Algorithmic Game Theory
Prof. Palash Dey
Department of Computer Science and Engineering
Indian Institute of Technology, Kharagpur

Lecture - 48
Example of VGC Mechanism

Welcome. In the last class we have seen groves mechanism and Clarke's mechanism and that constitutes the popular VGC mechanisms. In today's class we will see examples of VGC auction VGC mechanism in various applications.

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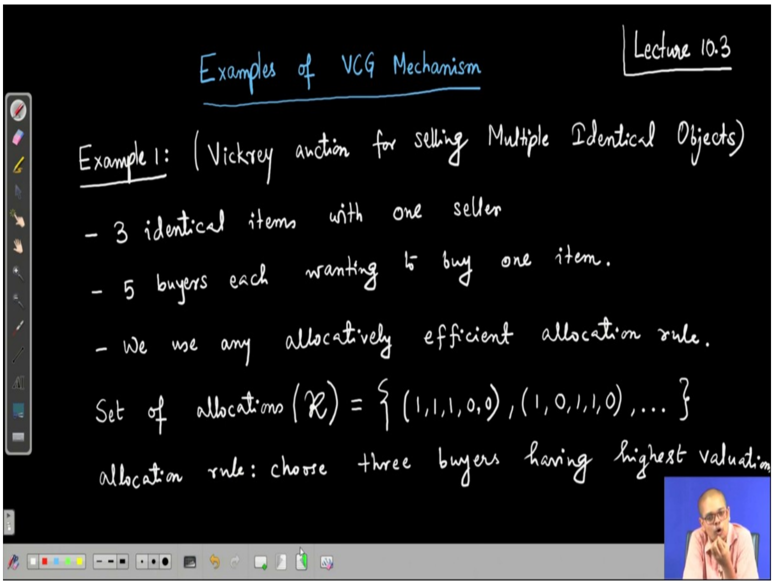

So, examples of VGC mechanism; so, our first example is Vickrey auction. So, example 1: So, we will apply VGC mechanism in an auction scenario for selling. Till now we have studied various auction scenarios, but always there only one item which is being sold or purchased.

Now, for the first time in this course we are looking at auction scenarios for selling multiple items and they are identical items. And we will see what auction will implement exposed efficient social choice function or a allocatively efficient allocation rule and the auction that we will get is called Vickrey auction. So, Vickrey auction for selling multiple identical objects.

So, for example, suppose we have 3 identical items and items with one seller we have 5 buyers each wanting to buy one item so, because the seller has 3 identical items. So, these 3 identical items will be sold to 3 buyers among the 5 buyers ok. Now here is the beauty. So, the allocation rule that we will use is an allocatively efficient allocation rule. So, we use any allocatively efficient allocation rule.

Now, what is recall what is allocatively efficient allocation rule it is an allocation rule which maximizes sum of valuations of the allocation of all the players. Now what are the possible allocations here? So, the set of allocations in this example set of allocations k is all 0 1 vectors of length 5 where, there are exactly 3 components having one. So, one possible vector is $(1,1,1,0,0)$. This allocation means that the first 3 buyers buy 1 item each and the last 2 buyers do not buy anything.

Similarly, another one could be $(1,0,1,1,0)$ and so on. So, this is the set of all allocations and the allocatively efficient allocation rule simply means in this context that whichever buyers value the item highest pick the top 3 buyers who value the item highest and give the item to each of the buyer; that is what the allocatively efficient allocation rule means. So, what is that rule? So, allocation rule; choose three buyers having highest valuations. So, that is what the allocation rule is.

Now, we will design a payment rule and the payment will be Clarke's payment rule. Now because of Clarke's theorem because of groves theorem you know the Clarke payment rule if I combine Clarke's payment rule with this allocation rule because this allocation rule is allocatively efficient the mechanism that we get is dominant strategy incentive compatible, which means that reporting true valuation is a dominant strategy equilibrium is a very weakly dominant strategy equilibrium.

