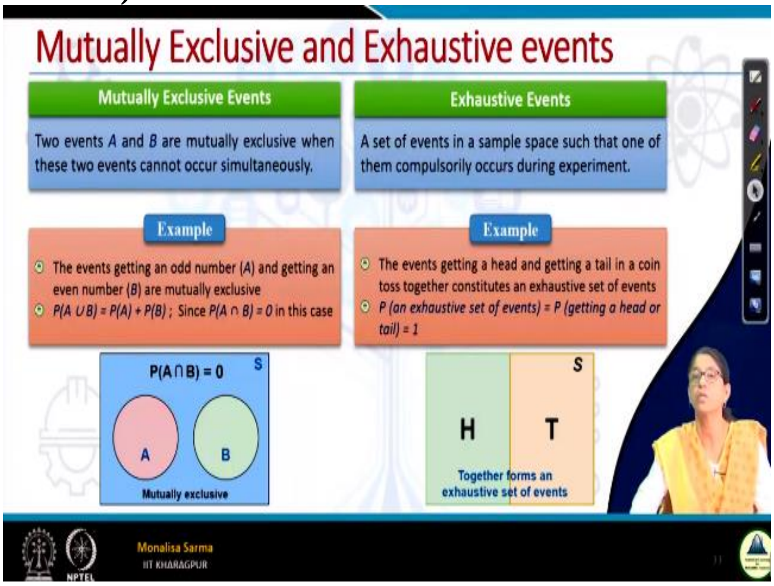


Mutually exclusive event, mutually exclusive events are when 2 events cannot occur simultaneously when we toss a coin at the same time head and tail both cannot occur. When you toss a tail at the same time 1 and 2 both cannot occur, any 2 numbers cannot occur at the same time. So, those we called as a mutually exclusive event. So, in case of mutually exclusive events like when we have done A union B.

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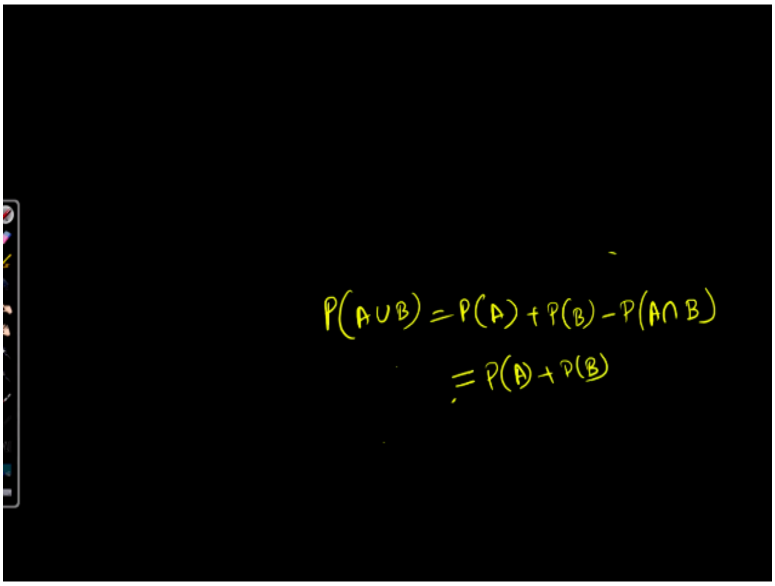


So, what we have got initially probability of A union B = probability of A + probability of B - probability of A intersection B. Now, if then mutually exclusive event, mutually exclusive events means both cannot happen together that means probability A intersection B is 0. So, if A and B are mutually exclusive events, so probability of A union B, we will just get probability A + probability of B.

So, the events getting an odd number and getting an even number, we at the same time we cannot get an odd number as well as you cannot get an even number. So, these are mutually exclusive event. So, probability of A union B is a simple probability of A + probability of B what we have just now discussed. Exhaustive events, exhaustive event is a set of events in a sample space such that one of them compulsorily occurs during experiment.

Actually when I when we talk about exhaustive events means all possible events that may happen in an experiment. In an experiment what events that can happen? So, this is we are considering all the events. So, when in an experiment when we are considering all the events, when we are doing an experiment one of the event has to occur. So, that exhaustive event is a set of events in a sample space such that one of them compulsorily occurs during an experiment.

