Algorithmic Game Theory
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Lecture - 55
Sponsored Search Auction

Welcome. In the last class, we have seen Myerson's Lemma which characterizes the implementable social choice functions in the single parameter domain. In today's class, we will see an Application of Myerson's Lemma in the Sponsored Search Auction.

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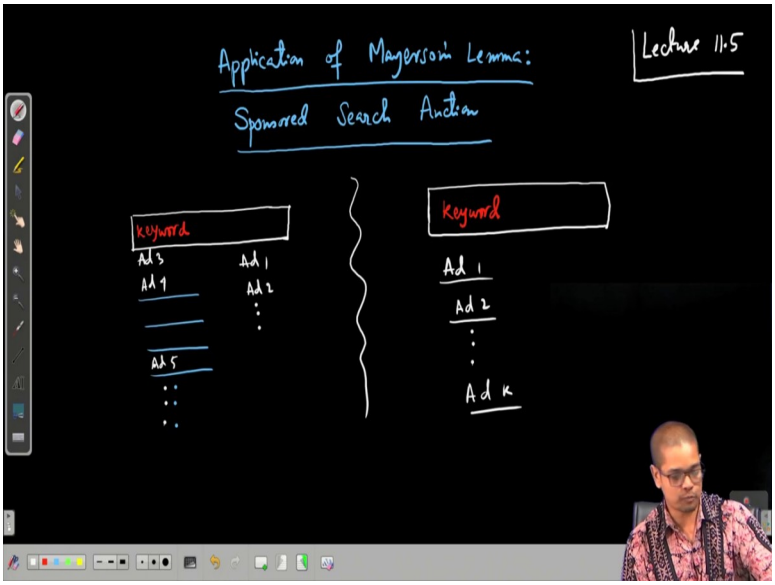

So, let us write application of Myerson's lemma. What application? Sponsored search auction. So, what is sponsored search auction? You know in typical search engine of say Google Yahoo or Bing or any search engine, when we search for a keyword, here is a we have a key word. There are search results some URLs comes which is a list of results. This this web pages are in some sense the search engines thinks that these web pages are most relevant for the search keyword.

But these are not all. There are many Ads come from where these companies earn revenue there are Ad slots, like Ad 1, Ad 2 and so on. I mean few years ago Ads used to come on the right, results used to come in the middle, but now Ads even come in the middle, here from here revenue comes, money comes. So, Ad 2, may be Ad 3, may be

some more Ad 4 and sometimes even inside the result also some Ad comes Ad 5 and so on.

Abstractly, what is happening? User is given a keyword to search. Here is user's keyword. And there are various Ads being shown. Of course, there are results, the relevant web pages, but there are various Ads from which the search companies Google, Yahoo and so on earn most of their revenue. This a revenue source. And we are interested in this part. This is the; this is called sponsored search auction.

So, this is sponsored search. So, here we have seen that just the search, next we will explain where come sponsored come and where is the auction. So, there is a search is happening and some Ads are shown. There are some predefined Ad slots, say Ad 1, slot for Ad 1, slot for Ad 2, and so on. Suppose, there are k slots in the entire page, at k various Ads will be shown.

Now, what the sponsors? Who are paying the search engines? The this advertisement companies will be are paying the search engine. And the deal is that you know search engine knows the users very much, they know which keywords they search, which things they buy and so on and so forth.

Based on all this information while user is using the search engine, the companies are trying to sell their product and Google comes or any search engine company comes in the middle and helps the companies to sell their product. And in turn, help the users who are also buyer, buy the correct product, the right product.

And so, in between of showing actual results they show Ads, and once the search engines job is to show those Ads to the user which is most likely that the user will click. And if the user clicks that Ad, then search engine gets the payment. How much payment? That is what sponsors of those Ads agreed them to pay. So, the deal is that, ok, here is my Ad, I am a company, here is my Ad and if per click of a user I will give you say some fixed amount of rupees.

Now, users from Google's perspective, when will Google earn? Google will earn only when or any search engine will earn only when it shows the most appropriate Ad. The most appropriate means that whichever Ad has the highest likely of user getting a click. That is one thing. And the second thing, it also needs to consider is that how much

amount of money the company or the advertising agency has agreed to pay the search engine company when user clicks.

So, that is where this explains the term sponsored. Each Ad is sponsored. And auction, where is auction? Because you know there are limited Ad slots, suppose we search for car there are so many or some mobile phone is or the search keyword is mobile phone, there are so many mobile phone companies, may be 100 mobile phone companies. But there are not 100 slots for this for users to show.

If we show 100 Ads, then user will not search because he also needs to get the result. So, there are limited slots, but say k slots which is much less than the number of advertiser who or who wants to show their Ads. And that is where the auction come into picture.