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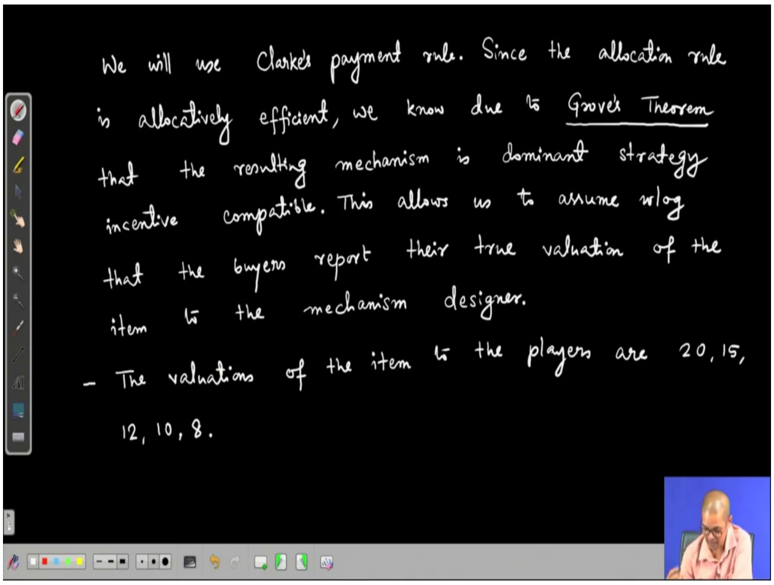

So, let me highlight that we will use Clarke payment rule. Since the allocation rule is allocatively efficient. We know due to groves theorem that the resulting mechanism is dominant strategy incentive compatible. So, whatever is the valuation of the item to each buyer we can assume without loss of generality that the buyer reveal their true type because this mechanism is dominant strategy incentive compatible.

So, let me write that this allows us to assume without loss of generality that the buyers report their true valuation of the item to the mechanism to the mechanism designer ok. So, and this is typical for all applications of VGC mechanism because the underlying mechanism is dominant strategy incentive compatible. We will simply state what are the private types are and we can we will know we will assume that without loss of generality of course, that players report their true type out of their best interest.

So, let us continue. Suppose the valuations of the item to the players are some numbers say 20, 15, 12, 10, 8. Suppose these are the evaluations of let me restate again these are private type this is typically not known, but because the mechanism is dominant strategy incentive compatible because of groves theorem we can assume without loss of generality that players report their true type voluntarily out of their own self interest ok.

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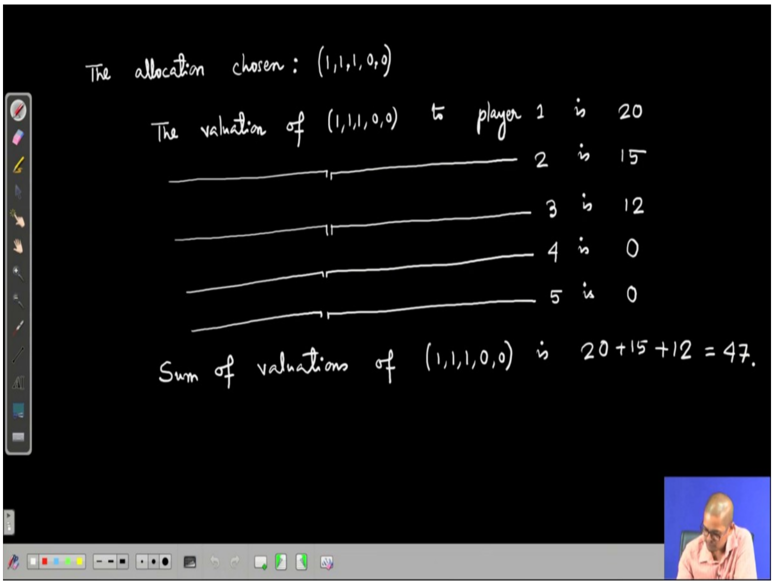

So, what are the payments? So, what is the allocation chosen first of all the allocation chosen is the first three players has the highest valuations. So, (1 , 1, 1, 0, 0). What is the sum of valuations? The valuation of this allocation to player 1 is 20 because player 1 is simply getting the object the valuation of this allocation to player 2 is 15 and the valuation of this allocation to player 3 is 12.