The events of getting a head and getting a tail in a coin toss together constitutes exhaustive event. So, what is the probability of an exhaustive event is always one because it is the whole sample space basically.

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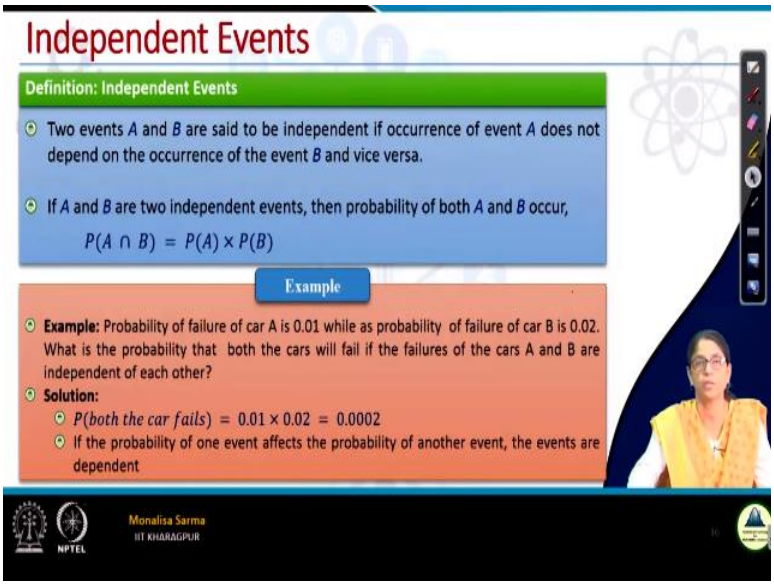


Now, the definition of independent events. So, if 2 events A and B are said to be independent, that is the occurrence of event A does not depend on the occurrence of event B that the event A has occurred, it will no way affect the occurrence of event B then we call as 2 events are independent. If then in that case, if you are interested in probability of occurring of the both the events is simply the multiplication of both the events.

So, probability of independent events independent probability of both events occurring we have seen when we try to find out probability about events occurring, it is intersection right . So, probability of A intersection B is probability of A x probability of B if they are independent events. A very good example is here, probability of failure of car A is 0.01 while the probability of failure of car B is 0.02 these 2 events cannot be in no way dependent.

They are totally independent events failures car A is failing, car B is failing what dependency can be there. So, what is the probability that both the cars will fail if the failures of car A and B are independent of each other. So, just a multiplication of both the probabilities, probability of both the car failures 0.01 and 0.02. If the probability of one event affects the probability of another event, then we call it a dependent event we will see dependent event later.

Now, see till now, how to find the sample space we have taken very small small example tossing a head, tossing a tail. So, we know the size of the sample where, but for bigger problems for real life problems, it is very difficult to find out the total size of the sample space to find the type and size of the sample space basically we have to use some methods so that we can count the sample points.

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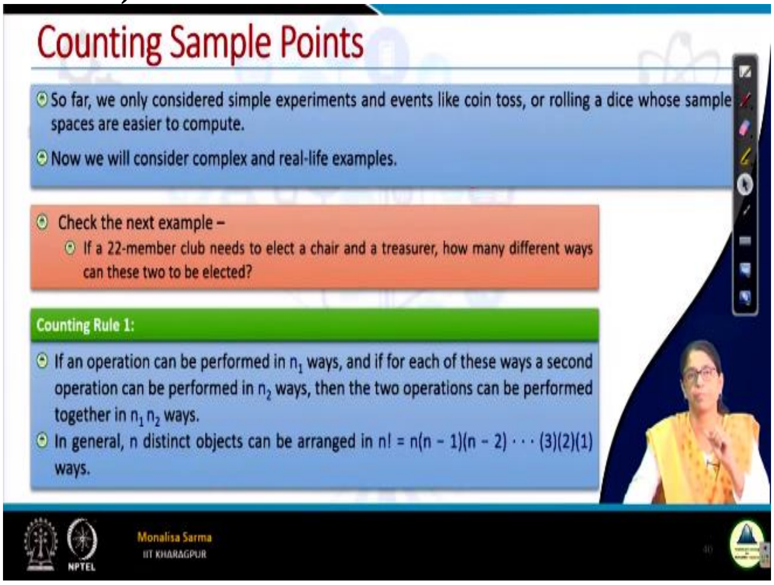


So, we will see that so like see this example, if a 22 member club needs to elect a chair and a treasurer how many different ways can these 2 be elected? We will learn, let us not talk about it definitely will be, we are finding this so that we can calculate some sort of probability. Now we are just interested in finding the sample space. Now, if you 22, member clubs needs to elect a chair and a treasurer, how many different ways can these 2 be selected?