Whichever advertiser pays highest and some sort of things, the search engine has to decide which among those potential Ads, which Ads it will show for which keyword. So, that its revenue is maximized.

When does it earns? It only when user clicks. How much it earns? It earns the agreed amount what the advertiser has agreed to pay the search engine when the user clicks. So, that is the fundamental problem and that is the setup of sponsored search auction. So, for the sponsor search auction there is a very useful term which is called click through rate.

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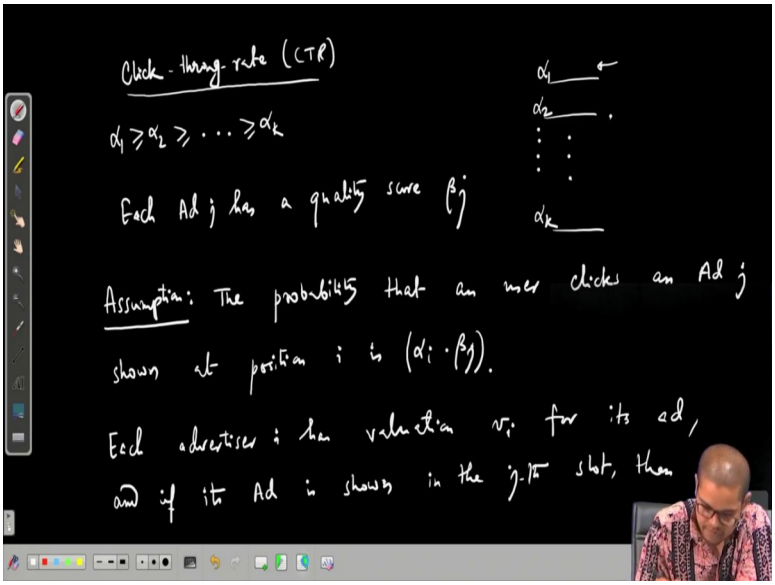

It is called CTR. What is CTR? It is just human behavior that we tend to read from top to bottom. If you are presented a page or anything any document we will start reading from top to bottom. So, although there are key slots for Ads from top to bottom, even if the same sort of quality wise the Ads are same the same user has more likelihood of clicking the top Ads than the bottom Ads.

So, if everything remains same, then what is the probability that the user I clicks the top Ad is say α_1, α_2 is the probability that it scrolls down i first Ad and clicks the second Ad and so on, so $\alpha_1, \alpha_2, \ldots, \alpha_k$.

And it is a valid reasonable assumption to make that $\alpha_1 \geq \alpha_2 \geq \ldots \geq \alpha_k$. This is because of the human behavior that we tend to read from top to bottom. So, Ads shown at the top of the page have more likelihood of user clicking it than Ads shown at the bottom of the page, even if everything else remains same, quality wise, Ad wise everything remains same.

Then, you know now each advertiser now what is the; so, whether the user clicks on a Ad or not it depends on two things. Of course, it depends on user that is third thing. But it other than the user it depends on two things, one is the position. Position of the page where the Ad is shown. If it is shown near the top of the page, it has more probability of getting clicked, that is quantified using this click through rates.

The second is quality of the Ad, how good is the Ad and so on. So, each Ad has a quality score. Each Ad say j has a quality score β_j. And it is like some parameter which the search engine algorithms maintain over time from the historic data, this Ad has been shown so many times and so on and so forth.

So, many users in what is the probability or in what fraction of times it has been clicked that should be that can be one parameter to judge the quality of the Ad. To begin with of course, we need to update it with various other parameters. But, that is not important.

So, how to maintain β_j, how to get this β_j, that is different thing. But that is not part of sponsored search auction. So, we will assume that each the search engine company knows the quality, inherent quality of the Ad β_j. And we will assume that the probability of user clicking the Ad due to purely because of its quality and because of its position

these two are independent. So, assume we; so it is a this is an assumption. The probability that a user clicks an Ad j shown at position i is $\alpha_i \beta_j$. It is a reasonable things to assume, and ok.

And the valuation like typical mechanism design setting, each advertiser, each advertiser i has a valuation v_i, ok. And it advertisers thinks that his Ad is the best. So, it does not need that quality score, although it also does not have also that data. This quality score data is typically with the search engine company.

And, but what it its concerned is that because it thinks it Ad is best, its Ad should be positioned as high as possible. So, each advertiser i has a valuation v_i for its Ad and if its Ad is shown in the j-th, slot there are k slot, then the advertiser has valuation, value of this allocation has valuation of this allocation, he has valuation $v_i \alpha_j$. So, this is the typical sponsored search auction scenario.