So, the let me write the valuation of this particular allocation to player 1 is 20 because, simply because player 1 is getting the object the valuation of this particular allocation to player 2 is 15. The valuation of this allocation to player 3 is 12. Valuation of this particular allocation to player 4 is 0 because player 4 is not getting the item and the valuation of this particular allocation to player 5 is 0.

So, as you can see this particular valuation; that means, (1, 1, 1, 0, 0) maximizes the sum of valuations. So, sum let me write sum of valuations of (1, 1, 1, 0, 0) is 20 + 15 + 12 which is 47. So, as you can see this particular allocation maximizes the sum of valuations which is exactly what any allocatively efficient allocation rule is supposed to do. Now, what are the payments?

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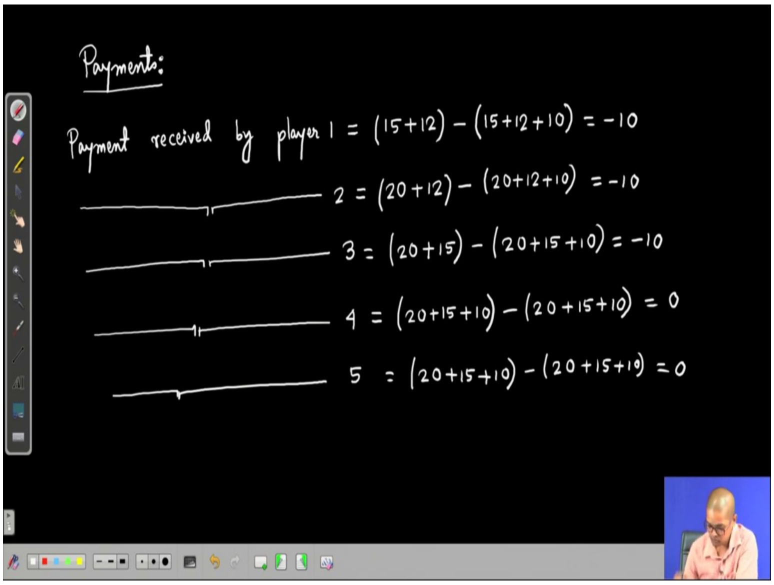

Payment received by player 1. What is it? How did we calculate? We add the valuation of all the players except 1 valuation of this allocation what is the valuation of this particular allocation (1, 1, 1, 0, 0) to all the players except player 1 it is 15 + 12. From this I should subtract, but I will first remove player 1 from the system. So, and think of a system with 4 players player 2 player 3 player 4 and player 5 and there I will find an allocatively efficient allocation.

Now, in the absence of player 1 sorry in the absence of player 1 the 3 identical objects will be given to player 2 player 3 and player 4 and in that case the sum of valuations of all the players of that particular allocation is 15 + 12 plus 8 sorry 10 this is minus 10. So, player 1 receives minus 10 which is equivalent to saying player 1 pays minus 10 player 1 pay pays 10, which sort of matches with your intuition. If player 1 is getting an item it should pay it should not receive any money.

Let us see how much is the payment received by player 2. What is the sum of valuations of all the players in this particular allocation (1, 1, 1, 0, 0) except player 2. So, I will leave player 2. So, the sum of valuations is 20 plus 12 20 plus 12 minus; now I would delete player 2 from the system now we have 4 players player 1 player 3 4 and 5.

And the allocation and allocatively efficient allocation will be objects the objects will be given to player 1 3 and 4 and their valuations sum of valuations will be 20 plus 12 plus

10 which is minus 10. So, player 2 also pays 10 20 rupees the same calculation will show you that player 3; 20 plus 15 minus 20 plus 15 plus 10 equal to minus 10.

