Algorithmic Game Theory
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Lecture - 42
Revelation Principle

Welcome. In the last lecture we have seen first price auction mechanism and second price auction, these two social choice functions. And we have observed that the direct mechanism for the first price auction is implementable in dominant strategy equilibrium, on the other hand the direct mechanism for second prize auction is not dominant strategy incentive compatible.

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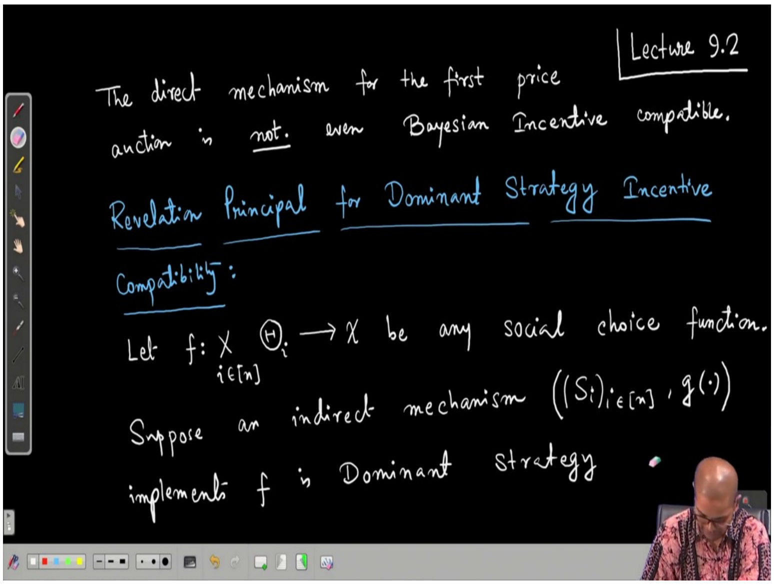

So, let us call. So, this is where we start that the direct mechanism for the first price auction is not even Bayesian incentive compatible, which simply means that revealing the true type of the players is not even a Bayesian Nash equilibrium.

And we left in, we ended in last class by saying that we will show that there cannot even exist a indirect mechanism, for implementing the first price auction, even in dominant strategy incentive compatibility or even in Bayesian incentive compatibility.

And that is a grand theorem because you know in indirect mechanisms, the mechanism designer has so much liberty, it can design new strategies and it can design the rules of the games and so on. And so how will we go about proving such a grand theorem means the mechanism designer can do anything.

And that is where the celebrated, the very important result called revelation principle comes into picture. So, what is revelation principle? That is our topic of discussion today. Now, we have two revelation principles, one is for dominant strategy incentive compatibility and the other is for Bayesian incentive compatibility and they are just mirrored image of one another.

So, let us first state and understand the revelation principle for dominant strategy incentive compatibility. So, what does it say? It says the following: let f be any social choice function; suppose an indirect mechanism S i i in n, the mechanism designer needs to decide what are the what is the strategies for each players and a function which we will map from strategy profile to the set of outcomes. Now, suppose there is an suppose an indirect mechanism this implements if in dominant strategy Nash equilibrium.

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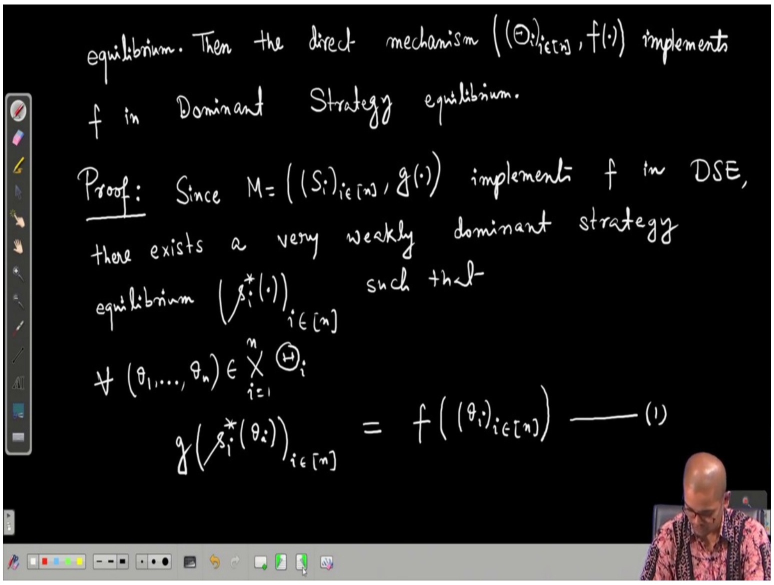

Then, then the revelation principle says that the direct mechanism, θ_i where the strategy set is simply their types and they are directly told to reveal their type. And the there is no g there is only f they are directly told that we are going to apply f on the type profile to find the outcome.