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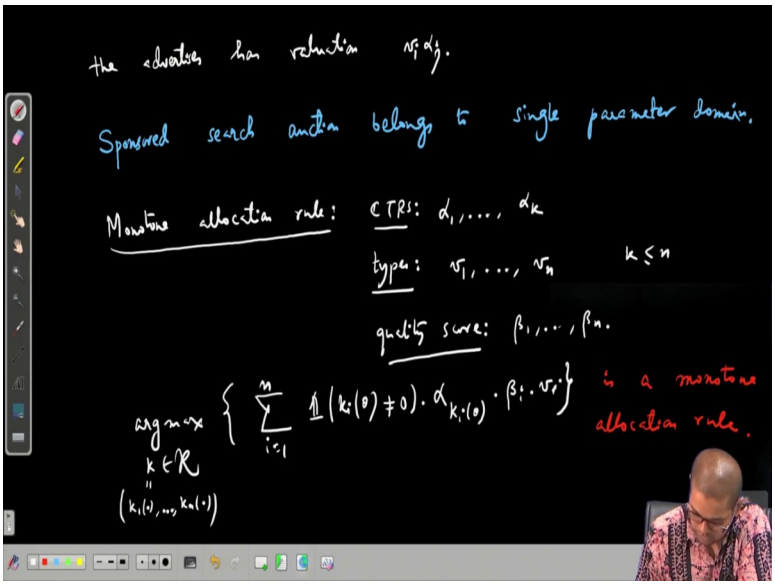

Next, we will see how using Myerson's lemma, we can implement a social choice function, a payment rule, the allocation we will see the what is the what is a monotone allocation rule. First of all observe that this is a single parameter domain because the type of each player which is v_i is a single, it is one real number. It is a single permanent domain.

So, first crucial observation is that sponsored search auction belongs to single parameter domain. Why? Simply because the types are only one real number. It is a single parameter v i, that is it. So, we will apply Myerson's lemma. Now, what is a monotone allocation rule? So, monotone allocation rule there are many, but the most natural one is that suppose there are, ok; what are the parameters let us write.

The click through rates CTRs alpha 1 to alpha K, and we have this Ads, this types which are like valuations $v_1, \ldots, v_n$. There are suppose there are n advertisers, β_j, k slots, $k \leq n$. And we have, what we have this goodness of a goodness of quality score of each Ad. Quality score is $\beta_1, \ldots, \beta_n$.

Now, what are the possible allocations? So, $arg\,max_{k \in K}$. Now, what does Google, what does any search engine wants? It wants to maximize its revenue. What is its revenue? So, for allocation, think of allocation is like its picking say k_1 is the; so, suppose k is like $(k_1, \ldots, k_n)$, and k_i says which position this Ad is shown. k_i is which position it is shown. If the Ad is not picked because only k slots are there, so $n-k$ Ads will not be shown, their position will be 0.

So, suppose k_i is the corresponding say, so if k_i is 0, so then they do not contribute. So, if k_i is 1 or k_i is k. So, only if this sum i equal to 1 to n, if $k_i(\theta)$ is not equal to 0, then only this expression will be calculated. If it is 0, then this sum and does not contribute. Now, if it is nonzero, then it shows it is a position where the Ad is shown. So, this is alpha of $k_i(\theta)$. Then what is the; what is the revenue that Google is earning times $\beta_i v_i$.

So, this is a monotone allocation rule and this one allocation rule, one allocation rule which Google wants to implement or any search engine want to implement because it will maximize the, it maximizes its revenue. So, this is monotone. Now, what is its payment? Let us compute the payment structure as given by Myerson's.

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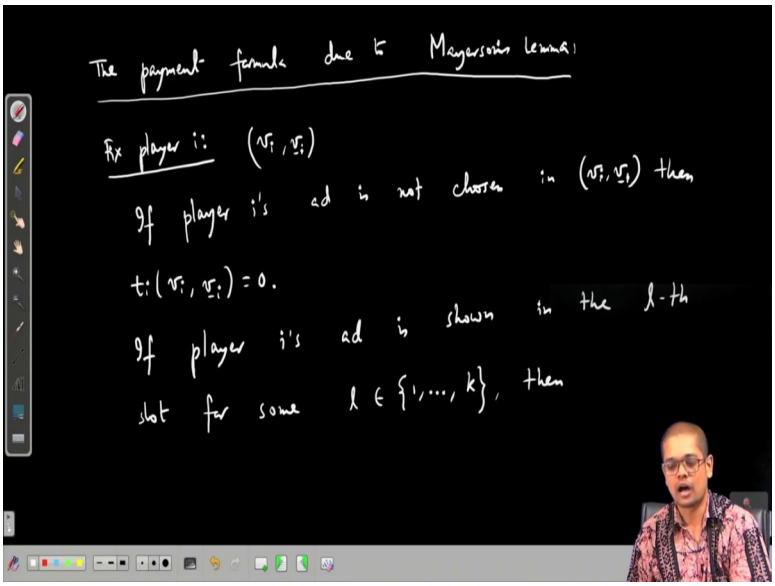

So, first observe that this is monotone allocation rule. So, next we find the payment formula due to Myerson's lemma is what, that we will compute next. But again first observe that this setting itself is complicated enough that it does not fall in the first part of our simple single permanent domain, where the allocations are like simple winning and losing.