Essentially, what are the different possible ways we can select means essentially, this is the sample space. In the sample space, how many elements will be there, so, in how many ways we can do so, there are 22 members, suppose we select a chair, this chair can be selected in 22 ways, from 22 person any person can be chair. So, chair can be selected 22 ways once his chair is selected, the rest remaining is 21.

So, now to select a treasurer we have to select any one from this 21 so basically, since both are again independent events, so it will be 22 x 21. So, event operation can be performed in n 1 ways. And if for each of these ways, a second operation can be performed in n 2 ways. Then these 2 perform operation can be performed together in n 1 x n 2 ways. So, how many if a 22 member clubs need to elect a chair and a treasurer?

How many different ways we can select these to be selected? That is basically 22 ways. Suppose we select a chair, then we seem to select to treasurer their 21, so 22 x 21. Now, if in general, if you are interested in n distinct object, how does distinct object can be arranged? So, then in general that n distinct object can be arranged in n factorial ways. Let me take a very small example suppose we have 3 letters a, b, c. So, in how many ways this all these 3 letters are distinct a, b and c, and how many ways we can have how many different elements we can have from this 3 letter?

How many ways we can so all are different? So it is abc, bca you see, you try to find out all the combination, you will totally you will be getting total 6 components and how do we get that. So basically, for the first position, we are considering arrangement of 3 digits for the first position so how many digits are there total 3 digits, for the second position once you have selected the first digit for the second position, how many digits remaining 2 digits remaining.

So, once we have selected the second position, now for the selection the third digit, how many options are remaining? Just 1, so it is 3 cross 2 cross 1. So, 3 cross 2 cross 1 is what basically, it is 3 factorial. So, in general n distinct object can be arranged in n factorial ways.

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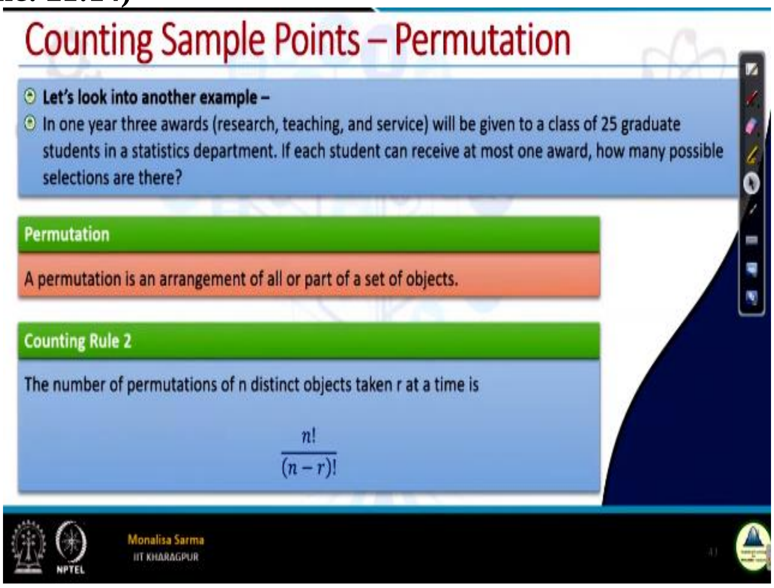


Now, something some different calculation from the one which we have just learned. So, in just see an example, in 1 year 3 awards will be given to a class of 25 graduate students in statistics department. There are 3 graduates, 3 awards will be given to a class of 25 students in a statistic, if each student can receive at most 1 award how many possible selection are there? Like, instead of doing that, we will take a simple example.

Like suppose there are 4 letters from this 4 letters, how many different ways we can have a combination of 2 letters? There are 4 letters a, b, c, d from these 4 letters in how many different combinations we can have 2 letters. So, basically, what so for 4 letters to have a combination of, we can take the help of the board here.