So, the direct mechanism this implements f in dominant strategy equilibrium; is Dominant Strategy Equilibrium not Nash equilibrium DSE. So, why this is so important? You know from this revelation principle it immediately follows suppose so it immediately follows that because the direct mechanism for the first price auction does not implement first price auction in dominant strategy equilibrium. There is no mechanism, which can implement the dominant the first price auction in dominant strategy equilibrium that is the revelation principle says.

And what is the revelation principle for BIC mechanisms Bayesian incentive compatible mechanisms. Same thing we replace dominant strategy equilibrium with Bayesian-Nash equilibrium. So, how does the revelation principle for Bayesian-Nash, Bayesian incentive compatibility we will read? Everything is same let f from type profiles to outcome be any social choice function.

Suppose an indirect mechanism $((S_i)_{i\in[n]}, g)$ implements f in Bayesian Nash equilibrium, then the direct mechanism implements f in Bayesian Nash equilibrium. That is the revelation principle for BIC mechanism. So, from the revelation principle for BIC mechanism it follows that there is no indirect mechanism which can implement the first price auction even in Bayesian-Nash equilibrium.

Why simply because the direct mechanism for the first price auction does not implement the first price auction in Bayesian-Nash equilibrium ok. So, very powerful theory, theorem with a very simple proof. So, let us see its proof ok. So, what are we given? We are given that the mechanism M, the indirect mechanism since mechanism m implements f in dominant strategy equilibrium DSE in short.

That just means that there exists very weakly dominant strategy equilibrium $(s_i^*(.))_{i\in[n]}$. Recall the strategies are functions from type sets to strategy sets. There exist equilibrium, there exists a very weakly dominant strategy equilibrium $(s_i^*(.))_{i\in[n]}$ such that for all type profile $(\theta_1,\ldots,\theta_n)$. You we look at what is the strategy plate under this type profile by the players in this equilibrium, the strategy profile plate is $s_i^*(\theta_i)$ this is the strategy, profile played $i\in[n]$.

And what is the outcome the outcome is when you apply g we will get the outcome this outcome should match with the desired outcome under this social choice function f under the type profile $(\theta_1,\dots,\theta_n)$.

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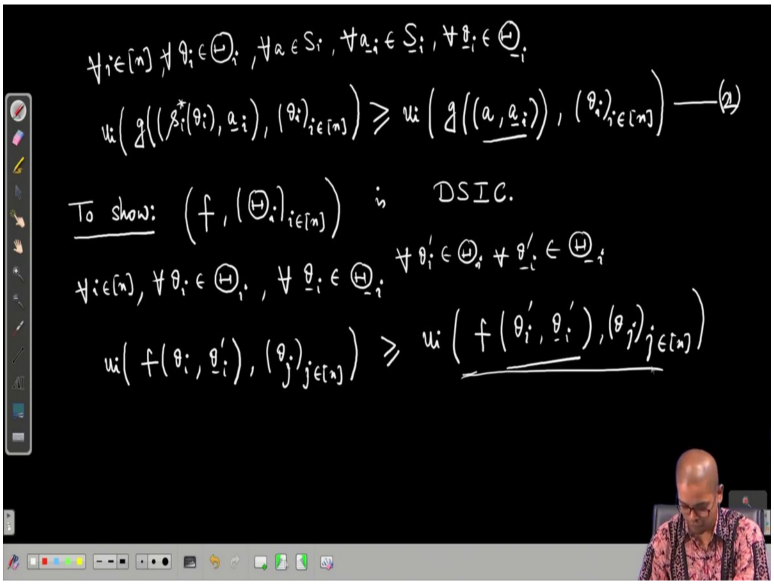

And because this is a very weakly dominant strategy equilibrium, we get that utility for all player $i\in[n]$. And if I look at any type θ_i of that player i and in that type theta i it is playing some strategy $s_i^*(\theta_i)$ that should be the best. So, it should be for every other strategy $a\in S_i$.