Player 3 pays 10 rupees. How much player 4 plays pays or receives? What is the sum of valuations in the presence of player player 4? 20 plus 15 plus 10 in the absence of player 4 the same allocation is picked 20 15 10 which is 0. So, player 4 does not pay anything or does not receive any money same with player 5 ok.

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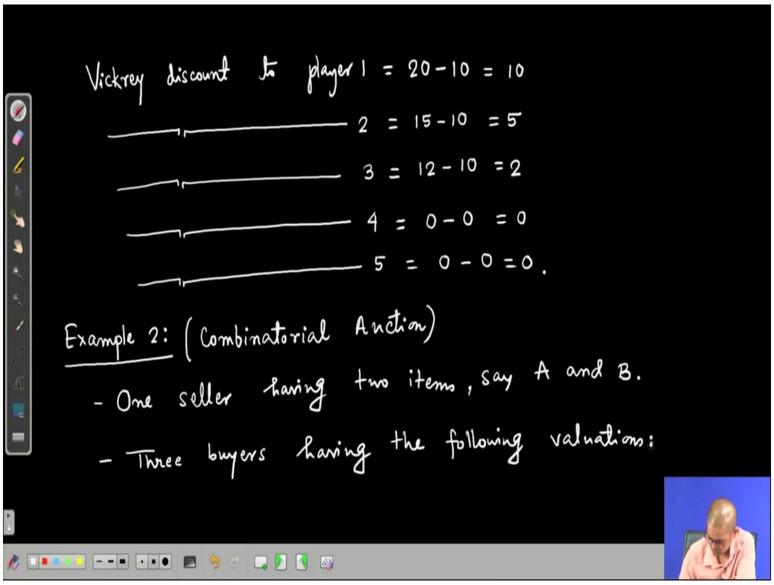

So, in the VGC payment losers does not pay anything what is Vickrey discount, Vickrey discount to player 1 is valuation 20 minus how much it is paying although player 1 is receiving an item which it values 20 it is paying only 10. So, the discount is 10. Vickrey discount to player 2 is 15 minus 10 which is 5. Vickrey discount to player 3; 12 minus 10 is 2. Vickrey discount to player 4; player 4 values the allocation to be 0 and it pays 0 0. Vickrey discount to player 5 0.

So, this is one interesting example of Vickrey mechanism VGC mechanism these also called Vickrey auction. Example 2; 2nd example: Our second example is a combinatorial auction. So, here we have one seller having two items say A and B. So, in our in our last example we had more than one item namely three item, but they are all identical items now they are not identical items. And we have three buyers having the following valuations.

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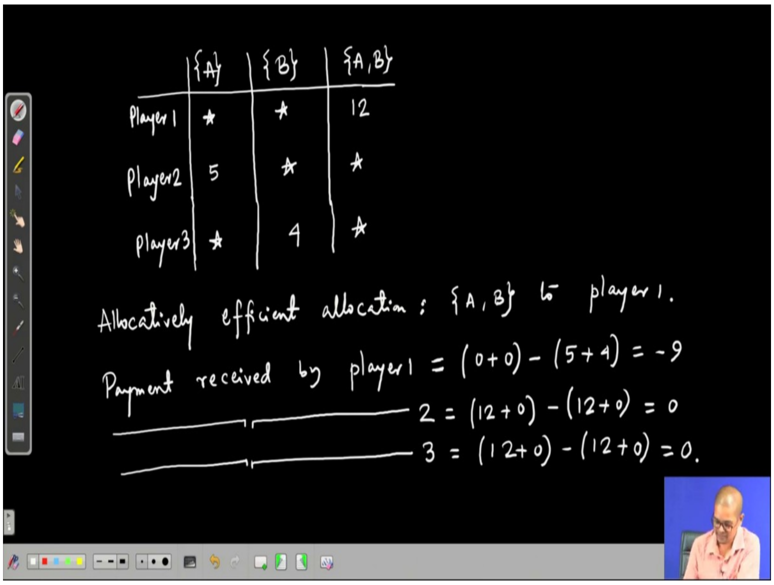

So, here are the three items A B and AB. So, because items are not identical they could be complementary they could be similar item or replacement type. So, each bundle each subset each player values differently it need not be value of a subset of items need not be the sum of the valuation of individual items.