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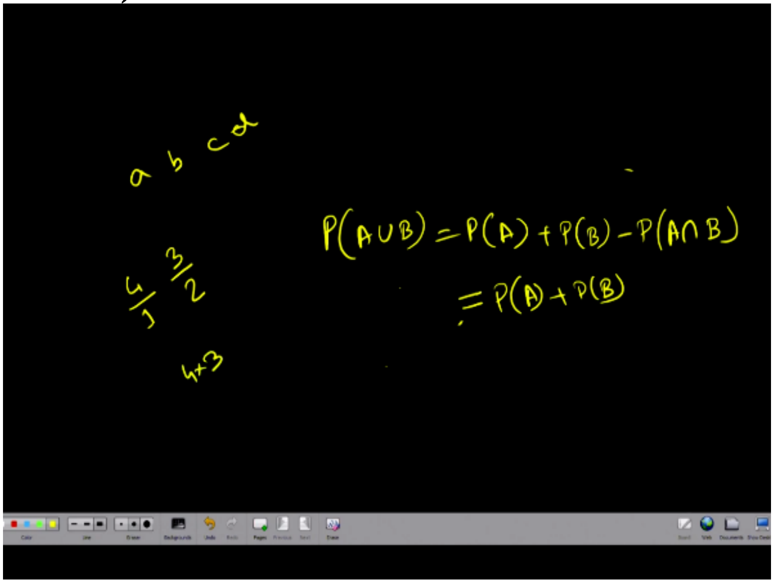


So first, let me rub this, that is not rub, let us just use it. So, we have 4 letters, a, b, c, d and we need a combination of 2 letters, to 2 letters just to position this is the first position, this is the second position. First position, we can fill it in how many ways we can fill it in 4 ways. Second position, we can fill it in how many ways second position we can fill it in 3 ways their total remaining 3 ways. So, this is 4 x 3. So, what is this 4 x 3?

So, this is 4 x 3 is nothing but n ! / n - r !, you see if you interested in finding out that is 4 x 3. So, from n if you are interested in selecting r objects, so, if you will do basically n into n - 1 into n - 2 this will go to out what level n - r + 1. So, n x n - 1 x n - 2 it will go to till n - r + 1. So, what is that, that is nothing but simple n ! / n - r !. So, now, this question in 1 year 3 awards will be given a class of 25 so, 3 of us will be given a class of 25 basically, so, this is nothing but 25 b 3.

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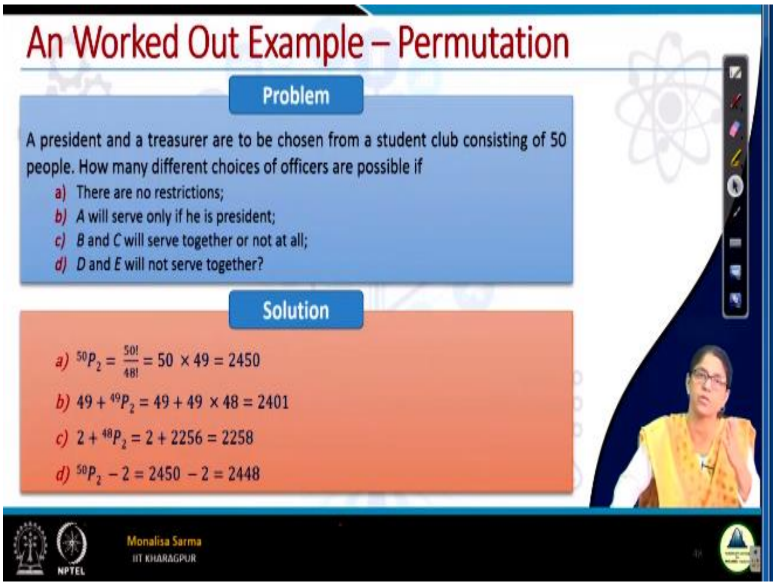


So, now this problem how will you solve it? A president and treasurer to be chosen from a student club consisting of 50 people. How many different choices of officers are possible if there are no restriction? So, we have to select from 50 people a president and a treasurer. How many different sizes of offices if there is first of all, there is no restriction we can any people can any person can be a president, any person can be a treasurer in how many ways we can do it using the counting rule.