So, for all these things what we have? Utility of player i, u_i is the utility function of player i. Now, g utility depends on outcome and the type profile, g what is the outcome? When it plays according to the dominant strategy profile $(s_i^*(\theta_i))_{i\in[n]}$ and the types are $(\theta_i)_{i\in[n]}$. Because it is a very weakly dominant strategy the strategy $s_i^*(\theta_i)$ should be the base strategy for player i irrespective of what other players play.

So, and for all $a_{-i}\in S_{-i}$, ith place is playing according to this s_i^* and other players may not need to follow s_i^* , because we are talking about very weakly dominant strategy equilibrium. This comma $(\theta_i)_{i\in[n]}$, this is the utility of player i when other players are playing actions a minus i strategies and the type profile is $(\theta_1,\ldots,\theta_n)$ and player i plays $s_i^*(\theta_i)$.

This particular utility should be greater than equal to $u_i(g(s_i^*(\theta_i))_{i\in[n]})$ let player play a and other players are playing a_{-i} and their type profile is $(\theta_i)_{i\in[n]}$. So, this should hold, ok. Now, we want to show that the same should, then we need to show that the direct mechanism f comma $(\theta_i)_{i\in[n]}$ is dominant strategy incentive compatible.

What do I mean by that? So, we need to show that if I pick any player $i\in[n]$ and pick any of its type $\theta_i\in\Theta_i$, here also there will be another $\theta_{-i}\in\Theta_{-i}$. And if I and player i should reveal θ_i and not anything else and for all $\theta_{-i}\in\Theta_{-i}$ utility of player I, when the outcome is $f(\theta_i,\theta_{-i}^{'})$.

Other players need not follow, need not be revealing truth, then also it does not matter to player i, comma $(\theta_i)_{i\in[n]}$; that means, $(\theta_j)_{j\in[n]}$ this is greater than equal to these two show $u_i(f(\theta_i^{'},\theta_{-i}^{'}),(\theta_j)_{j\in[n]})$. This should hold for all $\theta_i^{'}\in\Theta_i$ and for all $\theta_{-i}^{'}\in\Theta_{-i}$. So, this we need to show ok.

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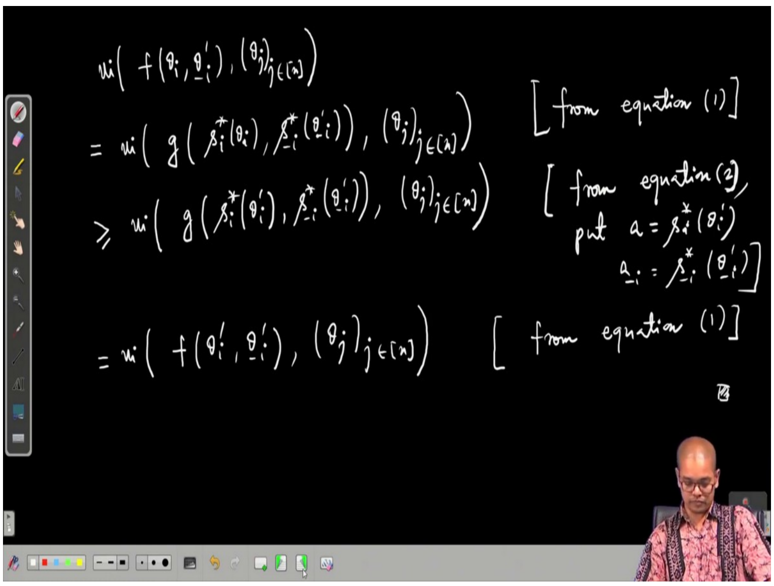

So, these two show; so, what we do is that we start from left hand side. So, what is what we have in left hand side? So, let us go to the next page $u_i(f(\theta_i, \theta'_{-i}), (\theta_j)_{j\in[n]})$. So, this we have ok. Now, we want to use, so this this equation 1, this is this we are given we need to use this. This we are given this, let this call equation 2 and we have also given this equation 1.