Here the allocation each player is not like either winning or losing. Among winning also at what position its Ad is shown, depending on that it values that allocation differently. So, we need that we need to use Myerson's lemma to get this, ok.

So, fix a focus on player i, fix player i and say type profile say (v_i, v_{-i}), ok. So, if player i is Ad is not chosen in (v_i, v_{-i}), then $t_i(v_i, v_{-i}) = 0$. This is because of the Myerson's payment formula.

Also, just this worth highlighting that you know because we are we will be using Myerson's payment formula and because the allocation rule is monotone, dominant strategy incentive compatibility is guaranteed. It comes for free.

We just need to ensure that the monotone, allocation rule is monotone and the payment is due to Myerson's payment formula. Then, then the Myerson's lemma tells that monotone social choice function is dominant strategy incentive compatible. So, you can assume for

free without loss of generality that each player reveals its true valuation v_i for its best interest.

Now, so, it if it is not picked, then we have nothing to worry, it is 0. So, if player i's Ad is shown in the l-th slot say, l-th slot for some l in, there are k slot, then we will calculate how what will be its payment, ok, good.

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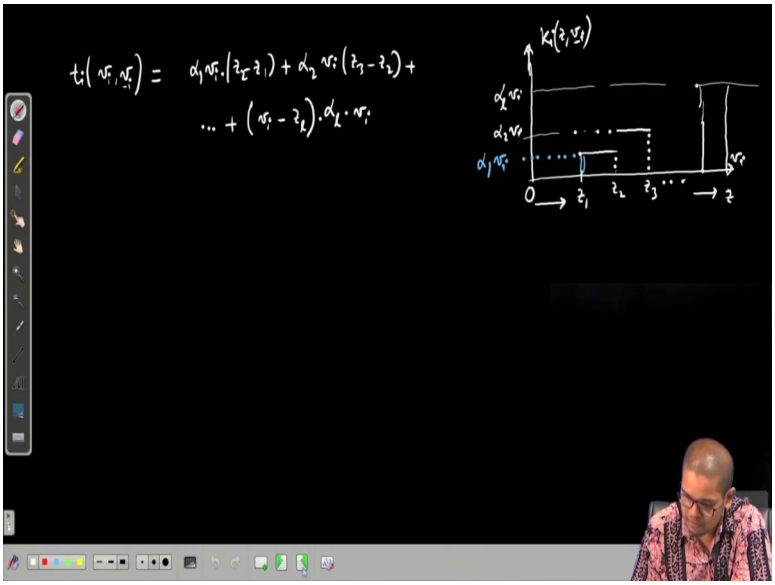

So, for that you again Myerson's formula, we that is z these are bits and this allocation $k_i(z, \theta_{-i})$ which is v_{-i}, now, and it is z_0. Now, you start increasing going from 0 to, start increasing z, and if it is 0, then it is not given any slot there will be one point say z_1, when it gets the first slot. And then this is its valuation.

And at some z_2, it is promoted to second slot because if its valuation increases, and the allocation rule is monotone, and you see look at the allocation rule if for if the valuation of other players remain fixed, and one player valuation keep on increasing it will its Ad will move up the slot. It is k slots. So, from z_2 to z_3, it is given higher slot, and higher slot is valued more and so on.

So, what is the payment? $t_i(v_i, v_{-i})$ is what is the valuation here? How much player i values the first slot? What is this valuation? This is the click through rate is α_1 and its valuation is v_i. So, this is $\alpha_1 v_i$.

And what is its length? $z_2 - z_1$. Plus, then what is for the next part, what is this? This is $z_3 - z_2$ and so on up to and its current position, it is in suppose this slot and this is in the lth slot, so this values is $\alpha_l v_i$ and we will go till v_i. We will not go beyond v_i. So, this will be the payment.

And this is what is the payment rule. And it looks complicated, but you know it all depends on understanding and this thing. You just implement this allocation rule and in this allocation only check where are those jumps, it will definitely make the jumps because of the structure it is, because of its monotonicity and wherever the jumps you draw you draw this structure and then you will you get it, ok.

So, we will conclude here today.

Thank you.