So, first if we select the president, president can be selected in 50 ways. And then if we select the treasurer, treasurer can be selected in 49 ways, because 1 already has become president. So, basically, what is that 50 P 2, is not it? So, it is 50 P 2, that second A will serve only if he is president, for then there are 2 possible one, we have taken one such choice where A is selected as a president or A is not selected because A will not take any other job when A is selected as a president then.

And next task we have to select a treasurer, treasurer will be selected can be selected how many ways treasurer can be selected out of the remaining 49 that is 49 way. So, 1 x 49 that is one option. Another option is A is not selected, A is not selected, then president and treasurer can be selected from any of the remaining 49 and that is from 49 P 2. So, it will be 49 + 49 P 2, 49 x 1 basically 49 into 1 + 49 P 2.

Now, next is B and C will serve together are not at all what does that mean B and C will serve together means B and C will be either B will be president C will be treasurer or B will be treasurer C will be president B and C will serve together means how many ways we can do that, we can do that in 2 ways or not at all then if they are not serving depends remaining is 48 so, that means 2 + 48 P 2. Cool right? So, next is D and E will not serve together, D and E will not serve to get this here we will be using the concept of complement.

First we will find out when how many ways D and E will serve together, D and E will serve together and how many ways if we can find out then 1 - of that will give us D and E will not serve together and D and E will serve together? How did you find out that we have just seen in C, B and C will serve together? So, in what way D and E will serve together there is only 2 ways and D and E will not serve together the total sample space is 50 P 2 - 2.

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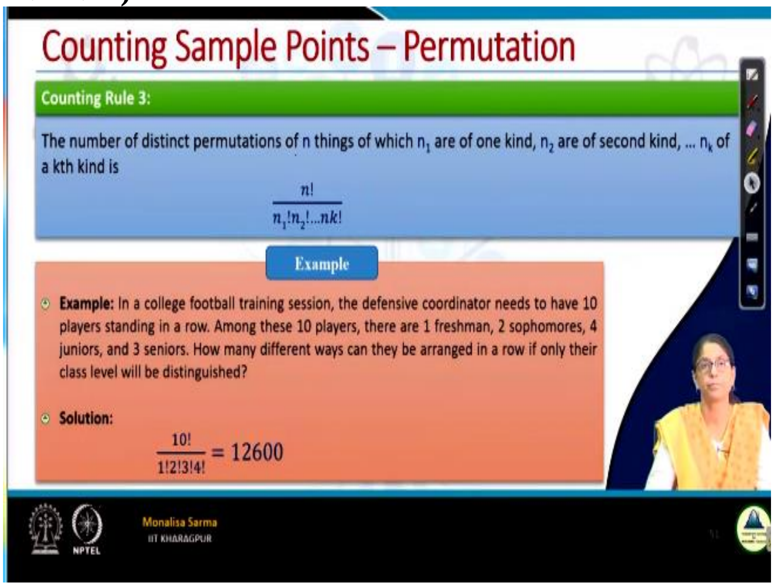


Now, the number of distinct permutations of n things of which n 1 are of one kind n 2 are of second kind and say n k of the kth kind, then what is the formula for that n ! / n 1 ! x n 2 ! … n k ! we will understand with an example. Suppose, in a college football training session, the defensive coordinator needs to have 10 players standing in a row they need 10 players.

Among these 10 players, there are 1 freshman, 1 first year student, 2 sophomores second year students, 4 juniors for third year students 3 senior students. So, these are not distinguishable that 1 freshman only 1, 2 second year students this 2 second year students are not distributable there we will consider both second year students only. Similarly 4 juniors are not distinguishable. So, basically there are 1 kind, 2 or 1 kind then 4 juniors are of 1 kind 3 seniors are 1 kind and among these I have to select total 10. How will I select, so, it is 10 ! divided by this 2 ! x 4 ! x 3 !.