So, what is $f(\theta_i)_{i\in[n]}$? This is $g(s_i^*(\theta_i))$. So, we apply this first applying equation one first what. We get is that this is equal to $u_i(g(s_i^*(\theta_i), s^*_{-i}(\theta'_{-i})), (\theta_j)_{j\in[n]})$ A, this we have from equation 1. Now, we apply equation 2 and by equation 2, this utility is greater than equal to $u_i(g(s_i^*(\theta_i), s^*_{-i}(\theta'_{-i})), (\theta_j)_{j\in[n]})$

Now, it this particular equality holds for all (a, a_{-i}), but I need to show that this is greater than or equal to $f(\theta'_i, \theta_{-i})$. So, I apply equation two, but replace put a equal to $f(\theta'_i, \theta_{-i})$. So, this is $s_i^*(\theta'_i)$. I put a equal to s i star of theta i prime let me write, this is from equation 2.

Put $a = s_i^*(\theta'_i)$ and $a_{-i} = s^*_{-i}(\theta'_{-i})$. $s^*_{-i}(\theta'_{-i}), (\theta_j)_{j\in[n]}$ ok. Now, we have got, but here you see that the right-hand side is in terms of f no g, in the right-hand side. So, from f to g to go back and forth we need to use this equation 1.

So, we have again apply equation 1 and from g we go to f, this is equal to $u_i(f(\theta'_i, \theta'_{-i}), (theta_j)_{j\in[n]})$. This again from equation 1, which exactly what we need to prove.

So, similarly you can we can prove the Bayesian revelation principle for Bayesian incentive compatible mechanisms. So, let us state and leave the proof to you as a homework.

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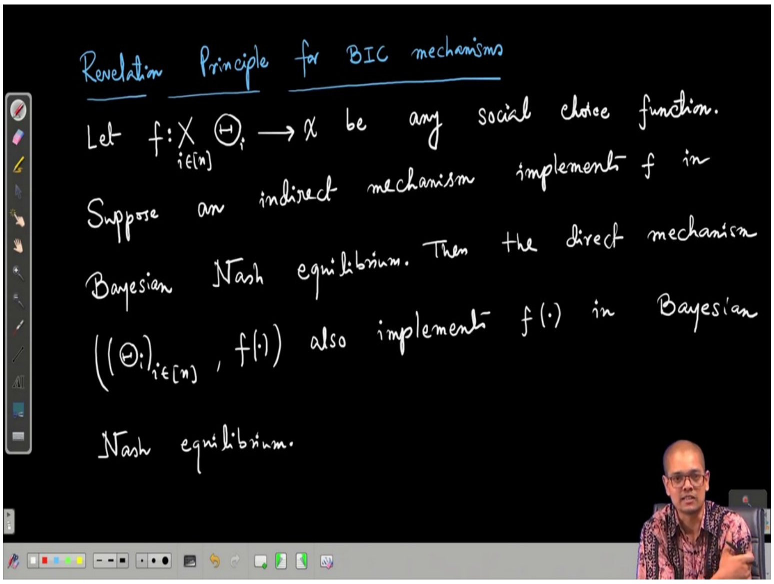

So, we write revelation principle for BIC mechanisms. As usual let if from $\times_{i\in[n]} \Theta_i \to X$ be any social choice function. Suppose an indirect mechanism implements f in Bayesian Nash equilibrium. Then the direct mechanism $((\Theta_i)_{i\in[n]}, f)$ also implements f in Bayesian Nash equilibrium ok.

So, let you prove it and the high-level structure is exactly as the revelation principle for DSIC mechanisms. You if you are stuck, you only it only shows that you have lack of clarity, lack of understanding in in Bayesian Nash equilibrium, Bayesian games and so on. So, this will give you a nice excuse to go back and clarify in your understandings, definitions of Bayesian Nash equilibrium, Bayesian games or indirect mechanisms.

And that is sort of the only difference from dominant revelation principle for dominant strategy incentive compatible mechanisms and the revelation principle for Bayesian incentive compatible mechanisms. So, we will stop here today. So, we will continue in the next class.