Say here we have 3 buyer's player 1, player 2, player 3. Suppose player 1 does not value anything. It is not interested in receiving only player only item is; that means, it gives assigns value 0 to it is not interested to get item B only also, but if it is interested to get both A and B and it values the bundle as 12.

Suppose player 2 is interested only in A it does not care whether it gets B it is also not it is not interested in A B. And suppose player 3 is only interested in B. So, star means that player is not interested in that bundle. Not interested means that what will be its valuation. So, for example, player 2 is not interested in the bundle A B. So, if you give him the bundle A B its valuation will be 5 because it has because it has item A and it values item A to be 5 ok.

What is an allocatively efficient allocation rule? Allocatively efficient allocation allocatively efficient allocation: So, the seller can give the bundle A B to player to player 1 and that is how it will maximize the sum of valuation. So, give A B to player 1. What are the payments?

Payment received by player 1 what is the sum of valuations of this allocation of all the players except player 1 of course, because both A and B player 1 is getting player 2 and player 3 is not getting anything their valuations is 0 so, 0 plus 0 minus. In the absence of player 1 the only allocatively efficient allocation is giving player A player 2 item A and giving item B to player 3.

So, in that case the sum of valuations will be 5 plus 4 so, minus 9. So, player 1 pays 9 rupees; that is what this minus sign indicates ok. Let us compute for other players. Payment received by player 2 sum of valuations of all the players its 12 plus 0 in the presence of player 2 and in the absence of player 2 allocation does not change it remains 12 plus 0 it is 0 player 2 does not pay anything same with player 3 it is not 10 12 plus 0 12 plus 0 which is 0.

So, players do not pay players 2 and 3 is not paying anything neither they are getting the item the Vickrey or the VGC mechanism allocates item a b to player 1 and player 1 pays 9 rupees.

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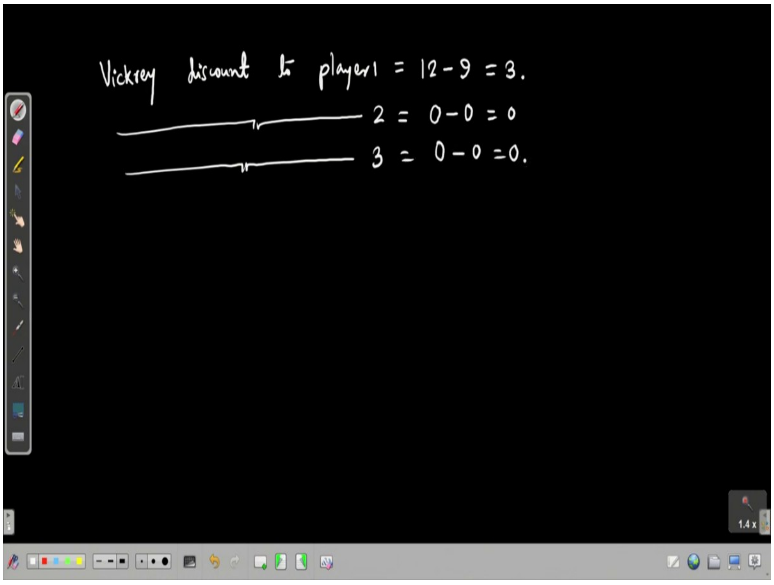

What is discount? Vickrey discount. Vickrey discount to player 1 is how much value how much player 1 values that bundle whatever it is getting 12 and it is paying 9. So, 12 minus 9 is 3. So, Vickrey discount is 3 Vickrey discount to player 2 whatever it is getting its values it values 0 and it is paying 0; so, 0 same with player 3. So, this is the payment ok. So, we will stop here today.