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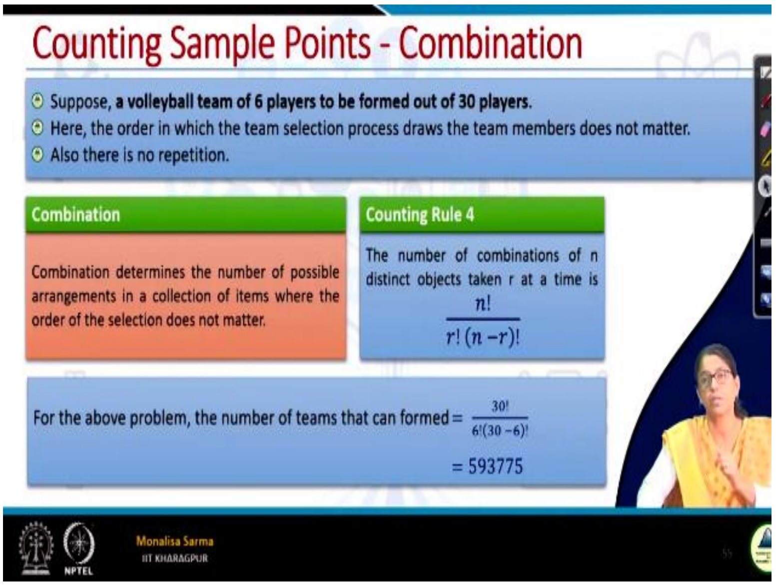


So now, below what we have discussed is permutation where the ordering methods abc and cba are 2 different things where we are distinguishing among the digits. So, now, if suppose the volleyball team of 6 players to be formed out of 30 players and the order will be the team selection process draws the team member does not matter, there are 30 players, we are not distinguishing the players from the 30 we need to the select 6, we are not distinguishing the players somehow.

Like there are 3 digits we are not distinguish that this is a, this is b, this is c we are not distinguish. So, there are 30 players from this 30 player we have just 2, we just need to say select c, then what we will be used combination and also there is no repetition. So, this is a concept of combination, combination determines the number of possible arrangement in a collection of items where the order of selection does not matter.

So, number of combination of n distinct objects taken r at a time. So, it is n distinct and the formula for n distinct n ! / r ! n - r ! all this actually it is like a quick recap for all of you because I am sure all of you can have done this in your school level. And if any of you have forgotten then we can consider this is a brush up basically. So, actual lecture will be start once we complete a probability theory basically.

From for the above question like when from the 30 players, we need to select 60 players. So, it is 30 c 3. So, how do we write the 30 c 3, 30 ! / 6 ! into 30 - 6 !.

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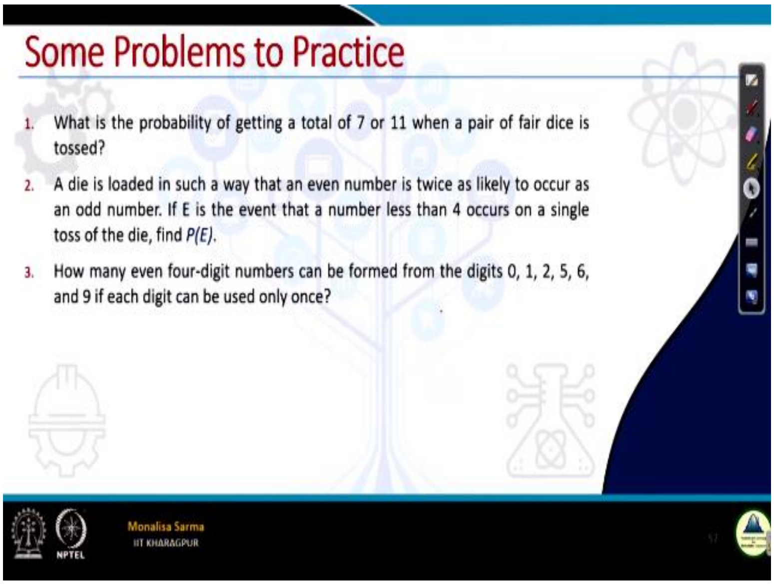


So now, there are a few problems which I have kept here of course I will be doing some tutorial classes also. But these 3 problems I have kept here I want you to solve these problems. So, if you have any problems, then we will be having doubt learning session then of course, we can clear this problem. But still, I want you to practice this problem. So, I will give you a hint on how to solve this problem?

So, first is what is the probability of getting a 7 or 11 when a pair of fair dice is tossed? I am tossing a pair of fair dice. What is the probability of getting a 7 or 11? See here that means 2 events. One is probability of getting a 7 if I consider it is an event A and probability of getting 11 it is considered is an event B basically that means I am interested in probability A union B A or B, but these 2 events are mutually exclusive both 7 and 11 cannot happen. So, it is a probability of A union B is just probability of A + probability of B.

So, now what is the probability of A from the sample space when we are tossing 2 dice? What will be the sample space size of the sample space will be 36 is not it 6 cross 6. Now, out of these 36 how many corresponds to 7 how many of this corresponds to a total of 7 like if I get 3 and 4, 4 and 3, 5 and 2, 2 and 5, 6 and 1, 1 and 6. So, these are the thing which corresponds to 7. So, I think around 6 of them will corresponds to a total of 7. So, what will be the probability of getting event A that is a total of 7 is 6 / 36.

Now, probability of getting 11 when will I get 11 when I get 5, 6 and 6, 5, so that is 2 that is 2 by 11. So, 6 / 36 + 2 by 36 is the answer for the first question. Second a die is loaded in such a way that an even number is twice as likely to occur as an odd number. So, we have seen when we toss a die we have considered as it is equally likely, equally likely means so, there are total 6 phases. So, what is the probability of occurrence of is as 1 by 6 since this sample size is 6 and it is equally likely so, it is occurrence of any one is 1 by 6.

Now, here it is different here it is not in fair die, if it is a fair die then it is equally likely that it is not in fair die, but it is given is loaded in such a way that an even number is twice as likely to occur as an odd number. So, what I will do? I will assume the probability of occurrence of odd number, let us take the probability of occurrence of odd number is w. So, now, what was the question? The question was probability of even number is a double the probability of getting an odd number.

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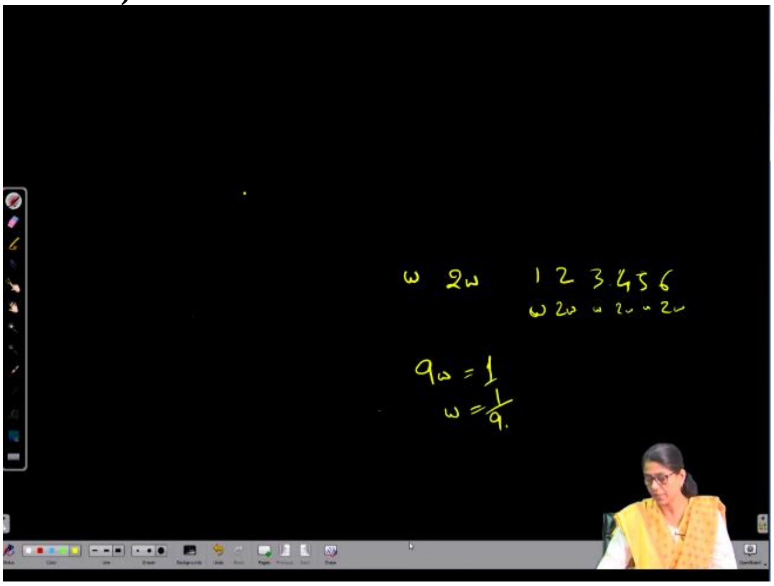


So, if the probability of getting an odd number is w, then I can take probability of getting an even number is 2w. So, I will take 1 as w and another as 2w. So, total there how many numbers in a die, so 1, 2, 3, 4, 5, 6 so, if 1 is w, this is 2w, so w 2w, w 2w so how much I got? I got 9w and this probability will be equals to 1 probability of getting this number total is 1. So, what is w? w will be 1 / 9. So, I got w 1 / 9.

Now let us come to the equation if E is an event number that a number less than 4 occurs on a single toss of a die less than 4 what is less than 4 that means 1, 2 and 3. So, what is the probability of occurrence of 1? Probability of occurrence of 1 is 1 / 9. Probability of occurrence of 2 is 2 / 9. Probability of occurrence of 3 is 1 / 9 so the probability is the sum of all this 3 I am not doing it just but I to explain you in fact you will be able to do it.

Now the next question how many even 4 digit number can be formed from the digit 0, 1, 2, 5, 6 and 9 if each digit is used only once? This will not be doing it I will just give you a hint up solve it. So, here we have to find out total even 4 digit number so the digit are 0, 1, 2, 5, 6 and 9 when we can see that number will be even if in the last digit we get either 0, 2 or 6. So, we will take 2 combinations one is when the last digit we got 0 another is when the last digit we did not get 0.

When the; last digit is 0 why we have separated because the first digit cannot be 0 in any of the cases. We are considering 4 digit numbers. So, first digit cannot be 0 in any of the cases. So, when the last digit is 0, then we will get a different way of finding out the probability because then the number of digits remaining will be total how many digits are here 1, 2, 3, 4, 5, 6 to let us 6 digit them for to filling out this other digit this there will be 6 digit.

But in the case of if the last digit is not 0 that means if the last digit is 2 or 6, then for filling the 3 position they will be different for filling the first position we cannot take 0. So, there will be 1 less number of points. So, now I will not explain further in this I want you to try and do this problem. Of course if you cannot do that in doubt clearing session we will be taking up and but only do not restrict to this problem, practice as many problem as you can.

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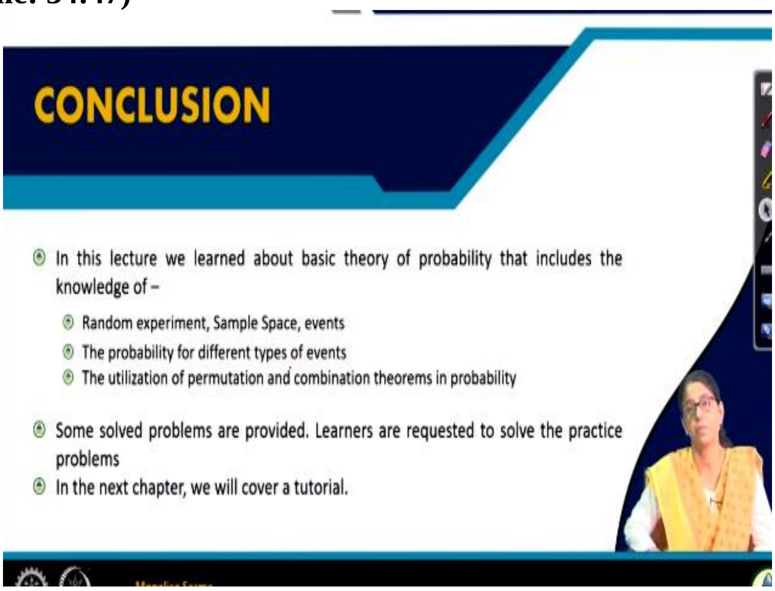


With this I am concluding this lecture. So, in this lecture what we learn? We learn what is an random experiment, what is a sample space, what are events then probability of different types of events also we have seen, then we have also seen the utilization of permutation combination theorems in probability. Then we have also solved some problems. And as I told you I request to solve as many problems as possible. In the next lecture, I will be covering the tutorial on the first 3 lectures.

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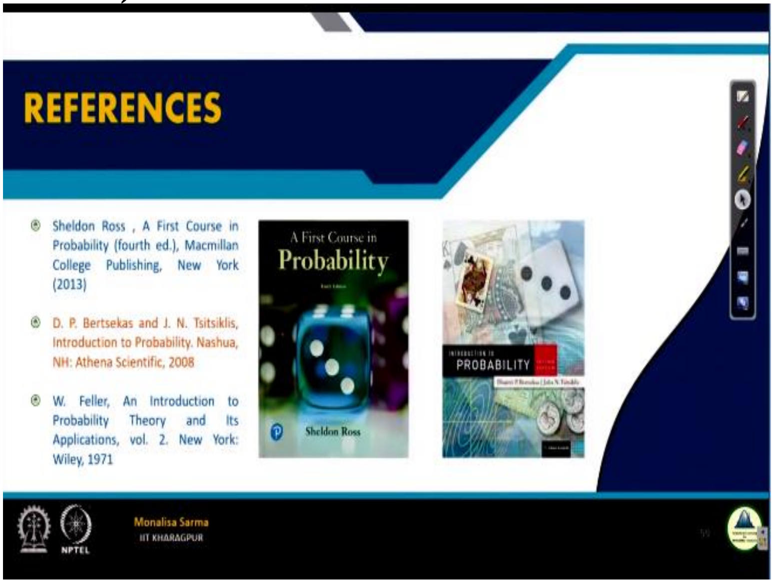


So, these are the basically 3 books which I am referring there do not restrict to this books they can walk from any books on probability and thank you.